_4-linearization-and-linearity

[AMv2 Ch 6]

goal: qualitative & quantitative analysis of linear system behavior & relation to nonlinear system behavior

topics:

1°. linear systems

1¹. linearization

1². linearity

1³. matrix exponential

1⁴. input/output response

1⁵. frequency response

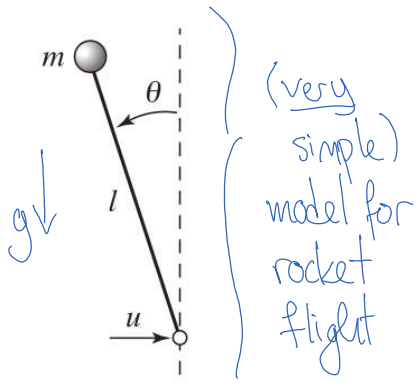
1°. linear systems

Few physical elements display truly linear characteristics. For example the relation between force on a spring and displacement of the spring is always nonlinear to some degree. The relation between current through a resistor and voltage drop across it also deviates from a straight-line relation. However, if in each case the relation is reasonably *linear, then it will be found that the system behavior will be very close to that obtained by assuming an ideal, linear physical element, and the analytical simplification is so enormous that we make linear assumptions wherever we can possibly do so in good conscience.*

Robert H. Cannon, *Dynamics of Physical Systems*, 1967 [Can03].

1! linearization

ex: inverted pendulum



(very simple) model for rocket flight

- previously determined dynamics $ml^2\ddot{\theta} = mgl\sin\theta - \gamma\dot{\theta} + lu\cos\theta$

have equilibria at: $\dot{\theta}_e = 0, \quad \theta_e = k\cdot\pi, \quad k\in\mathbb{Z}$

→ compute first-order Taylor series about equilibrium $(\theta_e, \dot{\theta}_e) = (k\pi, 0)$

1. linearity

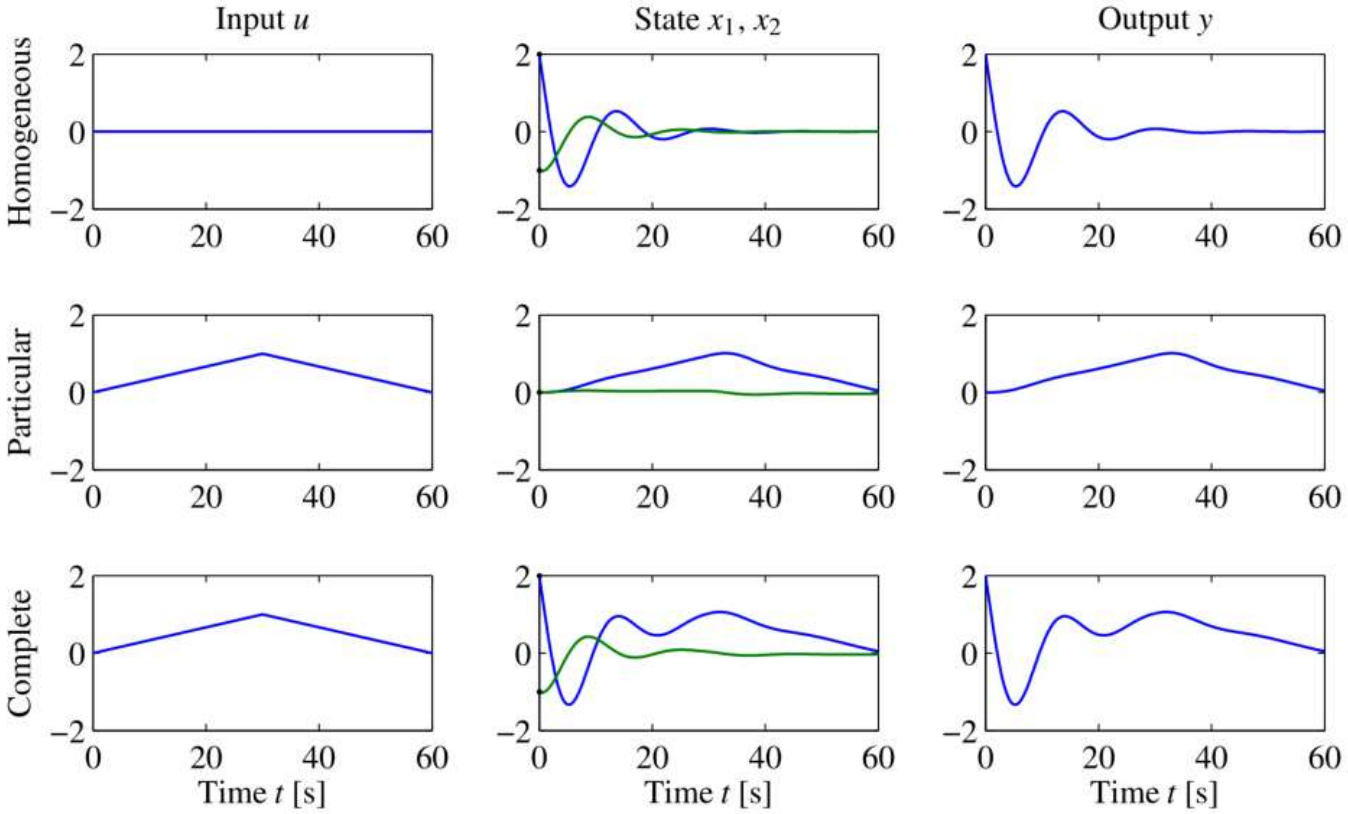


Figure 6.1: Superposition of homogeneous and particular solutions. The first row shows the input, state, and output corresponding to the initial condition response. The second row shows the same variables corresponding to zero initial condition but nonzero input. The third row is the complete solution, which is the sum of the two individual solutions.

1.3 matrix exponential (i.e. the homogeneous solution)

→ read [AMv2 "Eigenvalues and Modes"]

↳ interesting discussion of eigenvalues, eigenvectors, and coordinate choice

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14. input/output response

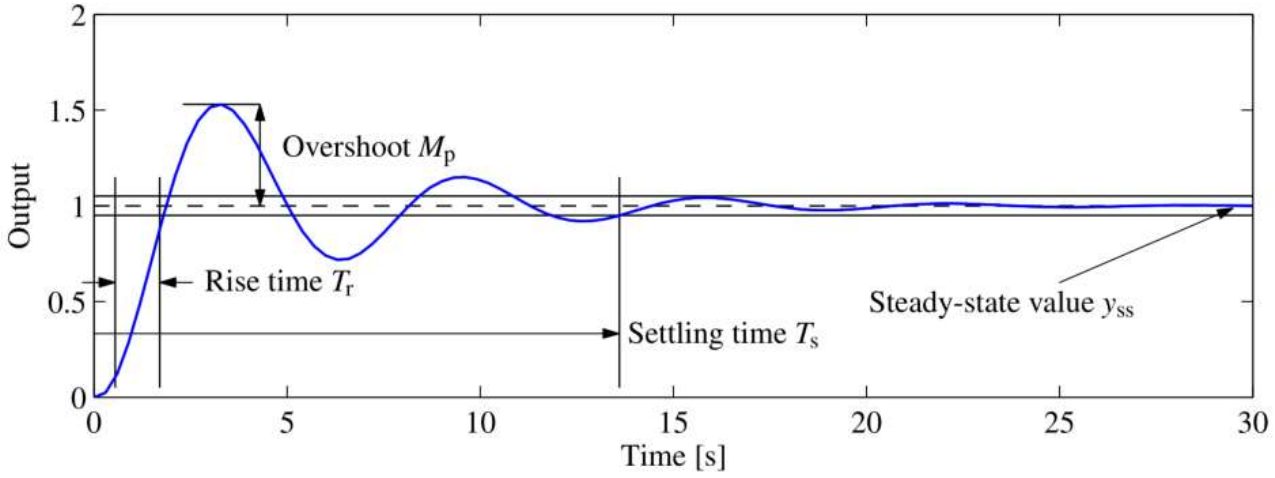

Figure 6.9: Sample step response. The rise time, overshoot, settling time, and steady-state value give the key performance properties of the signal.

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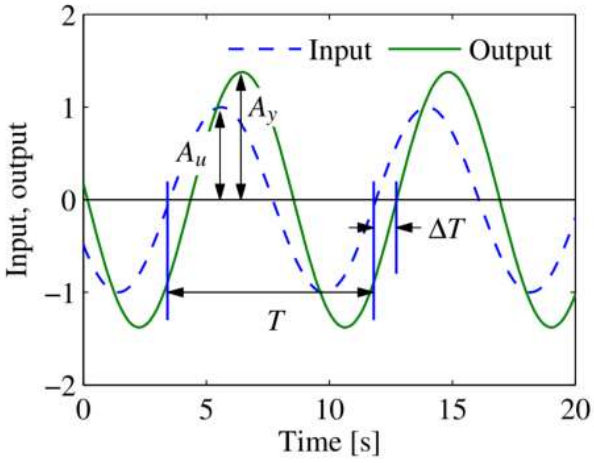
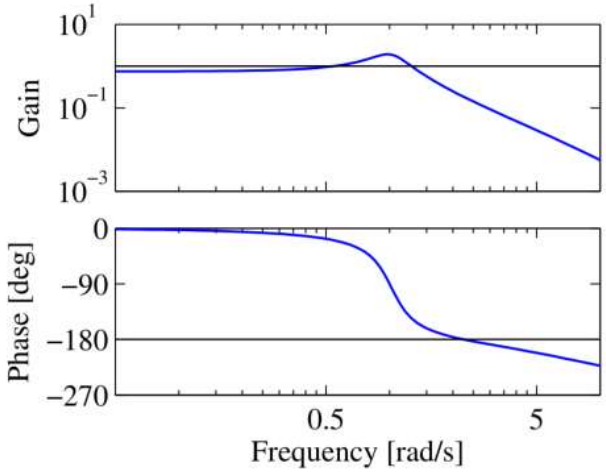
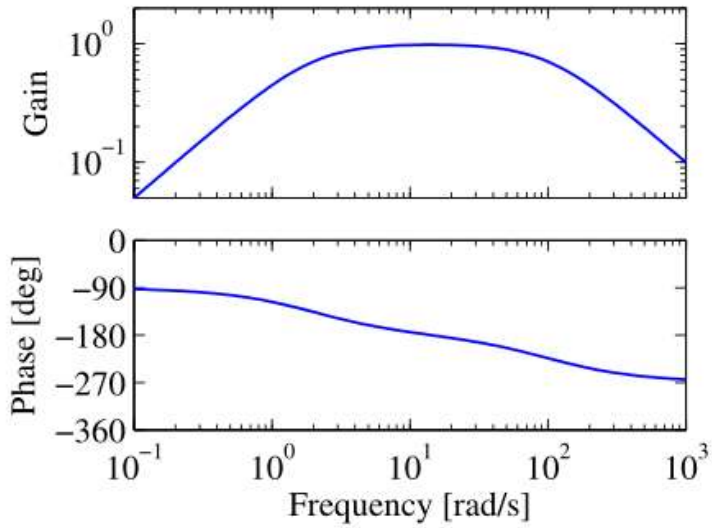
15. frequency response



(a) Input/output response



(b) Frequency response



(b) Frequency response