

goal: tools for analysis  
using transfer functions,  
root locus plots

1°. frequency domain analysis

1<sup>1</sup>. sensitivity functions

1<sup>2</sup>. root locus

[AMv2 ch 12.1, 12.2] [Nv7 not covered]

[AMv2 ch 12.5] [Nv7 ch 9]

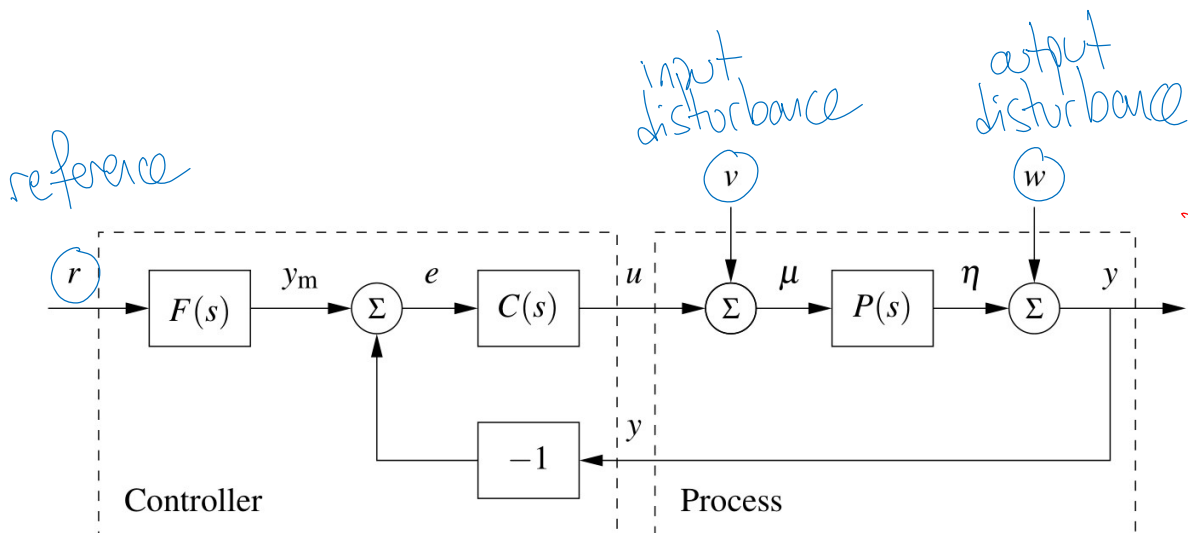
\* general comment: these techniques were  
developed before we had cheap computers,  
so there are many graphing heuristics  
that are traditionally taught;  
→ we'll rely on computers to graph,  
but still extract intuition

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1°. frequency domain analysis

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1<sup>1</sup>. sensitivity functions

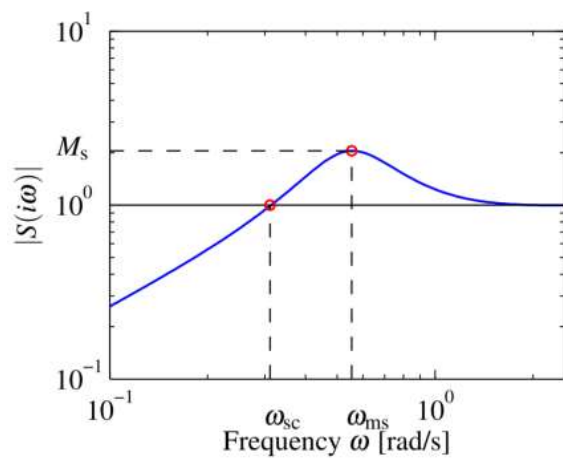


→ determine  $G_{uv}$  (how input disturbance affects input)

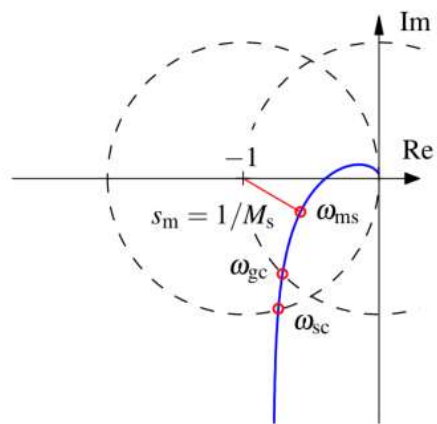
note: process  
input  $\mu$  and  
output  $y$  aren't  
what  $C$ :  
commands ( $u$ )  
or measures ( $y$ )

→ determine  $G_{uv}$  (how input disturbance affects input) or measures ( $y$ )  
 $\hat{z}$   $G_{yw}$  (how output disturbance affects output)

$y$	$u$	$e$	$\mu$	$\eta$	
$\frac{PCF}{1+PC}$	$\frac{CF}{1+PC}$	$\frac{F}{1+PC}$	$\frac{CF}{1+PC}$	$\frac{PCF}{1+PC}$	$r$
$\frac{P}{1+PC}$	$\frac{-PC}{1+PC}$	$\frac{-P}{1+PC}$	$\frac{1}{1+PC}$	$\frac{P}{1+PC}$	$v$
$\frac{1}{1+PC}$	$\frac{-C}{1+PC}$	$\frac{-1}{1+PC}$	$\frac{-C}{1+PC}$	$\frac{-PC}{1+PC}$	$w$



(a) Gain curves



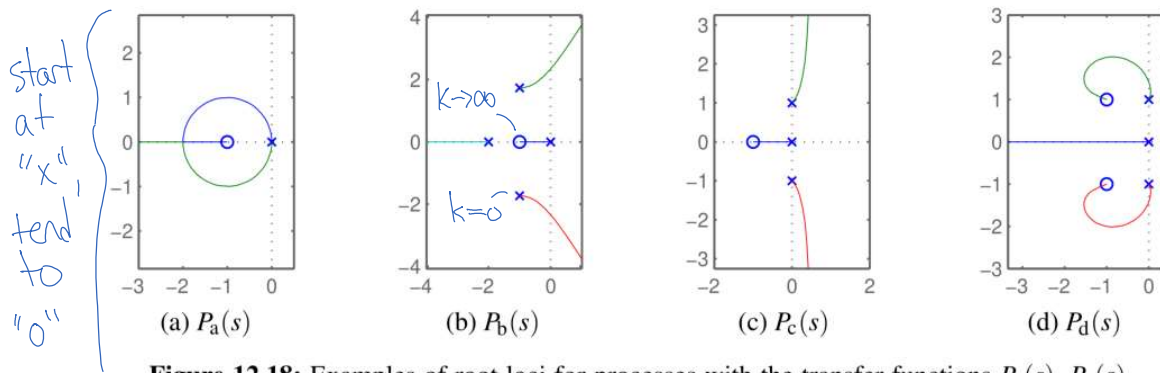
(b) Nyquist plot

\_9-frequency-domain

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1<sup>2</sup>. root locus

$$\begin{aligned}
 P_a(s) &= k \frac{s+1}{s^2}, & P_b(s) &= k \frac{s+1}{s(s+2)(s^2+2s+4)}, \\
 P_c(s) &= k \frac{s+1}{s(s^2+1)}, & P_d(s) &= k \frac{s^2+2s+2}{s(s^2+1)}.
 \end{aligned}
 \tag{12.18}$$



**Figure 12.18:** Examples of root loci for processes with the transfer functions  $P_a(s)$ ,  $P_b(s)$ ,  $P_c(s)$ , and  $P_d(s)$  given by equation (12.18).

→ which of these systems can be stabilized by proportional feedback?  
(can the gain be arbitrarily large?)

→ how would you use the root locus to determine whether a system can be stabilized with proportional feedback?

→ how would you use the root locus to determine whether a stable system can track a non-zero reference with integral feedback?