goal: tools for analysis using transfer functions, Nyquist / Bode plots

business: ∘ HW8 assigned – due Fri Nov 22 (/ Sun Nov 24)

* this is the last HW

→ project assigned Tue Nov 26, due Tue Dec 10

∘ Prof Burden traveling; TAs cover lec, OH

1°. frequency domain analysis

1¹. Nyquist stability criterion [AMv2 ch 10.1, 10.2] [Nv7 ch 10.3]

1². stability margins [AMv2 ch 10.3] [Nv7 ch 10.7]

1³. sensitivity functions [AMv2 ch 12.1, 12.2] [Nv7 not covered]

* general comment: these techniques were developed before we had cheap computers, so there are many graphing heuristics that are traditionally taught;

→ we'll rely on computers to graph, but still extract intuition

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1°. frequency domain analysis

key idea: assess stability, robustness, & sensitivity of closed-loop systems by studying open-loop systems

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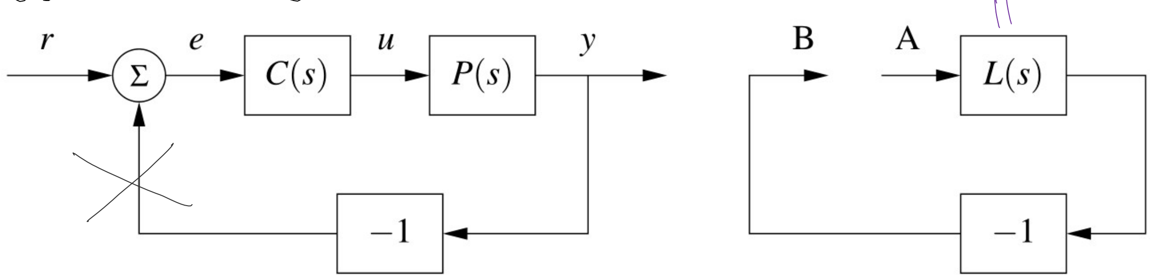
1¹. Nyquist stability criterion

$P(s)C(s)$

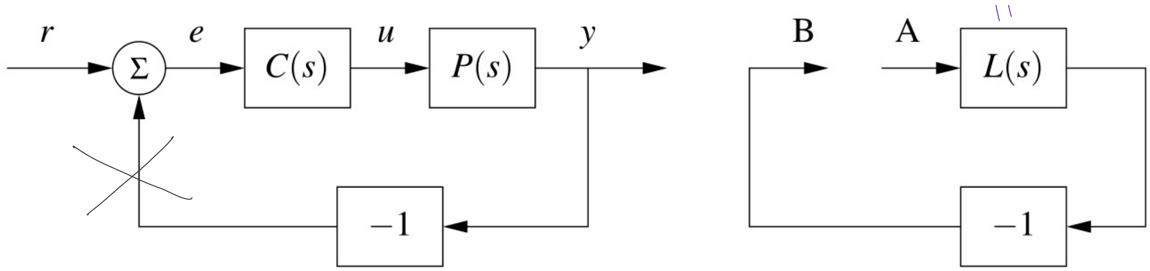
$\|$



(a) Closed loop system

(b) Open loop system

Figure 10.1: The loop transfer function. The stability of the feedback system (a) can be determined by tracing signals around the loop. Letting $L = PC$ represent the loop transfer



(a) Closed loop system

(b) Open loop system

Figure 10.1: The loop transfer function. The stability of the feedback system (a) can be determined by tracing signals around the loop. Letting $L = PC$ represent the loop transfer function, we break the loop in (b) and ask whether a signal injected at the point A has the same magnitude and phase when it reaches point B.

- we know the transfer function from r to y is $G_{yr} = \frac{PC}{1+PC}$

→ focus on what $L(s) = P(s)C(s)$ tells us about G_{yr}

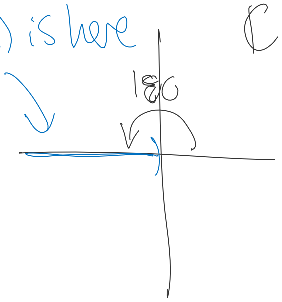
→ what value should $L(s) \in \mathbb{C}$ <u>never</u> take on?

* <u>never</u> want $L(s) = -1$ for any $s \in \mathbb{C}$

→ if $\exists\, s^* \in \mathbb{C}$ s.t. $L(s^*) = -1$,
what happens to $|G_{yr}(s)|$ as $s \to s^*$?

- $|G_{yr}(s)| = \left|\frac{P(s)C(s)}{1 + P(s)C(s)}\right| = \left|\frac{L(s)}{1+L(s)}\right| \xrightarrow{s \to s^*} \left|\frac{-1}{1-1}\right| \to \infty$

- practically speaking: system response is unbounded (i.e. unstable) for inputs $\simeq e^{s^* t}$



- thought experiment: suppose $\angle L(s) = 180°$

$$e^{st} \longrightarrow \boxed{L} \longrightarrow \boxed{-1} \longrightarrow -L(s)e^{st}$$

close feedback loop

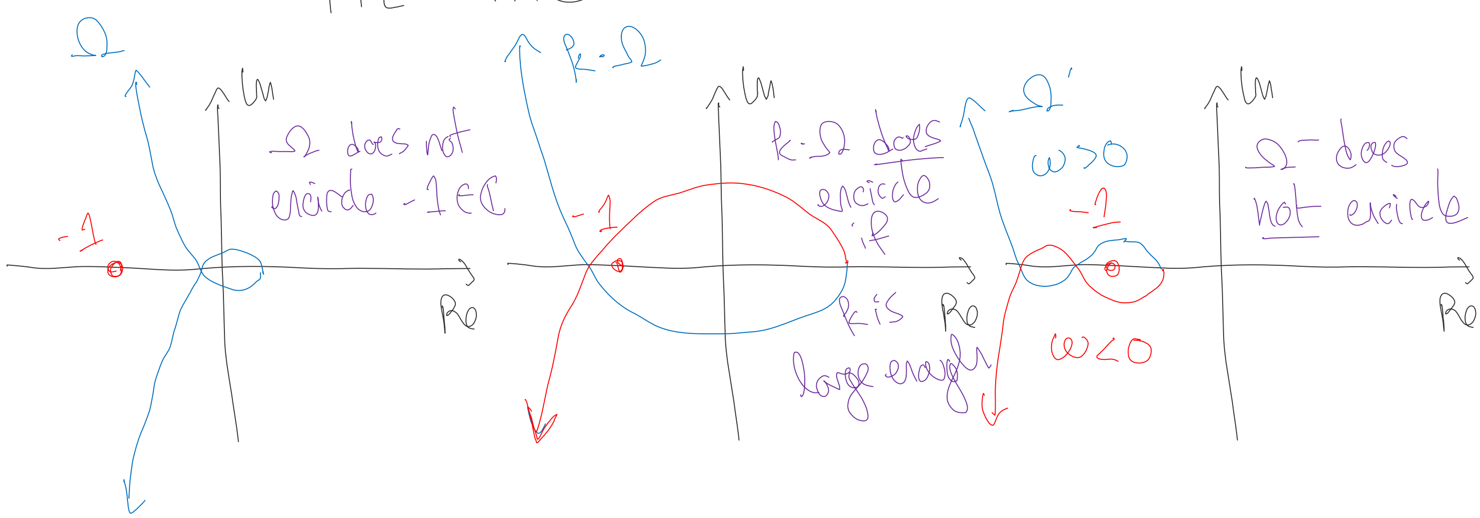
→ what happens to e^{st} if: 1°: $|L(s)| < 1$ – attenuates

→ what happens to e^{st} if: 1°: $|L(s)| < 1$ — attenuates
2°: $|L(s)| > 1$ — amplified
3°: $|L(s)| = 1$ — sustained

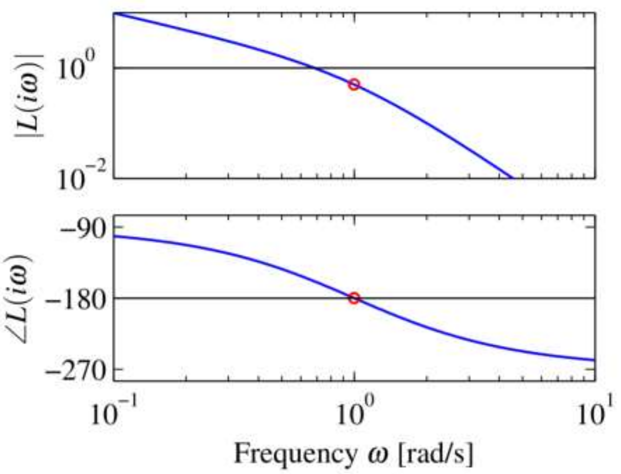
- with both lines of reasoning, conclude $L(s) = -1$ (i.e. $\angle L(s) = 180°$, $|L(s)| = 1$) is a critical point for the (open-) loop transfer function $L(s)$

— it turns out that graph of $L(j\omega)$ Nyquist plot
$\boxed{\Omega = \{L(j\omega) \in \mathbb{C} : -\infty < \omega < \infty\}}$

thm: (Nyquist stability criterion)
if L has no poles in the right-half plane (RHP), then $\frac{L}{1+L} = \frac{PC}{1+PC}$ is stable $\Longleftrightarrow$ Ω does not encircle $-1 \in \mathbb{C}$

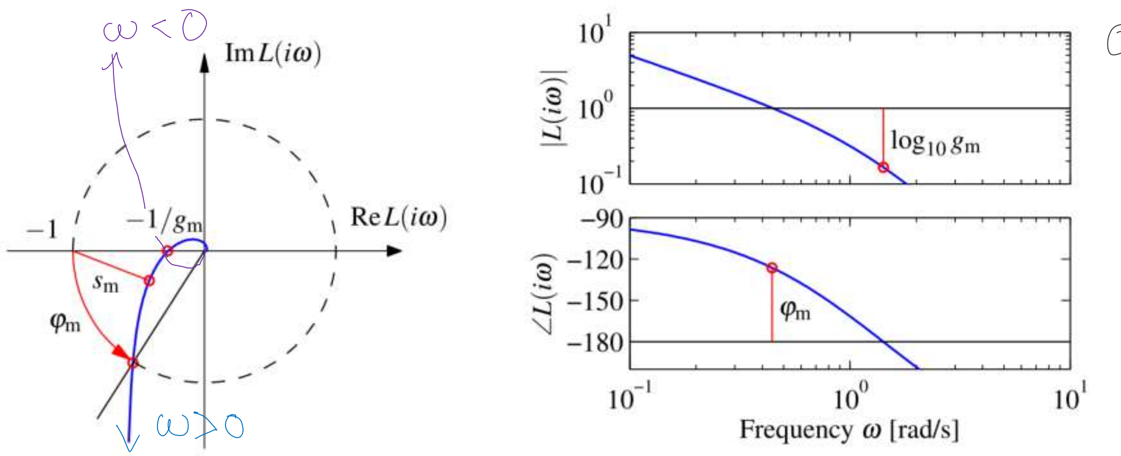


→ sketch the Nyquist plot (i.e. graph Ω) of transfer function L using Bode plot (what can you say about stability of closed-loop $\frac{L}{1+L}$?)



1°. stability margins

- in addition to providing a new technique for assessing stability, Nyquist's stability criterion gives a graphical tool for robustness



(a) Nyquist plot (b) Bode plot

Figure 10.11: Stability margins for a third-order loop transfer function $L(s)$. The Nyquist plot (a) shows the stability margin, s_m, the gain margin g_m, and the phase margin φ_m. The stability margin s_m is the shortest distance to the critical point -1. The gain margin corresponds to the smallest increase in gain that creates an encirclement, and the phase margin is the smallest change in phase that creates an encirclement. The Bode plot (b) shows the gain and phase margins.

Q1: if I want to implement $C(s)$ but instead get $k \cdot C(s)$, how large or small can k be while still ensuring stability?

Q2: if instead I get phase-shifted controller $e^{i\theta} C(s)$, how large or small can θ be while ensuring stable?

def: gain margin $g_m = \left(\begin{array}{c} \text{distance from } \Omega \text{ to} \\ -1 \text{ restricted to real axis} \end{array} \right)^{-1}$

↳ eg components / amplifiers / ADC has error / tolerances

def: phase margin φ_m = distance from Ω to -1 restricted to rotating Ω

↳ eg unmodeled passive RLC components / filtering or delay can cause phase shift (approximately)

def: stability margin s_m = distance from Ω to $-1 \in \mathbb{C}$

* takeaway: Nyquist plot can be used to assess how much process P or controller C can change while ensuring closed-loop stability

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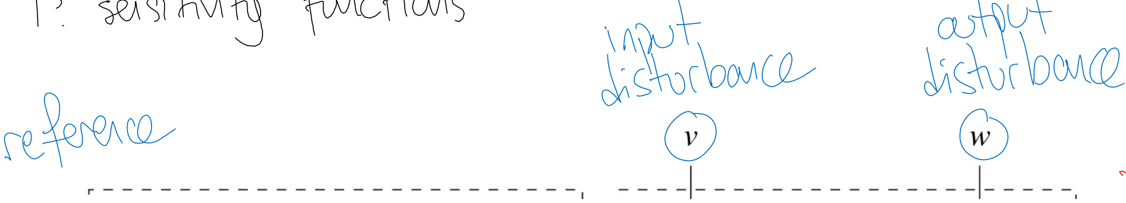
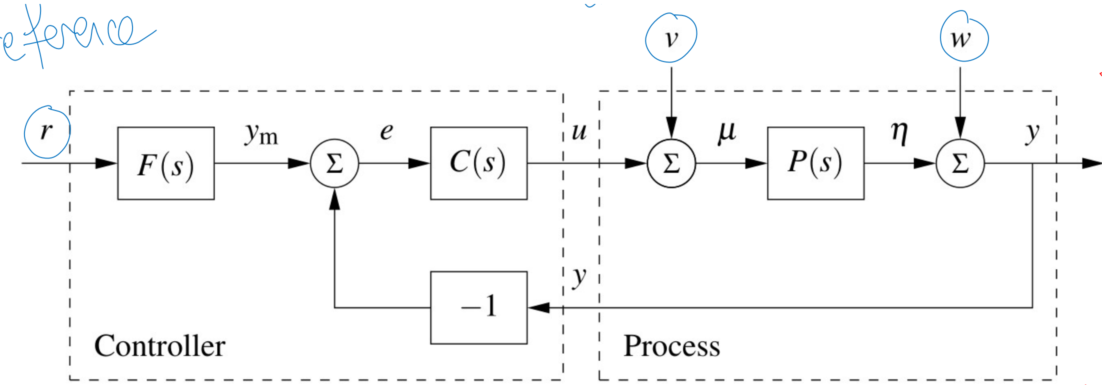
1.3. sensitivity functions



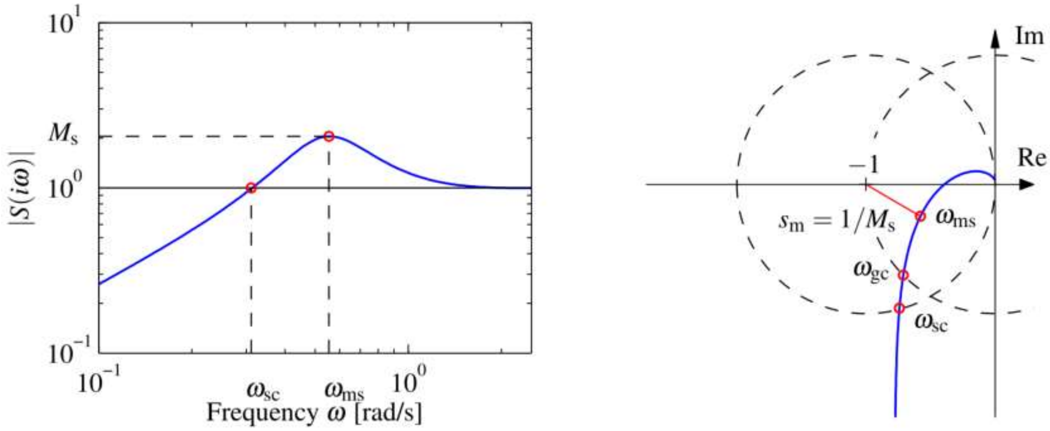
note: process

reference



note: process input μ and output η aren't what C: commands (u) or measures (y)

→ determine G_{uv} (how input disturbance affects input)
& G_{yw} (how output disturbance affects output)



(a) Gain curves

(b) Nyquist plot