[AMv2 Ch 3 & 4]

goal: further develop modeling tools
& apply them to physical phenomena

topics:

1°. modeling [AMv2 Ch 3] [Nv7 Ch 3,4,5]

1¹. concepts

1². state space models

1³. numerical simulation

2°. examples

2¹. RLC circuit

2². quadrotor

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1°. modeling

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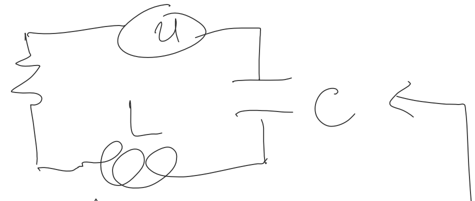
1¹. concepts

ex: spring-mass-damper: R



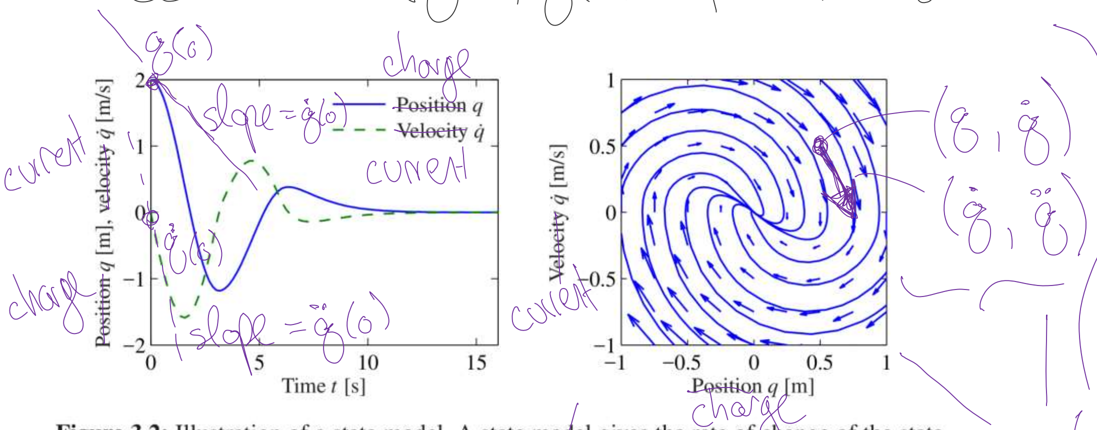

input $u$ - voltage  output $q$ - charge on C

KVL $\Rightarrow$ charge $q$ and current $\dot{q}$ interact over time

according to (DE) $L\ddot{q} + R\dot{q} + \frac{1}{C} q = v$

* note that initial condition $(q(0), \dot{q}(0))$ and input $u : [0,\infty) \to \mathbb{R}$ $: t \mapsto u(t)$

determine $(q(t), \dot{q}(t))$ for all $t \geq 0$




$u = 0$, i.e. homogeneous solution

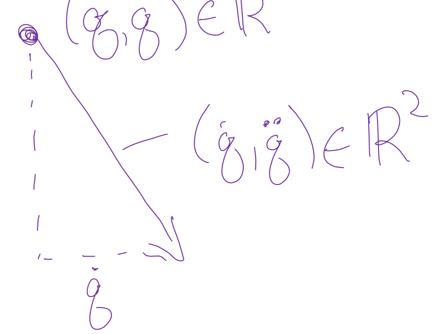
$\ddot{q} = \frac{1}{L}\left(v - R\dot{q} - \frac{1}{C} q\right)$

**Figure 3.2:** Illustration of a state model. A state model gives the rate of change of the state as a function of the state. The plot on the left shows the evolution of the state as a function of time. The plot on the right, called a *phase portrait*, shows the evolution of the states relative to each other, with the velocity of the state denoted by arrows.

$(q, \dot{q})$ plane is state space (or phase space)

↳ overlay trajectories ⇝ phase portrait

↳ overlay vectors ⇝ vector field




1.2 state-space models

∘ generalizing the preceding example,

ex: RLC circuit

- generalizing the preceding example,

let $x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n$ denote <u>state vector</u>

and $u = \begin{bmatrix} u_1 \\ \vdots \\ u_p \end{bmatrix} \in \mathbb{R}^p$ denote <u>input vector</u>

- then the state changes in time according to

(DE) $\frac{d}{dt} x = \dot{x} = \begin{bmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_n \end{bmatrix} = f(x, u)$

where $f : \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^n$
$: (x, u) \mapsto \dot{x}$

<u>ex:</u> RLC circuit

$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$, so $x_1 = q$
$x_2 = \dot{q}$

$u = [v]$, so $u_1 = v$
(voltage)

$L\ddot{q} + R\dot{q} + \frac{1}{C} q = v$

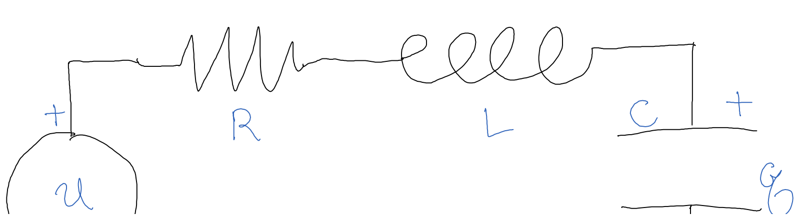
$\dot{x} = f(x, u) = \begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix}$

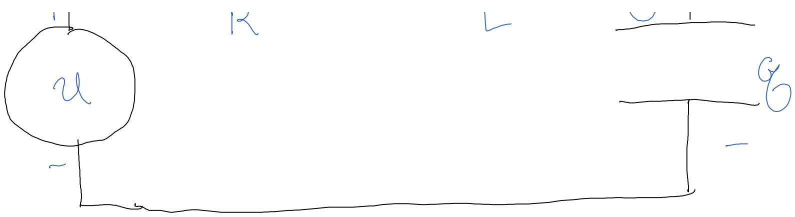
$= \begin{bmatrix} \dot{q} \\ \frac{1}{L}(v - R\dot{q} - \frac{1}{C} q) \end{bmatrix}$

1°. numerical simulation

2°. examples

$2^1$. RLC circuit






2². quadrotor

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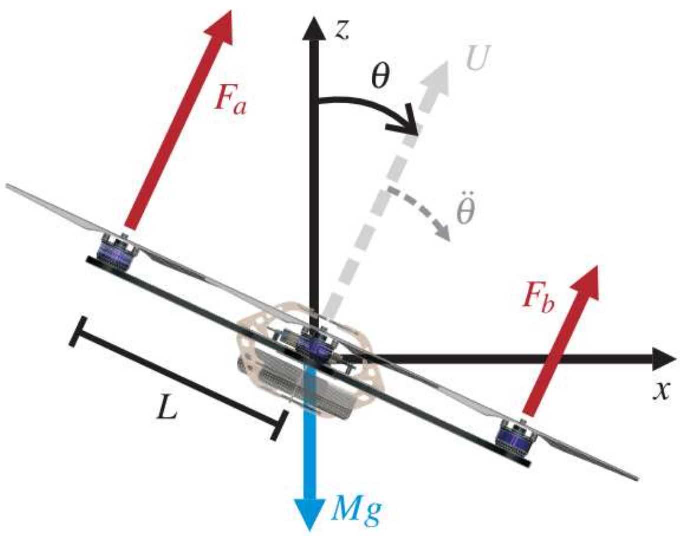
# A Simple Learning Strategy for High-Speed Quadrocopter Multi-Flips

Sergei Lupashin, Angela Schöllig, Michael Sherback, Raffaello D'Andrea




$$
\begin{aligned}
M\ddot{z} &= (F_a + F_b + F_c + F_d)\cos\theta - Mg && (1)\\
M\ddot{x} &= (F_a + F_b + F_c + F_d)\sin\theta && (2)\\
I_{yy}\ddot{\theta} &= L(F_a - F_b)\,, && (3)
\end{aligned}
$$