5-midterm-review

Friday, October 25, 2019 5:54 PM

- exam in-class 12:30 – 2:20p Thu Oct 31
- you can bring notes on one (1) sheet of 8.5 x 11in paper
- covers all material up through Thu Oct 24 (<u>except</u> programming)

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week 1: feedback principles

<u>goal</u>: introduce fundamental uses and properties of <u>feedback</u>

<u>topics</u>:

1°. mathematical models of systems [AMv2 Ch 2]

1¹. differential equations (DE) [Nise Ch 3,4,5]

$$\underbrace{\frac{d^n}{dt^n} y + a_1 \frac{d^{n-1}}{dt^{n-1}} y + \cdots + a_n y = b_1 \frac{d^{n-1}}{dt^{n-1}} u + \cdots + b_n u}$$

1². transfer functions

$$y = G u, \quad G(s) = \frac{b_1 s^{n-1} + \cdots + b_n}{s^n + a_1 s^{n-1} + \cdots + a_n}$$

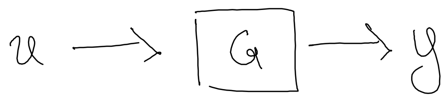
same a's & b's

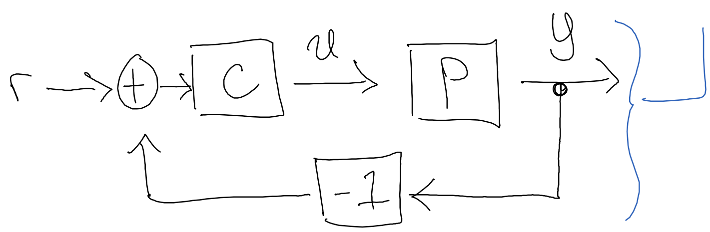
$$(1° \,\&\, 2°) \Rightarrow \quad y(t) = \sum_{k=1}^{n} c_k e^{s_k t} + G(s) e^{st} \text{ when } u = e^{st}, \ s \notin \{s_k\}$$

* $\{s_k\}_{k=1}^n$ are <u>roots</u> of <u>characteristic polynomial</u> $a(s) = s^n + a_1 s^{n-1} + \cdots + a_n$

* Routh-Hurwitz stability criteria — include $n=1,2,3$ case on notes

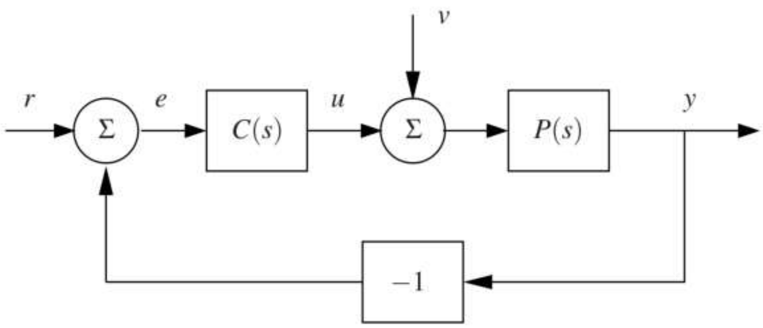
1³. block diagrams $\qquad G_{yr} = PC/(1+PC)$ } derive from block diagram





2°. effects of feedback [AMv2 Ch 2]

2¹. disturbance attenuation



$$G_{yv} = \frac{P}{1+PC}$$

$$C(s) = \boxed{k_p} \quad \text{proportional}$$

$$C(s) = k_p + \boxed{k_I/s} \quad \text{integral}$$

2². unmodeled dynamics
2³. reference tracking
} see HW 2 prob 1

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week 2: modeling and examples

goal: further develop modeling tools & apply them to physical phenomena

topics:

1°. modeling [AMv2 Ch 3]

1¹. concepts [Nv7 Ch 3,4,5]

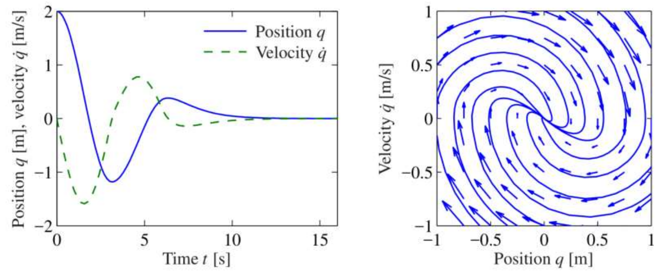

Figure 3.2: Illustration of a state model. A state model gives the rate of change of the state as a function of the state. The plot on the left shows the evolution of the state as a function of time. The plot on the right, called a *phase portrait*, shows the evolution of the states relative to each other, with the velocity of the state denoted by arrows.

1². state space models

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n \quad \text{– state vector} \qquad u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_p \end{bmatrix} \in \mathbb{R}^p \quad \text{– input vector}$$

$\dot{x} = f(x, u)$ – nonlinear

$f : \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^n$

$: (x, u) \mapsto \dot{x}$

$\dot{x} = Ax + Bu$ – linear

$A \in \mathbb{R}^{n \times n}$

$B \in \mathbb{R}^{n \times p}$

~~1³. numerical simulation~~ no programming on exam

2°. examples

2¹. RLC circuit

2². quadrotor

} review dynamics, phase portraits, equilibria, stability

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week 3: nonlinear dynamics & stability

goal: develop qualitative & quantitative tools to study nonlinear dynamics

topics:

tools to study nonlinear dynamics

topics:

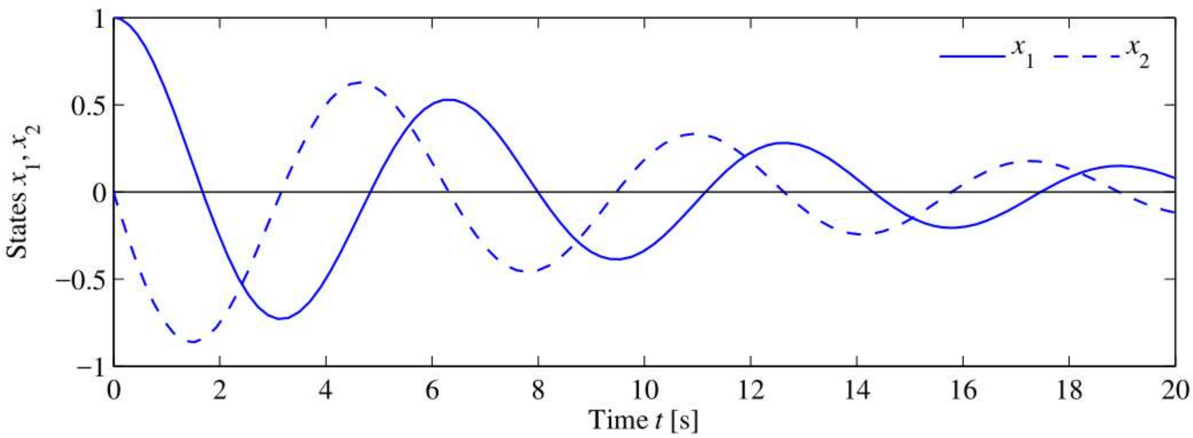
1°. nonlinear dynamics [Nv7 Ch 2] [AMv2 Ch 5]

1'. trajectories & visualization

$x : [0,\infty) \to \mathbb{R}^n$ is a trajectory of $\dot{x} = f(x,u)$

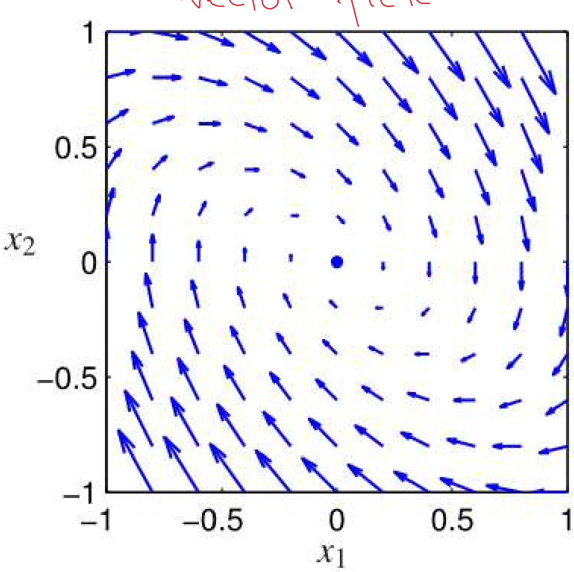
w/ initial state $x(0) \in \mathbb{R}^n$ and input $u : [0,\infty) \to \mathbb{R}^p$

if $\frac{d}{dt} x(t) = f(x(t), u(t))$ for all $t \in [0,\infty)$

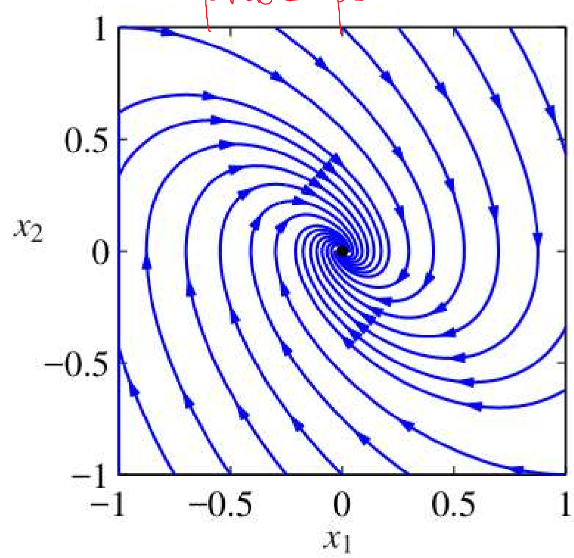


states x_1, x_2 vs time t

vector field



phase portrait



1^2. equilibrium trajectories

$x_e \in \mathbb{R}^n$, $u_e \in \mathbb{R}^p$ is equilibrium of $\dot{x} = f(x,u)$ if $f(x_e, u_e) = 0$

2°. stability * watch lec 3 video!

2^1. definition of stability [Nv7 Ch 6]

equilibrium x_e, u_e for $\dot{x} = f(x,u)$ is stable if

$\forall \varepsilon > 0 : \exists \delta > 0 :$

1°. $\|x(0) - x_e\| < \delta \Longrightarrow \|x(t) - x_e\| < \varepsilon$

↳ start close ⟹ stay close

2°. $x(t) \to x_e$ as $t \to \infty$

↳ get closer over time

* assuming x is trajectory w/ constant input u_e

2^2. stability of linear DE [AMv2 Ch 6]

$0 \in \mathbb{R}^n$ is always equilibrium of $\dot{x} = Ax$, $A \in \mathbb{R}^{n \times n}$

↳ stable if all eigenvalues of A have negative real part (i.e. all roots of characteristic polynomial $\det(sI - A)$ are in the left-half complex plane)

note: this is the same notion of stability we can test using Routh-Hurwitz!

2^3. parametric stability [Nv7 Ch 8]

see HW3 prob 3

week 4 : linearization & linearity

goal: qualitative & quantitative analysis of linear system behavior & relation to nonlinear system behavior

topics:

1°. linear systems

1¹. linearization [Nv7 Ch 2.11] [AMv2 Ch 6.4]

approximate $\dot{x} = f(x,u)$ near equilibrium $f(x_e, u_e) = 0$

using Jacobian derivatives $A = \frac{\partial}{\partial x} f(x_e, u_e) \in \mathbb{R}^{n\times n}$

$B = \frac{\partial}{\partial u} f(x_e, u_e) \in \mathbb{R}^{n \times p}$

$*$ $\delta\dot{x} = A\cdot\delta x + B \cdot \delta u$ ensures $x \simeq x_e + \delta x$

when $u \simeq u_e + \delta u$

1². linearity [Nv7 Ch 2.10, 3] [AMv2 Ch 6.1]

1³. matrix exponential [AMv2 Ch 6.2]

$\dot{x} = Ax + Bu$ has homogeneous solution $(u=0)$

$x(t) = e^{At}x(0)$ where $e^X = \sum_{k=0}^{\infty} \frac{1}{k!} X^k$ is matrix exponential