[AMv2 Ch 2]

goal: introduce fundamental uses and properties of feedback

topics:

1°. mathematical models of systems

1¹. differential equations (DE)

1². transfer functions

1³. block diagrams

2°. effects of feedback

2¹. disturbance attenuation

2². unmodeled dynamics

2³. reference tracking

* read [AMv2 Ch 2.2.5] to learn how positive feedback used in digital systems

* Nise on reserve?
→ ECE front desk copy

* HW0: watch Python videos

* Canvas Discussions
→ not Piazza

* work together on HW

* project = long homework

* welcome auditors / drop-ins

* post notes in advance

1°. mathematical models of systems

- we will work with multiple representations of linear control systems

1¹. differential equations

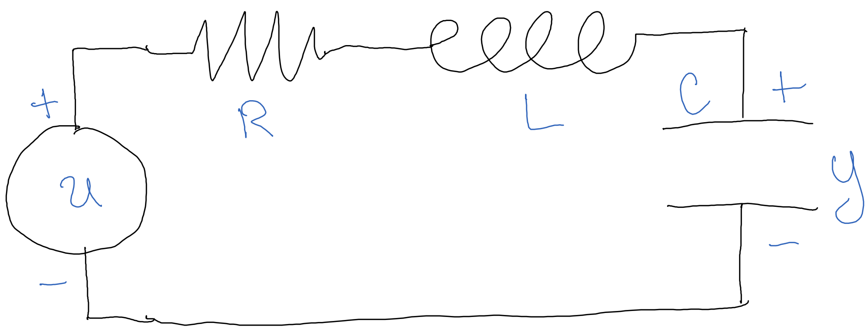
1². transfer functions

1³. block diagrams

each has unique advantages & provides novel insight

* what is a control system?

ex: consider an RLC circuit



* how does input voltage u relate to output charge y?

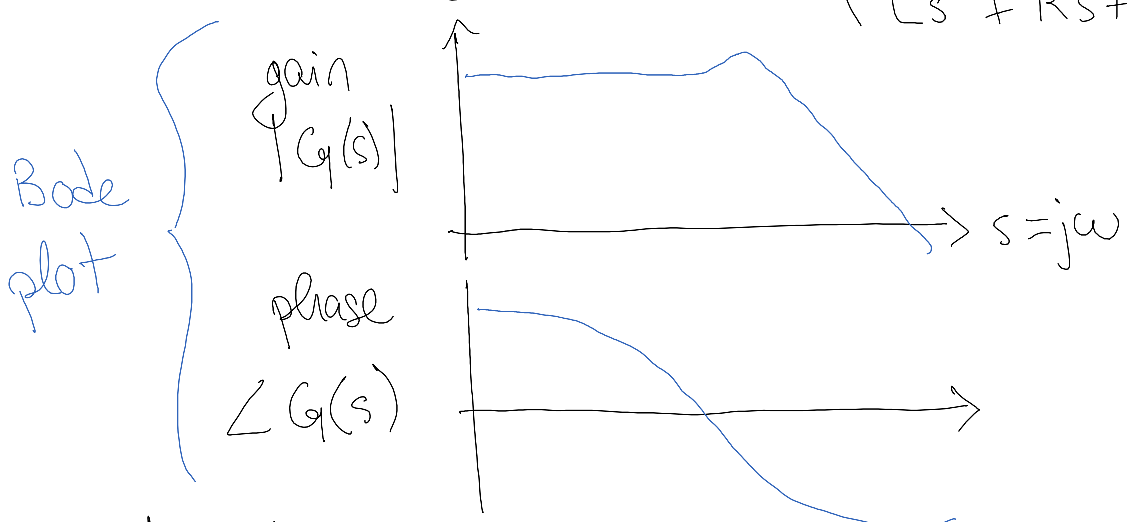
1°. differential equation

$$\text{KVL} \Rightarrow u = Ri + L\frac{d}{dt}i + \frac{1}{C}y$$
$$= R\frac{d}{dt}y + L\frac{d^2}{dt^2}y + \frac{1}{C}y$$

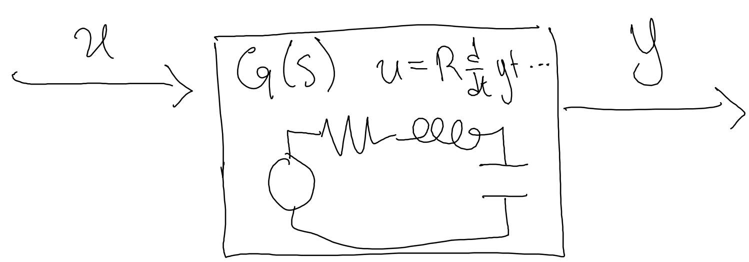
1². transfer function

$\in \mathbb{C}$; called the transfer function

$$u = e^{st} \Rightarrow y = G(s)\,u = \left(\frac{1}{Ls^2 + Rs + \frac{1}{C}}\right)u$$



1³. block diagram



1'. (linear) differential equation (DE): [AMv2 Ch 2] [Nv7 Ch 3,4]

→ if u, y solve (DE), show that

$u' : \mathbb{R} \to \mathbb{R}$ $\qquad$ $y' : \mathbb{R} \to \mathbb{R}$

$: t \mapsto u(t+\tau)$ $\qquad$ $: t \mapsto y(t+\tau)$

$u' : \mathbb{R} \to \mathbb{R}$ $\quad$ $y' : \mathbb{R} \to \mathbb{R}$

$: t \mapsto u(t+\tau)$ $\quad$ $: t \mapsto y(t+\tau)$

solve (DE) as well

↳ thus (DE) is <u>time-invariant</u>

→ if $(u_1, y_1), (u_2, y_2)$ solve (DE),

show that $(u_1 + \alpha u_2, y_1 + \alpha y_1)$

solve (DE) as well, where $\alpha \in \mathbb{R}$

↳ thus (DE) is <u>linear</u>

↳ (DE) is linear & time-invariant (LTI)

1º. transfer function:

[AMv2 Ch 2] [Nv7 Ch 2]

summary & synthesis of 1! & 1? :

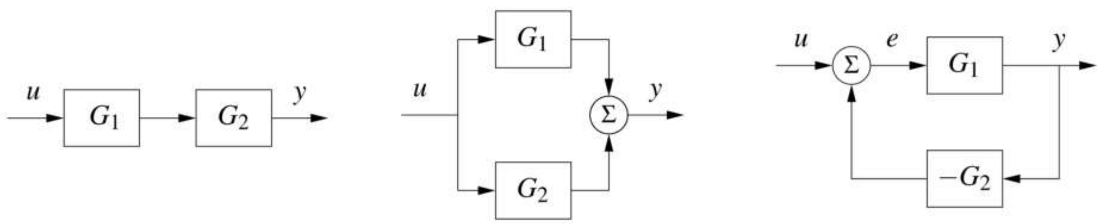
→ what happens when $x(t) = e^{s_k t}$, $a(s_k) = 0$?

- what does the transfer function tell us?
- " " (DE) tell us?

→ same Q's for $x(t) = e^{s_\ell t}$, $b(s_\ell) = 0$

→ s_k termed a <u>pole</u>, s_ℓ termed a <u>zero</u>

1³. block diagrams [AMv2 Ch 2] [Nv7 Ch 5]



(a) $G_{yu}(s) = G_2(s)G_1(s)$ (b) $G_{yu}(s) = G_1(s) + G_2(s)$ (c) $G_{yu}(s) = \dfrac{G_1(s)}{1 + G_1(s)G_2(s)}$

2°: effects of feedback

- there are many uses & types of feedback; we'll focus on these important cases:

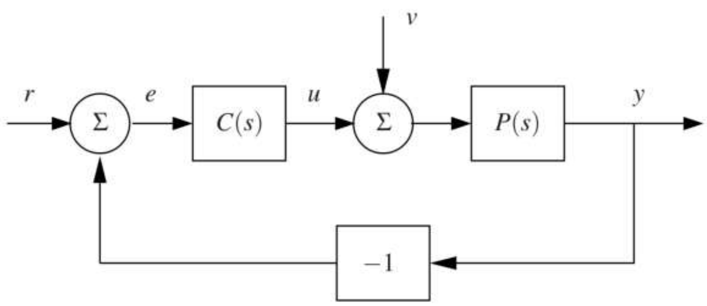
  2¹. disturbance attenuation

  2². unmodeled dynamics

  2³. reference tracking

*read [AMv2 ch 2] to learn about other uses & types of feedback

2¹. disturbance attenuation [AMv2 Ch 2.3]



v
r
Σ
e
C(s)
u
Σ
P(s)
y
−1

2^2. unmodeled dynamics [AMv2 Ch 2.4]

2^3. reference tracking [AMv2 Ch 2.5]