06 -- Tue Nov 10

ECE 447: Control Systems Fall 2020

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today: ☑ HW 4 overview

☑ week 6 lectures

☐ office hour / co-work time

this week: ☐ exam 1 results, solution, regrade process (later)

HW4 problem 2e

$$\dot{x} = Ax, \quad z = V^{-1}x \Rightarrow \dot{z} = V^{-1}\dot{x}$$

$$\Updownarrow \qquad\qquad = V^{-1}Ax$$

$$Vz = x \qquad\qquad = \underbrace{V^{-1}AV}_{\tilde{A}} z$$

$\tilde{A}$ ← diagonal b/c V contains eigenvectors

HW4 problem 3

$$\text{(NL)} \quad \ddot{q} = \frac{C}{M}\left(\frac{i}{q}\right)^2 - g, \; \dot{i} = \frac{1}{L}\left(-Ri + 2C\frac{i\dot{q}}{q^2} + u\right)$$

$$\text{(L)} \quad \dot{x} = Ax + Bu, \; y = Cx + Du.$$

$\in \mathbb{R}^{3 \times 1}$

$$\textbf{(L)} \quad \dot{x} = Ax + Bu, \; y = Cx + Du.$$

$\circ$ let $x = (q, \dot{q}, i) \in \mathbb{R}^3$ so $\dot{x} = \underbrace{\begin{bmatrix} \dot{q} \\ \ddot{q}(q, i) \\ \dot{i}(q, \dot{q}, i, u) \end{bmatrix}}_{\in \mathbb{R}} = f(x, u)$

$$y = q = Cx + Du = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \\ i \end{bmatrix} + 0 \cdot u$$

$\ast$ find (x_e, u_e) s.t. $f(x_e, u_e) = \text{"}\dot{x}_e\text{"} = 0$

$\delta \dot{x} = A \delta x + B \delta u$, $\quad \delta x = x - x_e$, $\quad \delta u = u - u_e$

$A = \partial_x f(x_e, u_e) \in \mathbb{R}^{3\times 3}$ $\qquad$ $B = \partial_u f(x_e, u_e) \in \mathbb{R}^{3\times 1}$

$x = (q, \dot{q}, i) \in \mathbb{R}^3$ so $\dot{x} = \begin{bmatrix} \dot{q} \\ \ddot{q}(q, i) \\ \dot{i}(q, \dot{q}, i, u) \end{bmatrix} = f(x, u)$

$$\partial_x f = \left[\partial_{x_j} f_i \right]_{i,j=1}^{n=3} = \begin{bmatrix} \cancel{\partial_q \dot{q}} \to 0 & \cancel{\partial_{\dot{q}} \dot{q}} \to 1 & \cancel{\partial_i \dot{q}} \to 0 \\ \partial_q \ddot{q} & \cancel{\partial_{\dot{q}} \ddot{q}} \to 0 & \partial_i \ddot{q} \\ \partial_q \dot{i} & \partial_{\dot{q}} \dot{i} & \partial_i \dot{i} \end{bmatrix}$$

$$A = \partial_x f(\underbrace{x_e}_{= \begin{bmatrix} q_e \\ \dot{q}_e \\ i_e \end{bmatrix}}, u_e) \qquad B = \partial_u f(x_e, u_e)$$

$$\partial_u f = \begin{bmatrix} \cancel{\partial_u \dot{q}} \; 0 \\ \cancel{\partial_u \ddot{q}} \; 0 \\ \partial_u \dot{i} \end{bmatrix}$$