ECE 447: Control Systems (Fall 2020)

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* please submit mid-quarter course assessment ← ~50% response rate

* take-home exam next week

today:
- ☑ HW 3 assigned – due Fri Oct 30
- ☑ week 4 lectures posted (~2 hours)
- ☑ HW 2 solution
- ☐ office hour

TODO: lec 4a ±

$$\ddot{q}(q,\dot{q},u) = \frac{g}{\ell}\sin q - \frac{\alpha}{m\ell^2}\dot{q} + \frac{1}{m\ell}u\cos q \Rightarrow \partial_u \ddot{q} = \frac{1}{m\ell}\cos q$$

$$x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \Rightarrow \dot{x} = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta}(\theta, \dot{\theta}, u) \end{bmatrix} = f(x, u)$$

$$\frac{\partial}{\partial x} f(x,u) = \begin{bmatrix} \partial_{x_1} f_1 & \partial_{x_2} f_1 \\ \partial_{x_1} f_2 & \partial_{x_2} f_2 \end{bmatrix} = \begin{bmatrix} \partial_\theta \dot{\theta} & \partial_{\dot{\theta}} \dot{\theta} \\ \partial_\theta \ddot{\theta} & \partial_{\dot{\theta}} \ddot{\theta} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ \frac{g}{\ell}\cos\theta - \frac{1}{m\ell} u \sin\theta & -\frac{\alpha}{m\ell^2} \end{bmatrix}$$

$$A = \frac{\partial}{\partial x} f(x_e, u_e) = \left. \begin{bmatrix} \partial_{x_1} f_1 & \partial_{x_2} f_1 \\ \partial_{x_1} f_2 & \partial_{x_2} f_2 \end{bmatrix} \right|_{\substack{x = x_e \\ u = u_e}}$$

$\theta_e = k\pi$

$$= \begin{bmatrix} 0 & 1 \\ \frac{g}{\ell}\cos\theta_e = \pm 1 & -\frac{\alpha}{m\ell^2} \end{bmatrix}$$

$$B = \frac{\partial}{\partial u} f(x_e, u_e) = \left. \begin{bmatrix} \partial_u \dot{\theta} \\ \partial_u \ddot{\theta} \end{bmatrix} \right|_{\substack{x = x_e \\ u = u_e}} = \begin{bmatrix} 0 \\ \frac{1}{m\ell}\cos\theta_e = \pm 1 \end{bmatrix}$$