06 -- Thu Nov 12

ECE 447: Control Systems Fall 2020

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today: ☑ HW 4 – questions? HW4 p3 eq: $\boxed{+}$ or – ?

☑ week 6 lecture – questions?

☐ office hour / co-work time

Friday: ☐ exam 1 results, solution, regrade process

HW 3 p3 b

$$\text{(NL)} \quad \ddot{q} = \frac{C}{M}\left(\frac{i}{q}\right)^2 - g, \ \dot{i} = \frac{1}{L}\left(-Ri + 2C\frac{i\dot{q}}{q^2} + u\right)$$

(b.) Linearize the nonlinear system (NL) around the equilibrium from (a.) to obtain a linear system (L)

$$\text{(L)} \quad \delta\dot{x} = A\delta x + B\delta u, \ \delta y = C\delta x + D\delta u.$$

1°. choose state vector $x = \begin{bmatrix} q \\ \dot{q} \\ i \end{bmatrix}$

non-linear, i.e. $\neq A \cdot x + B \cdot u$

2°. determine nonlinear dynamics $\dot{x} = f(x,u) = \begin{bmatrix} \dot{q} \\ \ddot{q}(\ldots) \end{bmatrix}$

2°. determine nonlinear dynamics $\dot{x} = f(x,u) = \begin{bmatrix} \dot{q} \\ \ddot{q}(q,i) \\ \dot{i}(q,\dot{q},i,u) \end{bmatrix}$

3°. choose/find equilibrium $(\underset{\in\mathbb{R}^n}{x_e}, \underset{\in\mathbb{R}^q}{u_e})$ s.t. $\underbrace{f(x_e,u_e)=0}_{\in\mathbb{R}^n}$ = "$\dot{x}_e$"

3¹. $\dot{q}_e = 0$ 3². $\ddot{q}(q,i) = 0 \rightsquigarrow$ solve for q_e i.t.o. i_e
or i_e i.t.o. q_e

4°. linearize: with $\delta x = x - x_e$, $\delta u = u - u_e$,

$$\delta\dot{x} = \dot{x} = f(x,u) \simeq \cancelto{0}{f(x_e,u_e)} + \partial_x f(x_e,u_e)\cdot\delta x + \partial_u f(x_e,u_e)\cdot\delta u + \cancelto{0}{\text{stuff}}$$

so $\delta\dot{x} \simeq A\,\delta x + B\,\delta u$, $A = \partial_x f(x_e,u_e) \in \mathbb{R}^{3\times 3}$

$B = \partial_u f(x_e,u_e) \in \mathbb{R}^{3\times 1}$

$x = \begin{bmatrix} q \\ \dot{q} \\ i \end{bmatrix}$ so $A = \partial_x f(x_e,u_e) = \left.\begin{bmatrix} \cancelto{0}{\partial_q \dot{q}} & \partial_{\dot{q}} \dot{q} & \cancelto{0}{\partial_i \dot{q}} \\ \partial_q \ddot{q} & \cancelto{0}{\partial_{\dot{q}} \ddot{q}} & \partial_i \ddot{q} \\ \partial_q \dot{i} & \partial_{\dot{q}} \dot{i} & \partial_i \dot{i} \end{bmatrix}\right|_{\substack{q=q_e \\ \dot{q}=\dot{q}_e \\ i=i_e \\ u=u_e}}$

$x_e = \begin{bmatrix} q_e \\ \dot{q}_e \\ i_e \end{bmatrix}$

$$f(x,u) = \begin{bmatrix} \dot{q} \\ \ddot{q}(q, i) \\ \dot{i}(q, \dot{q}, i, u) \end{bmatrix}$$

HW4 p3d

$$\delta\dot{x} = A\,\delta x + B\,\delta u$$

proportional control $\delta u = -k_p \cdot \delta q$, $\delta q = q - q_e$

(on position q)

$$= -K \cdot \delta x, \quad K = [k_p \;\; 0 \;\; 0]$$

$$\delta x = \begin{bmatrix} \delta q \\ \delta\dot{q} \\ \delta i \end{bmatrix}$$

closed-loop dynamics:

$$\delta\dot{x} = A\delta x - BK\delta x$$

$$= \underbrace{(A - BK)}\cdot\delta x$$

$$= A_{cl}(k_p) \leftarrow$$ compute roots of characteristic polynomial of this matrix

i.e. eigenvalues

$$\parallel$$

$$\partial_x f_{cl}(x_e, u_e)$$

$$\Uparrow$$

$$u = -k_p(q - q_e)$$

$u = -k_p(q - q_e)$

$$\dot{x} = f_{cl}(x, \cancel{u}) = \begin{bmatrix} \dot{q} \\ \ddot{q}(q, i) \\ \dot{i}(q, \dot{q}, i, -k_p(q - q_e)) \end{bmatrix}$$

given $f(x_e, u_e) = 0$, $u(x)$ s.t. $u(x_e) = u_e$

then with $A = \partial_x f(x_e, u_e)$, $B = \partial_u f(x_e, u_e)$,

$K = \partial_x u(x_e)$, $A_{cl} = \partial_x f_{cl}(x_e)$, $f_{cl}(x) = f(x, u(x))$

we have $A_{cl} = A + BK$

$$* \; A_{cl} = \partial_x f_{cl} = \partial_x f(x, u(x))$$
$$= \partial_x f + \partial_u f \cdot \partial_x u$$