447 20fa exam 2 due 5p Fri Dec 11

You are welcome (and encouraged) to:

- use analytical and numerical computational tools -- specify the tool(s) in sourcecode and/or text;
- reuse example sourcecode and other materials provided in this course;
- consult textbooks, websites, and other publicly-available materials -- include full citation(s) with the URL and/or DOI.

You are not permitted to discuss the exam problems or share any part of your solutions with anyone other than the Professor or TA for this course.

- By submitting your exam solution on Canvas, you are affirming your understanding of and adherence to these restrictions.
- We will answer questions during the class Zoom meetings Tue Dec 8 and Thu Dec 10.
- We will also answer questions posted to the Canvas Discussion board until 5p Fri Dec 11.

The exam deadline is 5p Fri Dec 11 on Canvas.

- Final submissions received before this deadline will receive +2 bonus points (equal to one subproblem).
- Everyone automatically receives a deadline extension to 11:59p Sun Dec 11.
- ***Further deadline extensions must be requested by Fri Dec 11.***

problem (1.)

Consider the nonlinear system

$$\dot{x_1} = -x_1 + x^3,\ \dot{x_2} = -2x_2,$$

which has 3 equilibria: $(0,0)$, $(1,0)$, $(-1,0)$.

subproblem (a.)

Linearize the system at one of the equilibria.

subproblem (b.)

Determine whether the equilibrium chosen in (a.) is stable.

problem (2.)

A process model $\dot{y} = u$ relating scalar input u to scalar output y has state-space representation

$$\dot{x} = ax + bu,\ y = cx + du$$

where $a = 0, b = 1, c = 1, d = 0$.

An observer-based controller for the process has state-space representation

$$\dot{\widehat{x}} = a\widehat{x} + bu - \ell(y - \hat{y}),\ \hat{y} = c\widehat{x} + du,\ u = -k\widehat{x}$$

where $k, \ell \in \mathbb{R}$ are the controller's parameters.

subproblem (a.)

Determine the transfer functions of the process, $G_{yu}(s)$, and the controller, $G_{uy}(s)$.

subproblem (b.)

Verify that the closed-loop system obtained by interconnecting the process and controller through y and u is stable for any $k > 0, \ell < 0$.

problem (3.)

Consider $L(s) = P(s)C(s) = 20/(s+1)^3$, the ***(open-)loop transfer function*** for a control system.

subproblem (a.)

Create the ***Nyquist diagram*** and determine whether the closed-loop system is stable using the ***Nyquist stability criterion***.

subproblem (b.)

Create one ***Bode Plot*** that visualizes three transfer functions: sensitivity $S = \frac{1}{1+L}$, complementary sensitivity $T = \frac{L}{1+L}$, and their sum $S + T$.

problem (4.)

Consider the process $P(s) = 1/s(s+1)^2$ and controller $C(s) = k$.

subproblem (a.)

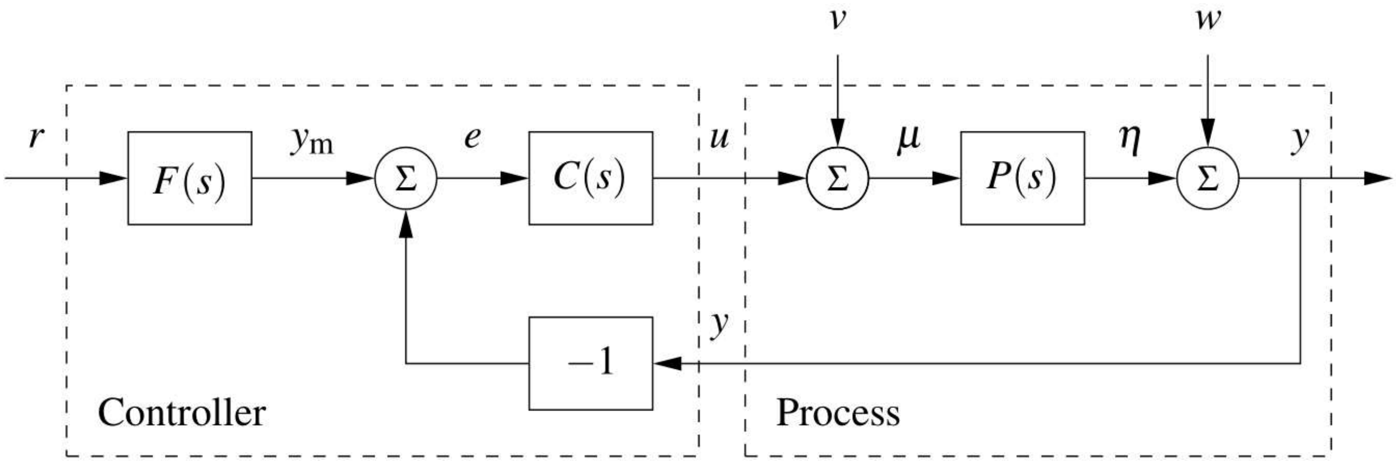
Create the ***root locus*** plot for the closed-loop system as k varies.

subproblem (b.)

Choose a value for k that results in a stable closed-loop system and determine the corresponding ***gain margin***.

problem (5.)

Consider the control system below with input disturbance v and output disturbance w.



You just started a new job as a ***Senior Control System Engineer*** at a startup company that is designing a quadrotor drone for power line inspection.

subproblem (a.)

At this morning's all-hands meeting, the mechatronics team announces that electromagnetic interference introduces significant input disturbance v and output disturbance w at $60\mathrm{Hz}$, so your boss tasks you with designing controller C to attenuate 90% of the effect of these disturbances on input u and output y, i.e. with G_{uv} denoting the transfer function from v to u and G_{yw} denoting the transfer function from w to y,

$$|G_{uv}(j\,60\mathrm{Hz})| \leq 0.1,\ |G_{yw}(j\,60\mathrm{Hz})| \leq 0.1.$$

Explain to your boss why it is not possible to satisfy this specification.

subproblem (b.)

During this afternoon's stand-up meeting, the aerodynamics team points out that turbulent wind conditions cause broad-band input disturbance v, so your boss tasks you with changing the controller design, from C to $\widetilde{C}$, to decrease the impact of these disturbances on the input u by 25% at all frequencies, i.e. with G_{uv} denoting the transfer function from v to u obtained with controller C and $\widetilde{G}_{uv}$ denoting the transfer function obtained with controller $\widetilde{C}$,

$$\forall \omega \geq 0 : |\widetilde{G}_{uv}(j\,\omega)| \leq 0.75|G_{uv}(j\,\omega)|.$$

Explain to your boss why it is not possible to satisfy this specification.