447 20fa exam 1 due 5p Fri Nov 6

You are welcome (and encouraged) to:

- use analytical and numerical computational tools -- specify the tool(s) in sourcecode and/or text;
- reuse example sourcecode and other materials provided in this course;
- consult textbooks, websites, and other publicly-available materials -- include full citation(s) with the URL and/or DOI.

You are not permitted to discuss the exam problems or share any part of your solutions with anyone other than the Professor or TA for this course.

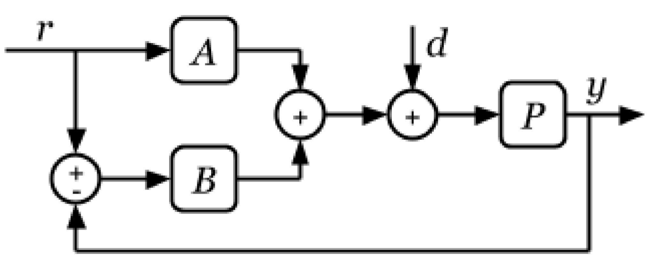
- By submitting your exam solution on Canvas, you are affirming your understanding of and adherence to these restrictions.
- We will answer questions during the class Zoom meetings Tue Nov 3 and Thu Nov 5.
- We will also answer questions posted to the Canvas Discussion board until 5p Fri Nov 6.

The exam deadline is 5p Fri Nov 6 on Canvas.

- Final submissions received before this deadline will receive +2 bonus points (equal to one subproblem).
- Everyone automatically receives a deadline extension to 11:59p Sun Nov 8. ***No further deadline extensions will be granted -- please plan accordingly.***

problem (1.)

Consider the following block diagram:



subproblem (1a.)

Determine the transfer function G_{yr} from r to y.

subproblem (1b.)

Determine the transfer function G_{yd} from d to y.

problem (2.)

Consider the following process model:

$$P(s) = \frac{b(s)}{a(s)} = \frac{b_1\, s^2 + b_2\, s + b_3}{a_0\, s^3 + a_1\, s^2 + a_2\, s + a_3}$$

where $b_1\,, b_2\,, b_3$ and $a_0\,, a_1\,, a_2\,, a_3$ are nonzero parameters.

subproblem (2a.)

Suppose $a_0 = 10$, $a_2 = 1$, $a_3 = 2$; what range of values for a_1 ensure P is stable?

subproblem (2b.)

Assume P is stable -- if a constant input is applied to P, what output is produced?

problem (3.)

Consider the following nonlinear system (NL):

$$\dot{x}_1 = x_2 + x_1(1 - x_1^2 - x_2^2),\ \dot{x}_2 = -x_1 + x_2(1 - x_1^2 - x_2^2).$$

subproblem (3a.)

Create a phase portrait of the nonlinear system (NL) on the square $x_1\,, x_2 \in (-1.5, +1.5)$: use `plt.quiver` or `plt.streamplot` as in the examples presented in lecture / provided on homework solutions.

subproblem (3b.)

Linearize the nonlinear system about the equilibrium $x_e = 0 \in \mathbb{R}^2$: provide matrix A such that $\dot{x} \simeq Ax$ for x near x_e.

problem (4.)

Consider the state-space linear system $\dot{x} = Ax + Bu$.

subproblem (4a.)

Suppose $\lambda \in \mathbb{C}$ is an eigenvalue of A, so that there exists a nonzero vector $v \neq 0$ such that $Av = \lambda v$.

Show that $e^{\lambda t}$ is an eigenvalue of e^{At} where $t \in \mathbb{R}$, and explain what this means about the trajectory initialized at $x(0) = v$.

subproblem (4b.)

If $A = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix}$, verify that $e^{At} = \begin{bmatrix} \cos \omega t & \sin \omega t \\ -\sin \omega t & \cos \omega t \end{bmatrix}$ by verifying that $x(t) = e^{At} x(0)$ solves $\dot{x} = Ax$ for all $x(0)$.

problem (5.)

Consider proportional-integral control of an unstable first-order process:

$$C(s) = k_P + \frac{1}{s} k_I,\ P(s) = \frac{1}{s-1},$$

and suppose k_P, k_I are chosen to make the roots of the characteristic polynomial of the closed-loop system $\frac{PC}{1+PC}$ equal to $-\sigma \pm j\omega$:

$$k_P = 2\sigma + 1,\ k_I = \sigma^2 + \omega^2.$$

In the following subproblems, we will hold $k_I = 1$ constant and let k_P vary.

subproblem (5a.)

Plot the root locus of the closed-loop characteristic polynomial as k_P varies.

subproblem (5b.)

Determine the range of k_P that results in a stable closed-loop system -- you may do this analytically (e.g. pen-and-paper) or numerically (e.g. using the root locus from (5a.)).