06 -- Mon Nov 8

ECE 447: Control Systems (Fall 2021)

Prof: Sam Burden TA: Sat Singh

today: ☑ logistics: HW 4 due Fri Nov 12

HW 6 due Fri Nov 19

☑ tutorial: linearization revisited – this time w/ outputs

☑ break: back ~ 1:25p

☐ office hour

---

tutorial: linearization with outputs

[AMv2 Ex 2.8, 5.12, 6.4, 7.4]

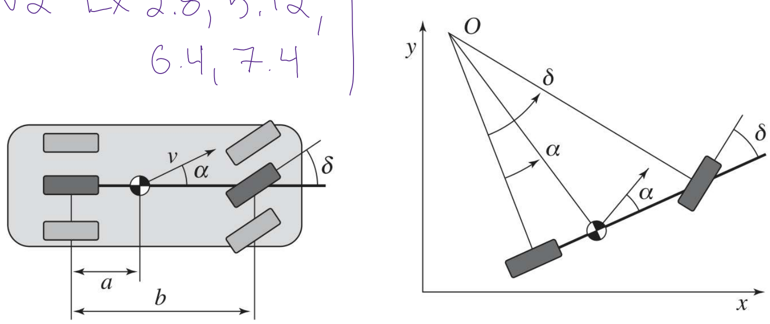


Figure 2.16: Vehicle steering dynamics. The left figure shows an overhead view of a vehicle with four wheels. The wheel base is b and the center of mass at a distance a forward of the rear wheels. By approximating the motion of the front and rear pairs of wheels by a single front wheel and a single rear wheel, we obtain an abstraction called the *bicycle model*, shown on the right. The steering angle is δ and the velocity at the center of mass has the angle α relative the length axis of the vehicle. The position of the vehicle is given by (x, y) and the orientation (heading) by θ.

$$\dot{x} = f(x,u), \quad y = h(x,u)$$

$$f(x,u) = \begin{pmatrix} v\sin(\alpha(u) + x_2) \\ \frac{v_0}{b}\tan u \end{pmatrix}, \qquad \alpha(u) = \arctan\left(\frac{a\tan u}{b}\right), \qquad h(x,u) = x_1.$$

$$A = \left.\frac{\partial f}{\partial x}\right|_{\substack{x=0\\u=0}} = \begin{pmatrix} 0 & v_0 \\ 0 & 0 \end{pmatrix}, \qquad B = \left.\frac{\partial f}{\partial u}\right|_{\substack{x=0\\u=0}} = \begin{pmatrix} av_0/b \\ v_0/b \end{pmatrix},$$

$$C = \left.\frac{\partial h}{\partial x}\right|_{\substack{x=0\\u=0}} = \begin{pmatrix} 1 & 0 \end{pmatrix}, \qquad D = \left.\frac{\partial h}{\partial u}\right|_{\substack{x=0\\u=0}} = 0,$$

$$\delta\dot{x} = A\cdot\delta x + B\cdot\delta u \qquad x \simeq x_e + \delta x$$

$$\delta y = C\cdot\delta x + D\cdot\delta u \qquad u \simeq u_e + \delta u$$

$$y \simeq y_e + \delta y$$

1°. compute derivatives

$$\frac{\partial f}{\partial x} = \begin{bmatrix} \partial_1 f_1 & \partial_2 f_1 \\ \partial_1 f_2 & \partial_2 f_2 \end{bmatrix} = \begin{bmatrix} 0 & v\cos(\alpha(u)+x_2)\cdot 1 \\ 0 & 0 \end{bmatrix}$$

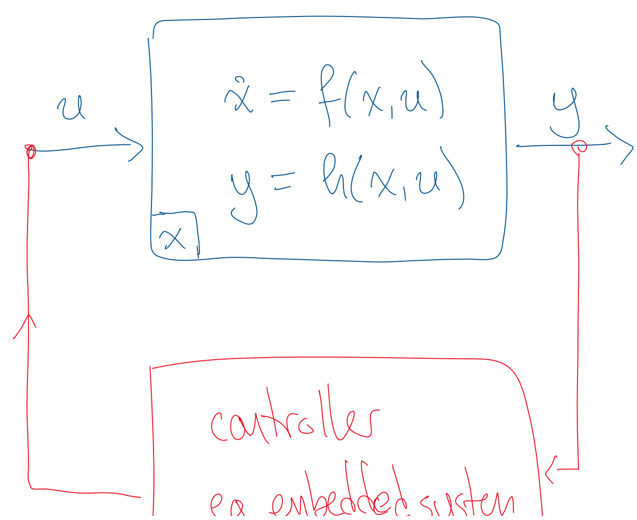
$$\frac{\partial f}{\partial u} = \begin{bmatrix} \partial_u f_1 \\ \partial_u f_2 \end{bmatrix} = \begin{bmatrix} v\cos(\alpha(u)+x_2)\cdot\partial\alpha(u) \\ \frac{v_0}{b}\sec^2 u \end{bmatrix}$$

$$C = \frac{\partial h}{\partial x} = \begin{bmatrix} \partial_1 h & \partial_2 h \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad \frac{\partial h}{\partial u} = 0 = D$$

given $\dot{x} = f(x,u)$ — dynamics of state $x \in \mathbb{R}^n$ driven by input $u \in \mathbb{R}^p$

$y = h(x,u)$ — "output" (i.e. sensor measurements) $y \in \mathbb{R}^o$

think about input/output perspective:



eg embedded system

given $\boxed{\begin{aligned} \dot{x} &= f(x,u) \\ y &= h(x,u) \end{aligned}}$ $\in \mathbb{R}^o$, $x_e \in \mathbb{R}^n$, $u \in \mathbb{R}^p$ and equilibrium $f(x_e, u_e) = 0$, $y_e = h(x_e, u_e)$

to linearize: 1°: compute $\partial_x f \in \mathbb{R}^{n\times n}$, $\partial_u f \in \mathbb{R}^{n\times p}$

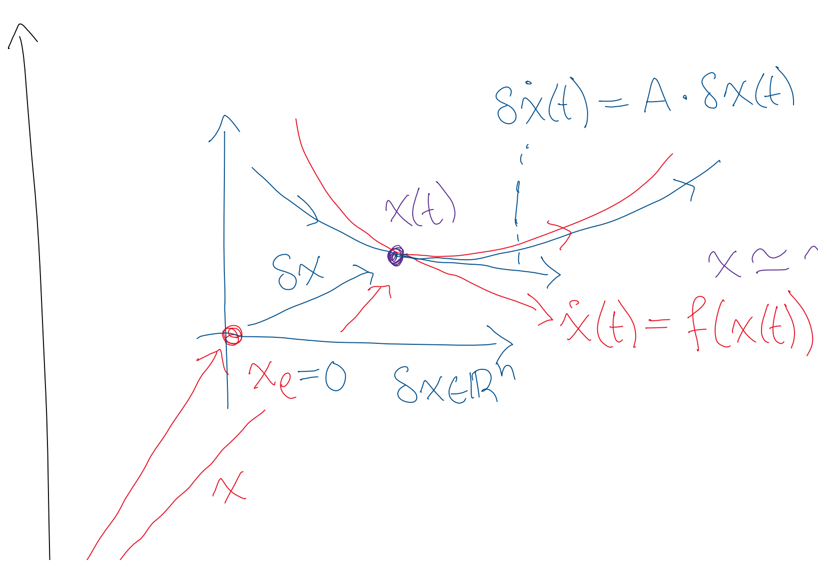
$\partial_x h \in \mathbb{R}^{o\times n}$, $\partial_u h \in \mathbb{R}^{o\times p}$

2°: evaluate @ (x_e, u_e): $A = \partial_x f(x_e, u_e)$, $B = \partial_u f(x_e, u_e)$

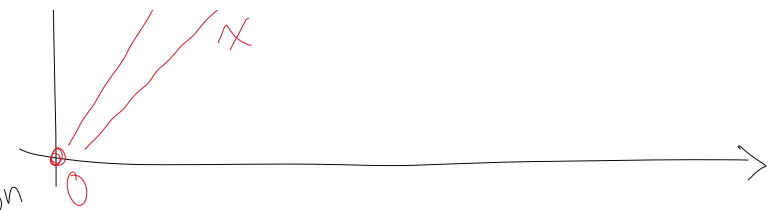
$C = \partial_x h(x_e, u_e)$, $D = \partial_u h(x_e, u_e)$

then: $\underset{\in \mathbb{R}^n}{x \simeq x_e + \delta x}$ when $\underset{\in \mathbb{R}^p}{u = u_e + \delta u}$, $\boxed{\begin{aligned} \delta\dot{x} &= A\cdot\delta x + B\cdot\delta u \\ \delta y &= C\cdot\delta x + D\cdot\delta u \end{aligned}}$

$\underset{\in \mathbb{R}^o}{y \simeq y_e + \delta y}$ when



$x \simeq x_e + \delta x \iff \delta x \simeq x - x_e$



$x \in \mathbb{R}^n$