09 -- Mon Nov 29

ECE 447: Control Systems (Fall 2021)

Prof: Sam Burden   TA: Sat Singh

today: ☑ logistics: HW 8 due Fri Dec 3

exam 2 will be assigned Fri Dec 3

☑ tutorial

~~☐ break~~

☐ office hour

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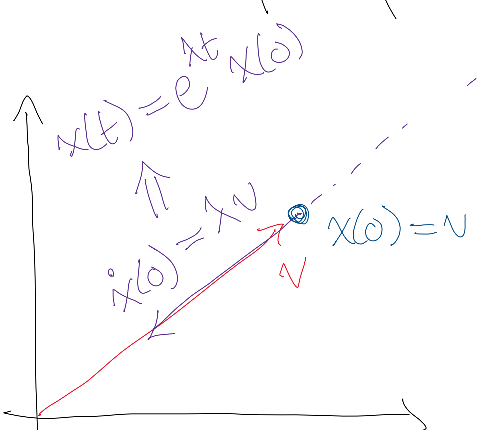
if λ, v is eigval/eigvec pair for A, i.e. $Av = \lambda v$, $v \neq 0$

then trajectory of $\dot{x} = Ax$ initialized at $x(0) = v$ is $x(t) = e^{\lambda t} v$

(because $\dot{x}(0) = A x(0) = Av = \lambda v = \lambda x(0)$

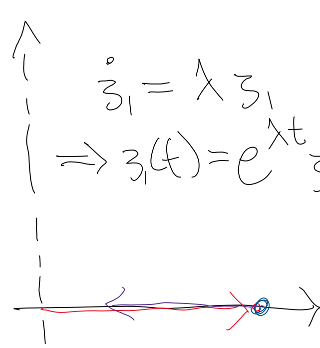
is "aligned" with v – so trajectory "stays on"

subspace spanned by v: $x(t) = e^{\lambda t} x(0)$)



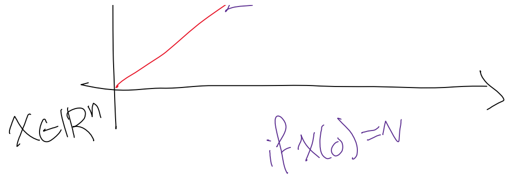
let $z = Tx$ be such that

$$T \cdot v = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

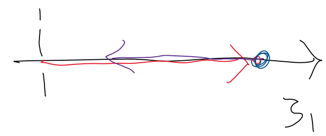


$\dot{z}_1 = \lambda z_1$

$\Rightarrow z_1(t) = e^{\lambda t} z_1(0)$



$$T v = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$$



$$x(t) = e^{At} x(0) \overset{!}{=} \underbrace{e^{At} v = e^{\lambda t} v}_{\text{can confirm } v \text{ is eigvec w/ eigval } e^{\lambda t} \text{ for } e^{At}}$$

complications:

$$M = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix} \qquad M \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \lambda \\ 0 \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \lambda v$$

$$Mw + \lambda v = \lambda w$$

$$M = \begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix} \rightsquigarrow \lambda^{\pm} = \sigma \pm j\omega$$

$v^{\pm}$ are complex

$$\dot{x} = Ax + Bu$$

$$z = Tx \implies \dot{z} = T\dot{x} = T[Ax + Bu]$$

$$\Downarrow$$

$$T^{-1} z = x$$

$$= TAT^{-1} z + TBu$$

$$= \tilde{A} z + \tilde{B} u$$