05 -- Mon Nov 1

ECE 447: Control Systems (Fall 2021)

Prof: Sam Burden TA: Sat Singh

today: □ logistics: HW4 due Fri Nov 12

□ exam 1 assigned Fri Oct 29

$\longrightarrow$ due Fri Nov 5

□ break

□ office hour

Prof Burden TODO: □ bonus points on HW

□ p1 b,c clarify P, A, B rational func's

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exam 1 p1 b, c

subproblem (1b.)

A sinusoidal disturbance can sometimes be perfectly rejected: give an example of P, B, and $\omega > 0$ such that y_d is zero when $d(t) = \sin(\omega t)$.

rephrase: $\omega > 0$ and P, A, B rational functions

subproblem (1c.)

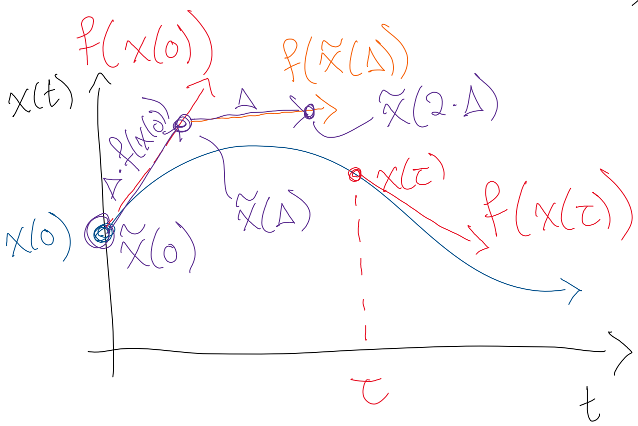
A sinusoidal reference can sometimes be perfectly tracked: give an example of P, A, B, and $\omega > 0$ such that $y_r = r$ when $r(t) = \sin(\omega t)$.

* specify P, A, B as rational functions with real coefficients.

$$P(s) = \frac{b_1 s^{n-1} + b_2 s^{n-2} + \cdots + b_n}{s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n} \qquad \left.\begin{matrix} b_i \in \mathbb{R} \\ a_i \in \mathbb{R} \end{matrix}\right\} \begin{matrix} \text{all} \\ i \in \{1, \dots, n\} \end{matrix}$$

numerical simulation

$$\dot{x} = f(x) \iff f(x(\tau)) = \lim_{\Delta \to 0} \frac{1}{\Delta}\left(x(\tau+\Delta) - x(\tau)\right)$$



$$\simeq \frac{1}{\Delta}\left(\tilde{x}(\tau+\Delta) - \tilde{x}(\tau)\right), \ \Delta > 0$$

$$\iff \tilde{x}(\tau+\Delta) = \tilde{x}(\tau) + \Delta f(\tilde{x}(\tau))$$

* if $\Delta > 0$ small ($\Delta << 1$, eg $\Delta = 10^{-4}$)
  then $\tilde{x}(\tau) \simeq x(\tau)$

exam 1 p3 $\quad \dot{x} = Ax \rightsquigarrow \tilde{x}^+ = \tilde{x} + \Delta \cdot A \cdot \tilde{x}$

$$\tilde{x}[1] = \tilde{x}(1\cdot\Delta) = \tilde{x}(0) + \Delta \cdot A\, \tilde{x}(0)$$

$$\tilde{x}[2] = \tilde{x}(2\cdot\Delta) = \tilde{x}(1\cdot\Delta) + \Delta \cdot A\, \tilde{x}(1\cdot\Delta)$$

$$\vdots$$

$$\tilde{x}[n+1] = \tilde{x}[n] + \Delta \cdot A \cdot \tilde{x}[n]$$

subproblem (4b.)

Consider the function

$$H(x_1, x_2) = -\frac{\lambda}{2}\left(x_1^2 + x_2^2\right) + \frac{1}{2}\left(x_1 x_2^2 - \frac{1}{3}x_1^3\right).$$

Show that H is constant along trajectories of (NL) by computing $\dot{H}$.

$H : \mathbb{R}^2 \to \mathbb{R}$

$: (x_1, x_2) \mapsto H(x_1, x_2)$

$\frac{\partial}{\partial x} H \in [\partial_{x_1} H \quad \partial_{x_2} H] \in \mathbb{R}^{1\times 2}$

$I : \mathbb{R} \to \mathbb{R}$

$: t \mapsto H(x_1(t), x_2(t)) = I(t)$

* compute $\frac{d}{dt} I(t) \in \mathbb{R}$