ECE 447: Control Systems (Fall 2021)

Prof: Sam Burden TA: Sat Singh

this week: ☑ HW 1 assigned → due Fri Oct 15

☑ week 2 lecture material

☑ office "Hour"

Prof Burden TODO: ☑ post HW0 solution

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week 2 tutorial

1°: quadrotor

A Simple Learning Strategy for High-Speed Quadrocopter Multi-Flips

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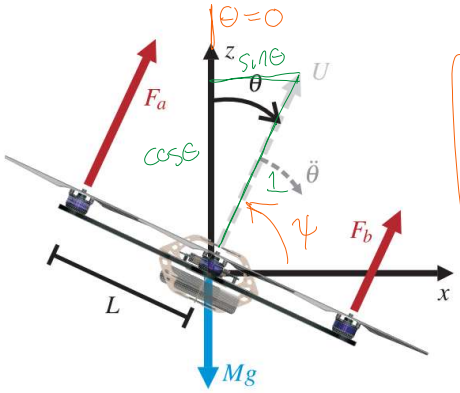

$$M\ddot{z} = (F_a + F_b + F_c + F_d)\cos\theta - Mg \quad (1)$$
$$M\ddot{x} = (F_a + F_b + F_c + F_d)\sin\theta \quad (2)$$
$$I_{yy}\ddot{\theta} = L(F_a - F_b), \quad (3)$$

↳ Fs are forces from rotors

$\psi = 90° - \theta$

↳ $\nu = z \Rightarrow \ddot{\nu} = \ddot{z} = \frac{1}{M}(F\cos\theta - Mg)$

1': state space η = horizontal, ν = vertical, θ = rotation,

inputs $F = F_a + F_b + F_c + F_d$ = net/total thrust

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$\tau = F_a - F_b$ = net torque in θ coordinate

state $x = \begin{bmatrix} \text{positions} \\ \text{velocities} \end{bmatrix} = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} \eta \\ \nu \\ \theta \\ \dot{\eta} \\ \dot{\nu} \\ \dot{\theta} \end{bmatrix} \Rightarrow \dot{x} = \begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} \dot{\eta} \\ \dot{\nu} \\ \dot{\theta} \\ \frac{F}{M}\sin\theta \\ \frac{1}{M}(F\cos\theta - Mg) \\ \frac{L}{I}\tau \end{bmatrix}$

position $q = \begin{bmatrix} \eta \\ \nu \\ \theta \end{bmatrix}$, velocity $\dot{q} = \begin{bmatrix} \dot{\eta} \\ \dot{\nu} \\ \dot{\theta} \end{bmatrix}$

1.2. continuous-time dynamics, discrete-time controller / simulation

state $x \in \mathbb{R}^6$ differential equation $\dot{x} = f(x,u) =$

input $u = (F, \tau) \in \mathbb{R}^2$

$f : \mathbb{R}^6 \times \mathbb{R}^2 \to \mathbb{R}^6$

$: (x,u) \mapsto \dot{x}$

continuous-time

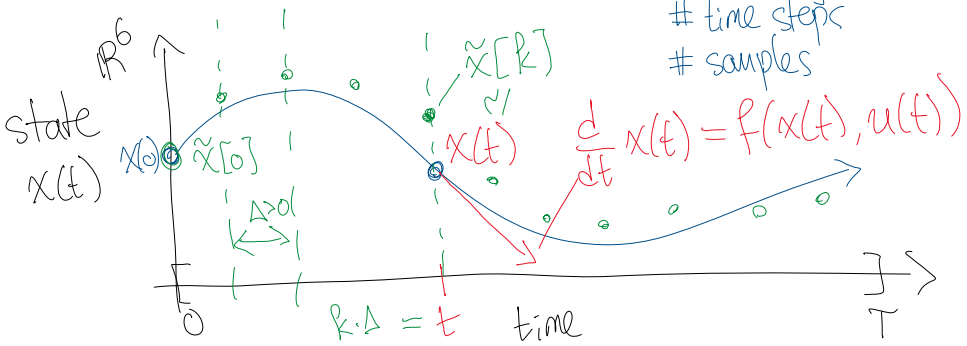
a trajectory $x : [0,T] \to \mathbb{R}^6$ satisfies $\frac{d}{dt}x(t) = f(x(t),u(t))$

$u : [0,T] \to \mathbb{R}^2$ for all $t \in [0,T]$

discrete-time

a simulation $\tilde{x} : [0,T] \to \mathbb{R}^6$ satisfies $\tilde{x}[k] \simeq x(k \cdot \Delta)$, $\Delta > 0$

eg. $\Delta = 0.01$ sec



1.3. linear vs nonlinear dynamics

$$f(x,u) = \dot{x} = \begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} \dot{\eta} \\ \dot{\nu} \\ \dot{\theta} \\ \frac{F}{M}\sin\theta \\ \frac{1}{M}(F\cos\theta - Mg) \\ \frac{L}{I}\tau \end{bmatrix}$$

these dynamics are not linear, i.e. they are nonlinear because:

1°: trigonometric functions (sin, cos)

2°: product of states & inputs ($F \cdot \sin\theta$, $F \cdot \cos\theta$)

approximation step called "linearization" (steps missing here)

$$\simeq \begin{bmatrix} \dot{\eta} \\ \dot{\nu} \\ \dot{\theta} \\ \frac{g}{M}\theta \\ \frac{F}{M} \\ \frac{L}{I}\tau \end{bmatrix} = Ax + Bu$$

$$x = \begin{bmatrix} \eta \\ \nu \\ \theta \\ \dot{\eta} \\ \dot{\nu} \\ \dot{\theta} \end{bmatrix} \qquad u = \begin{bmatrix} F \\ \tau \end{bmatrix}$$

$$A = \left[\begin{array}{cccccc} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \hline 0 & 0 & g/M & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \qquad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1/M & 0 \\ 0 & L/I \end{bmatrix}$$

2°. Colab demo — simulation, phase portrait, animation