p_newton-raphson

Wednesday, February 5, 2020 2:32 PM

consider $\min_{v \in \mathbb{R}^m} g \cdot v$ s.t. $\|v\|_H \leq \|g\|_H$

where $\|v\|_H = \sqrt{\frac{1}{2} v^T H v}$, $H = H^T > 0$

} e.g. $g = Dc(u)$

$H = D^2 c(u)$

$\longrightarrow$ $v^* = -H^{-1} g^T = -[D^2 c(u)]^{-1} \cdot Dc(u)^T$ $\boxed{\checkmark}$

let $\tilde{c}(v, \lambda) = g \cdot v + \lambda \cdot \underbrace{(v^T H v - g H g^T)}_{\|v\|_H \leq \|g\|_H \iff 2\|v\|_H^2 \leq 2\|g\|_H^2}$ $\longleftarrow$ note: $\lambda \in \mathbb{R}$

then necessarily at a stationary point:

$$D_v \tilde{c} = \underbrace{g}_{(1 \times m)} + \lambda \underbrace{v^T H}_{(1\times m)\times(m \times m) = (1 \times m)} = 0 \iff v^T H = -\frac{1}{\lambda} g \iff v_0 = -\frac{1}{\lambda} H^{-1} g^T$$

what is value of λ?

$$D_\lambda \tilde{c} = v^T H v - g H g^T = 0 \iff \|v_0\|_H = \|g\|_H$$

$$\|v_0\|_H^2 = \frac{1}{\lambda^2} g H^{-1} H H^{-1} g^T = \frac{1}{\lambda^2} g H g^T = \|g\|_H \iff \underbrace{\lambda^2 = 1 \iff |\lambda| = 1}$$

so $\lambda = \pm 1$; sign of λ determines whether v_0 points in direction of ascent vs descent