function approximation

goal: architectures & algorithms for approximating (optimal) value and policy functions

Bertsekas & Tsitsiklis Ch 3

- we'd like to consider MDP

$$\min_u E[c(x,u)] \text{ s.t. } x^+ \sim P(x,u)$$

with infinite state and action spaces, e.g. $X = \mathbb{R}^n$, $U = \mathbb{R}^m$

- now value $v : X \to \mathbb{R}$ and policy $\mu : X \to \Delta(U)$ cannot be represented with a finite-dimensional vector

* to use a digital computer,

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must approximate these functions
i.e. need: - architectures
- algorithms

architectures for approximation

- we seek to approximate $f : X \to \mathbb{R}$
using $\tilde{f} : X \times \Theta \to \mathbb{R}$
($e.g.\ f = v^*, \pi^*, \mu, v^\mu$)
where $\theta \in \Theta$ are approximator parameters

→ what properties are desirable?
- provide good approximation of f
- easy to choose parameters θ,
e.g. linear (convex, at least)

- Broadly, can consider architectures
that are linear or nonlinear

in parameters θ

we'll use semicolon to remind that θ is a parameter

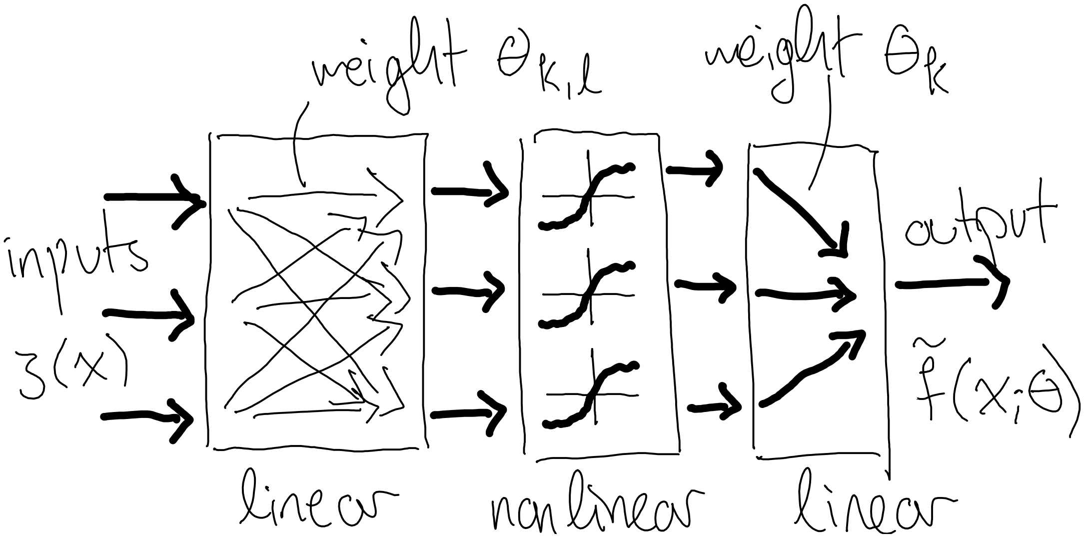
- linear: $\tilde{f}(x;\theta) = \sum_{k=0}^{K} \theta_k \cdot b_k(x)$

where $\{b_k\}_{k=0}^{K}$ are called <u>basis functions</u>

(though they might not form a vector space basis)

- nonlinear: huge variety of options; (so-called) "neural" networks are popular:

ex: (single-layer perceptron)

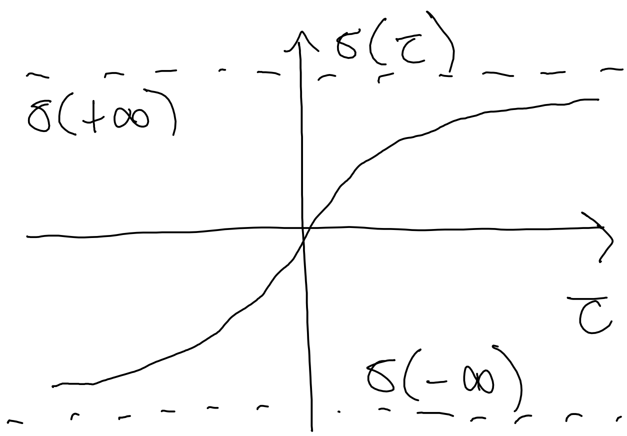


linear nonlinear (sigmoidal) linear

$$\tilde{f}(x;\theta) = \sum_{k=1}^{K} \theta_k \cdot \sigma\left(\sum_{l=1}^{L} \theta_{k,l} \cdot z_l(x)\right)$$

where $\sigma : \mathbb{R} \to \mathbb{R}$ is <u>sigmoidal</u>,
i.e. differentiable, monotonic,

$$-\infty < \lim_{\tau \to -\infty} \sigma(\tau) < \lim_{\tau \to +\infty} \sigma(\tau) < +\infty$$



$$\frac{e^{\tau} - e^{-\tau}}{e^{\tau} + e^{-\tau}}$$ hyperbolic tangent

$$\frac{1}{1+e^{-\tau}}$$ logistic

and $z : X \to \mathbb{R}^L$ is <u>feature vector</u>,
i.e. (partial) state observation/
(approximate) sufficient statistic in
a finite-dimensional vector space

<u>fact</u>: any continuous function over
a closed and bounded domain can

a closed and bounded domain can be approximated arbitrarily well by single-layer perceptron (with sufficiently large K, L and the right choice of weights)

* multiple layers are allowed; "in practice, the number of hidden layers is usually one or two, and almost never more than three"

algorithms for approximation

- assuming there exists $\Theta^* \in \Theta$ for which $\tilde{f}_{\Theta^*} \simeq f$, how to find Θ^*?

→ propose an algorithmic approach to compute/approximate Θ^* (specify algorithm inputs & any assumptions)

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- formulate an optimization problem:

$$\min_{\theta \in \Theta} \underbrace{\| \tilde{f}_\theta - f \|^2_{\mathbb{R}^X}}_{\text{norm on function space } \mathbb{R}^X}$$

- since norm $\| \cdot \|_{\mathbb{R}^X}$ can't be evaluated on infinite-dimensional $\mathbb{R}^X$, assume given a finite dataset

$$\{ (\xi_j, v(\xi_j)) \}_{j=1}^J \quad \text{where } \xi_j \in X$$

and $v(\xi_j) \simeq v^*(\xi_j)$ or $v^\mu(\xi_j)$, i.e. $v(\xi_j)$ is an estimate of optimal value or value of policy μ at state $\xi_j \in X$:

$$\min_{\theta \in \Theta} \sum_{j=1}^J \| \tilde{v}(\xi_j; \theta) - v(\xi_j) \|^2$$

- can then apply standard algorithms to this finite-dimensional (nonlinear) least-squares optimization problem least-squares optimization problem