interconnection

goal: derive time- and frequency-domain models for interconnections between systems

ref: Hespanha Ch 1 — LTI DE

[textbooks on "feedback systems"] — block diagram algebra

- we'll represent the interconnection between (sub)systems using <u>block diagrams</u>
  - a single block $\xrightarrow{u} \boxed{\text{system}} \xrightarrow{y}$ can stand in for a time- or frequency-domain model for [system], e.g.

$$\xrightarrow{u} \boxed{\begin{aligned} \dot{x}\,/\,x^+ &= Ax + Bu \\ y &= Cx + Du \end{aligned}} \xrightarrow{y} \quad \text{or} \quad \xrightarrow{u} \boxed{\hat{y} = T_{yu}\,\hat{u}} \xrightarrow{y}$$

  - given multiple blocks (ie systems), they can be interconnected to create a new system through 3 basic constructions:

$S_1 : \dot{x}_1 / x_1^+ = A_1 x_1 + B_1 u_1 \qquad S_2 : \dot{x}_2 / x_2^+ = A_2 x_2 + B_2 u_2$

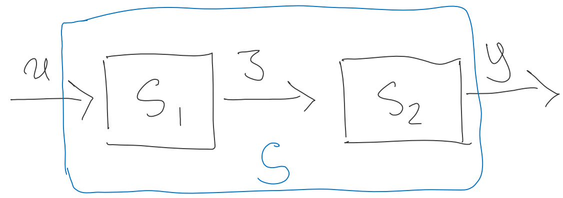
To create a new system through 3 basic constructions

$$S_1: \dot{x}_1 / x_1^+ = A_1 x_1 + B_1 u_1 \qquad S_2: \dot{x}_2 / x_2^+ = A_2 x_2 + B_2 u_2$$

$$y_1 = C_1 x_1 + D_1 u_1 \qquad y_2 = C_2 x_2 + D_2 u_2$$

$$T_1(s) = T_{y_1 u_1}(s) \qquad T_2(s) = T_{y_2 u_2}(s)$$

1°. cascade / series:



S can be represented in time- and freq.-domain:

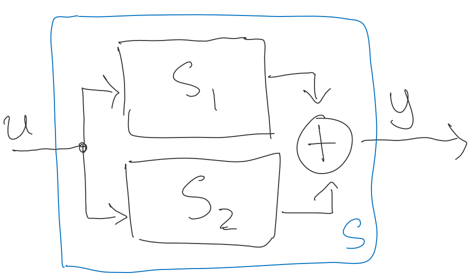
$$\hat{y}(s) = T(s)\,\hat{u}(s) = T_2(s)\,T_1(s)\,\hat{u}(s)$$

$$\begin{bmatrix} \dot{x}_1 / x_1^+ \\ \dot{x}_2 / x_2^+ \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ B_2 C_1 & A_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 D_1 \end{bmatrix} u$$

$$y = \begin{bmatrix} D_2 C_1 & C_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + D_2 D_1 u$$

→ when is this interconnection possible?
(what must be true of dimensions of (A,B,C,D)'s / T_{yu}'s?)

2°. parallel:



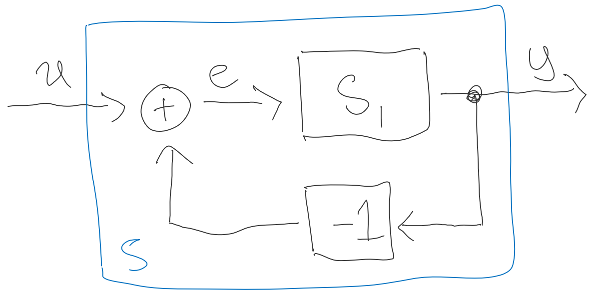
$$\hat{y}(s) = T(s)\,\hat{u}(s) = \left(T_1(s) + T_2(s)\right)\hat{u}(s)$$

$$\begin{bmatrix} \dot{x}_1 / x_1^+ \\ \dot{x}_2 / x_2^+ \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u$$

$$y = \begin{bmatrix} C_1 & C_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + (D_1 + D_2)u$$

→ when is this interconnection possible?
(what must be true of dimensions of (A,B,C,D)'s / T_{yu}'s?)

3°. feedback (negative)



→ show this is only possible if dim u = dim y

$$y = S_1 e = S_1(u-y) \iff y + S_1 y = S_1 u$$

"block diagram algebra"

$$\iff (I + S_1)y = S_1 u$$

$$\iff y = \underbrace{(I+S_1)^{-1}S_1}_{=S}\, u$$

freq. domain: $\hat{y}(s) = (I + T_1(s))^{-1}T_1(s)\,\hat{u}(s) = T(s)\cdot\hat{u}(s)$

time domain: $\dot{x} / x^+ = (A_1 - B_1(I+D_1)^{-1}C_1)x_1$

$$+ B_1(I - (I+D_1)^{-1}D_1)u$$

$$y = (I+D_1)^{-1}C_1 x_1 + (I+D_1)^{-1}D_1 u$$