tests -- eigenvalue

goal: characterize stability of LTI system using eigenvalues

ref: Hespanha Ch 8.3

- consider LTI DE $\dot{x}/x^+ = Ax$

set of eigenvalues of A

$\{s \in \mathbb{C} \mid \det(sI - A) = 0\}$

facts: (LTI DE) is:

0°. unstable if any eigenvalue of A: $\exists \lambda \in \operatorname{spec} A$

(CT) has positive real part (DT) has magnitude larger than 1

$\operatorname{Re} \lambda > 0$ $|\lambda| > 1$

1°. marginally stable if all eigenvalues of A: $\forall \lambda \in \operatorname{spec} A$

(CT) have non-positive real part (DT) have magnitudes no larger than 1

$\operatorname{Re} \lambda \leq 0$ $|\lambda| \leq 1$

$\mathrm{Re}\,\lambda \le 0$ than 1 $|\lambda| \le 1$

↳ * actually need additional condition for repeated eigenvalues (need concept of Jordan normal form)

2: exponentially (hence asymptotically) stable if all eigenvalues of A: $\forall \lambda \in \mathrm{spec}\,A$

(CT) have negative real part $\mathrm{Re}\,\lambda < 0$ (DT) have magnitude smaller than 1 $|\lambda| < 1$

* why should you believe these "facts"?

→ if $v \neq 0$, $\lambda \in \mathbb{C}$ are s.t. $Av = \lambda v$, i.e. λ is eigenval of A w/ eigenvector v

show that v is eigenvector of e^{At} (what is the eigenvalue?)

* $Av = \lambda v \implies A^k v = \lambda^k v$

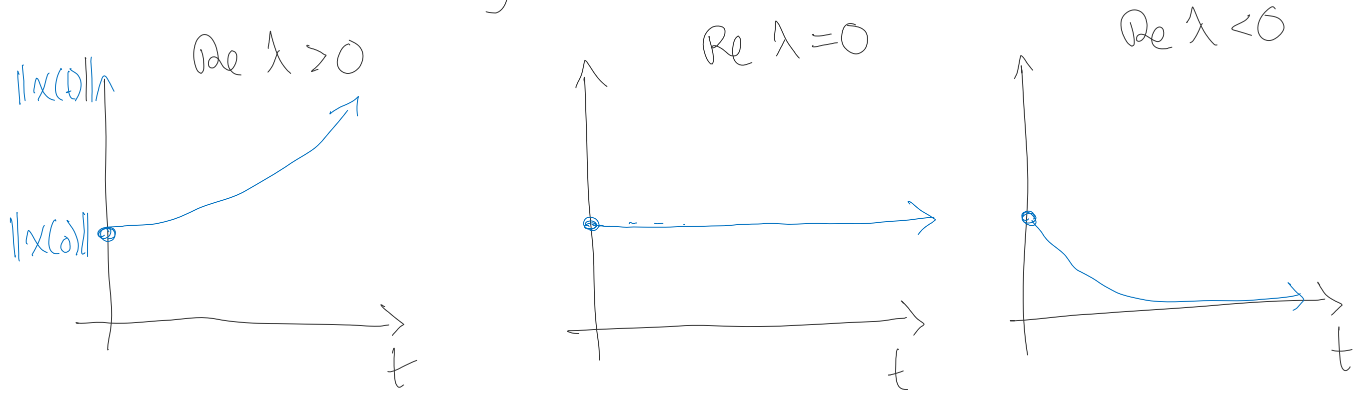
$$\implies e^{At} v = \left[\sum_{k=0}^{\infty} \frac{t^k}{k!} A^k\right] v = \sum_{k=0}^{\infty} \left[\frac{t^k}{k!} A^k v\right] = \left[\sum_{k=0}^{\infty} \frac{(\lambda t)^k}{k!}\right] v = e^{\lambda t} \cdot v$$

so $e^{\lambda t}$ is eigenval of e^{At} w/ eigenvector v

- so given $\underbrace{Av = \lambda v}_{v \neq 0}$ we know $x(0) = v \implies x(t) = e^{At} x(0)$
$= e^{\lambda t} x(0)$

↳ it's clear that $\lambda \in \mathbb{C}$, in particular $\mathrm{Re}\,\lambda$.

↳ it's clear that $\lambda \in \mathbb{C}$, in particular $\mathrm{Re}\,\lambda$, determines stability of $x(t)$:



* WARNING ! these eigenvalue tests for stability <u>do</u> <u>not</u> <u>apply</u> to LTV systems