LTV

goal: derive "solution" (i.e. trajectories and flow) for linear time-varying (LTV) systems

ref: Hespanha Ch 5

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linear time-varying (LTV) systems

- consider linear DE

$$\dot{x}(t) \,/\, x(t+1) = A(t)\,x(t) + B(t)\,u(t)$$
$$y(t) = C(t)\,x(t) + D(t)\,u(t)$$

$*$ the "solution" to DE has the same form as LTI case:

$$x(t) = \Phi(t,0)\,x_0 + \underbrace{\int_0^t \Phi(t,\tau)\,B(\tau)\,u(\tau)\,d\tau}_{}$$

interpret as $\sum_{\tau=0}^{t-1} \Phi(t,\tau+1)\,B(\tau)\,u(\tau)$ in DT case

where the state transition matrix $\Phi(t,\tau) \in \mathbb{R}^{d \times d}$ is defined:

(DT-LTV) $\Phi(t,\tau) = \begin{cases} I, & t=\tau, \\ A(t-1)A(t-2)\cdots A(\tau), & t>\tau. \end{cases}$

$$\underbrace{(\text{CT-LTV})\quad \Phi(t,\tau)}_{\text{(Peano-Baker series)}} = I + \int_\tau^t A(s_1)\,ds_1 + \int_\tau^t A(s_1)\int_\tau^{s_1} A(s_2)\,ds_2\,ds_1$$
$$+ \int_\tau^t A(s_1)\int_\tau^{s_1} A(s_2)\int_\tau^{s_2} A(s_3)\,ds_3\,ds_2\,ds_1 + \cdots$$

- output $y(t) = C(t)x(t) + D(t)u(t)$
  $= C(t)\Phi(t,0)x_0 + \int_0^t C(t)\Phi(t,\tau)B(\tau)u(\tau)\,d\tau + D(t)u(t)$

- flow $\phi(t,\tau,x_0,u) = \Phi(t,\tau)x_0 + \int_0^t \Phi(t,\sigma)B(\sigma)u(\sigma)\,d\sigma$

$\longrightarrow$ compute $D_{x_0}\phi$ $\quad$ * $D_{x_0}\phi(t,\tau,x_0,u) = \Phi(t,\tau)$

$\longrightarrow$ compute $D_u\phi$ (really: $D_{u(s)}\phi$)