output feedback

goal: synthesize a controller that stabilizes an LTI-DE using output feedback

ref: Hespanha ch 16.7

- consider LTI-DE $\dot{x} / x^+ = Ax + Bu$, $x \in \mathbb{R}^n$, $u \in \mathbb{R}^k$

$$y = Cx + Du, \quad y \in \mathbb{R}^m$$

and observer $\widehat{\text{LTI-DE}}$ $\dot{\hat{x}} / \hat{x}^+ = A\hat{x} + Bu - L(\hat{y} - y)$

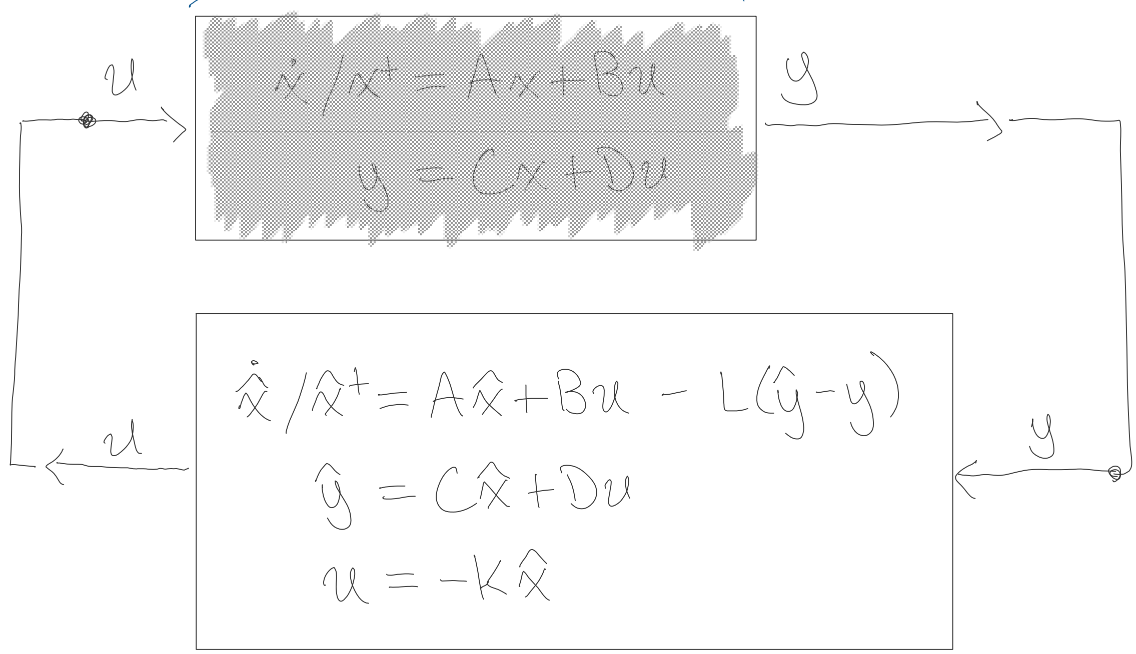
$$\hat{y} = C\hat{x} + Du$$

$$u = -K\hat{x}$$

- assuming $(A - LC)$ stable, know $\hat{x}(t) \to x(t)$ as $t \to \infty$
- assuming $(A - BK)$ stable, <u>we'll show</u> $\hat{x}(t) \to x(t) \to 0$ as $t \to \infty$

aside: look at LTI-DE & $\widehat{\text{LTI-DE}}$ from input/output perspective

* know input/output transformation $T(s) = C(sI-A)^{-1}B + D$



know everything: input/output, but also state

- consider dynamics of interconnected system:
  - ~~state~~ $\cancel{\begin{bmatrix} x \\ \hat{x} \end{bmatrix} \in \mathbb{R}^{2n}}$
  - actually, let's choose state $\begin{bmatrix} x \\ e \end{bmatrix}$ where $e = \hat{x} - x$

note: $$\begin{bmatrix} x \\ \hat{x} \end{bmatrix} = \begin{bmatrix} x \\ e + x \end{bmatrix} = \begin{bmatrix} I & 0 \\ I & I \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix}$$

- compute $$\begin{bmatrix} \dot{x} / x^+ \\ \dot{e} / e^+ \end{bmatrix} = \begin{bmatrix} Ax + Bu \\ (A - LC)e \end{bmatrix} = \begin{bmatrix} Ax - BK(e + x) \\ (A - LC)e \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} A-BK & -BK \\ 0 & A-LC \end{bmatrix}}_{\tilde{A}} \begin{bmatrix} x \\ e \end{bmatrix}$$

* since $A-BK$ stable and $A-LC$ stable: $\tilde{A}$ stable !

($\operatorname{spec} \tilde{A} = \operatorname{spec}(A-BK) \cup \operatorname{spec}(A-LC)$)

$$\boxed{\begin{array}{c} \text{* if } A-BK \text{ stable and } A-LC \text{ stable *} \\ \text{then } \hat{x}(t) \to x(t) \to 0 \text{ as } t \to \infty \text{ !} \end{array}}$$