motivation and definitions

goal: characterize when state of linear system can be uniquely determined from output

ref: Hespanha 15.1, 15.2

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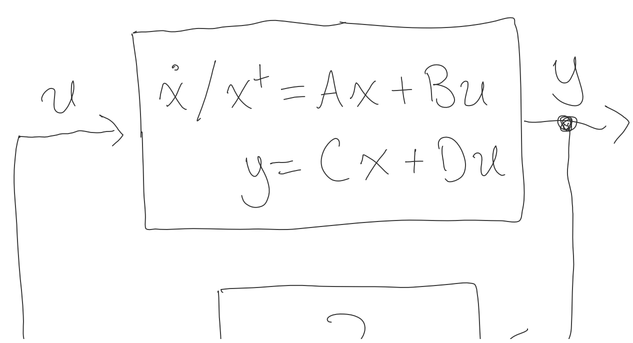
output feedback

- consider LTI-DE $\dot{x} / x^+ = Ax + Bu$, $x \in \mathbb{R}^n$, $u \in \mathbb{R}^k$
  $y = Cx + Du$ $\quad y \in \mathbb{R}^m$

- if (A, B) is controllable, can synthesize $K \in \mathbb{R}^{k \times n}$ such that $u = -Kx$ stabilizes (LTI-DE)

$\rightarrow$ how to determine x from y? (assume rank $C = n$)



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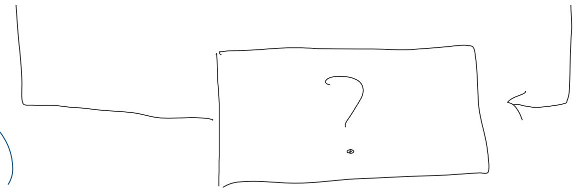

- if C invertible ($m = n = \text{rank}\, C$)

  then $x = C^{-1}(y - Du)$

- otherwise: $y = Cx + Du$

$$\Rightarrow \quad C^T y = \underbrace{C^T C}_{\in \mathbb{R}^{n \times n} \text{ \& rank } C^T C = n \text{, i.e. } C^T C \text{ invertible}} x + C^T D u$$

$$\Rightarrow \quad x = \underbrace{(C^T C)^{-1} C^T}_{= C^+ \text{, the "pseudo-inverse" of } C} (y - Du)$$

(non-)observability

- consider LTV-DE $\dot{x} / x^+ = A(t)x + B(t)u$, $x \in \mathbb{R}^n$, $u \in \mathbb{R}^k$

$$y = C(t)x + D(t)u, \quad y \in \mathbb{R}^m$$

- recall:

$$y(t) = C(t)\Phi(t, t_0)x_0 + \int_{t_0}^{t} C(t)\Phi(t, \tau)B(\tau)u(\tau)\,d\tau + D(t)u(t)$$

def: x_0 is <u>non-observable</u> on $[t_0, t]$ if $C(t)\Phi(t, t_0)x_0 = 0$

(called un-observable in Hespanha)

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i.e. $\mathcal{N}[t_0, t_1] = \mathcal{N}\left(C(t)\Phi(t_1, t_0)\right)$

note: if initial condition x_0 yields output $y(t_1)$ @ time t_1 and $\eta \in \mathcal{N}[t_0, t_1]$ then initial condition $(x_0 + \eta)$ <u>also</u> yields output $y(t_1)$ @ time t_1 since

$$C(t_1)\Phi(t_1, t_0)(x_0 + \eta) = C(t)\Phi(t_1, t_0)x_0 = y(t_1)$$

<u>no matter what input is applied</u>!

note: if $\mathcal{N}[t_0, t_1] = \{0\}$ then $y(t_1) \in \mathbb{R}^m$ uniquely determines $x(t_0)$ since $C(t_1)\Phi(t_1, t_0)x_0 = y(t_1)$ has a unique solution $x_0 \in \mathbb{R}^n$