goal: observe the fact that, in LTI systems, the definitions of controllability & observability are mathematically <u>dual</u>

ref: Hespanha 15.8

- consider the (CT-LTI) system

$$\dot{x} = Ax + Bu, \quad x \in \mathbb{R}^n, \quad u \in \mathbb{R}^k$$

$$y = Cx + Du, \quad y \in \mathbb{R}^m$$

- recall: 1°. (CT-LTI) controllable $\iff$ rank $W(t_0, t_1) = n$

where $W(t_0, t_1) = \int_{t_0}^{t_1} e^{A(\tau - t_0)} B B^T e^{A^T(\tau - t_0)} d\tau$

$\longrightarrow$ show that $\mathcal{R}(W(t_0,t_1)) = \mathcal{R}\left(\int_{t_0}^{t_1} e^{A(t_1 - \tau)} B B^T e^{A^T(t_1 - \tau)} d\tau\right)$

2°. (CT-LTI) observable $\iff$ rank $M(t_0, t_1) = n$

2°. (CT-LTI) observable $\iff$ rank $M(t_0,t_1)$ ...

where $$M(t_0,t_1)=\int_{t_0}^{t_1} e^{A^T(\tau-t_0)}C^TC\,e^{A(\tau-t_0)}\,d\tau$$

∘ consider the <u>dual</u> system

$(\overline{\text{CT-LTI}})$
$$\dot{\bar{x}}=A^T\bar{x}+C^T\bar{u},\quad \bar{x}\in\mathbb{R}^n,\ \bar{u}\in\mathbb{R}^m$$
$$\bar{y}=B^T\bar{x}+D^T\bar{u},\quad \bar{y}\in\mathbb{R}^k$$

note: 1°. $(\overline{\text{CT-LTI}})$ controllable $\iff$ rank $\overline{W}(t_0,t_1)=n$

where $$\overline{W}(t_0,t_1)=\int_{t_0}^{t_1} e^{A^T(\tau-t_0)}C^TC\,e^{A(\tau-t_0)}\,d\tau$$

2°. $(\overline{\text{CT-LTI}})$ observable $\iff$ rank $\overline{M}(t_0,t_1)=n$

where $$\overline{M}(t_0,t_1)=\int_{t_0}^{t_1} e^{A(\tau-t_0)}BB^T e^{A^T(\tau-t_0)}\,d\tau$$

i.e. $\quad W=\overline{M}\quad$ and $\quad M=\overline{W}$

so (CT-LTI) controllable $\iff$ $(\overline{\text{CT-LTI}})$ observable

(CT-LTI) observable $\iff$ $(\overline{\text{CT-LTI}})$ controllable

i.e. controllability matrix $\mathcal{C}$ for (CT-LTI) characterizes observability of $(\overline{\text{CT-LTI}})$ ($\mathcal{C}^T=\overline{\mathcal{O}}$ termed observability matrix)