LTI -- frequency domain

goal: derive input/output frequency-domain transformation determined by LTI DE

ref: Hespanha Ch 4 — LTI DE

[textbooks on "signals and systems"] — Laplace / Fourier / Z transform

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from time- to frequency-domain

- consider the LTI DE $\dot{x} / x^+ = Ax + Bu$ $\quad x \in \mathbb{R}^n,\ u \in \mathbb{R}^k$

$$y = Cx + Du \qquad y \in \mathbb{R}^m$$

- applying frequency-domain transformation (Laplace / Fourier in CT; "Z" in DT)

yields $s\hat{x} = A\hat{x} + B\hat{u}, \quad \hat{y} = C\hat{x} + D\hat{u}$

- rearranging: $\underbrace{(sI - A)\hat{x} = B\hat{u} \Rightarrow}$ $\hat{x} = (sI - A)^{-1} B\hat{u}$

assuming $(sI - A)$ invertible, i.e. $\det(sI - A) \neq 0$, i.e. $s \notin \operatorname{spec} A$

assuming $(sI-A)$ invertible, i.e. $\det(sI-A)\neq 0$, $\underbrace{\text{i.e. } s \notin \operatorname{spec} A}_{\text{i.e. } s \text{ isn't an eigenvalue of } A}$

- substituting: $\hat{y} = \underbrace{\left(C(sI-A)^{-1}B + D\right)}_{T_{yu}(s)\in\mathbb{R}^{m\times k}}\hat{u}$

$T_{yu}(s)\in\mathbb{R}^{m\times k}$ is the <u>transfer matrix</u>, i.e. a matrix of transfer functions

* note: $[T_{yu}]_{j,i}$ is transfer function from u_i to y_j

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from frequency- to time-domain

- given $G: \mathbb{C} \to \mathbb{C}^{m\times k}$, i.e. a matrix of transfer functions,
  $: s \mapsto G(s)$

  we say $\dot{x}/x^+ = Ax + Bu$, $y = Cx + Du$ is a <u>realization</u>
  if $G(s) = C(sI-A)^{-1}B + D$

* note that realizations are <u>not</u> unique!

→ given nonsingular $T\in\mathbb{R}^{n\times n}$, define $\tilde{x} = Tx$ and:

1°. derive LTI $\widetilde{DE}$ for state $\tilde{x}$

2°. show that transfer matrix is same for (DE) & $(\widetilde{DE})$

↳ we'll say (DE) & $(\widetilde{DE})$ are algebraically equivalent

→ find two DE w/ same transfer matrices but different state dimensions