definitions

goal: define different concepts for stability of linear systems

ref: Hespanha Ch 8

- consider LTV DE $\dot{x} / x^+ = A(t)x + B(t)u \quad x \in \mathbb{R}^n, u \in \mathbb{R}^k$

$$y = C(t)x + D(t)u \quad y \in \mathbb{R}^m$$

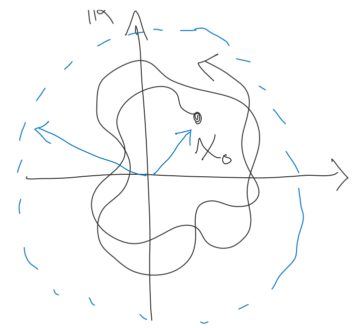
def: the (LTV-DE) is said to be:

0°: unstable if it is not marginally stable

1°: marginally (or internally) stable if, for every initial condition $x(t_0) = x_0$, the signal $x(t) = \Phi(t, t_0) x_0$, $t \geq t_0$, is uniformly bounded by an increasing function of $\|x_0\|$

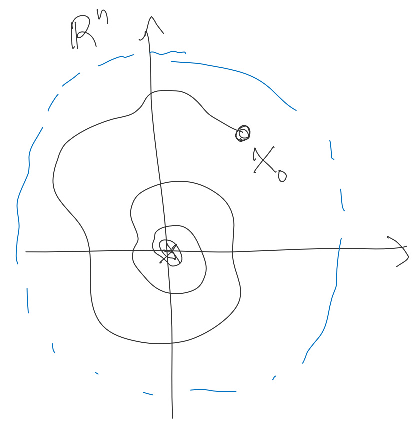
→ "start close, stay close"

↳ "start close, stay close"



2°. asymptotically stable if, in addition to (1°),
$x(t) \to 0$ as $t \to \infty$ for all x_0

↳ "goes to the origin"



↱ "goes to the origin exponentially fast"

3°. exponentially stable if, in addition to (1°), there are constants $c, \lambda > 0$ such that

$$\|x(t)\| \leq c\, e^{-\lambda(t-t_0)} \|x(t_0)\| \text{ for } \dot{x} \text{ (CT)}$$

$$\leq c\, \lambda^{(t-t_0)} \|x(t_0)\| \text{ for } x^+ \text{ (DT)} \quad * \ \lambda < 1$$

◦ note: these conditions only depend on $A(\cdot)$, so we'll use the same terms when referring to $\dot{x} / x^+ = A(t)x$

↳ in LTI case, we'll refer to matrix A as ___ stable