tests -- input/output

goal: characterize input/output stability of LTV & LTI systems

ref: Hespanha Ch. 9

∘ consider the linear DE $\dot{x} / x^+ = A(t)x + B(t)u \quad x \in \mathbb{R}^n, u \in \mathbb{R}^k$

$$y = C(t)x + D(t)u \quad y \in \mathbb{R}^m$$

* recall that $y(t) = C(t)\Phi(t,\tau)x(0)$

$$\tilde{y}(t) = \left\{ + \int_\tau^t C(t)\Phi(t,s)B(s)u(s)ds + D(t)u(t) \right.$$

def: linear DE is bounded-input/bounded-output (BIBO) stable if

$$\exists c > 0 \text{ s.t. } \forall u : \underbrace{\sup_{t \geq 0}}_{\text{"fancy maximum"}} \|\tilde{y}(t)\| \leq c \cdot \sup_{t \geq 0} \|u(t)\|$$

fact: linear DE is BIBO stable $\Longleftrightarrow$

$D_{ij}(\cdot)$ bounded and $\sup_{t \geq 0} \int_0^t |g_{ij}(t,\tau)| d\tau < \infty$

for every $i \in \{1, \cdots, m\}$, $j \in \{1, \cdots, k\}$ where

$$g_{ij}(t,\tau) = \left[C(t)\, \Phi(t,\tau)\, B(\tau) \right]_{i,j}$$

- in LTI case, $g_{ij}(t,\tau) = \left[C e^{A(t-\tau)} B \right]_{ij}$ is impulse response of output i to input j

fact: if LTI-DE is exponentially stable, it is BIBO stable

$\longrightarrow$ prove that the converse is false: BIBO stable $\not\Rightarrow$ exp stable

fact: LTI-DE exp. stable $\Longleftrightarrow$ all poles of transfer matrix are in open left-half plane