inverses

goal: characterize invertible LTI-DE & how to use inverses for control

ref: Hespanha 19.4, 19.5, 19.7

- consider the LTI-DE $\dot{x}/x^+ = Ax + Bu, \quad x \in \mathbb{R}^n, \; u \in \mathbb{R}^k$

$$y = Cx + Du, \quad y \in \mathbb{R}^m$$

def: $\overline{\text{LTI-DE}}$ $\dot{\bar{x}}/\bar{x}^+ = \bar{A}\bar{x} + \bar{B}\bar{u}, \quad \bar{x} \in \mathbb{R}^n, \; \bar{u} \in \mathbb{R}^m$

$$\bar{y} = \bar{C}\bar{x} + \bar{D}\bar{u}, \quad \bar{y} \in \mathbb{R}^k$$

is an <u>inverse</u> for (LTI-DE) if $\bar{G}(s)\,G(s) = I_{k\times k}$ } → left-inverse

$G(s)\,\bar{G}(s) = I_{m\times m}$ } → right-inverse

where $G(s) = C(sI - A)^{-1}B + D$

$\bar{G}(s) = \bar{C}(sI - \bar{A})^{-1}\bar{B} + \bar{D}$

in which case we'll write $\bar{G} = G^{-1}$

i.e. $\xrightarrow{u} \boxed{\text{LTI-DE}} \xrightarrow{y} = \xrightarrow{\bar{u}} \boxed{\overline{\text{LTI-DE}}} \xrightarrow{\bar{y}}$ $\Big\}$ $\bar{G}(s)\,G(s) = I \iff \bar{y} = u$

* implication holds for $x(0) = \bar{x}(0) = 0$

i.e. $\xrightarrow{\bar{u}} \boxed{\overline{\text{LTI-DE}}} \xrightarrow{\bar{y}} = \xrightarrow{u} \boxed{\text{LTI-DE}} \xrightarrow{y}$ $\Big\}$ $G(s)\,\bar{G}(s) = I \iff y = \bar{u}$

∘ note: if (LTI-DE) and ($\overline{\text{LTI-DE}}$) are stable and $\bar{G} = G^{-1}$

then $\lim_{t\to\infty} \| u(t) - \bar{y}(t) \| = 0 = \lim_{t\to\infty} \| y(t) - \bar{u}(t) \|$

regardless of initial states $x(0), \bar{x}(0) \neq 0$

thm: (LTI-DE) has an inverse $\iff$ D is (square and) nonsingular

in this case: $G^{-1}(s) = \bar{G}(s)$ where $\bar{G}(s) = \bar{C}(sI - \bar{A})^{-1}\bar{B} + \bar{D}$

and $\dot{\bar{x}} / \bar{x}^{+} = \bar{A}x + \bar{B}y, \quad \bar{A} = A - BD^{-1}C, \quad \bar{B} = BD^{-1}$

$u = \bar{C}x + \bar{D}y, \quad \bar{C} = -D^{-1}C, \quad \bar{D} = D^{-1}$

note: if D nonsingular: $y = Cx + Du \iff u = -D^{-1}Cx + D^{-1}y$

so $\dot{x}/x^{+} = Ax + Bu = (A - BD^{-1}C)x + BD^{-1}y$

i.e. $\dot{\bar{x}}/\bar{x}^{+} = (A - BD^{-1}C)\bar{x} + BD^{-1}y = \bar{A}\bar{x} + \bar{B}y$

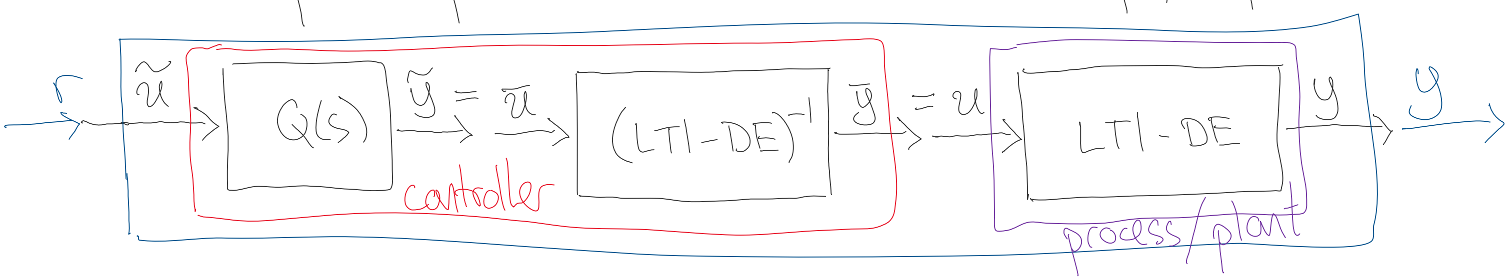
$u = -D^{-1}C\bar{x} + D^{-1}y \qquad = \bar{C}\bar{x} + \bar{D}y$

$$\dot{\bar{x}} = (A - BD^{-1}C)\bar{x} + BD^{-1}y = \bar{A}\bar{x} + \bar{B}y$$

$$u = -D^{-1}C\bar{x} + D^{-1}y = \bar{C}\bar{x} + \bar{D}y$$

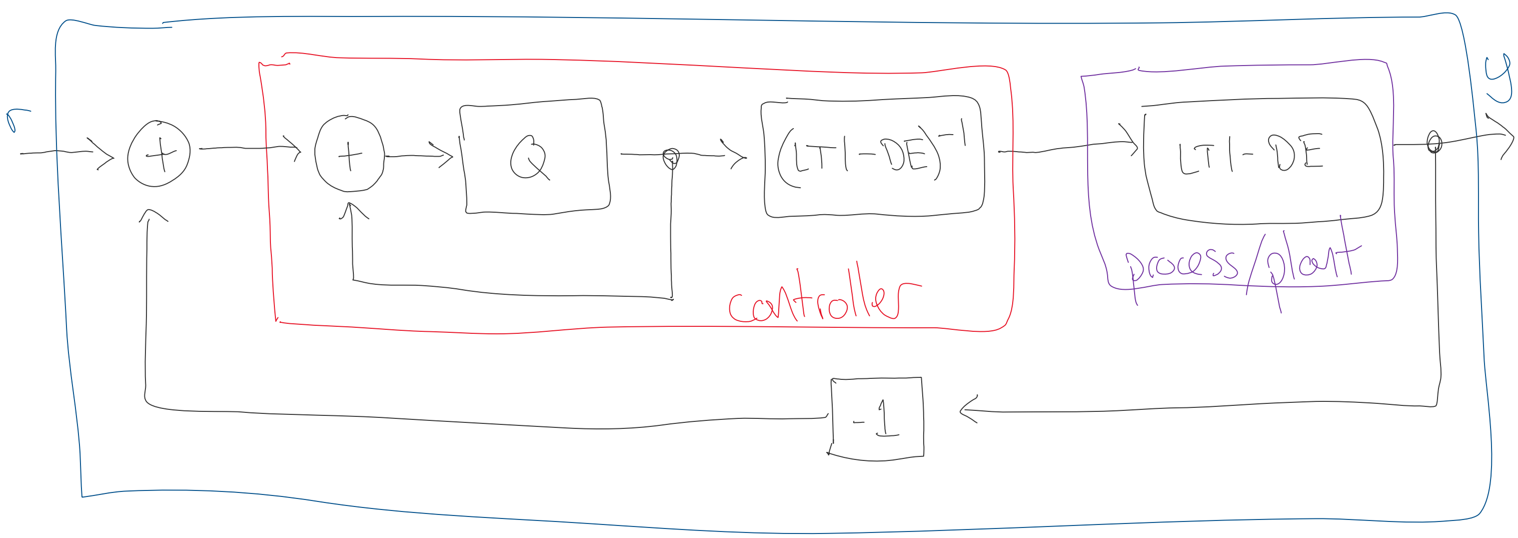
- given inverse $(\text{LTI-DE})^{-1}$ of (LTI-DE), what should we do w/ it?

idea 1: open-loop inversion to obtain desired input/output xform $Q(s)$



$$y = Q \cdot r$$

idea 2: closed-loop inversion to obtain I/O xform $Q(s)$



$$y = Q r$$