07 -- Tue May 12

AA ECE ME 548: Linear Multivariable Control

Prof Burden TA Tinu Spring 2020

today:
- ☑ exam 1 results, solution, regrade procedure
- ☑ HW 6 (optimal estimation) overview & Q's
- ☑ lec 7 (Kalman Filter) Q's
- ☐ ~~~10 min breakout discussion & follow-up~~
- ☐ Prof Burden OH

---

what is covariance?

- let $x : \Omega \to \mathbb{R}^2$ be a random vector

$E[x] \in \mathbb{R}^2$ – the mean (or average)

$$= \begin{bmatrix} E[x_1] \\ E[x_2] \end{bmatrix}$$

$$Cov[x]^T = E\left[(x - E[x])(x - E[x])^T\right]^T$$

$$\mathrm{Cov}[x]' = E\left[(x - E[x])(x - E[x])^T\right]'$$

$$= E\left[\left((x - E[x])(x - E[x])^T\right)^T\right]$$

$$= E\left[(x - E[x])(x - E[x])^T\right] = \mathrm{Cov}[x]$$

- let $z = x - E[x]$ so $\mathrm{Cov}[x] = \mathrm{Cov}[z]$

$\mathrm{Cov}[z] \in \mathbb{R}^{2\times 2}$ $\quad \mathrm{Cov}[z] = \begin{bmatrix} E[z_1 \cdot z_1] & E[z_1 \cdot z_2] \\ & E[z_2 \cdot z_2] \end{bmatrix}$

$$= \begin{bmatrix} \mathrm{Var}[z_1] & \mathrm{corr}[z_1, z_2] \\ & \mathrm{Var}[z_2] \end{bmatrix}$$

- if $\underbrace{z_1, z_2}_{\text{if independent}} \sim \mathcal{N}(0, \sigma^2)$, $\mathrm{Var}[z_1] = \mathrm{Var}[z_2] = \sigma^2$

  if independent: $\quad E[z_1 \cdot z_2] = 0$

  $\Rightarrow \mathrm{Cov}[z] = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix} = \sigma^2 \cdot I$

---

geometric intuition for covariance

- let $x : \Omega \to \mathbb{R}^n$ be random vector $E[x] = \mu$ $\quad \mathrm{Cov}[x] = \Sigma$

- let $x: \Omega \to \mathbb{R}^n$ be random vector, $E[x] = \mu$, $Cov[x] = \Sigma$

fact: $\Sigma^T = \Sigma \geq 0$ so $\exists\, U$ s.t. $U^T U = I$, i.e. $U^{-1} = U^T$

and $U \Sigma U^T = D$, diagonal

(i.e. $D = \mathrm{diag}\{\lambda_1, \ldots, \lambda_n\}$)

- define $z = U x$ so $E[z] = U\mu$, $Cov[z] = U \Sigma U^T = D$

i.e. $Var[z_i] = \lambda_i \geq 0$ and $\underbrace{corr[z_i z_j]}_{E[z_i \cdot z_j]} = 0$, $i \neq j$

---

least squares

given $z = Cx + \eta$, estimate $\hat{x}$ to minimize $\| z - C\hat{x} \|$

*if $E[\eta] = 0$ then $E[\hat{x}] = E[x]$

so $E[x - \hat{x}] = 0$

and $Cov[x - \hat{x}]$ is as small as possible using $Cov[\eta]$ as a weighting matrix