05 -- Thu Apr 30

AA ECE ME 548: Linear Multivariable Control

Prof Burden TA Tinu Spring 2020

$*$ $\boxed{\text{please fill out mid-quarter course evaluation}}$ $*$

$\hookrightarrow$ due tomorrow Fri May 1 11:59p

today: ☐ exam 1 questions

(no breakout discussion today – you aren't permitted to discuss exam w/o Prof or TA)

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p1 c, d

(c) compute first-order approximation of $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$:

$$D_x f \quad D_u f \quad \Big\{ \text{evaluate at } (x_0, u_0) = (0,0)$$

$$D_x f = \frac{\partial}{\partial x} f \qquad D_u f = \frac{\partial}{\partial u} f$$

(d) compute $\underbrace{\text{second-order approximation}}$ of $c(x^+, x, u)$

$$c \simeq \frac{1}{2} \begin{bmatrix} x^+ & x & u \end{bmatrix} D^2 c \begin{bmatrix} x^+ \\ x \\ u \end{bmatrix}$$

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$$D^2 c = \begin{bmatrix} D_{ij} c \end{bmatrix}_{ij} = \begin{bmatrix} \partial^2_{x_+} c & \partial_{x_+}\partial_x c & \partial_{x^+}\partial_u c \\ \cdot & \cdot & \cdot \end{bmatrix}$$

p1d $\longrightarrow$ what do I mean?

* use $c = \frac{1}{2} x_+^2 + \frac{1}{20} u^2$ to determine P, Q, R s.t.

$$c = \frac{1}{2} x_+^T P x_+ + \frac{1}{2} x^T Q x + \frac{1}{2} u^T R u$$

and solve the corresponding LQR problem

don't: use $c = \frac{1}{2}\arctan(x+u)^2 + \frac{1}{20}u^2$

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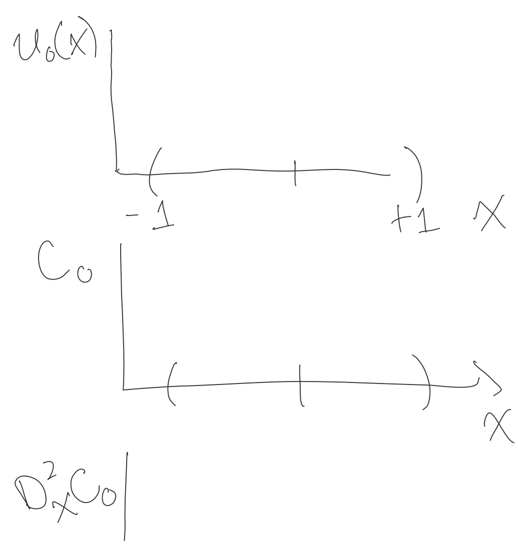
exam 1 p1(b)

plot:



p2

- Riccati differential equation:

$$\dot{P}_s = \lim_{\Delta \to 0} \frac{1}{\Delta}(P_{s+\Delta} - P_s) = -(A_s^\top P_s + P_s A_s - P_s B_s R_s^{-1} B_s^\top P_s + Q_s);$$

defines $P : [0,t] \longrightarrow \mathbb{R}^{d \times d}$

such that $u_s = -R_s^{-1} B_s^T P_s x_s$

minimizes $\frac{1}{2} x_t^T P_t x_t + \frac{1}{2}\int_0^t x_s^T Q_s x_s + u_s^T R_s u_s \, ds$

where $\dot{x}_s = A_s x_s + B_s u_s$

- letting $t \to \infty$ and restricting to time-invariant case:

$$0 = -(A^T P + PA - PBR^{-1}B^T P + Q)$$

defines $P \in \mathbb{R}^{d \times d}$

such that $u_s = -R^{-1} B^T P x_s$

minimizes $\frac{1}{2}\int_0^\infty x_s^T Q x_s + u_s^T R u_s \, ds$

where $\dot{x} = Ax + Bu$

→ solve_are   solve_algebraic_Riccati_Equation

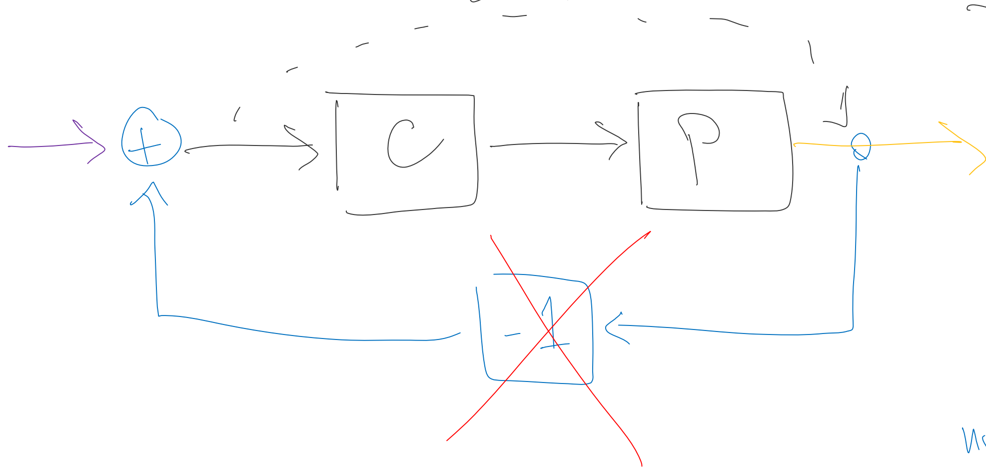
$\frac{1}{2} x^T P x = \min \frac{1}{2}\int^\infty x_s^T Q x_s + u_s^T R u_s \, ds$

$$\hookrightarrow \quad \frac{1}{2} x_0^T P x_0 = \min_u \frac{1}{2}\int_0^\infty x_s^T Q x_s + u_s^T R u_s \, ds$$

$$= \left(\frac{1}{2} x_0^T P x_0 - \underbrace{\frac{1}{2} x_t^T P x_t}_{\to 0 \text{ as } t \to \infty}\right) + \frac{1}{2} x_t^T P x_t$$

$L = PC$

$T_o = (I + PC)^{-1} PC$



I suggest doing this numerically for each $j\omega$

(d) Bode plot (η, θ) & (ν, σ) entries in $S = (I + PC)^{-1}$

(e) (Nyquist / Bode) plot (η, θ) & (ν, θ) entries in $L = PC$ $\rightarrow$ to determine g_m, φ_m

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$$\underbrace{\dot{x} = Ax + Bu \qquad u = -Kx}$$

$$\Updownarrow$$

$$\dot{x} = \underbrace{(A - BK)}x \qquad \bar{B}$$

$=$

$$= \bar{A} x + 0 \cdot u$$

Bode plot of $T_{kl}(s)$:

$$T(s) = (I + P(s)C(s))^{-1}$$