08 -- Thu May 21

AA ECE ME 548: Linear Multivariable Control

Prof Burden TA Tinu Spring 2020

<u>today</u>:
- ☐ HW 7 (Kalman filtering)
- ☐ lec 8 (robust control parts 1 & 2)
- ☐ ~10 min breakout discussion & follow-up
- ☐ Prof Burden OH

$$J(x) = \sum_{s=0}^{t} \delta_s^T \bar{Q}_s^{-1} \delta_s + \eta_s^T \bar{R}_s^{-1} \eta_s \qquad \begin{aligned} x_{s+1} - A_s x_s - B_s u_s &= \delta_s \\ y_s - C_s x_s &= \eta_s \end{aligned}$$

$$x : [0,t] \to \mathbb{R}^d$$

$$J(x) = \sum_{s=0}^{t} \Big[(x_{s+1} - A_s x_s - B_s u_s)^T \bar{Q}_s^{-1} (x_{s+1} - A_s x_s - B_s u_s) + (y_s - C_s x_s)^T \bar{R}_s^{-1} (y_s - C_s x_s) \Big]$$

- KF $\rightsquigarrow$ $\hat{x} : [0,t] \to \mathbb{R}^d$, so want to show: $DJ(\hat{x}) = 0 \in \mathbb{R}^{(t+1)\cdot d}$

$$J(x) = \sum_{s=0} \Big[(x_{s+1} - A_s x_s - B_s u_s)^T Q_s^{-1} (x_{s+1} - A_s x_s - B_s u_s) + (y_s - C_s x_s)^T \bar{R}_s^{-1} (y_s - C_s x_s) \Big]$$

- KF $\rightsquigarrow \hat{x} : [0,t] \to \mathbb{R}^d$, so want to show: $DJ(\hat{x}) = 0 \in \mathbb{R}^{(t+1)\cdot d}$

$$\hookrightarrow (x_1 - A_0 x_0 - B_0 u_0)^T \bar{Q}_0^{-1} (\dots) + (y_0 - C_0 x_0)^T \bar{R}_0^{-1} (\dots)$$

$$+ (x_2 - A_1 x_1 - B_1 u_1)^T \bar{Q}_1^{-1} (\dots) + (y_1 - C_1 x_1)^T \bar{R}_1^{-1} (\dots)$$

– note that $E[\hat{x}] = (C^T S^{-1} C)^{-1} C^T S^{-1} E[z]$,

$$E[z] = E[Cx] + E[\eta]$$

so
$$E[\hat{x}] = \underbrace{\cancel{\left((C^T S^{-1} C)^{-1} (C^T S^{-1} C)\right)}}_{I} E[x] + (C^T S^{-1} C)^{-1} C^T S^{-1} E[\eta]$$

$$= E[x] + (C^T S^{-1} C)^{-1} C^T S^{-1} E[\eta]$$

$\to$ so estimate bias is proportional to measurement bias