05 -- Tue Apr 28

AA ECE ME 548: Linear Multivariable Control

Prof Burden TA Tinu Spring 2020

* ~~please fill out mid-quarter course evaluation~~ *

↳ I need to fix the link - announcement soon!

today: ☐ exam 1 questions

(no breakout discussion today — you aren't permitted to discuss exam w/o Prof or TA)

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p1d → what do I mean?

* use $c = \frac{1}{2} x_t^2 + \frac{1}{20} u^2$ to determine P, Q, R s.t.

$$c = \frac{1}{2} x_t^T P x_t + \frac{1}{2} x^T Q x + \frac{1}{2} x^T R x$$

and solve the corresponding LQR problem

don't: use $c = \frac{1}{2} \arctan(x+u)^2 + \frac{1}{20} u^2$

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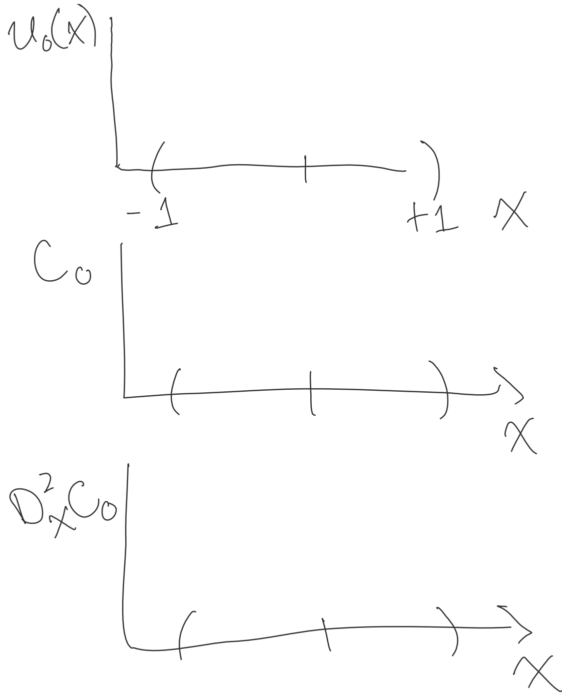
exam 1 p1(b)

plot: $u_0(x)$

plot:



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p2

- Riccati differential equation:

$$\dot{P}_s = \lim_{\Delta \to 0} \frac{1}{\Delta}(P_{s+\Delta} - P_s) = -(A_s^\top P_s + P_s A_s - P_s B_s R_s^{-1} B_s^\top P_s + Q_s);$$

defines $P : [0,t] \to \mathbb{R}^{d \times d}$

such that $u_s = -R_s^{-1} B_s^\top P_s x_s$

minimizes $\frac{1}{2} x_t^\top P_t x_t + \frac{1}{2}\int_0^t x_s^\top Q_s x_s + u_s^\top R_s u_s \, ds$

where $\dot{x}_s = A_s x_s + B_s u_s$

- letting $t \to \infty$ and restricting to time-invariant case:

$$0 = -(A^\top P + PA - PBR^{-1}B^\top P + Q)$$

defines $P \in \mathbb{R}^{d \times d}$

such that $u_s = -R^{-1} B^\top P x_s$

such that $u_s = -R^{-1}B^T P x_s$

minimizes $\frac{1}{2}\int_0^\infty x_s^T Q x_s + u_s^T R u_s \, ds$

where $\dot{x} = Ax + Bu$

→ solve_are   solve_algebraic_Riccati_Equation

m re gener lly:

to solve DRE: – example in HW3 solution

$\frac{d}{dt}$ $f(x)$

if $\dot{x} = f(x)$ then $\frac{d}{dt} x = f(x)$ $\frac{d}{dt}x = -f($

so with $\tau = -t$, $\frac{d}{d\tau}x = \left(\frac{dt}{d\tau}\right)^{-1} f(x)$

$q = (\eta, \nu, \theta)$ $x = (q, \dot{q}) \Rightarrow x = \begin{bmatrix} \eta \\ \nu \\ \theta \\ \dot{\eta} \\ \dot{\nu} \\ \dot{\theta} \end{bmatrix} \in \mathbb{R}^6$