04 -- Tue Apr 21

# AA ECE ME 548: Linear Multivariable Control

Prof Burden    TA Tinu    Spring 2020

**today:**
- ☑ ~10 min breakout to discuss
  - ☑ HW2 solution Q's
  - ☑ HW3 overview & Q's
  - ☑ lectures on state estimation & observers
- ☐ Prof Burden OH

- use chat to ask Q's
- leave breakout room if discussion ends early

**next time:** ☐ format / logistics for next week's take-home exam

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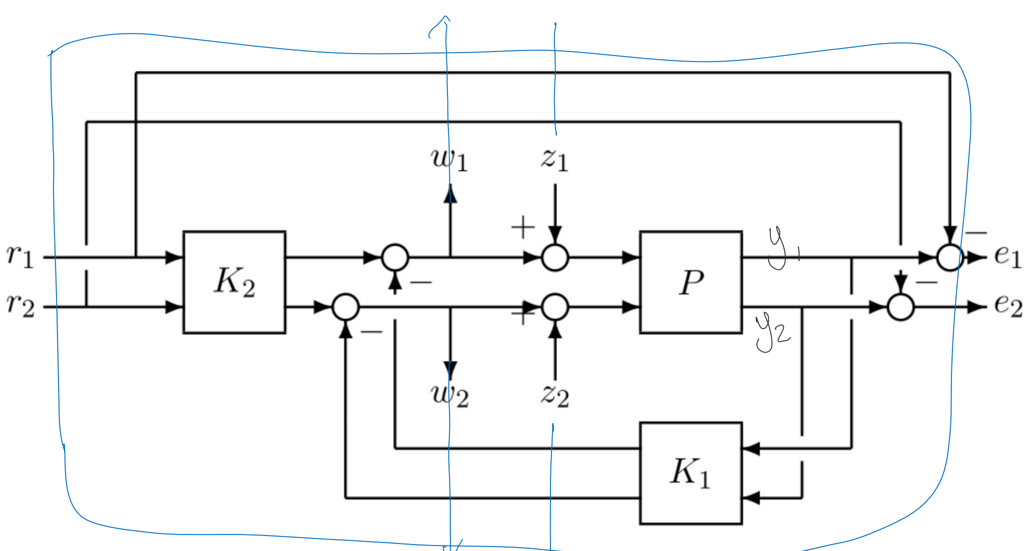


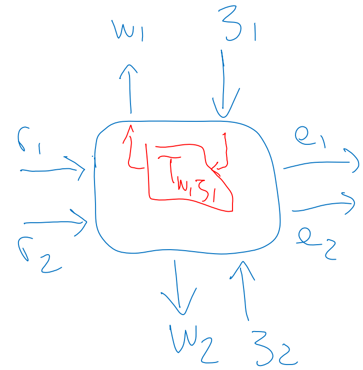


(b) *Analytically show that* $T_{w_1,z_1} = T_{w_2,z_2} = -\frac{1}{s+1}$.

$$w_1 = \cancel{[K_2]_{1,1}} \cdot r_1 - \underbrace{[K_1]_{1,1}}_{1} \cdot \underbrace{y_1}_{[P]_{1,1}(w_1+z_1) + [P]_{1,2}(w_2+\cancel{z_2})}$$

$$\dot{x} = Ax \qquad y = Cx$$

$$x(t+\Delta) = e^{\Delta \cdot A} \cdot x(t) \implies y(t+\Delta) = C \cdot e^{\Delta \cdot A} \cdot x(t)$$

$$= \bar{A}\, x(t)$$

$$y(t) = C \cdot x(t)$$

$$y(t+\Delta) = C \cdot \underbrace{e^{\Delta \cdot A}}_{\bar{A}}\, x(t)$$

$$\vdots$$

$$y(t+k\cdot\Delta) = C \cdot \underbrace{e^{k\cdot\Delta\cdot A}}_{\bar{A}^k}\, x(t)$$

$$\Updownarrow$$

$$y = \underbrace{\begin{bmatrix} C \\ C\bar{A} \\ \vdots \\ C\bar{A}^k \end{bmatrix}}_{\bar{O}_k} x(t)$$

I can solve for $x(t)$

$$\iff \operatorname{rank} \bar{O}_k = d$$

$$\overset{?}{\iff} \operatorname{rank} \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^k \end{bmatrix} = d$$

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what happens if $\operatorname{rank} \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{d-1} \end{bmatrix} < d$ ?

fact: given $\dot{x} = Ax$, $y = Cx$, let $1 \leq r = \operatorname{rank}\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{d-1} \end{bmatrix} < d$

there exists invertible $T$ s.t.

with $z = Tx$ $(\Rightarrow x = T^{-1}z)$

we have $\dot{z} = T\dot{x} = TAx = TAT^{-1}z = \tilde{A}z$

$$y = Cx = CT^{-1}z = \tilde{C}z$$

and

$$\tilde{A} = \begin{array}{c} \left[\begin{array}{c|c} \tilde{A}_{11} & 0 \\ \hline \tilde{A}_{21} & \tilde{A}_{22} \end{array}\right] \\ \begin{array}{cc} r & \quad d-r \end{array} \end{array}, \quad C = \begin{array}{c} \left[\begin{array}{c|c} C_1 & 0 \end{array}\right] \\ \begin{array}{cc} r & d-r \end{array} \end{array}$$

$\operatorname{rank} C_1 = r$