07 -- Thu May 14

AA ECE ME 548: Linear Multivariable Control

Prof Burden TA Tinu Spring 2020

today: ☑ exam 1 results, solution, regrade procedure
due midnight Sun May 17

☑ HW 6 (optimal estimation) overview & Q's

☑ lec 7 (Kalman Filter; duality & separation)

☐ ~10 min breakout discussion & follow-up

☐ Prof Burden OH

HW6 ple

- have: $w \sim \mathcal{N}(0, I)$
- want: $z \sim \mathcal{N}(0, Q)$ i.e. $\mathrm{Cov}[z] = Q$
- if $y = Sw$ then $y \sim \mathcal{N}(0, SS^T)$
- so we want S s.t. $Q = SS^T$, i.e. "$S = \sqrt{Q}$"

* recall that $Q = Q^T \geq 0$, $Q = \mathcal{U} \cdot D \cdot \mathcal{U}^T$, $\mathcal{U}^T \mathcal{U} = I$,
$D = \mathrm{diag}\{\sigma_1^2, \ldots, \sigma_n^2\}$

HW6 p2

$$\dot{x} / x^+ = Ax \quad , \quad \text{so} \quad x(t) = \Phi(t,0)\, x(0) \in \mathbb{R}^d$$

$$y = Cx$$

$$\text{so} \quad y(t) = C\,\Phi(t,0)\,x(0) = \sum_{j=1}^{d} \underbrace{x_j(0)}_{c_j} \cdot \underbrace{\left[C \cdot \Phi(t,0)\right]_j}_{b_j}$$

$$= \sum_{b \in B} c_b \cdot b(t)$$

$$z = Hc + \eta \qquad \hat{c} = (H^T H)^{-1} H^T z$$

$$\eta \sim \mathcal{N}(0, \Sigma) \Rightarrow E[\hat{c}] = (H^T H)^{-1} H^T E[z]$$

$$= \underbrace{\cancel{(H^T H)^{-1} (H^T H)}}_{I} E[c] + \cancelto{0}{E[\eta]} = E[c]$$

$$e = z - H\hat{c} \Rightarrow E[e] = E[z] - HE[\hat{c}] = HE[c] - HE[c] = 0$$

$$\mathrm{Cov}[e] = E\left[(e - \cancelto{0}{E[e]})(e - \cancelto{0}{E[e]})^T\right] = E[e e^T]$$

$$= E\left[(z - H\hat{c})(z^T - \hat{c}^T H^T)\right]$$

$$= E\left[z \cdot z^T - z\hat{c}^T H^T - H\hat{c} z^T + H\hat{c}\hat{c}^T H^T\right]$$

$$= E[z z^T] - E[z\hat{c}^T] H^T - H E[\hat{c} z^T] + H E[\hat{c}\hat{c}^T] H^T$$

$n \times d$

why $E[Ax] = A \cdot E[x]$? , $A \in \mathbb{R}^{n \times d}$

$$\text{recall} \quad E[Ax] = \int_\Omega \underbrace{A \cdot \overbrace{x(\omega)}^{\in \mathbb{R}^d}}_{\in \mathbb{R}^n} \cdot p(\omega)\, d\omega$$

$$= A \cdot \int_\Omega \underbrace{x(\omega)}_{\in \mathbb{R}^d} \cdot p(\omega)\, d\omega = A \cdot \begin{bmatrix} \int_\Omega x_1(\omega) \cdot p(\omega)\, d\omega \\ \vdots \\ \int_\Omega x_d(\omega) \cdot p(\omega)\, d\omega \end{bmatrix}$$

$$= A \cdot E[x]$$

HW 6 p3

$P(x=0) = P(y<0) = 1/2$