H2 and Hoo system norms -- computation

goal: computation of $\mathcal{H}_2$ and $\mathcal{H}_\infty$ system norms

refs: Doyle, Glover, Khargonekar, Francis 1989

State-Space Solutions to Standard $\mathcal{H}_2$ and $\mathcal{H}_\infty$ Control Problems

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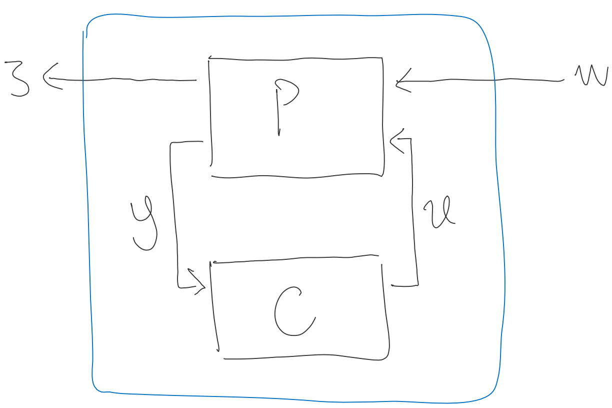
- consider the following feedback block diagram between process P and controller C:



- w contains disturbances (i.e. both input & output disturbances)
- z contains "errors" to be minimized (i.e. tracking error, control effort)

$$T_{zw} = P_{zw} + P_{zu}(I - C P_{yu})^{-1} C P_{yw}$$ → verify this formula

* our goal as control engineers is to minimize $\|T_{zw}\|$

↳ will focus on $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms

↳ we'll focus on H_2 and H_∞ norms

- assume given causal LTI stable transformation $z = Tw$ and let (A, B, C, D) be minimal realization:

$$\hat{T}(s) = C(sI - A)^{-1}B + D \quad \text{and} \quad (A, B) \text{ controllable, } (A, C) \text{ observable}$$

$\boxed{H_2}$ let L_c, L_o denote controllability & observability Gramians

i.e. $AL_c + L_cA^T + BB^T = 0, \quad A^TL_o + L_oA + C^TC = 0$

then: $$\|T\|_2^2 = \operatorname{tr}(CL_cC^T) = \operatorname{tr}(B^TL_oB)$$

– importantly, L_c & L_o can be obtained by solving linear equations

$\boxed{H_\infty}$ assuming A stable, $\gamma > 0$, define $H_\gamma = \underbrace{\begin{bmatrix} A & \frac{1}{\gamma^2}BB^T \\ -CC^T & -A^T \end{bmatrix}}_{\text{"Hamiltonian matrix"}}$

then: $\|T\|_\infty < \gamma \iff j\mathbb{R} \cap \operatorname{spec} H_\gamma = \emptyset$

(i.e. H_γ has no eigenvalues on imaginary axis)

– so we need to do a line search to find smallest γ