04 -- Thu Apr 23

AA ECE ME 548: Linear Multivariable Control

Prof Burden TA Tinu Spring 2020

<u>today</u>: ☑ ~10 min breakout to discuss

- ☑ exam 1 instructions
- ☑ HW2 solution Q's
- ☑ HW3
- ☑ lectures on state estimation & observers
- ☐ Prof Burden OH

– use chat to ask Q's

– leave breakout room if discussion ends early

<u>next week</u>: ☐ exam 1 (solo HW) ☐ mid-quarter course eval

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HW3 p2(a): P_s or $P_{s+\Delta}$

$$\dot{x}/x^+ = Ax + Bu \xleftarrow{\text{nominal}} \dot{\hat{x}}/\hat{x}^+ = A\hat{x} + Bu$$

actual: $= \tilde{A}x + \tilde{B}u = (A + \delta A)x + (B + \delta B)u,$

$$\|\delta A\|, \|\delta B\| < \varepsilon$$

$\rightarrow$ $e = x - \hat{x}$

$$- \dot{e}/e^+ = \dot{x}/x^+ - \dot{\hat{x}}/\hat{x}^+$$

$$= \tilde{A}x + \tilde{B}u - (A\hat{x} + Bu)$$

$$= \tilde{A}x - A\hat{x} + \tilde{B}u - Bu$$

$$= \tilde{A}x - A\hat{x} + \tilde{B}u - Bu$$

$$= Ax - A\hat{x} + \underbrace{Bu - Bu}_{\to 0} + \delta A \cdot x + \delta B \cdot u$$

$$= Ae + \underbrace{\delta A \cdot x + \delta B \cdot u}$$

assume $\|x\| \le \bar{X}$, $\|u\| \le \bar{U}$

$$= Ae + \delta(t) \quad \text{where } \|\delta(t)\| \le \varepsilon(\bar{X} + \bar{U})$$

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given $\dot{x}(s) / x^+(s+1) = A(s)\, x(s)$

we know $x(t) = \Phi(t,\tau)\, x(\tau)$ where $\frac{d}{dt}\Phi(t,\tau) = A(t)\cdot\Phi(t,\tau)$

$\Phi(\tau,\tau) = I$

so $\phi(t,\tau,x(\tau)) = \Phi(t,\tau)\, x(\tau)$

specifically for $\dot{x} / x^+ = Ax$, $\Phi(t,\tau) = e^{A(t-\tau)}$

know $x(t) = e^{At}x(0)$

so $\phi(t, x(0)) = e^{At}x(0)$, i.e. $\Phi(t,0) = e^{At}$

i.e. $D_2\phi(t, x(0)) = e^{At} = \Phi(t,0)$

$\dot{x} = f(x)$

$x(t) = \phi(t, x(0))$

let $X : [0,t] \to \mathbb{R}^{d\times d}$ satisfy

$\frac{d}{ds}X(s) = A(s)X(s)$, $X(0) = I$

$A(s) = Df(x(s))$

$= Df(\phi(s, x(0)))$

then $D_2\phi(t, x(0)) = X(t)$