Thu Apr 2 2020

today: ☑ course logistics

- ☑ TA is scheduling OH (office hours)
- ☑ Prof will hold OH in the last 30 min of Zoom

☑ Q's from lectures (solution of DE)

☑ Q's from HW0 (simulation, linearization, visualization)

$\boxed{\square \text{ Prof OH}}$

Q: how does the concept of "trajectory" differ from "solution"?

A: "solution" could imply many things, so I wanted specific terminology that's unambiguous

given $\dot{x} / x^+ = f(x)$ ← what does it mean to "solve" this (differential/difference) eq?

→ a "solution" is a <u>function</u> (not just one vector)

eg trajectory $x : [0,t] \to \mathbb{R}^d$ "solves" (DE) for initial state $x(0)$

if $\frac{d}{ds} x(s) = f(x(s))$ or $x(s+1) = f(x(s))$

- combine all trajectories into single function (called the flow)

$\phi : [0,t] \times \mathbb{R}^d \to \mathbb{R}^d$

$: (s, x(0)) \mapsto x(s)$ where x is trj starting at $x(0)$

now consider $\dot{x} / x^+ = f(x,u)$.

then to specify a trajectory, need $\underbrace{x(0) \; \& \; u:[0,t] \to \mathbb{R}^m}$

then $x:[0,t] \to \mathbb{R}^d$ is the traj with

if $\forall s \in [0,t)$: $\frac{d}{ds} x(s) = f(x(s), u(s))$

or $x(s+1) =$ " "

then the analogous concept of flow needs additional argument:

$$\phi : [0,t] \times \mathbb{R}^d \times \mathcal{U} \to \mathbb{R}^d$$

$: (s, x(0), u) \mapsto x(s)$ where x is traj for $x(0), u$

where $\mathcal{U} = \{ u:[0,t] \to \mathbb{R}^m \mid u \text{ is reasonable} \}$

ex: for LTI system $\dot{x} = Ax + Bu$:

$$x(t) = \underbrace{e^{At} x(0) + \int_0^t e^{A(t-\tau)} B u(\tau) d\tau}$$

$\Rightarrow \phi(t, x(0), u) =$!

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linearization of CT flow

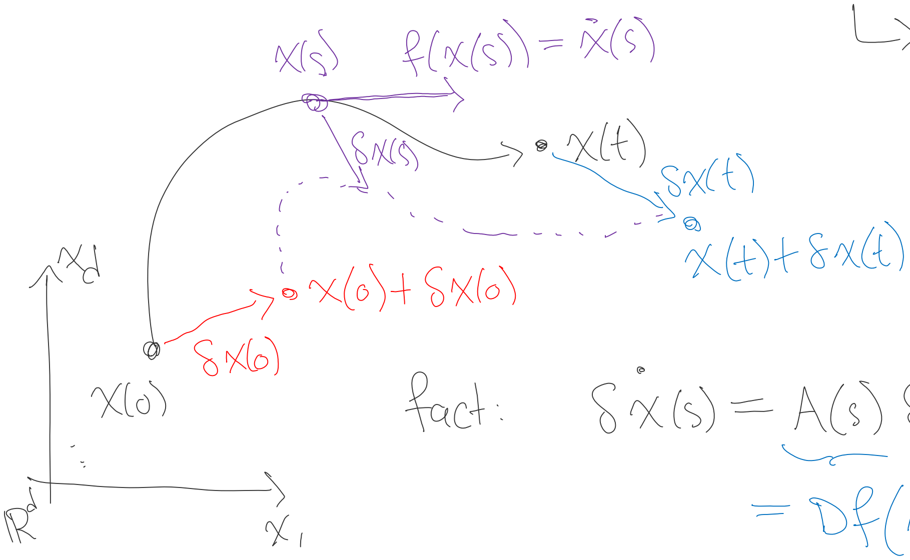
$\dot{x} = f(x) \rightsquigarrow \phi : [0,t] \times \mathbb{R}^d \to \mathbb{R}^d$

$: (s, x(0)) \mapsto x(s)$

Q: how does $x(s)$ depend on $x(0)$?

i.e. if $x(0) \mapsto x(0) + \delta x(0)$, then $x(s) \mapsto x(s) + \delta x(s)$

↳ what is $\delta x(s)$?



$*$ if $f(x(0)) = 0$

(i.e. $x(0)$ is equilibrium)

so that $x(s) \equiv x(0)$,

fact: $\delta \dot{x}(s) = \underbrace{A(s)} \delta x(s)$ then $A(s) \equiv A(0)$

$= Df(x(s))$ so $\delta x(t) = e^{A(0)t} \delta x(0)$

$\parallel$

$\frac{\partial}{\partial x} f \in \mathbb{R}^{d \times d}$

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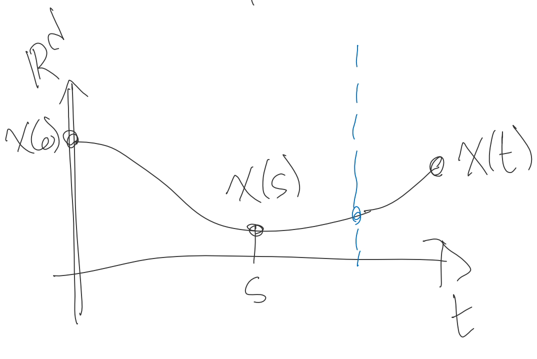
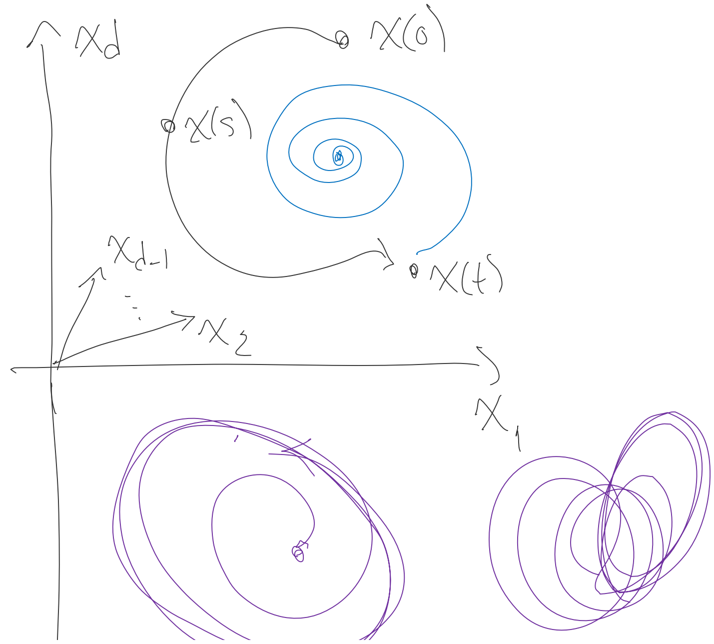
visualization

given traj $x : [0,t] \to \mathbb{R}^d$, can visualize

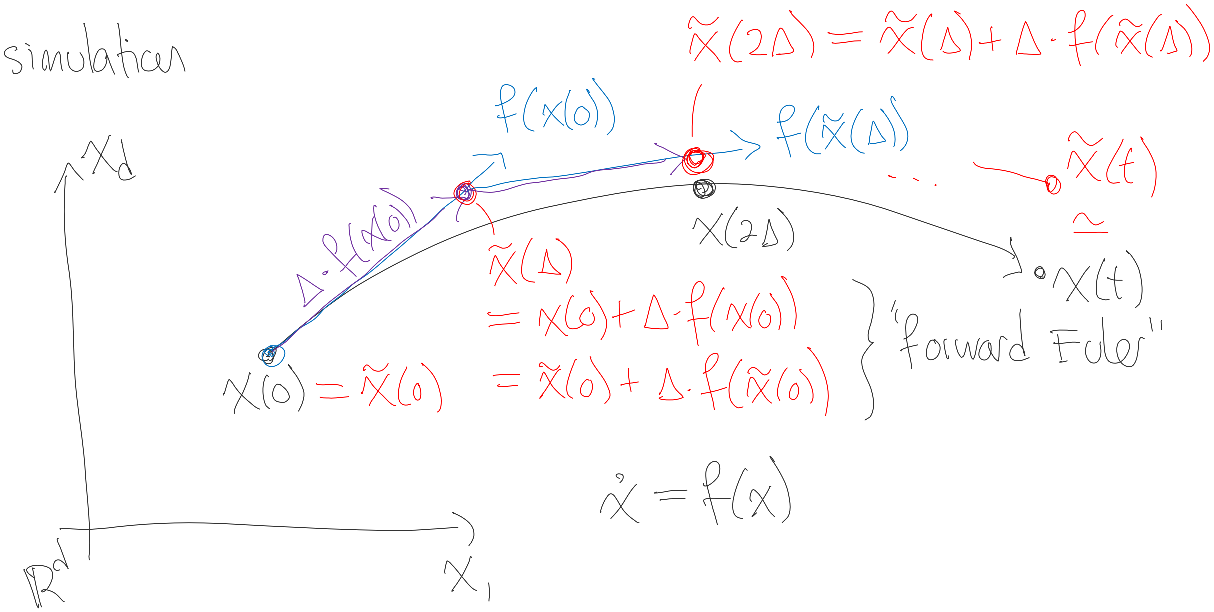
time plot



state-space / phase-space plot



simulation



forward Euler: $\tilde{x}(t) = x(t) + O(\Delta)$

Runge-Kutta (4,5): $= x(t) + O(\Delta^4)$