05 -- Tue Apr 14

AA/ECE/ME 548: Linear Multivariable Control

Prof Burden    TA Tinu    Spring 2020

if/when possible: keep video on; unmute to ask Q's

* specify your prefered name at identity.uw.edu
  (Zoom should use this name – can't change in Zoom profile)

today:
- ☑ HW 1 solution & self-assessment → due Sun Apr 19
- ☑ HW 2 Q's → due Fri Apr 17
- ☑ lec 3 Q's on optimization, dynamic programing, LQ regulation
- ☐ Prof OH

---

HW 2 P1 (c, d)

∘ sensitivity $S = \frac{1}{1+PC}$

↓

trades off w/
$T = 1 - S = \frac{PC}{1+PC}$

∘ open-loop xfer func $L = PC$ → gain/phase/stability margins $g_m, \varphi_m, s_m$

$S = \frac{1}{1+L} \iff L = \frac{1-S}{S}$

---

optimization

$\min_{u \in \mathbb{R}^m} J(u)$ objective/cost function

optimal control

$\min_{u} c(x,u)$ s.t. $x^+ = f(x,u)$

ex: fuel-efficient car

$u$ – design parameters like mass, geometry, engine parts, ...

$J(u)$ – \$/mi (economy)

– \$ (cost to manufacture/maintain)

Q: how do I evaluate $u \in \mathbb{R}^m$?

→ if $DJ(u) \neq 0$

$$\nabla J(u) = \frac{\partial}{\partial u} J(u) \in \mathbb{R}^{1 \times m}$$

then I can do better:

$$u^+ = u - \alpha DJ(u)$$

has lower cost

→ if $DJ(u) = 0$, check whether $D^2 J(u) > 0$ min

or $\frac{\partial^2}{\partial u^2} J(u) < 0$ max

ex: fuel-efficient driving

$u : [0, t] \to \mathbb{R}^m$ – throttle/brake

$x \in \mathbb{R}^d$ – car position, velocity, engine speed/temp

$c(x, u)$ – fuel consumed

- $J(u) = J(u_0) + b^T(u - u_0) + \frac{1}{2}(u - u_0)^T C (u - u_0)$, $C = C^T$

→ verify $DJ(u) = \left[\frac{\partial}{\partial u_1} J(u) \cdots \frac{\partial}{\partial u_m} J(u)\right]$ – why row?

$\rightarrow$ verify $\quad DJ(u) = \left[\frac{\partial}{\partial u_1} J(u) \cdots \frac{\partial}{\partial u_m} J(u)\right]$ — why row?

$$= b^T + (u-u_0)^T C$$

$DJ(u)$ is operator that tells me $DJ(u)\cdot v$ is rate of change of $J$ in $v$ direction

- $b^T(u-u_0) = b^Tu - b^Tu_0 \quad D_u\left(b^T(u-\cancelto{c}{u_0})\right) = D_u\left(b^Tu\right)$
- $b^Tu \in \mathbb{R} \quad b^Tu = b_1u_1 + b_2u_2 \cdots b_m u_m$

$$D_{u_k} b^Tu = D_{u_k}(b_k u_k) = b_k$$

$$DJ(u) = \left[\frac{\partial}{\partial u_1} J(u) \cdots \frac{\partial}{\partial u_m} J(u)\right] = \left[b_1 \quad b_2 \quad \cdots \quad b_m\right]$$
$$= b^T$$

- $D_u\left[\overbrace{(u-u_0)^T C (u-u_0)}^{y^TCy}\right]$

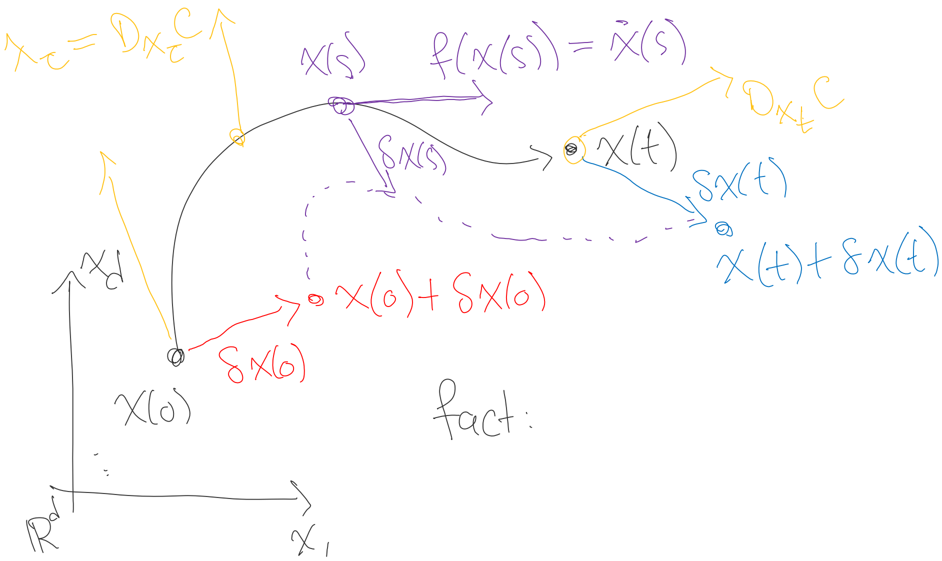
$$= \underbrace{\Big(\underbrace{\big[\underbrace{D_u(u-u_0)}_{I\in\mathbb{R}^{m\times m}}\big]^T \underbrace{C}_{\in\mathbb{R}^{m\times m}} \underbrace{(u-u_0)}_{\mathbb{R}^{m\times 1}}}_{\in\mathbb{R}^{m\times 1}}\Big)^T}_{\in\mathbb{R}^{1\times m}} + \underbrace{(u-u_0)^T C \big[\underbrace{D_u(u-u_0)}_{I}\big]}_{\in\mathbb{R}^{1\times m}}$$

$$= (u-u_0)^T C$$

---

- $C(x,u) = l_t(x_t) + \int_0^t \mathcal{L}_\tau(x_\tau,u_\tau)\,d\tau, \quad x^+ = f(x,u)$

$\longrightarrow \quad D_{u_\tau} c = \lambda_\tau \cdot D_u f$




fact: