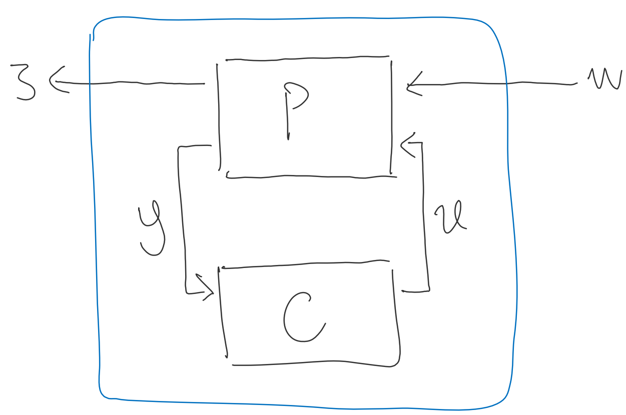
uncertain systems

goal: guarantee system stability in the presence of model uncertainty

ref: Dullerud & Paganini 2013 Ch 8

- consider the following feedback block diagram between process P and controller C:



- w contains disturbances (i.e. both input & output disturbances)
- z contains "errors" to be minimized (i.e. tracking error, control effort)

$$T_{zw} = P_{zw} + P_{zu}(I - C P_{yu})^{-1} C P_{yw}$$

* we know how to minimize $\|T_{zw}\|$ using H_2 or H_∞ norms

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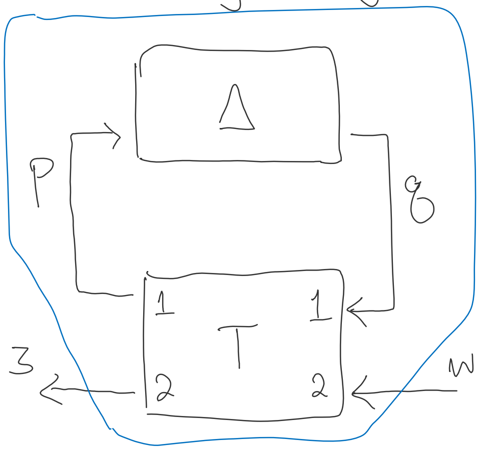
→ doing so gives robustness with respect to disturbance signals w



$$q = \Delta p$$

$$\begin{bmatrix} p \\ z \end{bmatrix} = T \begin{bmatrix} q \\ w \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} q \\ w \end{bmatrix}$$

* so long as $I - T_{11}\Delta$ is nonsingular,

the map $w \mapsto z$ is given by $\underbrace{T_{22} + T_{21}\Delta(I - T_{11}\Delta)^{-1}T_{12}}_{\star(T,\Delta)}$

→ determine $\star(T,\Delta)$ when $T = \begin{bmatrix} 0 & I \\ I & G \end{bmatrix}$, $T = \begin{bmatrix} 0 & G \\ I & G \end{bmatrix}$

$-\ T = \begin{bmatrix} 0 & I \\ I & G \end{bmatrix} \Rightarrow \star(T,\Delta) = G + I\Delta(I-0)^{-1}I$

$= G + \Delta$, i.e. additive uncertainty

$-\ T = \begin{bmatrix} 0 & G \\ I & G \end{bmatrix} \Rightarrow \star(T,\Delta) = G + I\Delta(I-0)^{-1}G$

$= (I + \Delta)G$, i.e. multiplicative uncertainty

thm: (D&P Thm 8.2) if $\boldsymbol{\Delta} = \{\Delta : \Delta \text{ is LTI}, \|\Delta\|_{L_2 \to L_2} \le \beta\}$

then $I - T_{11}\Delta$ nonsingular for all $\Delta \in \boldsymbol{\Delta} \iff \|T_{11}\|_{L_2\to L_2} < \frac{1}{\beta}$

$$\underset{\|\hat{T}\|_\infty}{\parallel}$$

read more / learn more: 594 (Robust Control)

Dullerud & Paganini 2013

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