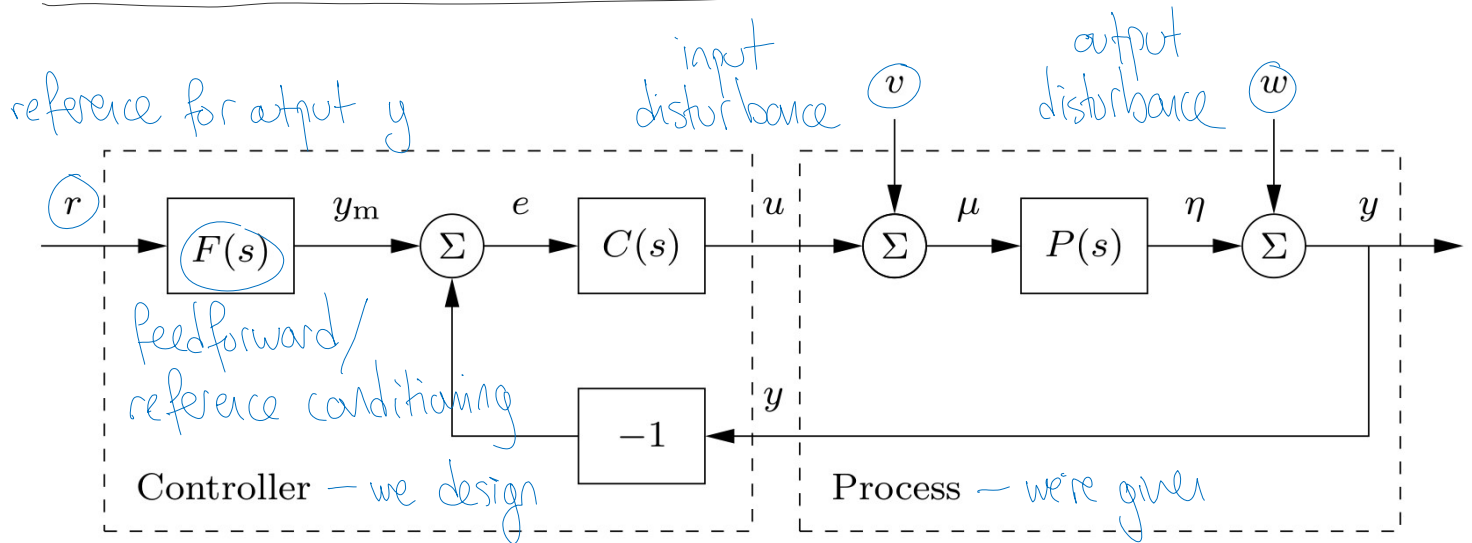


goal: performance tradeoffs & fundamental limits

refs: Astram & Murray ch 12 (SISO case) [AM]

Zhou, Doyle, Glover ch 6 (MIMO case) [ZDG]



want: $y \approx r$, i.e. $T_{yr} \approx 1$ - output tracks reference

$T_{yv}, T_{yw} \approx 0$ - output unaffected by disturbances

$T_{uv}, T_{uw} \approx 0$ - control input unaffected by disturbances

→ derive $T_{yr}, T_{yv}, T_{yw}, T_{uv}, T_{uw}$

$$\begin{aligned}
 y &= w + \eta = w + P\mu = w + P(v + u) = w + Pv + PCe \\
 &= w + Pv + PC(Fr - y)
 \end{aligned}$$

$$\Leftrightarrow (I + PC)y = w + Pv + PCFr$$

* assume nonsingular

$$\Leftrightarrow y = (I + PC)^{-1}w + (I + PC)^{-1}Pv + (I + PC)^{-1}PCFr$$

$$\Leftrightarrow y = \underbrace{(I+PC)^{-1}}_{T_{yw}} w + \underbrace{(I+PC)^{-1}P}_{T_{yv}} v + \underbrace{(I+PC)^{-1}PC}_{T_{yr}} r$$

$$= T_{yw} \cdot w + T_{yv} \cdot v + T_{yr} \cdot r$$

$$- u = Ce = C(Fr - y) = CFr - C(w + y) = CFr - w - CPu$$

$$= CFr - Cw - CP(v + u) = CFr - Cw - CPv - CPu$$

$$\Leftrightarrow \underbrace{(I+CP)}_{\substack{* \text{ assume} \\ \text{nonsingular}}} u = CFr - Cw - CPv$$

$$\Leftrightarrow u = \underbrace{(I+CP)^{-1}CFr}_{T_{ur}} - \underbrace{(I+CP)^{-1}Cw}_{T_{uw}} - \underbrace{(I+CP)^{-1}CPv}_{T_{uv}}$$

$$= T_{ur} \cdot r + T_{uw} \cdot w + T_{uv} \cdot v$$

• in SISO case: $T_{yw} = \frac{1}{1+PC}$ $T_{uv} = \frac{-PC}{1+PC}$

- recall: we want ≈ 0 to reject output disturbance ≈ 0 to reject input disturbance

* but: $T_{yw} - T_{uv} = \frac{1}{1+PC} + \frac{PC}{1+PC} = \frac{1+PC}{1+PC} = \underline{\underline{1}}$ ✓

→ so at any frequency $\omega \in \mathbb{R}$,

if $T_{yw}(j\omega) \approx 0$ then $T_{uv}(j\omega) \approx 1$

i.e. there's a tradeoff between rejecting output & input disturbances

ex: $P = \frac{1}{s - 1/\tau}$ ← first-order process w/ time constant τ

$r = k$ ← proportional control

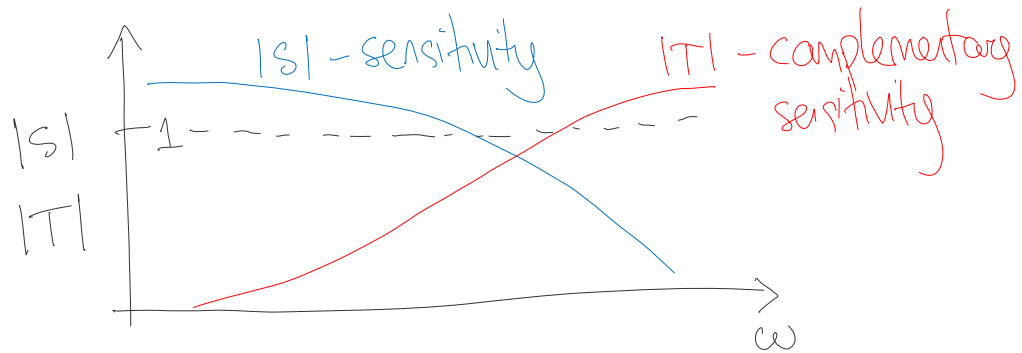
$$C = K \leftarrow \text{proportional control}$$

then Bode plots

$$\text{of } S = T_{gw}$$

$$\& T = T_{uv}$$

are:



* this is an example of a fundamental tradeoff that exists between competing performance specifications