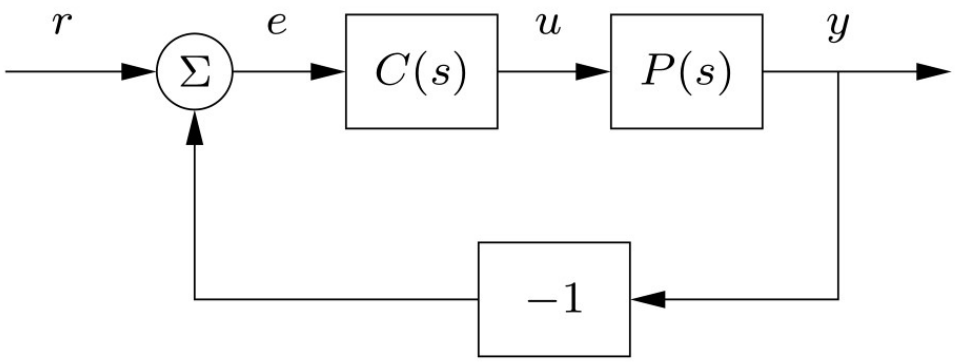
limits

goal: performance tradeoffs & fundamental limits

refs: Astrom & Murray Ch 12 (SISO case) [AM]
Zhou, Doyle, Glover Ch 6 (MIMO case) [ZDG]



- letting $S = \frac{1}{1+PC}$ denote sensitivity transfer function (SISO):

thm: [Thm 14.1 in AM] [Thm 6.2 ZDG] (Bode integral formula)

$$\int_0^\infty \log|S(j\omega)|\,d\omega = \pi \cdot \sum \{\mathrm{Re}\, p \mid p \text{ is a pole of } P \text{ in right half-plane}\}$$

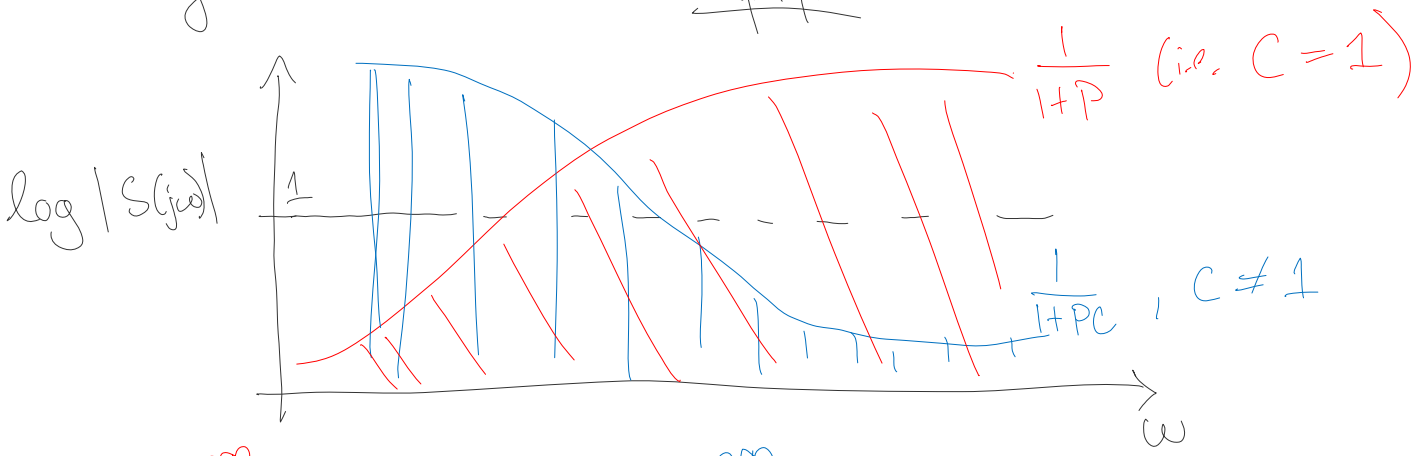
$$= 0 \iff \text{process } P \text{ is stable}$$

* importantly, the integral's value depends only on the (in)stability of the process P — invariant to controller C

- since $\log|S(j\omega)| < 0 \iff |S(j\omega)| < 1$,
any range of frequencies where controller attenuates output disturbance (since $S = T_{yw}$) must be compensated by
some where controller amplifies disturbance

disturbance (since $S = \frac{y}{w}$) must be compensated by

range where controller amplifies disturbance



$$\int_0^\infty \log|S(j\omega)|\,d\omega = \int_0^\infty \log|S(j\omega)|\,d\omega$$

* as control engineers, all we can do is shift area under curve (never decrease the area)