solution of DE, part 1: trajectories

goal: understand "solution" of differential/difference equations (DE)

$$\dot{x} \,/\, x^+ = f(x,u)$$

ref: Strogatz Ch 2 – geometric perspective

1°. trajectories

- we're considering differential or difference equations (DE)

$$\underbrace{\dot{x} = f(x,u)}_{\text{time rate of change of each component of } x} \quad \text{or} \quad \underbrace{x^+ = f(x,u)}_{\text{specifies the "next" } x}$$

f specifies: time rate of change of each component of x or specifies the "next" x

$x \in \mathbb{R}^d$, $u \in \mathbb{R}^m$, $f : \mathbb{R}^d \times \mathbb{R}^m \to \mathbb{R}^d$

$: (x,u) \mapsto \dot{x}$ or x^+

Q: what does it mean to "solve" a DE?

def: $x : [0,t] \to \mathbb{R}^d$ is a trajectory (traj) for (DE) if $x(s)$ satisfies (DE) at every time $s \in [0,t] \subset \mathbb{R}$ for $\dot{x}$, $\subset \mathbb{N}$ for x^+

- for a differential eqn:

$[0,t] \subset \mathbb{R}$ and $\forall s \in [0,t)$: $\underbrace{\dot{x}(s) = f(x(s),u(s))}$

$\to$ continuous-time (CT) system

x is differentiable at time s & its derivative is $f(x(s),u(s))$

– for a difference eqn:

$[0,t] \subset \mathbb{N}$ and $\forall s \in [0,t)$: $x(s+1) = f(x(s), u(s))$

→ discrete-time (DT) system

→ $x(s) = e^{As} x(0)$ is traj for what DE? $(s \in \mathbb{R})$

– $\frac{d}{ds} x(s) = \dot{x}(s) = \frac{d}{ds}\left[e^{As} x(0)\right] = A \cdot \underbrace{e^{As} x(0)}_{x(s)} = A \cdot x(s)$

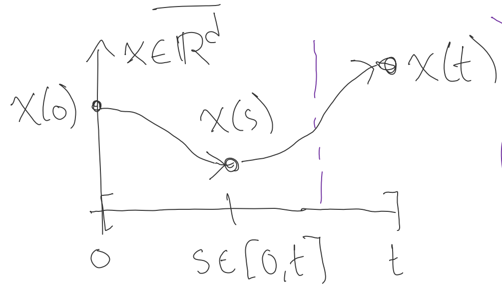
→ $x(s) = A^s x(0)$ is traj for what DE? $(s \in \mathbb{N})$

– $x(s+1) = A^{s+1} x(0) = A \cdot \underbrace{A^s x(0)}_{x(s)} = A \cdot x(s)$

* in both cases, the "solution" of DE is a signal, i.e. a function from a time domain into a vector space

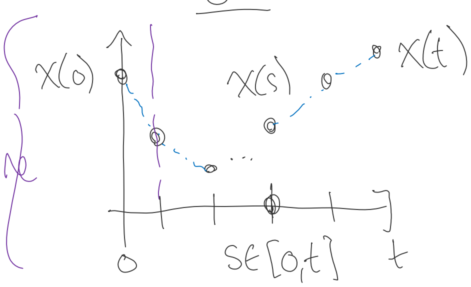
→ can visualize these functions in two main ways:

CT

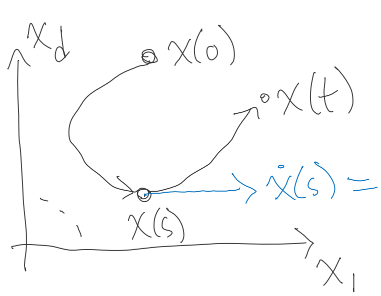


graph must pass "vertical-line test"

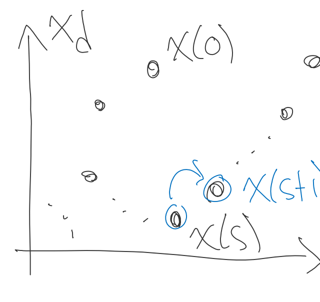
DT



time plot



$\underbrace{\dot{x}(s) = f(x(s), u(s))}_{\text{"vector field"}}$



$x(s+1) = f(x(s), u(s))$

state- or phase-space