linear process

goal: synthesize controllers by combining full-state feedback with observer

refs: Hespanha 2009 Ch 16.7

Aström & Murray 2019 Ch 8.3

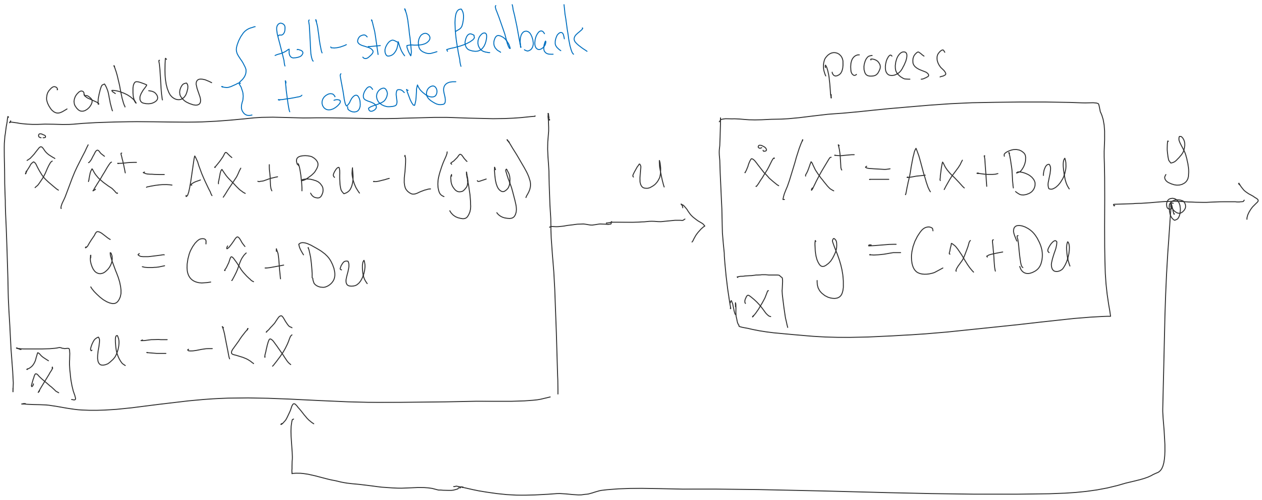
- suppose given LTI DE $\dot{x} / x^+ = Ax + Bu$
  - synthesize K s.t. $\underbrace{u = -Kx}_{\text{full-state feedback}} \rightsquigarrow \dot{x}/x^+ = \underbrace{(A - BK)x \text{ stable}}_{\dot{x}:\ \mathrm{Re}\,\lambda < 0;\ x^+ : |\lambda| < 1}$

    ↳ e.g. using pole placement, ∞-horizon LQ optimal control problem
  - synthesize L s.t. $y = Cx + Du$, $\dot{\hat{x}} / \hat{x}^+ = A\hat{x} + Bu - L(\hat{y} - y)$

    $\hat{y} = C\hat{x} + Du$

    ↳ using pole placement; ∞-horizon LQ estimation problem (i.e. Kalman filter)

$$e = x - \hat{x} \Rightarrow \dot{e}/e^+ = Ax + Bu - \left(A\hat{x} + Bu - L(\hat{y} - y)\right)$$
$$= (A - LC)(x - \hat{x}) = (A - LC)e \quad \text{stable}$$



$$\begin{bmatrix} \hat{x} \\ e \end{bmatrix}^{\circ/+} = \begin{bmatrix} A\hat{x} + \overset{-BK\hat{x}}{Bu} - L\underbrace{(\hat{y} - y)}_{=-Ce} \\ (A - LC)e \end{bmatrix} = \underbrace{\left[\begin{array}{c|c} (A - BK) & LC \\ \hline 0 & (A - LC) \end{array}\right]}\begin{bmatrix} \hat{x} \\ e \end{bmatrix}$$

I can independently design controller & observer: $\Leftarrow$ eigenvalues: $\mathrm{spec}(A - BK) \cup \mathrm{spec}(A - LC)$

$e \to 0$, so $x \to \hat{x}$ & $\hat{x} \to 0$, so $x \to 0$

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how to relate state-space & frequency-domain / transfer function representations of LTI system?

given $\dot{x}/x^+ = Ax + Bu$
$y = Cx + Du$

$\xrightarrow{\text{Laplace transform}}$ $sX = AX + BU \Leftrightarrow (sI - A)X = BU$
$Y = CX + DU$

$X = (sI - A)^{-1}BU$

$\Updownarrow$

$Y = \left[C(sI - A)^{-1}B + D\right]U$

want $\xrightarrow[U]{u} \boxed{G} \xrightarrow[Y]{y}$

want $\xrightarrow[U]{u} \boxed{\boxed{x}\ G} \xrightarrow[Y]{y}$

$$Y = \underbrace{\left[C(sI-A)^{-1}B + D \right]}_{G\text{ : transfer function from } U \text{ to } Y} U$$