Kalman filtering (KF)

goal: derive optimal state estimator for linear dynamics, Gaussian noise

ref: Stengel Ch 4.3

- consider DT-LTV system

$$x_{s+1} = A_s x_s + B_s u_s + \delta_s$$

$$y_s = C_s x_s + \cancel{D_s}^{\,0} u_s + \eta_s$$

where δ_s is process noise (or input disturbance)

η_s is measurement noise (or output disturbance)

- assume $\{\delta_s\}$, $\{\eta_s\}$ are zero-mean, independent, Gaussian:

$$E[\delta_s] = 0, \quad Cov[\delta_s] = Q_s, \quad E[\delta_s \eta_s^T] = 0$$

$$E[\eta_s] = 0, \quad Cov[\eta_s] = R_s, \quad E[\delta_s \delta_\tau^T] = 0, \; s \neq \tau$$

$E[\eta_s \eta_\tau^T] = 0, \; s \neq \tau$

* <u>want</u>: minimize "estimation energy" $\sum_{s=0}^{t} \delta_s^T Q_s^{-1} \delta_s + \eta_s^T R_s^{-1} \eta_s$

(i.e. least-squares estimation of $\{\hat{x}_s\}$ given $\{y_s\}$)

<u>solution</u>: $\hat{x}_s = \tilde{x}_s + K_s(y_s - C_s \tilde{x}_s) = (I - K_s C_s)\tilde{x}_s + K_s y_s$

<u>where</u>: $x_{s-1} \sim \mathcal{N}(\hat{x}_{s-1}, P_{s-1})$

$\boxed{\text{Kalman filter}}$

$\tilde{x}_s = A_{s-1}\hat{x}_{s-1} + B_{s-1}u_{s-1}$ ← state propagation

$K_s = \tilde{P}_s C_s^T (R_s + C_s \tilde{P}_s C_s^T)^{-1}$

$\tilde{P}_s = A_{s-1} P_{s-1} A_{s-1}^T + Q_s$ ← prediction covariance

$\tilde{P}_s = \mathrm{Cov}[\tilde{x}_s]$

$P_s = (I - K_s C_s)\tilde{P}_s (I - K_s C_s)^T + K_s R_s K_s^T$

$= \mathrm{Cov}[\hat{x}_s]$ ← estimate covariance