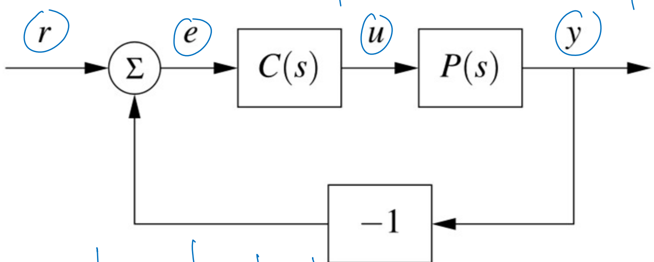
block diagram algebra

goal: recall how to translate block diagrams $\longleftrightarrow$ equations

refs: Astrom & Murray Ch 10 (SISO case)
Zhou, Doyle, Glover Ch 5 (MIMO case)

- consider the following block diagram:

reference error inputs output, i.e. observations/measurements



i.e. desired output

P(s) – process / plant, i.e. the system we're controlling

C(s) – controller / compensator, i.e. the system we're designing

* this block diagram is not (just) a pretty picture; if C & P are LTI, it's a formal mathematical model

ex: $y = Pu, \quad u = Ce \implies y = Pu = \underbrace{PCe = T_{ye}}\cdot e$

$\rightarrow$ derive T_{yr}, i.e. the transfer function from r to y so that $y = T_{yr}\cdot r$

$$- \; y = Pu = PCe = PC(r-y) = PCr - PCy$$

$$\iff y + PCy = PCr \iff (I+PC)\,y = PCr$$

$$\overset{(*)}{\iff} y = \underbrace{(I+PC)^{-1}}PCr$$

* assuming invertible. $T_{yr} = (I+PC)^{-1}PC$

* assuming invertible, $T_{yr} = (I + PC)^{-1} PC$

* important: in multi-input/multi-output systems (MIMO), transformations don't generally commute (in particular, dimensions don't match!)

ex: $y = Pu \qquad u = Ce = C(r - y)$

so if $y \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $m \neq n$

then $P \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{m \times n}$,

so PC makes sense, but $\cancel{CP}$ does not