margins for LQ(+KF) regulators

goal: guaranteed stability margins for LQ & LQ + KF = LQG regulators

refs:

[SA77] IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. AC-22, NO. 2, APRIL 1977

Gain and Phase Margin for Multiloop LQG Regulators

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[D78] IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. AC-23, NO. 4, AUGUST 1978

Guaranteed Margins for LQG Regulators

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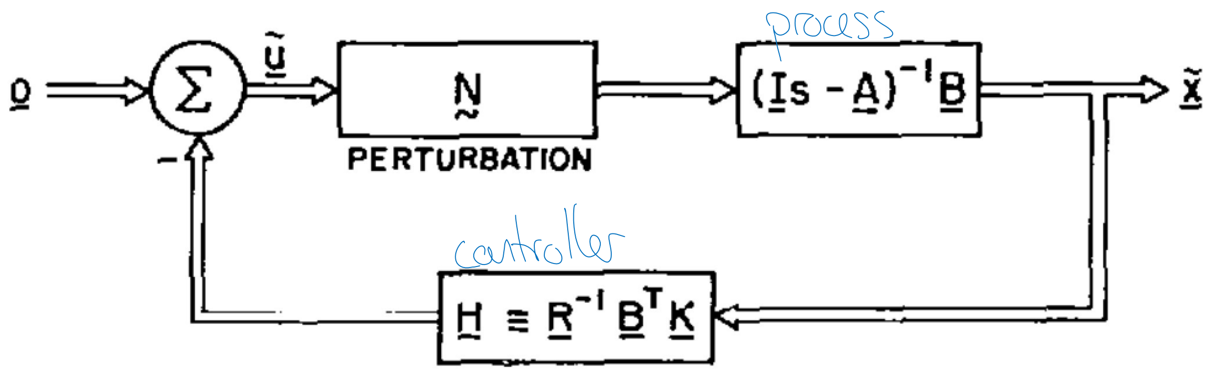
- consider the linear-quadratic (LQ) controller for $\dot{x} = Ax + Bu$,

  $u = -Kx$, $K = -R^{-1}B^TPx$, $0 = PA + A^TP - PBR^{-1}B^TP + Q$,

  which is a <u>static</u> compensator relying on <u>full-state</u> feedback,

  and which minimizes $\int_0^\infty x^TQx + u^TRu\,dt$, $Q, R > 0$



* [SA77] analyzed robustness of this LQ controller to perturbations finding the following gain/phase margins for each loop:

$$\text{gain margin} \in \left(\tfrac{1}{2}, \infty\right), \quad \text{phase margin} \in (-60^\circ, +60^\circ)$$

$\Rightarrow$ full-state LQ feedback is quite robust!

◦ one year later, [D78] reported on gain/phase margins of the linear-quadratic-Gaussian ($LQG = LQ + KF$) controller,

$$u = -K\hat{x}, \qquad \dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$$

$$\hat{y} = C\hat{x} + Du, \quad y = Cx + Du$$

— Kalman-Bucy filter

which is a dynamic compensator using output feedback, and which minimizes $E\left[\int_0^\infty x^T Q x + u^T R x \, dt\right]$, finding an example with 2-dimensional state where the gain margin can be arbitrarily small ...

$\Rightarrow$ so $LQG = LQ + KF$ regulators aren't guaranteed to be robust!