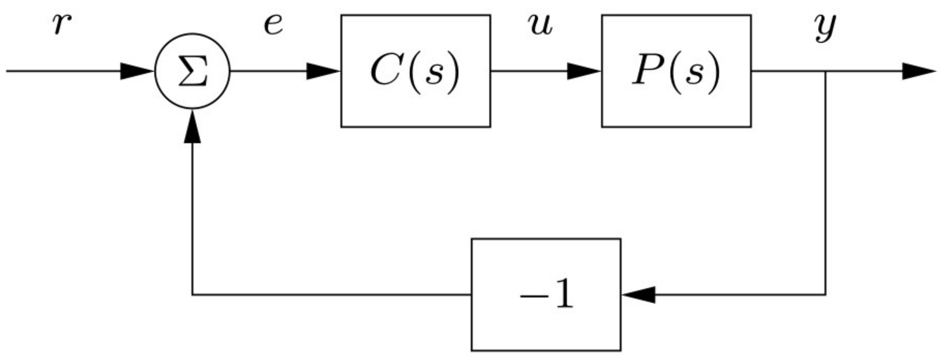
stability margins

goal: quantitatively and qualitatively assess robustness of feedback loops

refs: Astrom & Murray Ch 10 (SISO case)
Zhou, Doyle, Glover Ch 5 (MIMO case)



- recall that the transfer function from r to y is

$$T_{yr} = \underbrace{(I+PC)^{-1}PC}_{\text{MIMO}} = \underbrace{\frac{PC}{1+PC}}_{\text{SISO}}$$

- letting $L = PC$ denote the (open-) loop transfer function, we note that $I+PC = I+L$ shouldn't be singular:

SISO: $1+PC = 1+L \simeq 0$ i.e. $L \simeq -1 \in \mathbb{C}$ MIMO: $\det(I+L) \simeq 0$

→ in this case, small changes in reference r lead to large changes in outputs (hence, system states)

→ from another perspective, small errors in model or implementation ($\tilde{P}\tilde{C} \sim PC$ ...) can cause large tracking error

→ from another perspective, small errors in model or implementation ($\tilde{L} = \tilde{P}\tilde{C} \simeq PC = L$) can cause large tracking error

- in fact, we know L governs stability of feedback system:

<u>thm:</u> (MIMO Nyquist stability criterion)

closed-loop feedback system asymptotically stable

$\iff$

the # of clockwise encirclements of $0 \in \mathbb{C}$ by the locus of $\det(I + L(s))$ on a Nyquist contour*

*large enough to encircle all poles of L in RHP

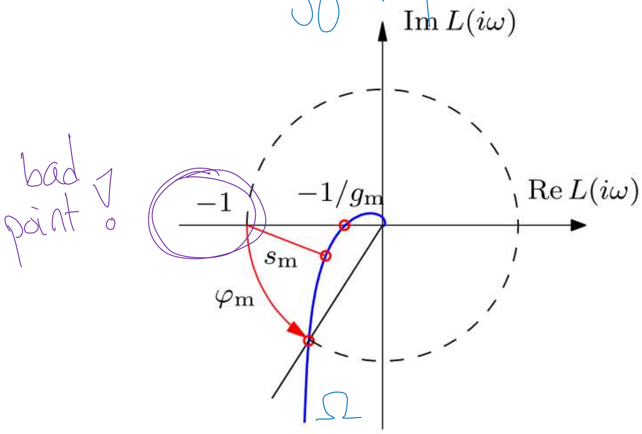
equals the # of poles of L in the closed right-half plane (RHP)

($\iff$ (SISO) # encirclements of $-1 \in \mathbb{C}$ by locus of L)

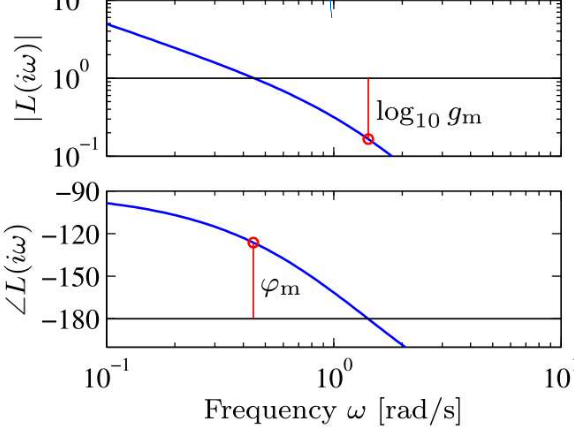
- thus, if controller C stabilizes process P then the locus of $\det(I + P(j\omega)C(j\omega))$ doesn't cross $0 \in \mathbb{C}$

→ so the distance to $0 \in \mathbb{C}$ is a robustness criterion termed a <u>stability margin</u>

Nyquist plot



Bode plot



three common ways to quantify robustness in terms of "distance" to $-1 \in \mathbb{C}$:

1°: s_m = distance from $\Omega = \{L(j\omega) : \omega \in (-\infty, \infty)\}$ to -1

"gain margin" 2°: $g_m = 1/$ distance from Ω to -1 restricted to scaling

"phase margin" 3°: φ_m = distance from Ω to -1 restricted to rotation