H2 and Hoo system norms -- computation

goal: computation of $\mathcal{H}_2$ and $\mathcal{H}_\infty$ system norms

refs: Doyle, Glover, Khargonekar, Francis 1989

State-Space Solutions to Standard $\mathcal{H}_2$ and $\mathcal{H}_\infty$ Control Problems

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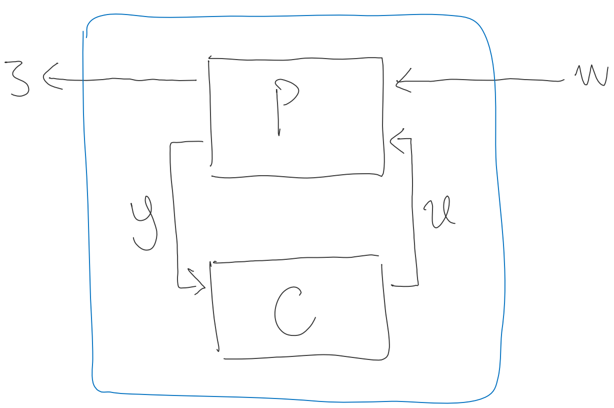
- consider the following feedback block diagram between process P and controller C:



- w contains disturbances (i.e. both input & output disturbances)
- z contains "errors" to be minimized (i.e. tracking error, control effort)

$$T_{zw} = P_{zw} + P_{zu}(I - C P_{yu})^{-1} C P_{yw}$$ → verify this formula

* our goal as control engineers is to minimize $\|T_{zw}\|$

↳ we'll focus on $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms

↳ we'll focus on H_2 and H_∞ norms

- assume given causal LTI $\boxed{\text{stable}}$ transformation $\boxed{z = Tw}$
  and let (A, B, C, D) be minimal realization: $\Rightarrow$ A stable

$$\hat{T}(s) = C(sI-A)^{-1}B + D \quad \text{and} \quad (A,B) \text{ controllable, } (A,C) \text{ observable}$$

$$\Gamma \uparrow \quad \downarrow \Gamma^{-1}$$

$$y \longleftarrow \boxed{\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}} \longleftarrow u$$

$$y(t) = (Tu)(t)$$

$$\hat{y}(s) = \hat{T}(s) \cdot \hat{u}(s)$$

$\boxed{H_2}$ let L_c, L_o denote controllability & observability Gramians

i.e. $AL_c + L_cA^T + BB^T = 0, \quad A^TL_o + L_oA + C^TC = 0$

then: $\|T\|_2^2 = tr(CL_cC^T) = tr(B^TL_oB)$

- importantly, L_c & L_o can be obtained by solving linear equations

$\boxed{H_\infty}$ ~~assuming A stable~~ (redundant), $\gamma > 0$, define $H_\gamma = \underbrace{\begin{bmatrix} A & \frac{1}{\gamma^2}BB^T \\ -C^TC & -A^T \end{bmatrix}}_{\text{"Hamiltonian matrix"}}$

then: $\|T\|_\infty < \gamma \Leftrightarrow j\mathbb{R} \cap \operatorname{spec} H_\gamma = \emptyset$

(i.e. H_γ has no eigenvalues on imaginary axis)

Hamiltonian matrix

(i.e. H_γ has no eigenvalues on imaginary axis)

- so we need to do a line search to find smallest γ