AA/ECE/ME 548 Linear Multivariable Control Sp 22 Prof Burden

today: ☑ course logistics, Canvas, etc

Thu Mar 31 ☑ HW 0

☑ week 1 lectures

☐ questions / office hours

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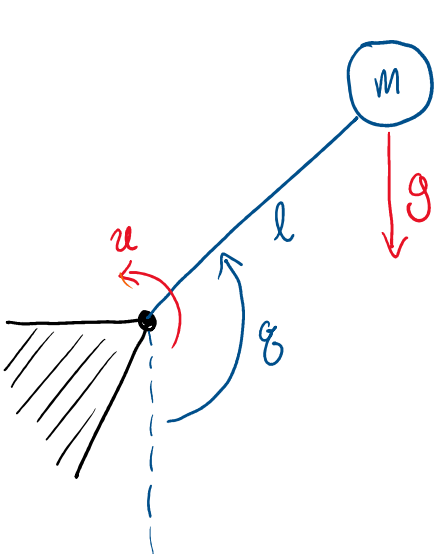
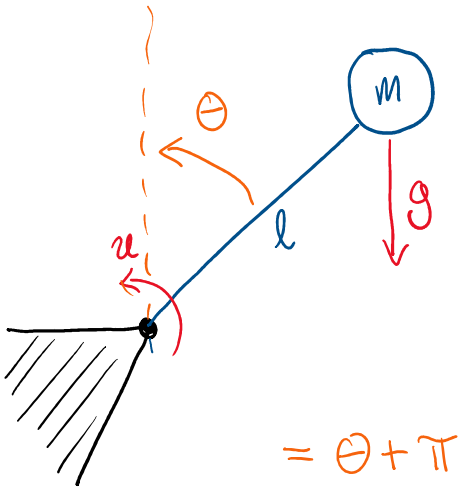
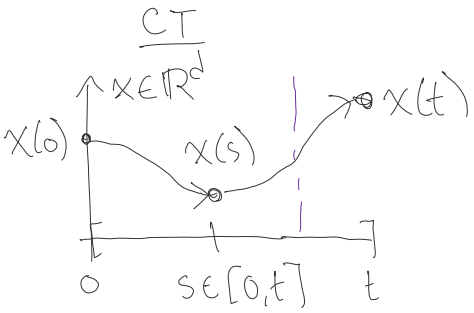
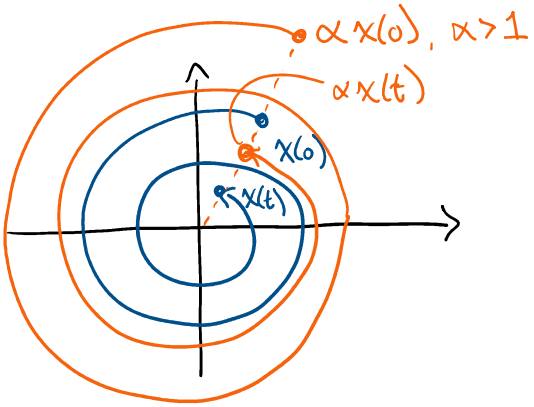
ex: inverted pendulum $ml^2\ddot{\theta} = -mg\sin\theta + u - b\dot{\theta}$

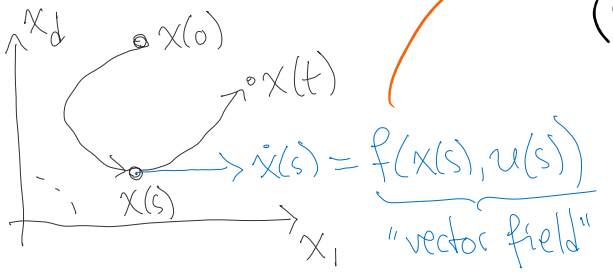






$$\dot{x} = Ax \iff x(t) = e^{At}x(0)$$





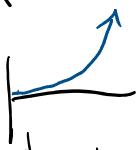
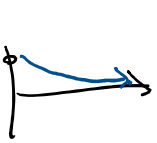
what if f is linear? (eg LTI?)

Linear systems:

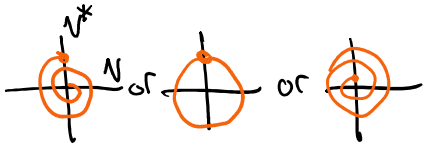
$$3(0) = x(0) + y(0)$$
$$\downarrow$$
$$3(t) = x(t) + y(t)$$

Q: what can the flow of an LTI system do?

$\dot{x} = Ax$: if $Av = \lambda v$ then $x(0) = \alpha \cdot v \Rightarrow x(t) = \alpha \cdot e^{\lambda t} v$, $\alpha \in \mathbb{R}$

so there are 2 cases: 1° if $\lambda \in \mathbb{R}$ then $v \in \mathbb{R}^d$ and $e^{\lambda t}$ looks like  or 

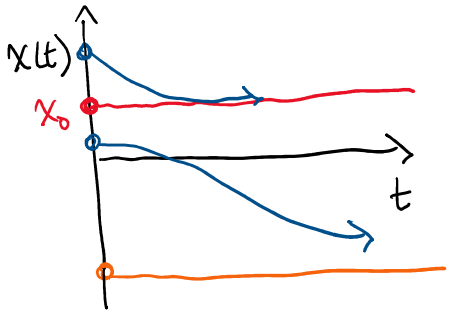
2° if $\lambda \in \mathbb{C}$ then $v \in \mathbb{C}^d$ and $Av^* = \lambda^* v^*$

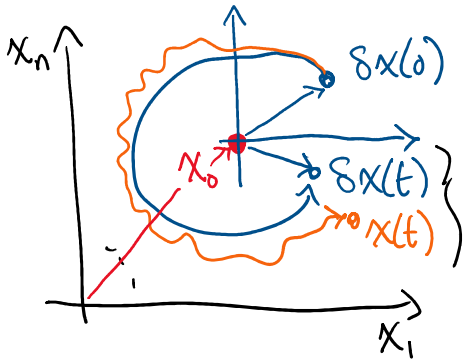
so $e^{\lambda t}$ looks like 

(NL) $\dot{x} = f(x,u)$, $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$

def: $(x_0, u_0) \in \mathbb{R}^n \times \mathbb{R}^m$ is an equilibrium for (NL) if $f(x_0, u_0) = 0$

↳ if $x(0) = x_0$, $u(t) = u_0$ then $x(t) = x_0$





* letting $\delta\dot{x} = A \cdot \delta x + B \cdot \delta u$ where $A = \partial_x f(x_0, u_0)$, $B = \partial_u f(x_0, u_0)$

then $x \simeq x_0 + \delta x$ if $u = u_0 + \delta u$

this approx. gets better as $\|\delta x\|$ gets smaller