Tue May 3

AA/ECE/ME 548 Linear Multivariable Control Sp22 Prof Burden

today:
- ☑ course logistics, Canvas, etc
- ☑ exam 1 – solution done, working on grading
- ☑ HW 3 self-assessment – due next Monday
- ☑ HW 4 – due this Friday
- ☐ week 6 lectures
- ☐ questions / office hours

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* connection between $\underbrace{\text{stability}}_{\text{i.e. Lyapunov}}$ & $\underbrace{\text{optimal control}}_{\text{i.e. LQR}}$

consider $\min \int_0^\infty x^T Q x + u^T R u$ s.t. $\dot{x} / x^+ = Ax + Bu$

* know that optimal $u^* = -Kx$ where $K = R^{-1} B^T P$ ($\dot{x}$)

or $K = (B^T P B + R)^{-1} B^T P A$ (x^+)

where $P = P^T > 0$ solves a Riccati equation

fact: closed-loop dynamics $\dot{x} / x^+ = Ax + Bu = (A - BK)x$

are (exponentially) stable and $V(x) = x^T P x$ is a Lyapunov function are (exponentially) stable and $\underbrace{V(x) = x' P x}$ is a Lyapunov function

* also the (optimal) cost-to-go

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ex: $\ddot{y} = u \longleftrightarrow x = \begin{bmatrix} y \\ \dot{y} \end{bmatrix} \Rightarrow \dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

$= Ax + Bu$

so $u = -Kx = -\begin{bmatrix} k_P & k_D \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$

* if we choose K to $\min \int_0^\infty x^T Q x + u^T R u$

then we've synthesized the "optimal" PD controller

ex: $\ddot{y} = u$, $\dot{z} = y \longleftrightarrow x = \begin{bmatrix} y \\ z \\ \dot{y} \end{bmatrix} \Rightarrow \dot{x} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ z \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$

so $u = -Kx = -\begin{bmatrix} k_P & k_I & k_D \end{bmatrix} \begin{bmatrix} y \\ z \\ \dot{y} \end{bmatrix}$

* if we choose K to $\min \int_0^\infty x^T Q x + u^T R u$

then we've synthesized the "optimal" PID controller

(alternatively cf Zeigler-Nichols)

given nonzero time-varying reference r to track,

might want to choose K to $\min \int_0^\infty q\|r-y\|^2 + r\|u\|^2$

$= (r-y)^T \cdot q \cdot (r-y)$

but this problem doesn't have a solution!

$$= \begin{bmatrix} r-y \\ z \\ \dot{y} \end{bmatrix}^T \begin{bmatrix} q & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} r-y \\ z \\ \dot{y} \end{bmatrix}$$

let $e = r - y$

$$\ddot{e} = \ddot{r} - u$$
$$\dot{z} = e$$

$$\longleftrightarrow \quad x = \begin{bmatrix} e \\ z \\ \dot{e} \end{bmatrix} \Rightarrow \dot{x} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e \\ z \\ \dot{e} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \ddot{r}$$