Tue May 24

AA/ECE/ME 548 Linear Multivariable Control Sp22 Prof Burden

today:
- ☐ course logistics, Canvas, etc
- ☑ results from "check-in" survey Sat Jun 4 (for +2 bonus)
- ☑ exam 2 next week – due ~~Fri Jun 3 (?)~~
- ☑ HW 7 self-assessment – due ~~next Monday~~ Mon Jun 6
- ☑ HW 8 – due this Friday
- ☐ week 9 lectures
- ☐ questions / office hours

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ex: $L_p(-\infty,\infty) = \{ u : \underbrace{(-\infty,\infty)}_{= \mathbb{R} \text{ or } \mathbb{Z} = \{\dots,-2,-1,0,1,2,\dots\}} \to \mathbb{C}^n \mid \|u\|_p < \infty \}$

given $u, v \in L_p(-\infty,\infty)$, $\alpha \in \mathbb{C}$

define $w = u + \alpha \cdot v$ where $\forall t \in (-\infty,\infty) : \underbrace{w(t) = u(t) + \alpha \cdot v(t)}_{\in \mathbb{C}^n}$

ex: $W = \{ f : S \to V \}$ where V is vec space, S is set

eg $V = \mathbb{C}^n$ eg $S = (-\infty,\infty)$

$u, v \in W$, $\alpha \in \mathbb{C}$: $w = u + \alpha \cdot v$ defined $\forall a \in S : w(a) = u(a) + \alpha \cdot v(a)$

$$\|u\|_p = \left(\int_{-\infty}^{\infty} \|u(t)\|_p^p \, dt \right)^{1/p} \qquad \|u\|_\infty = \operatorname*{ess\,sup}_{t\in\mathbb{R}} \|u(t)\|_\infty = \max_{t\in\mathbb{R}} \|u(t)\|_\infty$$

<u>ex:</u> $V = \{ u : \underbrace{\{0,1,2,\dots,t-1\}}_{=: S} \to \mathbb{R}^n \}$

given $u \in V$ define $\bar{u} \in \mathbb{R}^{t\cdot n}$ by $\bar{u} = \begin{bmatrix} u(0) \\ u(1) \\ \vdots \\ u(t-1) \end{bmatrix}$

then $\|\bar{u}\|_2^2 = \bar{u}^T \cdot \bar{u} = \sum_{s=0}^{t-1} u(s)^T \cdot u(s)$

so I might propose $\|w\|_{\cancel{2}2} := \sqrt{\int_{-\infty}^{\infty} w^T(s) \cdot \overset{\in \mathbb{R}^n}{w(s)} \, ds} = \sqrt{\int_{-\infty}^{\infty} \|w(s)\|_2^2 \, ds}$

for any $w : (-\infty, \infty) \to \mathbb{R}^n$

more generally, $\|w\|_p = \left(\int_{-\infty}^{\infty} \|w(s)\|_p^p \, ds \right)^{1/p}$

$p \in [1, \infty)$

$\|T\|_{L_2 \to L_2}$ is the induced norm of LTI system T

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"disturbances" $\quad w \to \boxed{T} \to z \quad$ "errors"

$w \in L_2 \qquad\qquad z \in L_2$

$w(t) \in \mathbb{R}^m \qquad z(t) \in \mathbb{R}^{o}$

$\hookrightarrow$ quantifies the maximum amount of "power" (quantified as $\|w\|_2$) that T transfers from w to z (quantified as $\|z\|_2$)

* in contrast, $\|\hat{T}\|_2$ quantifies power in output ($\|z\|_2$) produced by <u>one</u> <u>specific</u> <u>input</u> ($\|w^*\|_2$)