Thu Apr 14

AA/ECE/ME 548 Linear Multivariable Control Sp22 Prof Burden

today:
- ☑ course logistics, Canvas, etc
- ☑ HW 1 self-assessment – due next ~~Monday~~ thursday
- ☑ HW 2 – due this Friday
- ☑ week 3 lectures
- ☐ questions / office hours

TODO:
- ☐ HW1 solution ⇝ Canvas
- ☐ 2π in p1(a)
- ☐ link notebooks / point to Python intro
- ☐ ECE Colloq
- ☐ Robotics Colloquium
- ☐ Northwest Robotics Symp.

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* how does continuous-time LQR relate to discrete-time?

$$\dot{x} = Ax + Bu \quad \rightsquigarrow \quad x^+ = e^{\Delta A} x = A_\Delta x, \quad \Delta > 0 \text{ timestep}$$

$$A_\Delta = (I + \Delta A)$$

$$c = \int_0^{k\cdot\Delta} x^T Q x + u^T R u \quad \rightsquigarrow \quad \sum_{\ell=0}^{k-1} x_\ell^T Q_\Delta x_\ell + u_\ell^T R_\Delta u_\ell = c_\Delta$$

LQR

$$u^* = -Kx \qquad\qquad u^* = -K_\Delta x$$

then: $\lim_{\Delta \to 0} K_\Delta = K, \quad \lim_{\Delta \to 0} c_\Delta = c$

assuming $A_\Delta = A + O(\Delta)$

$\Delta \to 0$

assuming $A_\Delta = A + O(\Delta)$

$Q_\Delta = Q + O(\Delta)$

$R_\Delta = R + O(\Delta)$

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* compare finite-horizon vs infinite-horizon

finite-horiz: $\dot{x}(t)$ or $x(t+1) = A_t x(t) + B_t u(t)$

cost is $\int_0^T$ or $\sum_{t=0}^{T} x_t^T Q_t x_t + u_t^T R_t u_t$

$\rightsquigarrow$ opt ctrl is $u_t^* = -K_t x_t$

∞-horiz: $\dot{x} / x^+ = Ax + Bu$

cost $\int_0^\infty$ or $\sum_{t=0}^{\infty} x^T Q x + u^T R u$

$\rightsquigarrow$ opt ctrl is $u^* = -Kx$

$$K_s = (B_s^T P_{s+1} B_s + R_s)^{-1} B_s^T P_{s+1} A_s$$

$$P_s = (A_s - B_s K_s)^T P_{s+1} (A_s - B_s K_s) + K_s^T R_s K_s + Q_s$$

$$u_s^* = -K_s x_s, \quad V_s^* = \frac{1}{2} x_s^T P_s x_s$$

$$K = (B^T P B + R)^{-1} B P A$$

(LTI) $P^- = (A - BK)^T P (A - BK) + KRK + Q$

↳ (LTI) $P^- = (A-BK)^T P (A-BK) + KRK + Q$

* not linear, but: 1°. preserves symmetry & definiteness

2°. has unique GES equilibrium P^*

* P^* is solution of ∞-horiz LTI LQR !