

AA/ECE/ME 548 Linear Multivariable Control Sp22 Prof Borden

today: ☐ course logistics, Canvas, etc

☒ results from "check-in" survey Sat Jun 4 (for +2 bonus)

☒ exam 2 next week - due ~~Fri Jun 3 (?)~~

☒ HW 7 self-assessment - due ~~next Monday~~ Mon Jun 6

☒ HW 8 - due this Friday

☐ week 9 lectures

☐ questions / office hours

$$\text{ex: } L_p(-\infty, \infty) = \left\{ u : \underline{(-\infty, \infty)} \rightarrow \mathbb{C}^n \mid \|u\|_p < \infty \right\}$$

$$= \mathbb{R} \text{ or } \mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$$

given $u, v \in L_p(-\infty, \infty)$, $\alpha \in \mathbb{C}$

define $w = u + \alpha v$ where $\forall t \in (-\infty, \infty) : w(t) = u(t) + \alpha \cdot v(t)$
 $\in \mathbb{C}^n$

ex: $W = \{ f : S \rightarrow V \}$ where V is vec space, S is set
 eg $V = \mathbb{C}^n$ eg $S = (-\infty, \infty)$

$u, v \in W$, $\alpha \in \mathbb{C}$: $w = u + \alpha \cdot v$ defined $\forall a \in S : w(a) = u(a) + \alpha \cdot v(a)$

$$\|u\|_p = \left(\int_{-\infty}^{\infty} \|u(t)\|_p^p dt \right)^{1/p}$$

$$\|u\|_{\infty} = \operatorname{ess\,sup}_{t \in \mathbb{R}} \|u(t)\|_{\infty} = \max_{t \in \mathbb{R}} \|u(t)\|_{\infty}$$

ex: $V = \{ u: \underbrace{\{0, 1, 2, \dots, t-1\}}_{=:s} \rightarrow \mathbb{R}^n \}$

given $u \in V$ define $\bar{u} \in \mathbb{R}^{t \cdot n}$ by $\bar{u} = \begin{bmatrix} u(0) \\ u(1) \\ \vdots \\ u(t-1) \end{bmatrix}$

then $\|\bar{u}\|_2^2 = \bar{u}^T \cdot \bar{u} = \sum_{s=0}^{t-1} u(s)^T \cdot u(s)$

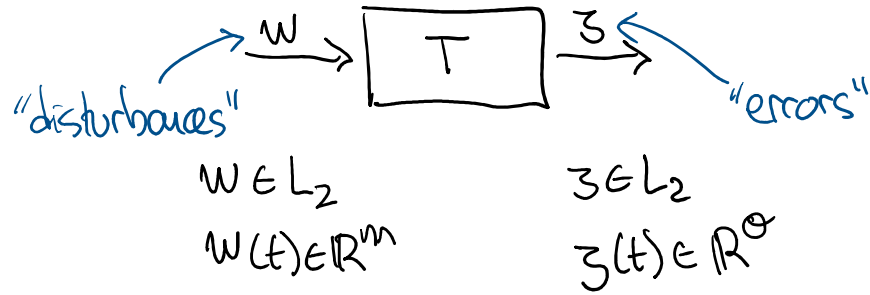
so I might propose $\|w\|_2^2 := \int_{-\infty}^{\infty} \overbrace{w^T(s) \cdot w(s)}^{s \in \mathbb{R}^n} ds = \int_{-\infty}^{\infty} \|w(s)\|_2^2 ds$

for any $w: (-\infty, \infty) \rightarrow \mathbb{R}^n$

more generally, $\|w\|_p = \left(\int_{-\infty}^{\infty} \|w(s)\|_p^p ds \right)^{1/p}$
 $p \in [1, \infty)$

$\|T\|_{L_2 \rightarrow L_2}$ is the induced norm of LTI system T

$\| \cdot \|_{L_2 \rightarrow L_2}$ is the induced norm of LTI system 1



→ quantifies the maximum amount of "power" (quantified as $\|w\|_2$) that T transfers from w to z (quantified as $\|z\|_2$)

* in contrast, $\|\hat{T}\|_2$ quantifies power in output ($\|z\|_2$) produced by one specific input ($\|w^*\|_2$)