Tue Apr 5

AA/ECE/ME 548 Linear Multivariable Control Sp22 Prof Burden

today:
- ☑ course logistics, Canvas, etc
- ☑ HW0 self-assessment – due next Monday
- ☑ HW1 – due this Friday
- ☐ week 2 lectures
- ☐ questions / office hours

TODO: ☐ post notes

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consider $\dot{x} = Ax$, $A \in \mathbb{R}^{n \times n}$

fact: if A has n distinct eigenvalues $\lambda_1, \dots, \lambda_n \in \mathbb{C}$
with corresponding eigenvectors $v_1, \dots, v_n \in \mathbb{C}^n$
then $D = VAV^{-1} = \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix} \Rightarrow A = V^{-1} D V$

where $V = [v_1, \dots, v_n]$

A is diagonalizable

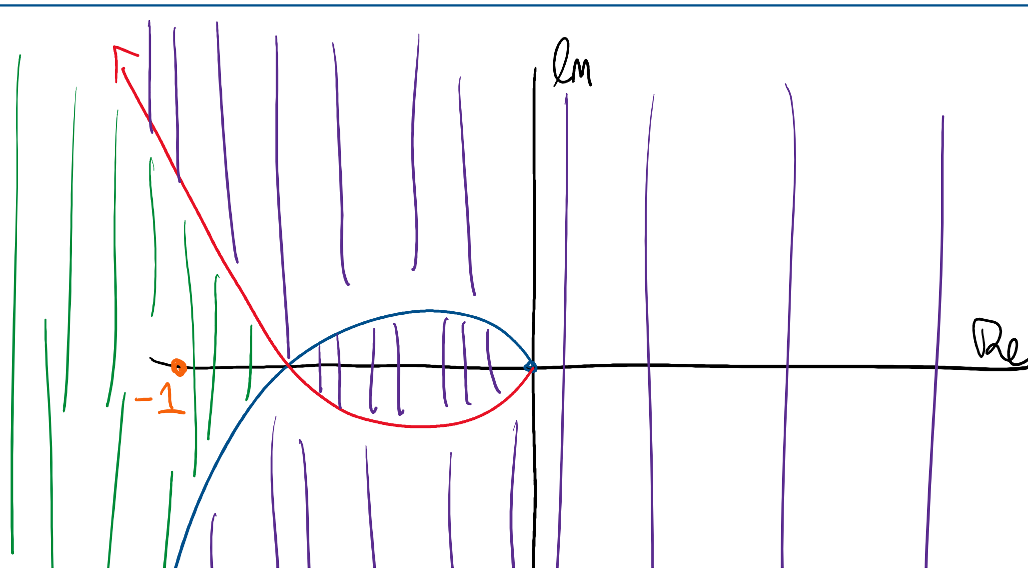
so $e^{At} = e^{V^{-1}DVt} = V^{-1} e^{Dt} V$

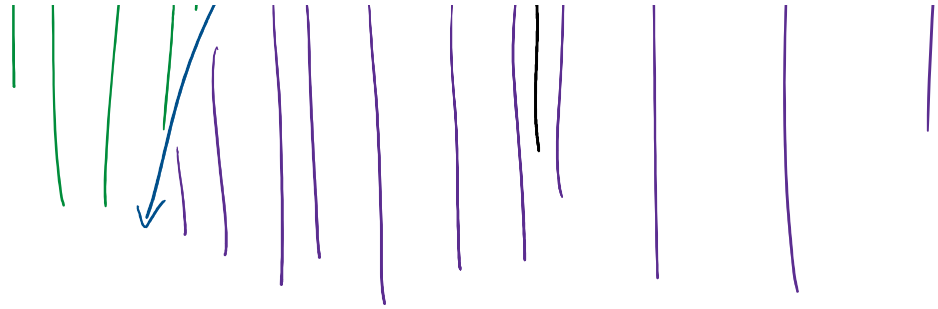
so $e^{At} = e^{V^{-1}DVt} = V^{-1}e^{Dt}V$

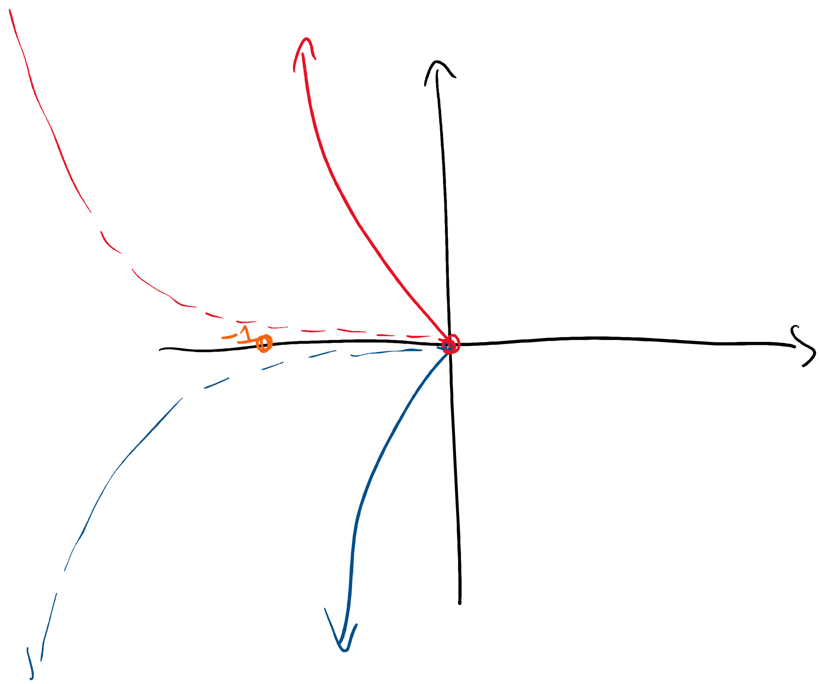
$$e^{Dt} = \begin{bmatrix} e^{\lambda_1 t} & & 0 \\ & \ddots & \\ 0 & & e^{\lambda_n t} \end{bmatrix}$$

more generally, for any A there exists T^{-1} s.t.

$$J = TAT^{-1} = \begin{bmatrix} \Lambda_1 & & 0 \\ & \ddots & \\ 0 & & \Lambda_n \end{bmatrix}, \quad \Lambda_1 = \begin{bmatrix} \lambda_1 & 1 & & 0 \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ 0 & & & \lambda_1 \end{bmatrix}$$







$\xi = xi$

$\phi = phi$

$$D_2\phi(s+1,\xi) = D_\xi\left[f(\phi(s,\xi))\right]$$
$$= Df(\phi(s,\xi)) \cdot D_2\phi(s,\xi)$$

$$\begin{array}{ccccccc} \phi(0,x) & & \phi(1,x) & & \phi(2,x) & & \\ \| & & \| & & \| & & \\ x & \mapsto & f(x) & \mapsto & f(f(x)) & \mapsto & \end{array}$$

$$\phi : \mathbb{R}\times\mathbb{R}^d \to \mathbb{R}^d \qquad f : \mathbb{R}^d \to \mathbb{R}^d$$
$$: (t, x_0) \mapsto x_t \qquad : x \mapsto x^+$$

$\hookrightarrow Df(x) : \mathbb{R}^d \to \mathbb{R}^d$ linear
$\in \mathbb{R}^{d\times d}$

$$\phi(s+1,x) = f(\phi(s,x))$$

$$D_x\left[\phi(s+1,x)\right] = D_x\left[f(\phi(s,x))\right]$$

$$D_x[\phi(s+1, x)] = D_x[f(\phi(s, x))]$$
$$= Df \cdot D_2\phi$$
$$= Df(\phi(s, x)) \cdot D_2\phi(s, x)$$