Nearly Efficient Tuitions and Subsidies in American Public Higher Education

Abstract

A two-stage setting for determining subsidies and tuitions in a public university context is developed where fixed costs introduce an efficiency-enhancing role for taxpayer-financed appropriations. The optimal subsidy per enrollment is shown to be proportional to students' maximum net willingness to pay. This result extends a well-known result associated with Ramsey pricing to include endogenous appropriations to public higher education. Realistic restrictions are imposed on the subsidy structure, and multiple scenarios for determining tuitions are addressed analytically and illustrated numerically, using budget data for the University of Iowa and the University of Michigan.

Key Words: Higher Education Finance, Tuitions, Subsidies, Distributional Transfers

JEL: D61, L38

I. Introduction

As state appropriations for public higher education have declined, universities have used tuition increases to maintain expenditures (College Board, 2013 and SHEEO, 2013).[1] Universities have generally not, however, adjusted their tuition structures to reflect changes in program costs and shifts in student demand.[2] As a result, tuition revenue from low-cost programs increasingly subsidizes high-cost graduate programs, higher-income students subsidize lower-income students (WSJ, 2014), and instructional revenues support research. In an increasingly contested national market, the viability of a more tuition-dependent approach to financing public higher education is challenged by focused low-cost educational providers and potentially threatened by scalable Internet instructional technologies.

Why are universities reluctant to embrace differentiated tuitions, and why do they persevere in supporting innumerable cross-subsidies? A traditional answer, which loses traction in a high-technology environment, is that

[1] Reductions in state support per student in public higher education have been ongoing for nearly three decades; see Duderstadt and Womack (2003), Ehrenberg (2006), and Fethke and Policano (2012).

[2] Economists have long recognized the importance of considering willingness to pay and costs in the determination of tuitions (Hoenack and Weiler, 1975, Siegfried and Round, 1997).

charging differential tuitions to reflect significant discipline-based differences in instructional costs is not "practical" (Middaugh et al, 2003). Another explanation is that centrally-administered budget allocations to individual programs are anchored in tradition—what a program gets this year is based on last year's allocation plus some percentage increment, which is the same for all programs. While supported by distinctly funding streams, the traditional allocation typically co-mingles tuition revenues and the state appropriation. Even though the proportions have significantly changed over time, greater tuition revenue has offset the declining subsidies thus perpetuating existing cross-subsidies.[3] A third explanation for cross subsidies derives from the assertion that a student's choice of career should not be influenced by charging different tuitions. Said differently, a more flexible tuition structure has enrollment implications that many do not support—even if overall average tuition might be lower or even if the current tuition structure inefficiently distributes value among programs.[4]

Notwithstanding these observations, there have been modifications to the tuition structure. State nonresidents are charged substantially higher tuition than residents; undergraduates in business, engineering, and nursing are charged a premium; and graduate and professional students, especially those in dentistry, law and medicine, are charged more (Yanikaski and Wilson, 1984, Nelson, 2008, CHERI, 2011 and Stange, 2013). Emergent decentralized budgeting also helps to focus attention on the tuition structure. Specifically, the adoption of resource-centered management (RCM), which allocates tuition revenue to the units producing it, stimulates interest in making more informed tuition-structure adjustments.

This paper explores the implications of making differentiated tuition and subsidy choices in the framework of public university budgeting. There has been considerable empirical research on setting resident and nonresident tuitions that treat subsidies as exogenous; see Rizzo and Ehrenberg (2004) and Epple, et al (2013).[5] A more limited

[3] Inertia is the usual explanation given for the historical percentage distribution of legislative funds to three Iowa universities by the Board of Regents. Recent rejection of the historically-based allocation of state support prompted development of performance-based criteria that recognizes outcome measures: resident enrollments, student progress, access, research funding, and the graduate program mix (Agnew, 2014 and Rivard, 2014). It also appears the overcoming inertia was the motivating factor that led to the "rebenching" exercise for the University of California System (Kiley, 2013).

[4] The Academic Senate of the University of California opposed even the consideration of differential fees because they indicated a threatening move toward the "privatization" of public higher education (University of California, Academic Senate, 2010).

[5] Rizzo and Ehrenberg (2004) argue that for flagship public universities, resident and nonresident tuitions both increase in response to reductions in state support; consequently, they assert that nonresident tuition is not being used to shelter residents from tuition increases. Epple et al (2013) examine the effects of a reduction in an exogenous state subsidy, accompanied by increases in resident and non-resident tuition. In their general equilibrium model, a $2000 reduction in the state appropriation per student accompanied by a $2000 increase in tuitions drops total enrollment substantially in public universities.

literature treats both tuitions and subsidies as jointly determined; see (Fethke, 2005 and 2011) and (Lucca, et al, 2015). The primary intent here is to extend, both analytically and empirically, assessments of the impact on program enrollments and university budgets of optimal changes in the structures of tuitions and subsidies.[6] We examine unrestricted subsidy and tuition structures. Then, we consider cases where subsidies are restricted to account for residency status, lack of willingness to pay, and higher program costs. Restrictions on the subsidy structure, apparently imposed for reasons of fairness, introduce enrollment inefficiencies. We illustrate the various cases using comprehensive budget data from the University of Iowa and University of Michigan.

We use a multi-stage decision process for legislatures, university governing boards, and students whereby the legislature determines the structure of the subsidy, the university establishes tuitions, and students enroll in a mixture of academic programs that feature different tuitions, costs, and subsidies. In this game-theoretic formulation, the legislature is viewed as the leader that can credibly commit to the subsidy structure, and the university is viewed as a follower that makes tuition decisions based on student demands and the subsidies provided by the legislature. The goal of the university is to maximize student net consumer surplus subject to a break-even constraint that incorporates tuition revenues, program-specific variable costs, shared fixed costs, and the state appropriation.[7] The academic programs (colleges) are linked by shared fixed costs, and the university is constrained to break even. An implication of the fixed cost structure, even with subsidy support, is that tuitions and enrollments can only achieve quasi-efficiency. Roughly speaking, as long as fixed costs exceed the appropriation, the university sets tuitions that minimally deviate from marginal costs.

The subsidy structure used throughout can accommodate a mixture of enrollment subsidies and direct offsets against fixed expenditures, and we show that the highest achievable value occurs when the state appropriation is used to offset fixed costs. This same outcome can also be achieved when enrollment subsidies are unrestricted and there is no direct offset. With optimal unrestricted subsidies, enrollments are shown to be quasi-efficient; this result implies that changes in the state appropriation will have no effect on the ratio of subsidized

[6] In a review of the implementations of differential tuitions across public universities, it was noted that: "None of the schools that had implemented differential tuition reported that it affected enrollment patterns in significant ways, but there were no data cited to back up these claims." (University of Washington, 2011).

[7] This preference structure assumes that legislatures, governing boards, and university administrations all seek to maximize students' value. Admittedly, the objective of universities is a widely debated issue. For example, Epple, et al. (2013) claim that private universities seek to maximize quality, which depends on student ability and university expenditure, while public universities seek to maximize the "achievement" of resident students, this later approach to resident preferences appears sympathetic to that adopted here.

enrollments between any two programs. If marginal costs do not depend on residency status (as we assume), an implication of quasi-efficient enrollments is that resident tuitions will differ from nonresident tuitions only by differences in their demand elasticities. Optimal a*d valorem* subsidies (subsidies per enrollment relative to maximum net willingness to pay) take the form of efficient "Ramsey subsidies," with every enrollment receiving the same percentage subsidy.

While closed-form solutions are developed for the unrestricted subsidy cases, the restricted cases are not analytically tractable. Nevertheless, we reduce the restricted-subsidy cases to the solution of two nonlinear equations in two unknowns and use an iterative procedure to solve them. The solutions provide the optimal unrestricted *ad valorem* subsidy rates and the proportional degree of efficiency.

We illustrate the results using decentralized budget data (FY2014) for the University of Iowa (UI) and the University of Michigan (UM). Demand curves, marginal costs, fixed costs, and enrollment subsidies are calibrated to match publicly available base-case allocations for each university. We numerically determine tuitions, enrollments, and subsidies for all academic colleges (or programs) of the two universities. Subsides facilitate the setting of lower tuitions, usually for residents, by effectively reducing marginal costs. An appealing feature of the formulations is the ability to evaluate alternative tuition-subsidy options using standard budgeting templates. Three scenarios are presented: 1) unrestricted subsidy structures; 2) resident-only subsidies; and 3) differentiated subsidies applied to selected high-cost resident programs.

Our results for the calibrated budgets for the UI and the UM reveal that placing restrictions on the structure of the subsidy and/or adjusting the size of the state appropriation primarily involve equity (fairness) rather than efficiency considerations. Given the appropriations, we find that the percentage gains in welfare associated with efficiently adjusting tuition and subsidy structures are eight percent at the UI and four percent at the UM. There is, however, a substantial redistribution from residents to nonresidents. For the UI, when subsidies are not restricted by residency status, resident value declines from the base case by 34 percent and nonresident value increases 37 percent. Similarly, a 22 percent reduction in the UI state appropriation leads to only a one percent reduction in welfare, but there is a 21 percent decrease in student value offset by the increase in taxpayer value.

The paper is organized as follows: Section II discusses the specifications of demand and costs. Section III considers the second-stage problem of setting tuitions based on a predetermined subsidy structure. Section IV contains the main analytical results of the paper, solving the first-stage problem of determining the unrestricted and restricted subsidy structures. Section V presents calibrated demand and cost formulations that replicate the decentralized budget allocations of the two public universities, and, using those frameworks, it numerically analyzes three tuition-subsidy scenarios. Section VI concludes.

II. Demand and cost specifications

Now distinguishing between different colleges (programs) at the university and between resident and nonresidents, let the set of colleges be $I \equiv \{1,...,n\}$, indexed by i, each with a per credit hour variable cost of c_i, and let $J \equiv \{1,2\}$ be the enrollment types within each college (resident and nonresident), indexed by j. For each pair $(i,j) \in IxJ$, there are linear demand curves $E_{ij} = a_{ij} - b_{ij}T_{ij}$ for given parameters a_{ij} and b_{ij}, where E_{ij} are semester-credit-hour enrollments (SCHs) of type j in college i, and T_{ij} are the corresponding tuitions. The parameters a_{ij} and b_{ij} reflect maximum enrollment and the tuition responsiveness for each program, and a_{ij} / b_{ij} is maximum willingness to pay. The tuition elasticity is $\eta_{ij} = -b_{ij}(T_{ij} / E_{ij})$, and the expression for student net consumer surplus is $E_{ij}^2 / 2b_{ij}$.

The university cost structure exhibits constant marginal costs, c_i, and shared fixed costs, F, with total cost $C = \sum_{i=1}^{n} c_i \sum_{j=1}^{2} E_{ij} + F$. The total subsidy consists of a (linear) subsidy, s_{ij}, applied to SCHs in each program plus a lump-sum offset against fixed cost, S, and is given by $\sum_{i=1}^{n}\sum_{j=1}^{2} s_{ij}E_{ij} + S \leq M$, where M is the appropriation determined exogenously by state tax revenues. The ultimate purpose of subsidizing public higher education is to facilitate the setting of reduced tuitions. Public universities face a break-even budget constraint, whereby tuition revenue plus the appropriation equals total expenditures: $\sum_{i=1}^{n}\sum_{j=1}^{2}(T_{ij} - c_i + s_{ij})(a_{ij} - b_{ij}T_{ij}) + S - F = 0,$

where $E_{ij} = a_{ij} - b_{ij}T_{ij}$. Linear enrollment subsidies permit lower tuitions by reducing a program's marginal cost, while a lump-sum subsidy acts to offset fixed costs. We require throughout that $s_{ij} \geq 0$.

III. A two-stage decision problem for the legislature and the university

This description for setting tuitions in a high fixed-cost public university environment is motivated by formulations that seek to maximize a general measure of consumer preferences subject to a constraint on producer revenue (Baumol and Bradford, 1970, and Goldman, et al, 1984). The key contextual extensions incorporated here are: inclusion of the break-even constraint on university net revenue, addition of a flexible subsidy structure, and development of a realistic sequential decision process.

We initially presume a two-stage decision process for determining subsidies and setting tuitions that draws upon Fethke (2011, 2014). In the first stage (the upper level), the legislature, acting as the leader, determines subsidies that meet its budget constraint and maximize the net value students get from their education minus the total university subsidy. Since the legislature moves first, it can credibly impose subsidies that determine the tuitions set by the university. In the second stage (the lower level), the university governing board, acting as the follower, sets tuitions that account for the predetermined subsidies. The university chooses tuition and consequently enrollments that are quasi-efficient in the sense that fixed costs must be covered to achieve a break-even budget. We solve the problem by starting at stage two and moving back to stage one.

A. Stage 2: The university governing board's problem

In the second stage, the governing board takes the legislature's subsidy structure (s, S) as given and selects tuitions to maximize net consumers' surplus subject to demand equations and the break-even requirement:

(1a) $$\max_{T,E} \sum_{i,j} \frac{E_{ij}^2}{2b_{ij}}$$

s.t.

(1b) $$E_{ij} = a_{ij} - b_{ij}T_{ij}$$

(1c) $$\sum_{i,j} (T_{ij} + s_{ij} - c_i)E_{ij} + S \leq F$$

To economize on notation, we here and subsequently define $\sum_{i=1}^{n} \sum_{1}^{2} \equiv \sum_{i,j}$. Stage 2 decisions are predicated on a predetermined subsidy structure. Enrollment subsidies can be unrestricted, applying to all programs and students, or they can support particular programs, for example, by subsidizing only residents. Enrollment subsidies are generally intended to decrease tuition and increase enrollments by reducing net marginal costs or, equivalently, by increasing maximum willingness to pay. Restricted subsidies introduce inefficiencies into the tuition structure by increasing tuition in some programs above marginal cost to accommodate tuitions elsewhere that are below marginal cost. Alternatively, the subsidy can be applied as a lump sum, S, which offsets a portion of shared fixed costs (an "offset"). An offset increases consumers' wellbeing without distorting tuitions.

Some suggest that nonresidents are not viewed symmetrically with residents. In the extreme case, when nonresident net consumer surplus is removed from the objective function, our formulation implies that nonresident tuitions will be determined at the full monopoly tuition levels. With the independent demand functions, maximum nonresident net revenue is then subtracted from fixed costs in the break-even constraint, and the resulting formulation reduces to the case of determining quasi-efficient resident tuitions given the associated structure of the resident subsidy. Thus, eliminating nonresidents from the objective function raises no additional analytical or computational issues. We will briefly consider this case in the subsequent examination of University of Iowa budget data.

Given the subsidies (s, S), the "quasi-efficient" enrollments, as determined by the university (Fethke, 2014), are

(2) $$\widetilde{E}_{ij}(s,S) = \rho(s,S)b_{ij}(d_{ij} + s_{ij}),$$

where

(3) $$d_{ij} \equiv (a_{ij}/b_{ij} - c_i),$$

(4) $$\kappa(s,S) \equiv \frac{(F - S)}{\sum_{ij} b_{ij}(d_{ij} + s_{ij})^2/4},$$

(5) $$\rho(s,S) = (1/2)(1 + \sqrt{1 - \kappa(s,S)})$$

satisfy $\kappa(s,S) \le 1$ and $1/2 \le \rho(s,S) \le 1$. A sufficient condition for positive enrollments is $d_{ij} \equiv a_{ij}/b_{ij} - c_i > 0$.

In the absence of subsidies, a quasi-efficient enrollment structure requires a proportionate changes in all enrollments from levels that would occur if tuitions were set at marginal costs; see (2) above, and Baumol and Bradford (1970, p. 271). The scalar $\rho(s,S)$ reflects a common divergence of enrollments from those occurring when tuitions equal net marginal costs. The interpretation of $\rho(s,S)$ involves using (4) and (5). The numerator of $\kappa(s,S)$ in (4) is fixed cost net of the offset, and the denominator is the maximum net revenue the university can realize by setting (subsidized) monopoly tuitions in every program. The degree of efficiency can vary between two extremes: i) $\kappa(s,S)=0$ where $\rho(s,S)=1$ and tuition equals net marginal cost: and ii) $\kappa(s,S)\to 1$ where $\rho(s,S)\to 1/2,$ and tuition equals one-half net marginal cost (monopoly pricing). Enrollment subsidies, s_{ij}, have two ways to increase enrollments: i) they effectively increase the net maximum willingness to pay; and ii) they increase $\rho(s,0)$ by increasing the denominator of $\kappa(s,0)$. A direct offset, S, increases enrollments by reducing net fixed cost.

Given the demand curves and (2), the relative tuition margin is

$$\frac{\widetilde{T}_{ij}+s_{ij}-c_i}{\widetilde{T}_{ij}}=-\left[\frac{1-\rho(s,S)}{\rho(s,S)}\right]\frac{1}{\eta_{ij}}.$$

Here, $\eta_{ij}\equiv -b_{ij}\widetilde{T}_{ij}/\widetilde{E}_{ij}$ is the tuition elasticity of demand. When there are no subsidies, (2) implies that all enrollments will adjust by the same proportion, $\rho(0,0)$, in response to changes in fixed cost. To accommodate fixed costs, unsubsidized tuitions are set to minimally exceed marginal costs, with the highest relative tuition markups over marginal cost associated with programs that feature the least elastic demands: high- willingness- to-pay students pay higher tuitions relative to marginal cost than do low- willingness- to- pay students. An increase in the subsidy leads to a decrease in tuition, that is,

$$\frac{\partial\widetilde{T}_{ij}}{\partial s_{ij}}=-\frac{1}{2}\left[1+\sqrt{1-\kappa(s,S)}+\frac{\kappa(s,S)b_{ij}(d_{ij}+s_{ij})^2}{\sqrt{1-\kappa(s,S)}\sum_{i,j}b_{ij}(d_{ij}+s_{ij})^2}\right]<0.$$

An increasing subsidy increases welfare when $\widetilde{T}_{ij} \geq c_i - s_{ij}$ and $\partial T_{ij} / \partial s_{ij} > -1.$[8]

B. Stage one: The legislature's problem

At stage one, the legislature determines the structure of the subsidies to satisfy a budget constraint. Welfare, which accounts for the second-stage problem, is:

(6) $$\widetilde{W}(s,S) \equiv \sum_{i,j} \frac{\widetilde{E}_{ij}(s,S)^2}{2b_{ij}} - (\sum_{i,j} s_{ij}\widetilde{E}_{ij}(s,S) + S) \ .$$

The legislature seeks to

(7a) $$\max_{s.S} \widetilde{W}(s,S)$$

s.t.

(7b) $$\sum_{ij} s_{ij}\widetilde{E}_{ij}(s,S) + S \leq M$$

where $M \leq F$ is exogenous. Specifically, the legislature balances students' welfare against the taxpayers' appropriation. At stage one, the legislature in determining the structure of the subsidies anticipates state-two enrollment responses. In Fethke (2014), it is demonstrated in a similar two-stage setting that welfare is increasing in S for all $M < F$. Numerical results are also developed by iterating on a common resident subsidy rate.

IV. An endogenous subsidy structure

In this paper, we consider a general subsidy structure $(s,S),$ where the legislature selects a mix of a direct offset and enrollment subsidies. When enrollment subsidies are unrestricted, the optimal subsidy per enrollment is shown to be proportional to maximum net willingness to pay, with the factor of proportionality being the same for all programs. We then examine the implications of imposing restrictions on the subsidy structure,

[8] Rizzo and Ehrenberg (2004) estimate the tuition elasticity of a per unit subsidy for flagship public universities, and find the (absolute value) estimate to be "far less" than one.

including limiting subsidies to supporting only residents. Restricting the subsidy structure both reduces and redistributes value.

A. Unrestricted subsidies and the k-ratio rule ("Ramsey subsidies")

In this section, optimal subsidies are determined for *every* program. Consider the following optimization problem, which is a representation of the two-stage problem (7) with the exception that ρ is not forced to equal $\rho(s,S)$:

(8a) $$\max_{s,S,T,E,\rho} \sum_{i,j} \frac{E_{ij}^2}{2b_{ij}} - \left(\sum_{ij} s_{ij} E_{ij} + S \right)$$

s.t.

(8b) $$E_{ij} = a_{ij} - b_{ij} T_{ij}$$

(8c) $$\sum_{ij} (T_{ij} + s_{ij} - c_i) E_{ij} = F - S$$

(8d) $$\sum_{i,j} s_{ij} E_{ij} + S \leq M$$

(8e) $$E_{ij} = \rho b_{ij} (a_{ij} / b_{ij} - c_i + s_{ij})$$

(8f) $$\frac{1}{2} \leq \rho \leq 1$$

Formal developments of Results 1-3 are provided in Appendix A.

Result 1: *Suppose (s, S, T, E,* ρ *) satisfy, (8e). Then (8c) and (8f) hold if and only if* $\rho = \rho(s,S)$.

Even though ρ is not required to equal $\rho(s,S)$, it does equal $\rho(s,S)$ if the stage-two rule (8e) holds. Given the subsidy structure specified by the break-even constraint (8c) and the university's rule for determining enrollments (8e), Result 1 implies that any assignment of the efficiency scalar will be consistent with the subsidy structure actually implemented.

Optimal unrestricted subsidies can be shown to be proportionate to maximum net willingness to pay, with the factor of proportionality denoted as "k" being the same for all unrestricted programs:

Result 2: T*here exists some scalar variable k such that* $s_{ij}^{*} = kd_{ij}$ *for all of the otherwise unrestricted* s_{ij}.

The k-ratio subsidies imply that optimal enrollments are proportional to efficient enrollments, where the scalar variable ρ $(1/2 \leq \rho \leq 1)$ reflects the degree of efficiency. Thus, k-ratio subsidies maintain the quasi-efficiency property. The k-ratio subsidies can therefore be called "Ramsey subsidies," since they represent the subsidy-equivalent to "Ramsey taxes." (Ramsey, 1927).[9] Since the subsidy structure for unrestricted s_{ij} ensures corresponding quasi-efficient enrollments, a sufficient condition for the legislative budget constraint to bind, with positive unrestricted subsidies, is for the appropriation plus the maximum net revenue received from the enrollments on restricted subsidies to not exceed fixed cost: $M \leq F - \frac{1}{4}\theta_r$, where $\theta_r \equiv \sum_r b_{ij} d_{ij}^2$.

Subsidies per unit (SCH or headcount) are difficult to compare across programs-a subsidy of $500 per SCH may be significant in liberal arts but insignificant in medicine. An a*d valorem* subsidy permits standardized comparisons across programs, and is conveniently provided by $k = s_{ij} / d_{ij}$. Here, for example, $k = .33$ implies a 33 percent subsidy for every program. The optimal *total* subsidy for a particular program is

$$\tilde{s}_{ij}\tilde{E}_{ij} = \frac{b_{ij}(a_{ij}/b_{ij} - c_i)^2}{\sum_{i,j} b_{ij}(a_{ij}/b_{ij} - c_i)^2}(M - S).$$

A program's total subsidy is given by the ratio of maximum net consumer surplus for that program relative to that for all programs, times the net appropriation. Programs that display higher consumer value receive higher subsidies.

When no restrictions are placed on the structure (k, S), a continuum of feasible k and S are consistent with quasi-efficient enrollments:

Result 3: *When the subsidy structure is unrestricted, the legislature's optimization problem (7) simplifies to a one-dimensional strictly-convex programming problem, which has a unique optimal solution* x^*. *Given* x^*,

[9] An insightful discussion of optimal Ramsey taxes when demand curves are linear and marginal costs are constant is provided by Casey Mulligan, accessible at: http://home.uchicago.edu/~cbm4/econ260/E203rams.pdf

$\rho \in [1/2, 1]$ can be selected arbitrarily, and $k = x^* / \rho - 1$. Then, we can determine $s_{ij} = kd_{ij}$ and

$S = F - \theta\rho(1-\rho)(1+k)^2$. By Result 1, it follows that $\rho = \rho(s, S)$.

The primary insight provided by Result 3 is that the optimal degree of efficiency, x^*, is independent of the unrestricted subsidy structure. In the unrestricted case, it makes no difference whether the appropriation is used as a direct offset against fixed cost or is applied, in any combination, as a direct offset and unrestricted enrollment subsidies. This implies a continuum of (s, S) that yield the optimum (quasi-efficient) tuitions. This will not be the case below when the subsidy structure is restricted, for example, to favor only residents.

B. Computation with restricted subsidies

Notions of fairness and entitlement influence the determination of (s, S). Nonresidents are typically not subsidized, while residents are differentially subsidized, often with high-cost programs being favored. As a result, subsidized resident tuitions are less than marginal costs, while nonresident tuitions exceed marginal costs. Similarly, residents in high-cost programs often pay about the same tuitions as do residents in low-cost programs; this also leads to distortions in the relationships between tuitions and marginal costs. With welfare represented as total consumers' surplus net of the appropriation, we have a benchmark against which to measure the losses associated with restricted subsidies.

In developing the optimization, our computational strategy is to presume that ρ^* is given and then determine (s^*, S^*). As demonstrated in Appendix B, this exercise results in expressions for k^* and S^* in terms of ρ^*:

$$k^* = \frac{1}{2\rho^*}[1 - 2\rho^* + \sqrt{\frac{4(\rho^*(1-\rho^*)\theta_r - F + M) + \theta_u}{\theta_u}}], \tag{9}$$

$$S^* = F - \rho^*(1-\rho^*)[(1+k^*)^2\theta_u + \theta_r], \tag{10}$$

and, after substituting for k^* and S^* in the objective,

$$W(\rho^*) = \frac{1}{4}(-2F - 2M + 2\rho^*\theta_r + \theta_u + \sqrt{\theta_u}\sqrt{-4F + 4M + 4(1-\rho^*)\rho^*\theta_r + \theta_u}). \tag{11}$$

Here, $\theta_l \equiv \sum_l b_{ij} d_{ij}^2$, with $l \in \{u, r\}$.

Selection of the relative degree of efficiency, ρ^*, will determines the subsidy structure (k^*, S^*), then s_{ij}^* is determined by $s_{ij}^* = k^* d_{ij}$. For example, with an unrestricted subsidy structure, the critical point of (11) is:

$$\rho^*(0, M) = \frac{1}{2}[1 + \sqrt{1 - \frac{4(F - M)}{\theta_u + \theta_r}}], \text{ with } S^* = M \text{ and } k^* = 0.$$

For the restricted cases, a closed-form solutions for ρ^* is not available. Nevertheless, we can provide numerical results by imposing restrictions on the subsidy structure and then iterating on ρ to identify the optimal ρ^*. For example, when only residents are subsidized, we impose the restriction $S = 0$ and solve the breakeven constraint for the associated k^*. For this case, (16) provides an equation in $(1 + k^*)^2$, with the relevant (positive) root:

(12) $$k^* = -1 + \sqrt{\frac{F / \rho^*(1 - \rho^*) - \theta_r}{\theta_u}}.$$

To find the optimal numerical solution, (12) is inserted into the objective and we iterate on ρ to find the optimal ρ^*. Result 1 implies that

(13) $$\rho^* = \frac{1}{2}[1 + \sqrt{1 - \frac{4F}{(1 + k^*)^2 \theta_u + \theta_r}}].$$

The remaining check is to ensure that the legislative budget constraint is not violated.

V. Examples of determining optimal tuitions for three subsidy structures

Using 2014 budget data for the University of Iowa and the University of Michigan, this section numerically evaluates the possibilities associated with varying the structure of the subsidy to allocate resources among multiple academic programs. Demand and cost parameter estimates are constructed, and optimal tuitions, enrollments, and subsidies are developed. Altering the structure of the subsidy imposes constraints on tuitions that have implications. Three situations are considered. Case 1 examines the unrestricted subsidy structure. Case 2 restricts the structure to providing support only to resident enrollments. Case 3 provides higher subsidies to selected high-cost resident programs.

A. University of Iowa (UI)

Traditionally, tuitions for the three Iowa regent universities are determined by a governing board (Board of Regents), with tuition revenues collected and retained by each of the system's universities. To support low resident tuition, a taxpayer-financed appropriation is provided.[10] At the UI, tuition revenue and the appropriation are combined centrally, then base-adjusted budget allocations are distributed to the colleges and shared service units. In Table 1, the actual 2014 UI budget allocations and revenues (UI, 2014) are displayed in Column 1, and resident and nonresident SCHs for each college are presented in Columns 2 and 3.

In Column 4 of Table 1, variable cost per SCH for each program is measured as the amount distributed to each college divided by student credit hours produced.[11] All expenditures for the shared-service units (net of indirect costs recovered to support research) are treated as fixed costs. While the average variable cost per SCH is $424, there is considerable variation across programs. For example, the College of Liberal Arts and Sciences (CLAS), which accounts for 60 percent of the total credit hours, receives an allocation of $281 per SCH, while the College of Dentistry receives $1,842. Column 5 contains the estimate for fully-allocated cost per SCH, which is

[10] Historically, an amount is appropriated by the governor and the legislature and allocated to the regents. Once the appropriation is known, resident and nonresident undergraduate tuitions are adjusted based on the proposed (incremental) budgets submitted by each campus. Periodically, requests are made to adjust the tuition structure, with some increments added to tuitions in professional programs, particularly undergraduate programs in business, engineering, nursing, and professional graduate programs. This budgeting-tuition process motivates the formal two-stage model used here, where the legislature initially determines the appropriation and the regents allocate that appropriation and then interacts with the three public universities to set tuitions based on the appropriation.

[11] Similar approaches to measuring variable cost are adopted in moving from centralized to decentralized budget-allocation models; see the University of Florida (2014) for the determination of relative-costs weights for undergraduate and graduate programs. Relative costs for lower division and upper division undergraduate programs are also examined in four states by SHEEO (2010) and calculated in a definitive study of direct instructional costs for twenty-four academic disciplines (Middaugh et al, Table 3, 2003). As points of comparison with the UI, the variable-cost estimates reported in Table 1 for Liberal Arts, Business, and Engineering are $281, $237, and $559, respectively. The corresponding estimates for the University of Florida are $216, $207, and $571 (Fethke, 2014, Table 3), and those (calculations done by the authors) for Iowa State University are $244, $300, and $572. At the UI, in developing a recent incremental scheme to accommodate variable instruction costs, all colleges other than the professional colleges (Dentistry, Law, Medicine and Pharmacy), are allocated $100,000 for each incremental change of 485 SCH that occurs relative to the FY2015 base-year total; this implies a variable-cost estimate of $206.19 for these colleges (email to the authors from the UI Office of the Provost, March 19, 2015).

marginal cost plus average fixed cost calculated at the optimum total enrollment. On the revenue side, average resident tuition revenue per SCH actually collected is $561, and the appropriation per resident SCH is $570.

Table 1 here

Linear demand curves, $E_{ij} = a_{ij} - b_{ij}T_{ij}$, are developed for residents and nonresidents, where $\eta_{ij} \equiv -b_{ij}(T_{ij} / E_{ij})$ is the tuition elasticity expression. We assume that the demand elasticity for every resident programs is $-.25$, and the demand elasticity for every nonresident programs is $-.5$.[12] To establish net tuitions (what students actually pay net of subsidies), list tuitions for all colleges are measured relative to resident list tuition in the Colleges of Liberal Arts and Sciences (CLAS). These normalized tuitions are weighted by enrollment shares, and the weighted resident tuition in CLAS is determined to match actual total tuition revenue per SCH, which is $561. In other words, the tuition revenue using the calculated net tuitions is measured to equal actual tuition revenue. The calculated tuitions derived from this estimate are presented in Columns 6 and 7. To compute demand-curve parameters for each program, we use: $b_{ij} = -\eta_{ij}E_{ij} / T_{ij}$, $a_{ij} = E_{ij} + b_{ij}T_{ij}$, the presumed elasticities (-.25 and -.5), actual SCHS, and the calculated net tuitions. The resulting demand-curve parameters are presented in Columns 8-11. Typically, calculated resident net tuitions are below marginal costs, while calculated net nonresident tuitions are above marginal costs, reflecting the tuition reductions for residents. Nonresident tuitions in CLAS, Business, and Pharmacy exceed fully-allocated costs, but are less for all others. Average nonresident tuition is less than average fully allocated cost.[13] Net consumers' surpluses by residency status, presented in Columns 12 and 13, are: $\sum_{i=1}^{n} E_{i1}^2 / 2b_{i1} = \$242m$ and $\sum_{i=1}^{n} E_{i2}^2 / 2b_{i2} = \$292m$, respectively. Subtracting the appropriation of $\$222m$ from their sum yields welfare of $\$312m$.

[12] A commonly reported ("consensus") estimate of demand responsiveness in higher education is that a $1,000 change in tuition in real dollars is associated with a 3-5 percentage decrease in enrollment (Kane 2006). In 2014, UI undergraduate tuition was $8061, so the standard estimate implies a tuition elasticity of between -.25 and -.4. More specifically to the Iowa context, the Board of Regents asserts that a one percent increase in tuition will yield a $4.5 m increase in system-wide tuition revenue (http://www.iptv.org/iowapress/episode.cfm/3416.) This relationship translates into an estimate for the UI tuition elasticity of demand of about -.5.

[13] Data provided by the Office of the Provost at the UI to the authors indicates that average net resident tuition is $392 per SCH and average nonresident tuition is $750 per SCH, implying that average nonresident tuition is even further below average fully-allocated cost of $862.

We recognize that the calibration approach we employ to obtain demand parameters at the college level, while consistent with net tuitions and enrollments in each college, does not rely on direct estimation. Furthermore, while the elasticity assumptions we use to develop linear demand curve parameters do have empirical support from studies using aggregate data, no attempt is made to calibrate cross-price elasticities. This omission implies that changes in tuition in one college do not affect the enrollment decisions made elsewhere. Introducing this desirable enrichment, however, presents formidable analytical and empirical issues that we have not been able to overcome. Nevertheless, engaging even the simple independent demand and constant marginal cost structures into a familiar public university budgeting environment facilitates the asking of interesting "what if" type questions that we believe provides insight to those making budgeting and resource allocation decisions.

1. Unrestricted subsidy structure and quasi-efficient enrollments

When quasi-efficient enrollments are presumed and no restrictions are placed on the structure of the subsidy (s, S), optimal solutions to are provided in Columns 1-6 of Table 2. As indicated by Result 3, enrollments in this case depend only on the size of the appropriation, M. Using either a direct offset, (0, M), or unrestricted subsidies, $(k,0)$, provides the same enrollment outcomes. A total welfare of $\$338m$ in Case 1 is the largest attainable with $M < F$. Optimal total enrollment is lower than that for the base case, with resident enrollments declining and nonresident enrollment increasing. There are increases in enrollments for CLAS and Business, and decreases for all other colleges, with a total decrease of 9,145 SCHs. All tuitions exceed marginal costs, and the enrollment adjustments are the expected result of a narrowing of resident-nonresident tuition differentials, with higher nonresident tuitions reflecting higher willingness to pay. There is considerable variability in the optimal tuition structure. For Liberal Arts, optimal resident tuition is \$342 per SCH, while optimal nonresident tuition is \$452, which compare to the base-case estimates of \$206 and \$787, respectively. For Medicine, optimal resident tuition is \$1,768 and optimal nonresident tuition is \$1,729, while those base-case estimates are \$997 and \$1505. A Laspeyres tuition index, computed as tuition revenue using Case 1 tuitions and the base enrollments relative to base tuition revenue, indicates that a decline in average weighted tuitions occurs in moving from the base case to Case 1.

With unrestricted enrollment subsidies, optimal subsidies as indicated by Result 2 are given by $s_{ij} = kd_{ij}$. In Column 2 of Table 2, we report the scalars: $x^* = .9178$, $\rho^* = .7769$, $k^* = x^*/\rho^* - 1 = .1814$, and

$S^* = F - \theta\rho^*(1-\rho^*)(1+k^*)^2 = 0.$ With unrestricted subsidies, enrollments are 92 percent of the efficient enrollments (those determined where tuitions equal marginal costs). Every program receives support of 18 percent per SCH, with the highest subsidies assigned to programs with the highest net maximum willingness to pay. The highly subsidized programs are nonresident programs generally and the graduate professional programs specifically.

Moving to Case 1, unrestricted subsidies increases total welfare by \$26*m*, which is an eight percent increase in efficiency. With the same appropriation, \$222*m,* there is a decrease in resident net consumer surplus of -\$82*m* (thirty four percent) and an increase in nonresident net consumer surplus of \$107*m* (thirty seven percent). This adjustment represents a considerable redistribution from residents toward nonresidents, and it likely a primary reason why an unrestricted tuition-subsidy structure is politically untenable in public higher education. Subsidies restricted to supporting only residents, while sacrificing efficiency, are generally considered to be fair.

If nonresidents are not considered in the objective functions of either the legislature or the university, nonresident tuitions are determined at full monopoly rates, with the net revenue realized applied as an offset against fixed costs. Using the demand and cost parameters presented in Table 1, this case requires an increase in average nonresident tuition of nearly \$600 per SCH. Then, resident tuitions can then be set at marginal costs, with a supporting appropriation of just \$85m. Welfare declines from \$339m in the unrestricted case, with symmetric valuation of residents and nonresidents, to \$224m.

Table 2 here

2. Subsidies applied to resident enrollments

The results for Case 1 conflict with actual practice where subsidies are typically restricted to favor residents. When the entire appropriation is used to support only resident enrollments, equations (12) and (13) provide the solution for k^* and ρ^*. Tuitions enrollments and the resident subsidies are provided in Columns 7-11 of Table 2. As expected, resident tuitions decline substantially from those in Case 1, while nonresident tuitions increase. The optimal k-ratio subsidy applied to residents is increasing in the appropriation, M, decreasing in the efficiency scalar, ρ^*, and decreasing in maximum net tuition revenue associated with the unrestricted programs, which is $\theta_1/4.$ For Case 2, $\rho^* = .7682,$ $\theta_1 = \$380.9m,$ $M = \$221m$, $k_1^* = .504$ (residents only).

Restricting the subsidy to residents introduces a pattern of enrollment inefficiency, with resident tuitions below marginal costs and nonresident tuitions above marginal costs. Resident enrollments exceed efficient enrollments by factor of 1.16, while nonresident enrollments are below efficient enrollments by a factor of .77. Average resident tuition is \$268, average nonresident tuition is \$859, and the average resident subsidy is \$554; not surprisingly, since subsidized resident tuition are set substantially below nonresident tuitions, these outcomes align closely to the base case of average resident tuition, average nonresident tuition and the average resident subsidy of \$206, \$787, and \$570, respectively. Total enrollment is lower than that achieved with the unrestricted subsidy structure, but is closer to the base case. Welfare of $\$312.4m$ is below the $\$338m$ in the unrestricted subsidy case, but is close to the $\$311.8m$ base case.

Case 2 is a restricted version of Case 1, with the restriction placed on the use of the offset (S = 0). If the restriction is relaxed, the efficiency scalar will increase, the k-ratio will decrease, and welfare will increase, as resident tuitions increase and nonresident tuitions decrease. For example if $S^* = \$100m,$ then $k^* = .297,$ $\rho^* = .8317,$ with $SW^* = \$327.9m.$ This adjustment response will continue until $S^* = \$222m,$ and $k^* = 0,$ which are optimal for Case 1.

3. Subsidies applied differentially to dentistry, law, and medicine

Introducing flexibility into the tuition structure has substantial enrollment implications. As shown in Case 2, optimal resident tuitions for dental and medical students are nearly eight times those for liberal arts students. The enrollment reductions in these high-cost programs associated with this kind of tuition increase are often deemed unacceptable. To address this issue, public universities have moved toward differentially assigning levels of support for students in academic programs based on program cost. For example, the University of California System's "rebenching" exercise recommends applying different funding weights (Pitts, 2010, Kelly, 2012 and Kiley, 2013). Under their approach, each UC campus retains its tuition revenue, with the state appropriation allocated as follows: undergraduate, post baccalaureate, graduate professional and graduate academic master's students are weighted at 1, doctoral students at 2.5, and health sciences students at 5. Similarly, the University of Florida at Gainesville implements a budgeting process whereby credit hours generated by each program are weighted by their respective "cost of delivery" to determine budget allocations (University of Florida, 2012).

To illustrate the implications of targeted subsidies, Columns 12-17 in Table 2 present the results of assigning differential subsidies of \$2,000 *per SCH* to resident enrollments in Dentistry, Law and Medicine, with all other resident programs receiving unrestricted subsidies and nonresidents not subsidized. The assigned subsidy is selected to exceed those determined for these programs in the unrestricted case. In this illustration, all unrestricted resident programs receive a 46 percent subsidy, nonresidents receive zero, Dentistry receives 54 percent, Law receives 86 percent, and Medicine receives 57 percent. The targeted subsidies increase enrollment and consumers' surplus in the targeted programs, but they reduce consumer's surplus in all the other programs. Welfare of $\$311.8m$ close to the base case of $\$312.8m$ but lower than the $\$388m$ associated with unrestricted subsidies. Weighted-average resident tuition is $276 and nonresident average tuition of $857, compared to the base tuitions of $313 and $840.

The targeting of subsidies to selected programs requires budget reallocations that increase average tuitions for other resident programs, while decreasing the tuitions of the targeted programs. For example, in moving from Case 2 to Case 3, resident tuition in Law declines from $1,362 to $726, which is the largest reduction for the targeted programs. This adjustment occurs because the maximum net willingness to pay in Law of $2,323 is substantially below, for example, Dentistry's $3,692; thus a $2,000 subsidy in Law has a larger effect on enrollment than does a similar subsidy in Dentistry. The net increase in value realized by students in the targeted programs does not compensate for that lost by students in the non-targeted programs. Here, the less than $1m loss encountered is modest. If the maximum willingness to pay across programs is nearly uniform, targeted subsidies will have much larger effects.

B. University of Michigan (UM)

Using budget and enrollment data for the University of Michigan for FY2014 (UM, 2014a and 2014b), we consider the same cases presented for the UI. The results for the UM base case are presented in Table 3. Columns 1 and 2 are *head-count enrollments* for the winter semester, by residency status. In Column 3, variable cost per enrollment is determined by dividing the college budget allocation by its total enrollment. The enrollment-weighted average variable cost per student is $24,125.[14] Fixed costs of $\$503.4m$ are measured as all expenditure not directly allocated to the colleges, minus indirect cost recoveries from research grants and "other" income, which is mostly interest income earned on General Education Funds. Fully allocated total costs per enrollment are given in Column 4. Again, there is considerable variation in cost among programs. For example, the variable cost per enrollment in Liberal Arts is $18,694, while that in Medicine is $67,485.

In calibrating linear demand curves for the UM base case, it is assumed the elasticity of demand used for every resident program is -.25, and the elasticity used for every nonresident program is -.5. Total tuition revenue is

[14] The UM has a resource-centered-management program in place, whereby tuition revenue earned by each college is allocated based on a formula that equally weights credit hours by source of teaching and by major areas (University of Michigan, 2007).

$\$1{,}218m.$ To determine net tuitions, list tuitions are measured relative to resident tuition in the Colleges of Literature, Arts and Sciences (LAS). Then, the enrollment share-weighted resident tuition in LAS is determined to match actual tuition revenue per enrollment, which is $29,589. The calculated resident and nonresident net tuitions are presented in Columns 5 and 6 in Table 3. Typically, net tuition for residents is below marginal cost, while net nonresident tuition, with the exceptions of Dentistry, Medicine, and Social Work, exceed marginal cost. Using the enrollments in Columns 1 and 2, the calculated net tuitions, and the elasticity assumptions, the parameter estimates for the demand curves are calculated in the same way as those for the UI, and they are presented Columns 7-10. Even though a standard approach to the parameterization of cost and demand functions is applied to every college, there is considerable variable in maximum net willingness to pay. Columns 11 and 12 provide net consumers' surpluses for residents and nonresidents. Welfare for the UM base case is calculated as $\$1.31bn.$

Table 3 here

1. An unrestricted subsidy structure with quasi-efficient enrollments

Outcomes for the three cases developed for the UM are presented in Table 4. The results for unrestricted subsidies are provided in Columns 1–6. The modest differences between resident and nonresident tuitions again reflect differences in willingness to pay. The scalar- variable solutions are: $x^* = .9365,$ $k^* = .0789,$ and $S^* = 0.$ Weighted-average tuitions of residents and nonresidents are $26,662 and $31,472, which differ from the base equivalents of $16,472 and $46,218. Measured against the UM base case, resident enrollment in the unrestricted case declines and nonresident enrollment increases, with total enrollment declining by two percent (904 students). The largest percentage declines in enrollment are in high-cost programs in Dentistry (24 percent) and Medicine (25 percent). While all enrollments receive a 7.89 percent subsidy, the highest per enrollment subsidies are directed at programs where students express the highest maximum net willingness to pay, which are nonresident students generally and the professional graduate programs specifically. Moving from the base case to accommodate quasi-efficiency enrollments and unrestricted subsidies increases welfare by $\$58.6m.$

Table 4 here

2. Subsidies applied to resident enrollments

With subsidies restricted to residents, the results are presented in Columns 7-11 of Table 4. These results more closely resemble the base case. Average resident tuition is $19,675 and average nonresident tuition is $39,432, as compared to the base case tuitions of $17,809 and $44,523. Total enrollment of 41,128 is slightly less than base-case total of 41,158, with lower resident enrollment and higher nonresident enrollment. The k-ratio subsidy, which applies only to resident programs, is 21.7 percent. With this restricted subsidy structure, nonresident enrollments are 13% lower than efficient enrollments and resident enrollments are 5% more than efficient enrollments. Even with the entire appropriation assigned to residents, quasi-efficient enrollments require reducing nonresident tuitions. Welfare of $\$1.359bn$ in this case is lower than the unrestricted total subsidy of $\$1.376bn$.

3. Subsidy differentially applied to dentistry, law, medicine, and social work

The results of applying a $30,000 subsidy to each resident student in Dentistry, Law, Medicine, and Social Work are presented in Columns 12-16 of Table 4. As expected, a differentiated subsidy leads to declines in resident tuitions in the selectively-subsidized programs. In the case of Law, however, the $30,000 subsidy for residents is less than $50,897 resident subsidy Law receives in Case 2, so Law's tuition actually increases. Since these selectively subsidized programs express higher willingness to pay, there is only a small reduction of value in moving from Case 2 to Case 3.

VI. Conclusions

The main purpose of this paper has been to develop and implement a model for determining the structures of tuitions and subsidies in a realistic, multi-program public university budgetary setting. Maximizing student value is the goal of the university governing board, while the legislature additionally considers the taxpayers' appropriation. Fixed costs, combined with the university break-even requirement, introduce a role for subsidies, which are used to offset the need to charge tuitions that excessively exceed marginal costs. Specifically, to accommodate fixed costs, an unrestricted subsidy structure supports enrollments that deviate minimally from those achievable when tuitions equal marginal costs.

We develop a two-stage decision process where the legislature initially determines the subsidy structure and a university governing board subsequently sets tuitions based on these subsidies. Any feasible mixture of unrestricted enrollment subsidies and a direct offset is compatible with unique quasi-efficient enrollments. A case of special interest occurs with the exclusive use of enrollment subsidies. Under the resulting k-ratio rule ("Ramsey" subsidies), value is increased by offering higher subsidies per enrollments in programs that express higher willingness to pay net of marginal cost. In this case, *ad valorem* subsidies provide the same percentage support to every enrollment. With restricted subsidies, the k-ratio rule still holds for unrestricted programs. Restrictions placed on the subsidy structure always reduce value, with the gains for those in favored programs unable to offset the losses in the not favored programs.

Economist William Baumol, an early advocate of using quasi-efficient prices, later argued that such pricing was difficult to implement (Baumol and Sidak, 1994). We show that representational demand and cost parameters can be calibrated to match the decentralized budget allocations of major public universities, and that the comparative results we obtain are not particularly sensitive to variation in demand elasticities.[15] We use the parameter estimates to calculate three subsidy-structures: unrestricted subsidies, subsidies restricted to residents, and differential subsidies for selected high-cost resident programs. A structure of unrestricted subsidies yields closed-form enrollment solutions, while restrictions placed on the subsidy require a numerically tractable iterative solution. These scenarios provide comparable budget allocations for two public universities, University of Iowa and University of Michigan, which satisfy break-even requirement and the legislative budget constraint. Tuitions, enrollments, and subsidies are calculated and compared, providing quantitative measures of the losses associated with restricting the subsidy structure. Restricting subsidies to supporting residents reduces efficiency, and the appropriation applied as a direct offset against fixed costs always leads to higher welfare. Differential subsidies that favor high-cost resident programs at the expense of high value-added programs lead to inefficient distortions in enrollment patterns.

The primary insight is that adding flexibility to achieve quasi-efficient tuition and subsidy structures will always increase welfare and, typically, will reduce average tuition, benefitting the (collective) interests of students, the university, and taxpayers. The practical implications of these predictions, however, will depend on the application. For the UI and the UM, using similar methods of calibration, the losses associated with placing restrictions on the subsidy structure are modest when compared to the magnitudes of redistributions between residents and nonresidents. Our numerical results for these public universities suggest that equity may trump efficiency in determining actual subsidies.

[15] Vogelsang and Finsinger (1979) develop an algorithm for implementing differential tuitions in a regulated environment that does not require regulators to have information about demand elasticities; see Fethke (2014) for an implementation of their algorithm in the context of tuition setting.

Appendix A: Unrestricted Tuition and Subidy Structures

Result 1: *Suppose (s, S, T, E, ρ) satisfy, (8e). Then (8c) and (8f) hold if and only if* $\rho = \rho(s, S)$.

Proof: Constraints (8c) and (8e) ensure

$$F - S = \sum_{i,j} ((a_{ij} - E_{ij}) / b_{ij} - c_i + s_{ij}) E_{ij}$$

$$= \sum_{i,j} ((a_{ij} - \rho b_{ij} (d_{ij} + s_{ij})) / b_{ij} - c_i + s_{ij}) \rho b_{ij} (d_{ij} + s_{ij})$$

$$= \rho \sum_{i,j} (d_{ij} + s_{ij} - \rho(d_{ij} + s_{ij})) b_{ij} \rho(d_{ij} + s_{ij})$$

$$= \rho(1-\rho) \sum_{i,j} b_{ij} (d_{ij} + s_{ij})^2$$

We have a quadratic equation such that $\rho(1-\rho) = \frac{1}{4}\kappa(s, S)$ from (4). So

$$\rho^{-} = \frac{1}{2}(1 - \sqrt{1 - \kappa(s,S)}) \quad \text{or} \quad \rho^{+} = \frac{1}{2}(1 + \sqrt{1 - \kappa(s,S)})$$

Since $\frac{1}{2} \leq \rho \leq 1$ by (8f), the second root is valid. Moreover, the second root equals $\rho(s, S)$ by (5). This proves the result.

Result 2: *Consider problem (A1)-(A4) with restrictions placed on certain* s_{ij} *and/or on S in such a way that the Lagrangian is separable. Then there exists some scalar variable k such that* $s_{ij}^{*} = kd_{ij}$ *for all of the otherwise unrestricted* s_{ij}.

Proof: Substitution for T_{ij} using (8b) and for E_{ij} using (8e), problem (8) can alternatively be expressed as:

(A1) $$\max_{s,S,\rho} \sum_{i,j} \frac{1}{2} \rho^2 b_{ij} (s_{ij} + d_{ij})^2 - \left[\sum_{i,j} \rho b_{ij} (s_{ij} + d_{ij}) s_{ij} + S \right]$$

s.t

(A2) $$\sum_{i,j} \rho b_{ij} (s_{ij} + d_{ij}) s_{ij} + S \leq M$$

(A3) $\quad \sum_{i,j} \rho(1-\rho) b_{ij}(s_{ij}+d_{ij})^2 = F - S$

(A4) $\quad 1/2 \le \rho \le 1$

Consider the restriction in which ρ is fixed to ρ^* but (s,S) remain variables. It is clear that (s^*, S^*) is optimal for this restriction, and we can examine the corresponding first-order KKT conditions. Let α be the Lagrange multiplier for (A2) and λ the multiplier for (A3). The corresponding Lagrangian is

$$L_{\alpha,\lambda}(s,S) = \sum_{i,j} b_{ij}[\frac{1}{2}(\rho^*)^2(s_{ij}+d_{ij})^2 - (1+\alpha)\rho^*(s_{ij}+d_{ij})s_{ij} - \lambda\rho^*(1-\rho^*)(s_{ij}+d_{ij})^2]$$

$$-(1+\alpha S+\lambda S)+\alpha M+\lambda M+\lambda F.$$

The Lagrangian is separable in (s,S). Focus on the summand

$$l_{\alpha,\lambda,ij}(s_{ij}) \equiv \frac{1}{2}(\rho^*)^2(s_{ij}+d_{ij})^2 - (1+\alpha)\rho^*(s_{ij}+d_{ij})s_{ij} - \lambda\rho^*(1-\rho^*)(s_{ij}+d_{ij})^2 .$$

Corresponding to a single unrestricted s_{ij}, the first-order conditions ensure $l'_{\alpha,\lambda,ij}(s_{ij}) = 0$, which simplifies to

$$s^*_{ij} = \frac{(1-\rho^*)(1+\rho^*+2\lambda\rho^*)+\alpha}{(\rho^*)^2 - 2(1+\alpha) - 2\lambda(1-\rho^*)\rho^*} d_{ij},$$

which proves the result.

Result 3: *When the subsidy structure is unrestricted, the legislature's optimization problem (7) simplifies to a one-dimensional strictly-convex programming problem, which has a unique optimal solution* x^*. *Given* x^*, $\rho^* \in [1/2,1]$ *can be selected arbitrarily, and* $k = x^*/\rho - 1$. *Then, we can determine* $s_{ij} = kd_{ij}$ *and* $S = F - \theta\rho(1-\rho)(1+k)^2$. *By Result 1, it follows that* $\rho = \rho(s,S)$.

Proof: When the subsidies are unrestricted, Result 2 allows us to replace every s_{ij} with kd_{ij}. Defining $\theta \equiv \sum_{i,j} b_{ij} d_{ij}^2 > 0$, problem (A1)-(A4) can be rewritten as

(A5) $$\max_{S,\rho,k}\frac{1}{2}\theta\rho^2(1+k)^2-\theta\rho(1+k)k-S$$

s.t

(A6) $$\theta\rho(1+k)k+S\leq M$$

(A7) $$\theta\rho(1-\rho)(1+k)^2=F-S$$

(A8) $$1/2\leq\rho\leq 1$$

Solving for S, substituting, and simplifying, we get

(A9) $$\max_{S,\rho,k}\frac{1}{2}\theta\rho(1+k)(2-\rho(1+k))-F$$

s.t

(A10) $$\theta\rho(1+k)(\rho(1+k)-1)+F\leq M$$

(A11) $$S=F-\theta\rho(1-\rho)(1+k)^2$$

(A12) $$1/2\leq\rho\leq 1$$

Introducing a new variable $x=\rho(1+k)$, we arrive at

(A13) $$\max_{S,\rho,k,x}\frac{1}{2}x(2-x)-F$$

s.t.

(A14) $$\theta x(x-1)+F\leq M$$

(A15) $$S=F-\theta\rho(1-\rho)(1+k)^2$$

(A16) $$1/2\leq\rho\leq 1$$

(A17) $$x=\rho(1+k)$$

Note that k and S can be derived from x and ρ. So, we arrive at the simplified convex-optimization problem that does not depend on ρ:

(A18) $$\max_{x}\frac{\theta}{2}x(2-x)-F$$

s.t

(A19) $$x(x-1)\theta+F\leq M.$$

The optimal x^* is either the critical point $\bar{x} = 1$ of the objective, or it is one of the endpoints:

$$x^- = \frac{1}{2}(1 - \sqrt{1 - \kappa(0, M)}) \text{ or } x^+ = \frac{1}{2}(1 + \sqrt{1 - \kappa(0, M)}), \text{ where } \kappa(0, M) \equiv \frac{4(F - M)}{\theta}$$

Since $M < F$, $\bar{x} = 1$ is infeasible. Since x^+ yields a higher objective, we assign $x^* = x^+$. Note that $x^* \in$[½, 1]. Once we have x^*, $\rho \in$ [½, 1] can be selected arbitrarily to calculate $k = x^* / \rho - 1$. Then, we can determine $s_{ij} = kd_{ij}$ and $S = F - \theta\rho(1 - \rho)(1 + k)^2$. By Result 1, it follows that $\rho = \rho(s, S)$. The resulting subsidy structure (s, S) is optimal for (7), which proves the result.

Appendix B: Restricted subsidies

If selected $s_{ij} = 0$ are restricted and $\rho = \rho^*$ is fixed, then the optimization occurs over the remaining unrestricted s_{ij} and S, that is

(B1) $$\max_{s_{ij}, S}\{\sum_u \frac{1}{2}\rho^{*2} b_{ij}(s_{ij} + d_{ij})^2 + \sum_r \frac{1}{2}\rho^{*2} b_{ij} d_{ij}^2 - [\sum_u \rho^* b_{ij}(s_{ij} + d_{ij})s_{ij} + S]\}$$

s.t.

(B2) $$\sum_u \rho^* b_{ij}(s_{ij} + d_{ij})s_{ij} + S \le M$$

(B3) $$\sum_u \rho^*(1 - \rho^*) b_{ij}(s_{ij} + d_{ij})^2 + \sum_r \rho^*(1 - \rho^*) b_{ij} d_{ij}^2 = F - S$$

(B4) $$\frac{1}{2} \le \rho^* \le 1$$

For the unrestricted subsidies, $s_{ij}^* = kd_{ij}$ by Result 2. The optimization can now be rewritten as

(B5) $$\max_{S,k}\{\frac{1}{2}\theta_u \rho^{*2}(1 + k)^2 - \theta_u \rho^*(1 + k)k - S + \frac{1}{2}\theta_r \rho^{*2}\}$$

s.t.

(B6) $$\theta_u \rho^*(1 + k)k + S \le M$$

(B7) $\theta_u \rho^*(1-\rho^*)(1+k)^2 = F - S - \rho^*(1-\rho^*)\theta_r$

where $\theta_l \equiv \sum_l b_{ij} d_{ij}^2$, with $l \in \{u, r\}$. After eliminating S in (B5)-(B7), the problem becomes:

(B8) $\max_k \{\frac{1}{2}\rho^{*2}(\theta_u(1+k)^2 + \theta_r) - \theta_u \rho^*(1+k)k - [F - \rho^*(1-\rho^*)(\theta_u(1+k)^2 + \theta_r)]\}$

s.t.

(B9) $\theta_u \rho^*(1+k)k + [F - \rho^*(1-\rho^*)(\theta_u(1+k)^2 + \theta_r)] \le M$

The optimal solution is that associated with the right endpoint of (B9):

(B10) $k^* = \frac{1}{2\rho^*}[1 - 2\rho^* + \sqrt{\frac{4(\rho^*(1-\rho^*)\theta_r - F + M) + \theta_u}{\theta_u}}].$

Equations (B10) and (B7) provide k^* and S^* for a fixed ρ^*.

The expression (B10) can be used to eliminate k^* in (B5), yielding the concave expression:

(B11) $W(\rho^*) = \frac{1}{4}(-2F - 2M + 2\rho^*\theta_r + \theta_u + \sqrt{\theta_u}\sqrt{-4F + 4M + 4(1-\rho^*)\rho^*\theta_r + \theta_u})$

The critical point is:

(B12) $\rho^*(0, M) = \frac{1}{2}[1 + \sqrt{1 - \frac{4(F-M)}{\theta_u + \theta_r}}].$

Substituting (B12) into (B10) reveals that $k^* = 0$, and (B6) implies $S^* = M$. Using the entire appropriation as a direct offset against fixed costs attains maximum achievable value.

Restrictions on the subsidy structure reduce economic value. While closed-form results are not available for the restricted cases, we develop numerical solutions by imposing restrictions on (B7) and then iterating on ρ to identify the maximum restricted objective. For example, with $S = 0$ imposed, we solve (B7) to determine: $k^* = -1 + \sqrt{\frac{F/\rho^*(1-\rho^*) - \theta_r}{\theta_u}}$. To determine ρ^* we iterate on ρ to determine the maximum objective.

For a given ρ^*, the expression for welfare that applies to all of the cases is:

$$W(\rho^*) = \frac{F\rho^*}{2(1-\rho^*)} - M.$$

Here, $\frac{d\rho^*}{dM} = \frac{2(1-\rho^*)^2}{F} > 0$ and $\frac{d\rho^*}{dF} = \frac{-2\rho^*(1-\rho^*)}{F} < 0.$

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Table 1

Budget Allocations, Revenues, Expenditures, and Demand and Cost Parameters for the University of Iowa, 2014

UI 2013-14	Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Col. 7	Col. 8	Col. 9	Col. 10	Col. 11	Col. 12	Col. 13
Colleges	Budget Allocations	Resident SCH	Nonres. SCH	Var. Cost/SCH	Full Cost/SCH	Cal. Res. Tuition	Cal. Nonres. Tuition	Res. Intercept	Nonres. Intercept	Res. Slope	Nonres. Slope	Res. NCS	Nonres. NCS
Liberal Arts & Sciences	$125,689,038	231,757	215,825	$281	$719	$206	$787	289,696	323,738	281.6	137.1	$95,358,408	$169,866,119
Business	$24,941,689	43,706	61,480	$237	$675	$237	$819	54,633	92,220	46.2	37.5	$20,676,031	$50,331,561
Dentistry	$23,937,910	9,133	3,866	$1,842	$2,280	$1,107	$1,808	11,417	5,798	2.1	1.1	$20,217,628	$6,990,463
Education	$15,482,702	22,506	14,719	$416	$854	$206	$787	28,133	22,078	27.3	9.4	$9,260,209	$11,584,801
Engineering	$19,525,894	17,104	17,803	$559	$997	$206	$787	21,380	26,705	20.8	11.3	$7,037,686	$14,011,753
Law	$18,723,326	6,172	4,693	$1,723	$2,161	$809	$1,456	7,715	7,039	1.9	1.6	$9,991,288	$6,831,755
Medicine	$63,678,621	28,273	14,785	$1,479	$1,917	$997	$1,505	35,341	22,177	7.1	4.9	$56,388,840	$22,251,194
Nursing	$9,127,182	8,564	5,059	$670	$1,108	$277	$861	10,706	7,588	7.7	2.9	$4,742,369	$4,358,342
Pharmacy	$8,863,313	9,778	4,154	$636	$1,074	$653	$1,217	12,222	6,231	3.7	1.7	$12,764,993	$5,055,111
University College	$2,471,461	12,506	4,379	$146	$584	$206	$206	15,633	6,569	15.2	10.6	$5,145,450	$900,802
Total Colleges	$312,441,136	389,500	346,763	$424	$862	$313	$840	486,875	520,144	413.7	218.2	$241,582,902	$292,181,902
Shared Services	$343,479,728												
Total Expenses	$680,369,954												
State Appropriation	$222,014,572	Appro / Res SCH	$570.00										
Tuition Revenue	$412,973,000	Tuitio Rev/SCH	$560.90										
Other Income	$46,240,428												
Total Income	$680,369,954												
Resident Elasticity	-0.25												
Nonresident Elasticity	-0.5												
Fixed Cost	$322,546,436												
Total Enrollment	736,263												
Social Welfare	$311,750,232												

Table 2

Tuitions, Enrollments, and Subsidy Structures for the University of Iowa

			Case 1							Case 2		
			Unrestricted Subsidies							Unrestricted Subsidy to Residents		
UI 2013-14	Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6		Col. 7	Col. 8	Col. 9	Col. 10	Col. 11
Colleges	Res. Tuition	Res. Enroll.	Nonres. Tuition	Nonres. Enroll.	Res. Subsidy	Nonres. Subsidy		Res. Tuition	Res. Enroll.	Nonres. Tuition	Nonres. Enroll.	Res. Subsidy
Liberal Arts & Sci.	$342	193,263	$452	261,742	$136	$378		$164	243,386	$763	219,112	$377
Business	$315	40,082	$420	76,454	$172	$404		$90	50,477	$751	64,002	$477
Dentistry	$2,146	6,991	$2,137	3,514	$672	$652		$1,267	8,804	$2,672	2,942	$1,863
Education	$466	15,377	$576	16,691	$111	$354		$321	19,365	$867	13,973	$309
Engineering	$598	8,950	$708	18,700	$85	$328		$486	11,271	$977	15,654	$237
Law	$1,915	4,065	$1,941	3,911	$423	$481		$1,362	5,119	$2,336	3,274	$1,172
Medicine	$1,768	22,811	$1,729	13,685	$638	$552		$933	28,728	$2,183	11,456	$1,769
Nursing	$729	5,070	$828	5,158	$130	$348		$559	6,385	$1,114	4,318	$360
Pharmacy	$853	9,030	$884	4,721	$478	$548		$227	11,371	$1,335	3,952	$1,325
University College	$219	12,304	$185	4,599	$160	$86		$9	15,496	$256	3,850	$445
Totals/Averages	$530	317,943	$541	409,175	$200	$387		$268	400,402	$859	342,532	$554
Total Enrollment (SCH)	727,118											
Appropriation	$222.01 M							$222.01 M				
Social Welfare	$338.01 M							$312.4 M				
x*	0.9176							NA				
Rho*	0.7764							0.7682				
k*	0.1819							0.5044				
Laspeyres Index	0.9895							0.9746				

				Case 3	
			Differentiated Subsidies to Residents		
UI 2013-14	Col. 12	Col. 13	Col. 14	Col. 15	Col. 16
Colleges	Res. Tuition	Res. Enroll.	Nonres. Tuition	Nonres. Enroll.	Res. Subsidy
Liberal Arts & Sci.	$191	236,007	$763	219,058	$343
Business	$123	48,946	$752	63,986	$434
Dentistry	$1,162	9,019	$2,673	2,941	$2,000
Education	$342	18,778	$867	13,969	$281
Engineering	$503	10,930	$977	15,650	$215
Law	$726	6,330	$2,337	3,273	$2,000
Medicine	$757	29,978	$2,183	11,453	$2,000
Nursing	$584	6,191	$1,114	4,317	$328
Pharmacy	$319	11,027	$1,336	3,951	$1,206
University College	$40	15,026	$256	3,849	$405
Totals/Averages	$276	392,232	$857	342,447	$550
Total Enrollment (SCH)	734,680				
Appropriation	$222.01 M				
Social Welfare	$311.8 M				
Rho*	0.7680				
Laspeyres Index	0.9801				

Table 3

Enrollment, Cost and Demand Parameters for the University of Michigan, 2014

UM 2013-14	Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Col. 7	Col. 8	Col. 9	Col. 10	Col. 11	Col. 12
Colleges	Res. Enroll.	Nonres. Enroll.	Var. Cost/E	Full Cost/E	Res. Tuition	Nonres. Tuition	Res. Slope	Nonres. Slope	Res. Inter.	Nonres. Inter.	Res. NCS	Nonres. NCS
Arc&Urban Planning	270	371	$ 26,581	$38,490	$14,308	$45,213	0.005	0.004	338	557	$7,726,483	$16,773,965
Art&Design	279	286	$ 19,282	$31,191	$14,308	$45,213	0.005	0.003	349	429	$7,984,033	$12,930,873
Dentistry	386	247	$ 49,802	$61,711	$25,837	$40,419	0.004	0.003	483	371	$19,946,283	$9,983,613
Education	327	198	$ 33,744	$45,653	$16,157	$48,412	0.005	0.002	409	297	$10,566,626	$9,585,584
Engineering	4,570	3,765	$ 20,886	$32,795	$15,326	$45,478	0.075	0.041	5713	5,648	$140,080,826	$171,225,329
Kinesiology	549	373	$ 13,927	$25,836	$16,157	$48,412	0.008	0.004	686	560	$17,740,299	$18,057,691
Lit, Arts &Sciences	11,714	7,220	$ 18,694	$30,603	$14,308	$45,213	0.205	0.080	14643	10,830	$335,214,910	$326,436,733
Music	381	669	$ 30,771	$42,680	$14,889	$45,822	0.006	0.007	476	1,004	$11,345,407	$30,654,784
Natural Resources	113	180	$ 38,404	$50,313	$14,308	$45,213	0.002	0.002	141	270	$3,233,676	$8,138,312
Nursing	735	160	$ 20,141	$32,050	$14,308	$45,213	0.013	0.002	919	240	$21,033,205	$7,234,055
Pharmacy	285	142	$ 30,482	$42,391	$25,046	$41,853	0.003	0.002	356	213	$14,275,941	$5,943,085
Public Policy	117	179	$ 34,305	$46,214	$16,157	$48,412	0.002	0.002	146	269	$3,780,719	$8,665,755
Business	1,207	2,012	$ 28,829	$40,738	$15,189	$45,983	0.020	0.022	1509	3,018	$36,666,522	$92,517,271
Information	137	289	$ 38,177	$50,086	$16,157	$48,412	0.002	0.003	171	434	$4,426,996	$13,991,080
Law	226	791	$ 42,026	$53,935	$55,437	$58,699	0.001	0.007	283	1,187	$25,057,428	$46,430,953
Medicine	679	455	$ 67,485	$79,394	$33,878	$53,134	0.005	0.004	849	683	$46,005,718	$24,175,801
Public Health	466	472	$ 35,308	$47,217	$27,177	$44,926	0.004	0.005	583	708	$25,328,831	$21,204,961
Rackman	249	197	$ 19,019	$30,928	$21,833	$44,926	0.003	0.002	311	296	$10,872,966	$8,850,376
Social Work	319	143	$ 43,753	$55,662	$26,203	$41,998	0.003	0.002	399	215	$16,717,213	$6,005,775
Totals	23,009	18,149	$ 24,126		$16,472	$46,218	0.370	0.197	28761	27,224	$758,004,082	$838,805,994
Total Tuition Revenue	$1,217,808,035											
Total Enrollment	41,158											
Average Tuition	$29,589											
Total Appropriation	$279,108,700											
Appro./Resident	$12,130											
Resident Elasticity	-0.25											
Nonresident Elasticity	-0.5											
Total Alloc. To Colleges	$992,967,086											
Fixed Cost	$503,449,609											
Social Welfare	$1,317,701,376											

Sources**:**

http://www.ro.umich.edu/report/14sp201.pdf

http://ro.umich.edu/enrollment/enrollment.php

Table 4

Tuitions, Enrollments, and Subsidy Structures for the University of Michigan

	Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Col. 7	Col. 8	Col. 9	Col. 10	Col. 11	Col. 12	Col. 13
Colleges	Budget Allocations	Resident SCH	Nonres. SCH	Var. Cost/SCH	Full Cost/SCH	Cal. Res. Tuition	Cal. Nonres. Tuition	Res. Intercept	Nonres. Intercept	Res. Slope	Nonres. Slope	Res. NCS	Nonres. NCS
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