Python Programming in Math

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Fractals and Dimension Finding

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In this project, we write algorithms to create Newton fractals - the fractals related to Newton's method - with function:

Newton_Fractal.py

containing many parameters that allow the user to change the colour schemes and specify various levels of detail in generating a fractal.

Numerous example functions along with their calculated roots are provided in:

functions.py

The dimension of each fractal is calculated with the Minkowski–Bouligand dimension method, or the box-counting method in:

Box_Count.py

Due to the logarithmic nature of the calculation, high accuracy beyond 1.0e-2 was difficult to achieve in the time-frame of the project. This is because the detail (pixel count) required in the fractals needed to increase exponentially in order to add additional data points (beyond n=9) to the dimension solver's linear regression plot. Several assumptions were made for the code:

- 1. The image loaded is a jpeg with greyscale colours and minimal compression issues.
- 2. The image size is $(2^k \times 2^k)$ in pixels
- 3. The image contains a fractal.

The following calculation was necessary to properly scale the Newton Fractals to an ideal detail.

$$f(x) = log_2 \left(2.56 \cdot 200 \cdot x \right)$$

where:

$$2.56 = \text{interval size}$$

 $200 = \text{dpi}$
 $x = \text{scalar}$
 $f(x) = \text{exponent to range found in n (located in Box Dim())}$