

# Tidal field anisotropy as a tracer of cosmic voids

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## ABSTRACT

Finding and characterizing underdense regions (voids) in the large scale structure of the Universe is an important task in cosmological studies. In this paper we present a novel approach to find voids in cosmological simulations. Our approach is based on algorithms that use the tidal and the velocity shear tensors to locally define the cosmic web. Voids are identified using the fractional anisotropy (FA) computed from the eigenvalues of each web scheme. We define the void boundaries using a watershed transform based on the local minima of the FA and its boundaries as the regions where the FA is maximized. This void identification technique does not have any free parameters and does not make any assumption on the shape or structure of the voids. We test the method on the Bolshoi simulation and report on the density and velocity profiles for the voids found using this new scheme.

**Key words:** Cosmology: theory - large-scale structure of Universe - Methods: data analysis - numerical - N-body simulations

## 1 INTRODUCTION

Since voids were found in the first compiled galaxy surveys they have been identified as one of the most striking features of the Cosmic Web (Chincarini & Rood 1975; Gregory & Thompson 1978; Einasto et al. 1980a,b; Kirshner et al. 1981; Zeldovich et al. 1982; Kirshner et al. 1987; Bond et al. 1996). However, due to the large volume extension of void regions ( $\sim 5 - 10 \text{ Mpc h}^{-1}$ ), statistically meaningful catalogs of voids (Pan et al. 2012; Sutter et al. 2012; Nadathur & Hotchkiss 2014) have only become available through modern galaxy surveys like the two-degree field Galaxy Redshift Survey (Colless et al. 2001, 2003) and the Sloan Digital Sky Survey (York et al. 2000; Abazajian et al. 2003). This advancement generated a great interest to study voids observationally during the last decade (Hoyle & Vogeley 2004; Croton & et al. 2004; Rojas et al. 2005; Ceccarelli et al. 2006; Patiri et al. 2006; Tikhonov 2006; Patiri et al. 2006; Tikhonov 2007; von Benda-Beckmann & Müller 2008; Foster & Nelson 2009; Ceccarelli et al. 2013; Sutter et al. 2014).

On the theoretical side the rough theoretical framework that explains the origin of voids was established in the seminal work of Zel'dovich (1970) and refined in the following decades. The first detailed theoretical models describing formation, dynamics and properties of voids (Hoffman & Shahan 1982; Icke 1984; Bertschinger 1985; Blumenthal et al. 1992) were complemented and extended by numerical stud-

ies (Martel & Wasserman 1990; Regos & Geller 1991; van de Weygaert & van Kampen 1993; Dubinski et al. 1993; Bond et al. 1996). Currently, the dominant tendency to study voids relies on using data from N-body simulations. For an extensive compilation of previous numerical works we refer the reader to Colberg et al. (2008).

The relevance of voids to cosmological studies can be summarized in three aspects (Platen et al. 2007). First, voids are a key ingredient of the Cosmic Web. They dominate the volume distribution at large-scales and additionally, they compensate overdense structures in the total budget of matter. Second, voids provide a valuable resource for inferring and probing cosmological parameters as their structure and dynamics are highly determined by these values. Third, they are a largely a largely pristine environment to test galaxy evolution.

Although a visual recognition of voids in galaxy surveys and simulations is possible in most cases, a formal systematic identification is desirable for statistical studies. However, the community does not have an unambiguous definition of cosmic voids. There are many different void finding techniques in the literature (for a detailed comparison of different schemes, see the Void Finder Comparison Project Colberg et al. (2008)). In spite of the diversity of existing schemes, they can be roughly classified into two main types. First, geometric schemes based on point spatial or redshift distribution of galaxies in surveys or dark matter halos in simulations (Kauffmann & Fairall 1991; Müller et al. 2000; Gottlöber et al. 2003; Hoyle & Vogeley 2004; Brunino et al.

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2007; Foster & Nelson 2009; Micheletti & et al. 2014; Sutter et al. 2014); second, schemes based on the smooth and continuous matter density field either from simulations or from reconstruction procedures on surveys (Plionis & Basilakos 2002; Colberg et al. 2005; Shandarin et al. 2006; Platen et al. 2007; Neyrinck 2008; Muñoz-Cuartas et al. 2011; Neyrinck et al. 2013; Ricciardelli et al. 2013).

Here we introduce a new tracer of the structure of cosmic voids that can be used once the continuous matter density or velocity distribution is known. The algorithm is based on two tensorial schemes used to classify the cosmic web. The first (the T-web) is based on the Hessian of the gravitational potential or tidal tensor (Hahn et al. 2007; Forero-Romero et al. 2009). The second (the V-web) is based on the velocity shear tensor (Hoffman et al. 2012). These web schemes classify the Cosmic Web into four different types of environment depending on the counting of the number of eigenvalues below an user-defined threshold ( $\lambda_{r_{\text{mth}}}$ ): voids, sheets, filaments and knots for 3, 2, 1 or 0 eigenvalues below  $\lambda_{\text{th}}$ , respectively.

This procedure allows a new description of the internal structure of voids that goes beyond a simple definition of a void as just an underdense regions in the large-scale matter distribution. The tidal and the shear tensors encode more information than the density field as they trace the collapsing or expanding nature of the matter field, which defines the dynamics of the Cosmic Web.

The tracer that we use to define the voids is the fractional anisotropy (FA) computed from the set of eigenvalues of the tensor under consideration. The FA was initially introduced by Bassler (1995) to quantify the anisotropy degree of the diffusivity of water molecules through cerebral tissue in nuclear magnetic resonance imaging. Libeskind et al. (2013) introduced this concept in the context of Cosmic Web classification schemes.

Once we establish the FA as a void tracer, we proceed to identify individual voids as basins of local minima. For this we implement a *watershed transform algorithm* (Beucher & Lantuejoul 1979; Beucher & Meyer 1993) which has been used to define as catching basins of local minima of the density field (Platen et al. 2007; Neyrinck 2008); we use instead minima of the FA field to find define the voids.

This paper is organized as follows. In Section 2 we describe the algorithms we use to find the cosmic web in N-body simulations. In Section 3 we describe the used simulation. Then in Section 4 we explain in detail our void finder based on the fractional anisotropy of the tidal and shear tensor fields. Our results are presented in 5 to finally comment and conclude about these findings in Section 6.

## 2 ALGORITHMS TO LABEL THE COSMIC WEB

### 2.1 The tidal web (Tweb)

This scheme was initially proposed by Hahn et al. (2007) as an alternative for classifying the Cosmic Web based on the tidal tensor. The tidal tensor allows a classification in terms of the orbital dynamics of the matter field. This approach extends to second-order the equations of motion around local minima of the gravitational potential. The second-order

term corresponds to the tidal tensor, which is defined as the Hessian matrix of the normalized gravitational potential.

$$T_{\alpha\beta} = \frac{\partial^2 \phi}{\partial x_\alpha \partial x_\beta}, \quad (1)$$

where the physical gravitational potential has been rescaled by a factor of  $4\pi G\bar{\rho}$  in such a way that  $\phi$  satisfies the following Poisson equation

$$\nabla^2 \phi = \delta, \quad (2)$$

with  $\bar{\rho}$  the average density in the Universe,  $G$  the gravitational constant and  $\delta$  the dimensionless matter overdensity.

Since the tidal tensor can be represented by a real and symmetric  $3 \times 3$  matrix, it is always possible to diagonalize it and obtain three real eigenvalues  $\lambda_1 \geq \lambda_2 \geq \lambda_3$  with its corresponding eigenvectors  $\mathbf{u}_1$ ,  $\mathbf{u}_2$ ,  $\mathbf{u}_3$ . This set of eigenvalues are indicators of the local orbital stability in each direction  $\mathbf{u}_i$ , which is to label the Cosmic Web. The number of positive (stable) or negative (unstable) eigenvalues allows to label a location into one of the next four types of environment: voids (3 negatives eigenvalues), sheets (2), filaments (1) and knots (0). A modification to this scheme was introduced by Forero-Romero et al. (2009) by means of a relaxation of the stability criterion. The relative strength of each eigenvalue is no longer defined by the sign, but instead by a threshold value  $\lambda_{\text{th}}$  that can be tuned in such a way that the visual impression of the web-like matter distribution is reproduced.

### 2.2 The velocity web (Vweb)

We also use a kinematic scheme to define the Cosmic Web environment in the simulation, which has been thoroughly described in Hoffman et al. (2012). This scheme has been used to study the shape, spin and peculiar velocity alignments of dark matter halos with the cosmic web (Libeskind et al. 2013; Forero-Romero et al. 2014). We refer the reader to these papers to find a detailed description of the algorithm, its limitations and capabilities. The Vweb scheme for environment finding is based on the local velocity shear tensor calculated from the smoothed dark matter velocity field in the simulation. This tensor is given by the following expression

$$\Sigma_{\alpha\beta} = -\frac{1}{2H_0} \left( \frac{\partial v_\alpha}{\partial x_\beta} + \frac{\partial v_\beta}{\partial x_\alpha} \right), \quad (3)$$

where  $v_\alpha$  and  $x_\alpha$  represent the  $\alpha$  component of the comoving velocity and position, respectively. Like the tidal tensor,  $\Sigma_{\alpha\beta}$  can be represented by a  $3 \times 3$  symmetric matrix with real values, making it possible to find three real eigenvalues and its corresponding eigenvectors.

In this case we also use the relative strength of the three eigenvalues with respect to a threshold value  $\lambda_{\text{th}}$  to classify the cosmic web in the four web types we have already described.

Usually, the threshold is a free parameter that is tuned to reproduce the visual appearance of the comic web. In this paper we take a different approach. We find the optimal value of the threshold based on the maximization of the

fractional anisotropy field in the locations label as filaments and walls.

### 3 NUMERICAL SIMULATIONS

We use the Bolshoi simulation to test the method describe in the paper. This simulation follows the non-linear evolution of a dark matter density field on a cubic volume of size  $250h^{-1}\text{Mpc}$  sampled with  $2048^3$  particles. The cosmological parameters in the simulation are  $\Omega_m = 0.27$ ,  $\Omega_\Lambda = 0.73$ ,  $h = 0.70$ ,  $n = 0.95$  and  $\sigma_8 = 0.82$  for the matter density, cosmological constant, dimensionless Hubble parameter, spectral index of primordial density perturbations and normalization for the power spectrum, respectively. This values are consistent with the seventh year of data of the Wilkinson Microwave Anisotropy Probe (WMAP) (Jarosik et al. 2011). For more detailed technical information about the simulation, see Klypin et al. (2011).

We find the cosmic web we estimate first the density and velocity fields using a *cloud-in-cell* (CIC) algorithm onto a grid of  $256^3$  cells, corresponding to a spatial resolution of  $0.98h^{-1}\text{Mpc}$  per cell side. These fields are smoothed with a gaussian filter with a width of  $\sigma = 0.98h^{-1}\text{Mpc}$ . We compute the tidal and shear tensors through finite-differences and Finally, the eigenvalues and eigenvectors of the tensor are obtained for each grid cell. The data we use here is publicly available through the MultiDark database <http://www.multidark.org/MultiDark/> which is described in Riebe et al. (2013).

## 4 A NEW VOID FINDING TECHNIQUE

### 4.1 The fractional anisotropy

The fractional anisotropy (FA), as developed by Basser (1995), was conceived to quantify the anisotropy degree of a diffusion process, e.g. the diffusivity of water molecules through cerebral tissue in nuclear magnetic resonance imaging. Here we propose the FA, much in the same way as Libeskind et al. (2013), as a tracer of cosmic voids.

The FA is defined as follows.

$$FA = \frac{1}{\sqrt{3}} \sqrt{\frac{(\lambda_1 - \lambda_3)^2 + (\lambda_2 - \lambda_3)^2 + (\lambda_1 - \lambda_2)^2}{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}}, \quad (4)$$

where the eigenvalues are taken from either the Tweb or the Vweb (FA-Tweb and FA-Vweb respectively). Such as it is defined,  $FA = 0$  corresponds to an isotropic distribution ( $\lambda_1 = \lambda_2 = \lambda_3$ ) and  $FA = 1$  with a highly anisotropic distribution.

In left panels of Fig. 1 we show the FA field for both web schemes over a slide of the simulation. From this Figure we can observe that both, voids and knots, display a highly isotropic expanding dynamic at their centres, becoming gradually more anisotropic at outer regions. On the other hand the filamentary structure is traced by high FA values. These characteristics are the key to use the FA as a tracer of cosmic voids.

### 4.2 Fractional anisotropy as a void tracer

Voids are regions where  $\lambda_3 \leq \lambda_2 \leq \lambda_1 \leq \lambda_{th}$ . This implies that a void is completely fixed by the relative strength of the  $\lambda_1$  eigenvalue with respect to the  $\lambda_{th}$ . As we increase/decrease the threshold value  $\lambda_{th}$ , voids increase/decrease progressively through contours of increasing/decreasing  $\lambda_1$ .

Voids are thus characterized by low values of both FA and  $\lambda_1$ . In Fig. 2 we show that these two values are indeed closely correlated. The left panel shows the correlation between  $\lambda_1$  and  $FA$  for all the grid cells in the simulation while the right panel shows the correlation between  $\lambda_1$  and the local overdensity  $\delta$ . This shows that the overdensity has a large scatter at fixed  $\lambda_1$ .

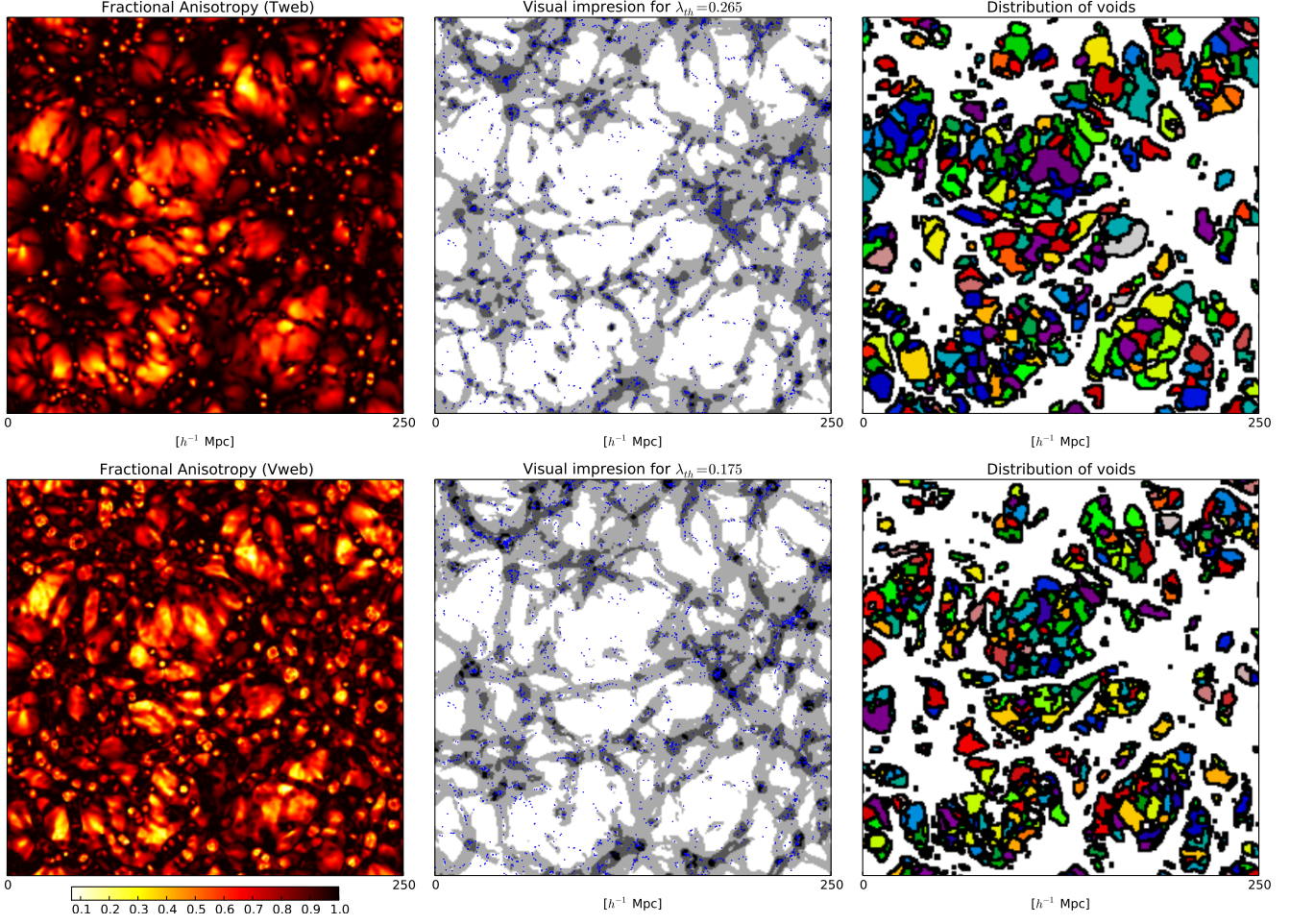
From Figs. 1 and 2 we conclude that the FA is a good tracer of voids as it is almost perfectly correlated with low values of  $\lambda_1$ . We propose that voids should be composed completely by regions of  $FA < 0.95$ . If we increase the values of  $\lambda_1$  from its minimum until it we reach  $FA = 0.95$  in 2 we find that this correspond to critical values of  $\lambda_1^T = 0.265$  and  $\lambda_1^V = 0.175$  for the Tweb and Vweb, respectively. This means that setting  $\lambda_{th}$  to either  $\lambda_1^T/\lambda_1^V$  automatically produces voids with all the cells  $FA < 0.95$ . The middle column in Fig. 1 shows the web classification for this choice of  $\lambda_{th}$ , demonstrating that it is a sensible choice to define voids.

### 4.3 Defining voids with a watershed algorithm

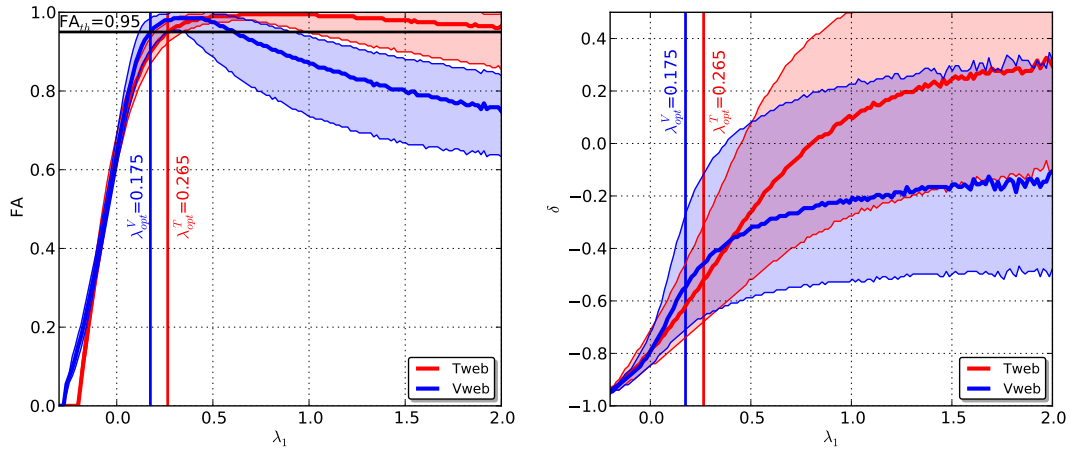
Once established the FA as a good tracer of voids, the next step is to find individuals voids in the overall distribution. For this purpose, we use the *watershed transform algorithm* (Beucher & Lantuejoul 1979; Beucher & Meyer 1993), in which a void is identified as the catching basin of a local minimum of the FA. The use of this algorithm is advantageous as it is parameter-free and does not require any assumption on the shape and morphology of voids.

Previous implementations of this technique as those of Platen et al. (2007); Neyrinck (2008) have shown remarkable results regarding voids finding, however they use the density field instead of the FA field. On the other hand, we use a *cloud-in-cell* (CIC) algorithm on a Cartesian mesh for estimating the density and tensor fields, instead of the more sophisticated *Delaunay tessellation for field estimator* (DTFE) technique (Schaap & van de Weygaert 2000), but our implementation of the watershed transform should not be significantly affected as we are interested in low density regions where the CIC gives similar estimations.

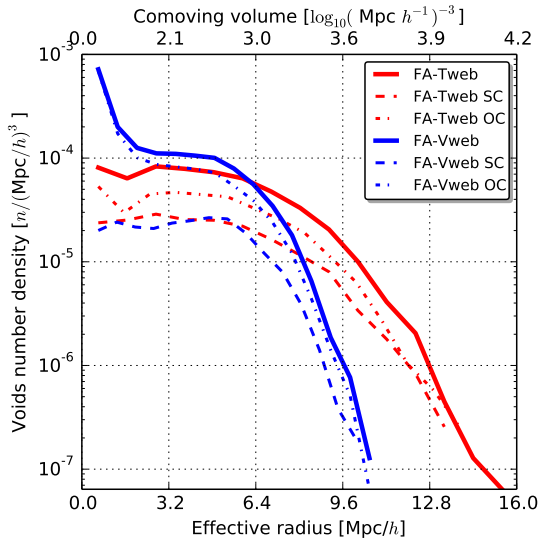
As suggested by Platen et al. (2007) and references therein, it is necessary to apply a threshold merger in order to reduce spurious features and to avoid void hierarchization. A natural choice is a threshold of  $\delta = -0.8$ , which means that any ridge between two voids with density below that value would be removed and the respective voids merged. As we are working with the FA, the equivalent threshold according to Fig. 2 is  $FA = 0.65$ . In right panels of Fig. 1 we show the resulting catalogs of voids for both schemes.



**Figure 1.** Left column. Visual impression of the FA field over a slide of the simulation for each web scheme (T-web, upper panels. V-web, middle panels). Middle column. Components of the Cosmic Web using  $\lambda_{th}$  values that put an upper bound of  $FA = 0.95$  to voids. Voids corresponds with white, sheets to gray, filaments to dark gray and knots with black regions. Right column. Sketches the distribution of the catalogued voids by our method.



**Figure 2.** Distributions of the FA (left panel) and the density field (right panel) with respect to the eigenvalue  $\lambda_1$  for each web scheme (Tweb, red lines. Vweb, blue lines) as calculated over all cells of the grid. Thick central lines correspond with the median and filled regions with the 50% of the distribution.



**Figure 3.** Volume size distribution of voids for both catalogs. FA-Tweb (red curves), FA-Vweb (blue curves). Continuous lines corresponds with the total number of voids, dot-dashed with subcompensated voids and dashed lines with over-compensated voids.

## 5 RESULTS

### 5.1 Enclosed mass

When we analyse individual density profiles of voids (see left panels of Fig. 4), they seem to be distributed in two different families, differentiated by the presence or absence of a compensation ridge (e.g. see also Fig. 14 of Colberg et al. (2005)). In order to discriminate each void in one of the families, we propose the compensation index  $\mathcal{C}$ , which is defined as the mass of a void enclosed in a spherical volume of radius  $R$  and normalized by the mass of the same volume as assuming the mean background density instead

$$\mathcal{C} = \frac{M_v}{M} = \frac{3}{2R^3} \int_0^R [\delta(r) + 1] r^2 dr \quad (5)$$

We choose an integration radius of  $R = 4r_{eff}$ , that is large enough to enclose the compensation ridge for a typical void in case there is one. This leads us to voids with  $\mathcal{C} > 1$  having more mass than expected, constituting the family of overcompensated voids. These voids generally exhibit a compensation ridge associated to dense nearby structures. In the same fashion, voids with  $\mathcal{C} < 1$  constitute the family of subcompensated voids.

### 5.2 The void size distribution

Since the shape of the voids exhibit a wide range of geometries, a common way to represent their size is through the equivalent spherical radius or effective radius, defined as  $r_{eff} = [3/(4\pi)V]^{1/3}$ , with  $V$  the total volume of the void. As mentioned before, we applied a threshold merger in order to avoid void hierarchization, what means we are working with the top hierarchy of voids. We limit our findings to voids with effective radius greater than the smoothing length of the density field, i.e.  $\sim 1h^{-1}\text{Mpc}$ . Below that scale numerical resolution effects becomes important. We find a volume

filling fraction of voids of 54.88% of the total volume of the simulation for the FA-Tweb and 47.06% for the FA-Vweb.

In Fig. 3 we show the void size distributions for both schemes and for the overall distribution of voids as well as for overcompensated and subcompensated void subsamples. The central region of the distributions ( $3h^{-1}\text{Mpc} - 6h^{-1}\text{Mpc}$ ), exhibit an approximately flat shape, contrary to expected from highly non-linear structures like dark matter halos, where the abundances increase for smaller structures. The distribution of the FA-Tweb is consistent with a two-barrier problem (Sheth & van de Weygaert 2004): formation of large voids is inhibited by the *void-in-void* problem (first barrier), where large voids are constituted hierarchically of smaller ones. In turn, formation of small voids is damped by the *void-in-cloud* problem (second barrier), where nearby collapsing structures limit the abundance of embedded small voids.

The FA-Vweb scheme does not tackle properly the *void-in-cloud* problem as it predicts large abundances of smaller voids. However, an interesting result is the difference between the overcompensated and subcompensated subsamples. Subcompensated voids, which are expected to be embedded in low-density regions like walls and larger voids, exhibit smaller abundances compatible with the result of the FA-Tweb. Then, the most dramatic contribution to the overabundance of small voids for the FA-Vweb comes from overcompensated voids. This implies that, although nearby non-linear structures squeeze out embedded smaller voids, the rapid and isotropic collapse in these zones, along with the more rugged nature of the velocity field make these voids still detectable by this scheme.

For both schemes overcompensated voids are more abundant, however at larger scales abundances of subcompensated voids become comparable. This is somehow expected as larger voids are more likely to abut other voids. A recent estimation of the radius of the local void gives  $r_{LV} \approx 10 \text{ Mpc}$  (Nasonova & Karachentsev 2011). Taking the dimensionless Hubble constant as  $h = 0.70$ , the size of the local void would be around  $r_{eff} = 7 h^{-1}\text{Mpc}$  in the shown distribution. For both schemes, this places the local void as a rather common void in terms of size, and more likely to be overcompensated.

### 5.3 Physical profiles of voids

## 6 CONCLUSIONS

## ACKNOWLEDGMENTS

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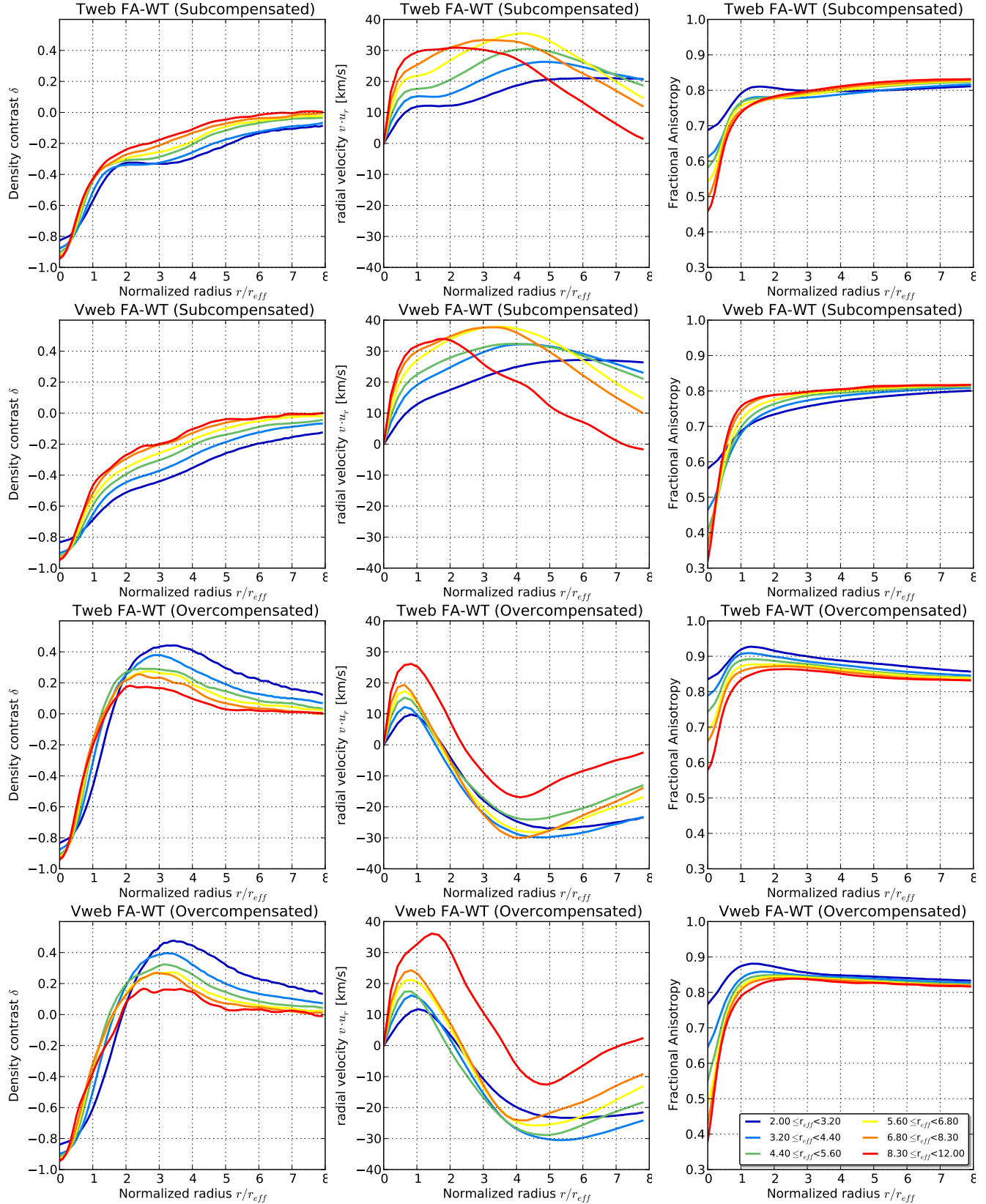


Figure 4. Density of voids for each finding scheme.



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