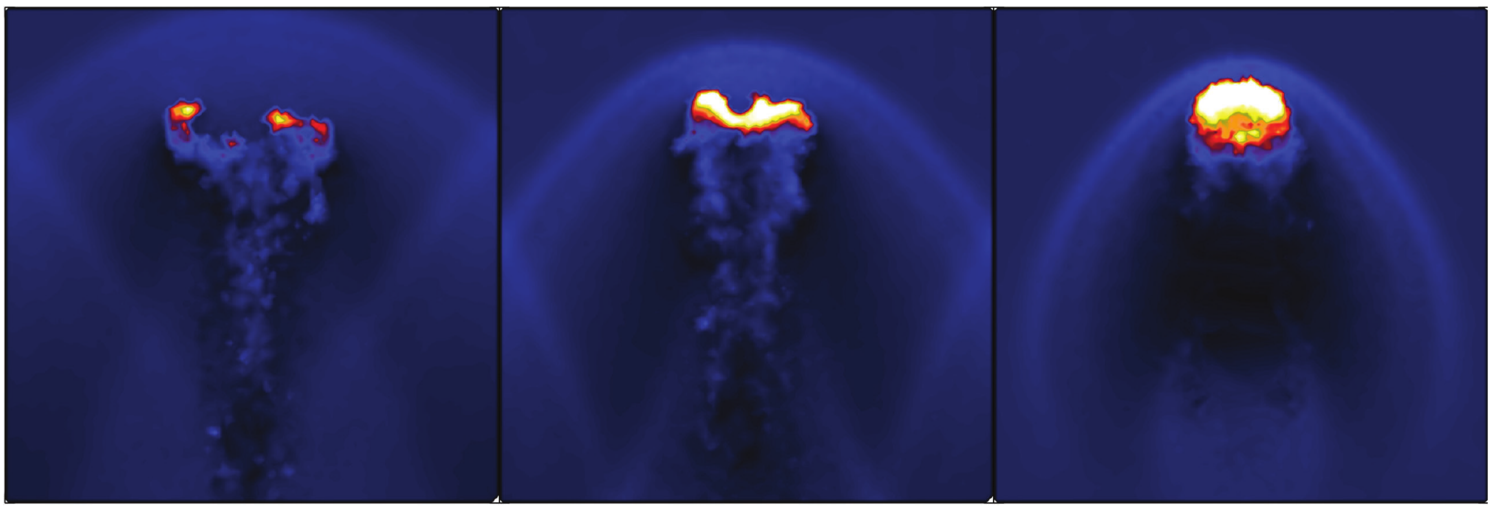
Verifying the Voronoi-Particle Hydrodynamics (VPH) scheme in Galaxy Formation



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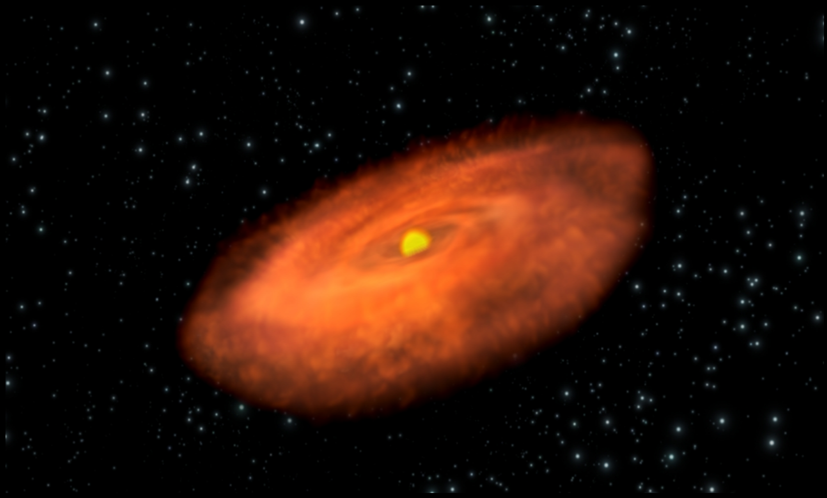
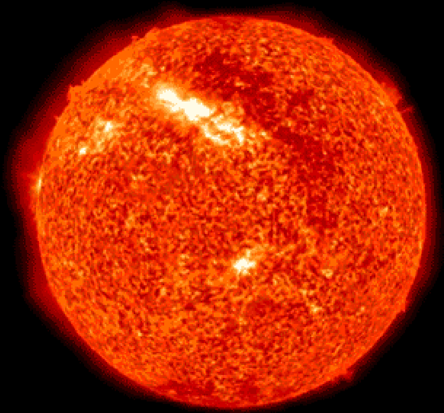
Motivation

- Understanding and simulating hydrodynamics processes with good accuracy in astrophysics is a very important task as many phenomena are modeled as fluid systems.

- Provided the complexity of the hydrodynamic equations, numerical solutions are usually the only feasible approach.

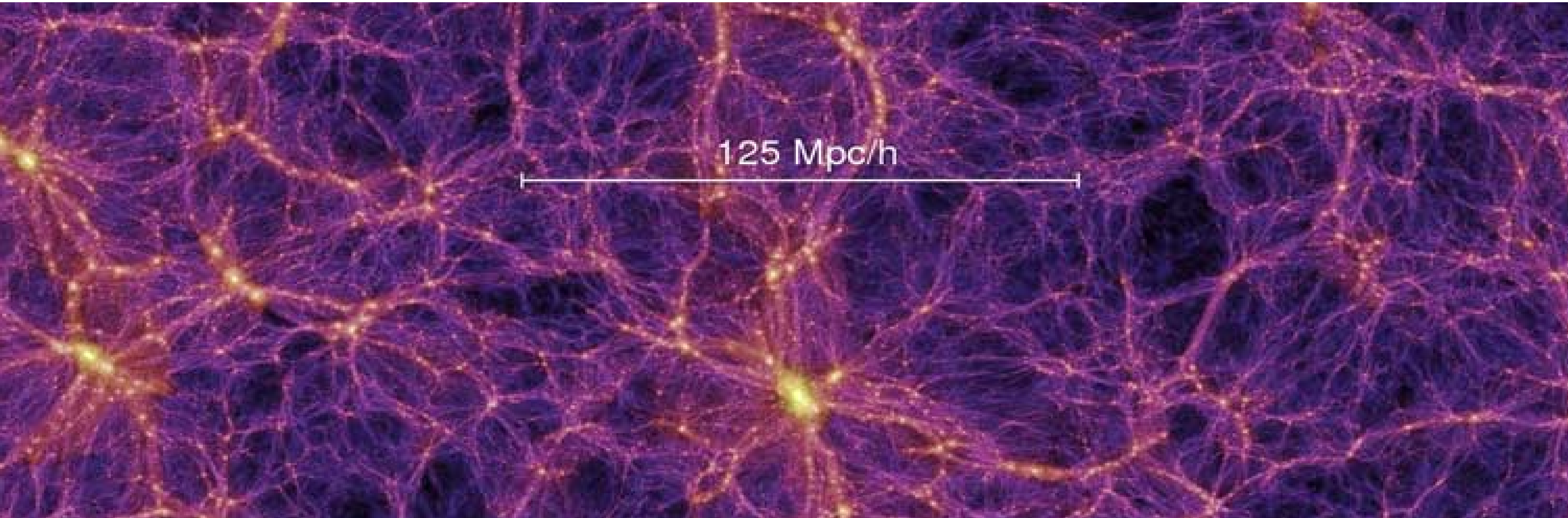
Motivation





Motivation

The large-scale Universe is a very rich scenario where a plethora of hydrodynamical processes occur. In this fashion, cosmological setups are excellent systems for comparing hydrodynamic solvers.



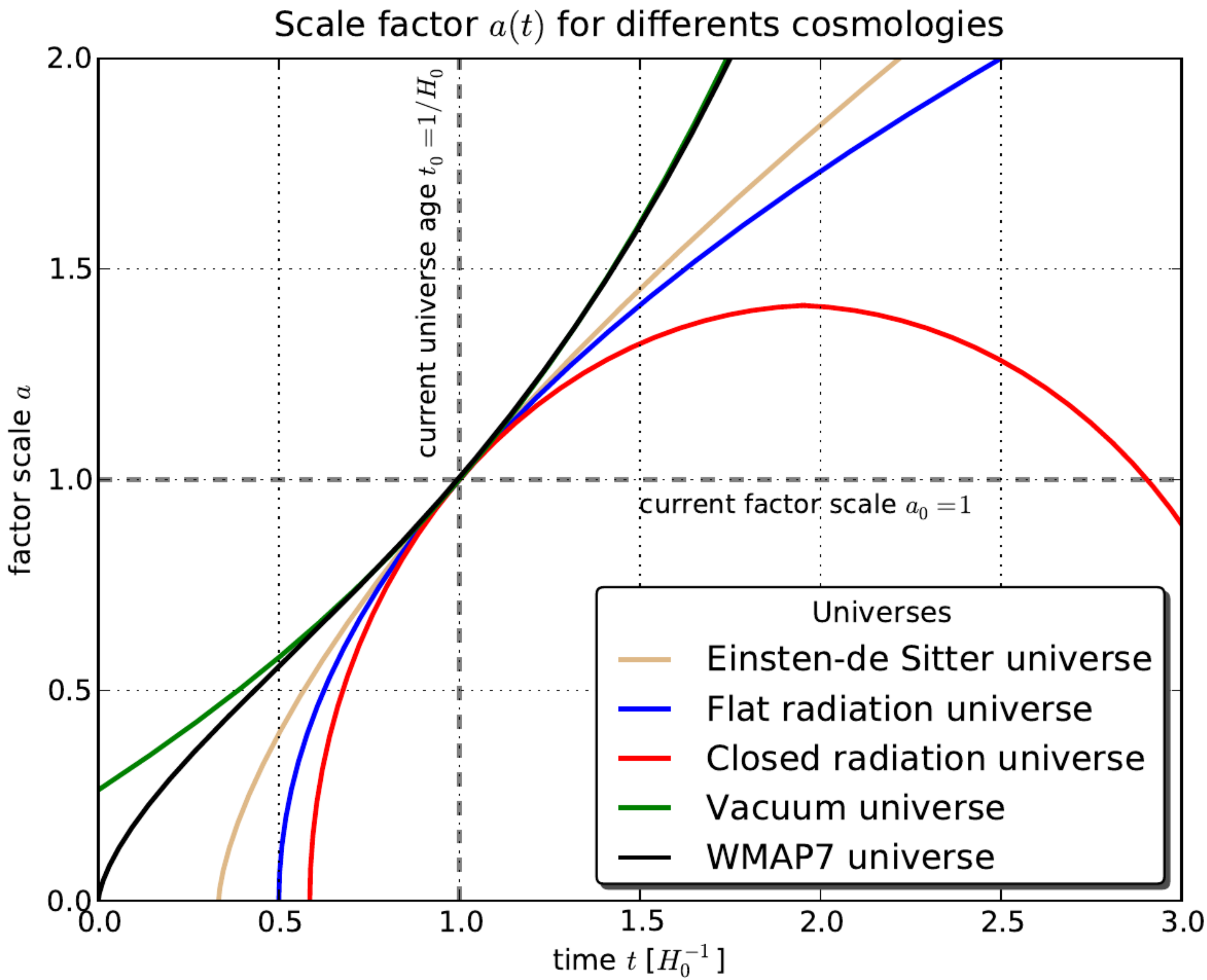
The cosmological setup

The Universe at very large scales can be modeled as a homogeneous and isotropic fluid, following the next equation

$$H^2(t) = H_0^2\left[(1-\Omega_0)\frac{1}{a^2} + \Omega_m\frac{1}{a^3} + \Omega_r\frac{1}{a^4} + \Omega_\Lambda\right]$$

where $a(t)$ is the scale factor, $H = a/\dot{a}$, H_0 is the current Hubble constant.

The cosmological setup



The cosmological setup

However, a homogeneous and isotropic Universe can not account for complex structures like galaxies, stars, planets and us!

Continuity equation

$$\frac{\partial \delta}{\partial t} = -\frac{1}{a}\nabla_r \cdot [(1+\delta)\boldsymbol{v}]$$

Euler's equation

$$\frac{\partial \boldsymbol{v}}{\partial t} + \frac{\dot{a}}{a}\boldsymbol{v} + \frac{1}{a}\left(\boldsymbol{v}\cdot\nabla_r\right)\boldsymbol{v} = -\frac{\nabla_r P}{a\bar{\rho}(1+\delta)} - \frac{1}{a}\nabla_r \Phi$$

Poisson's equation

$$\nabla_r^2 \Phi = 4\pi G \bar{\rho} a^2 \delta$$

$$\rho = \bar{\rho} + \delta\rho = \bar{\rho}(1+\delta)$$

The cosmological setup

When modes of the density field are small enough ($\delta \ll 1$), it is possible to decouple them, obtaining the next equation:

$$\frac{d^2\delta_{\boldsymbol{k}}}{dt^2} + 2\frac{\dot{a}}{a}\frac{d\delta_{\boldsymbol{k}}}{dt} = \left[4\pi G\bar{\rho} - \frac{c_s^2}{a^2}k^2\right]\delta_{\boldsymbol{k}}$$

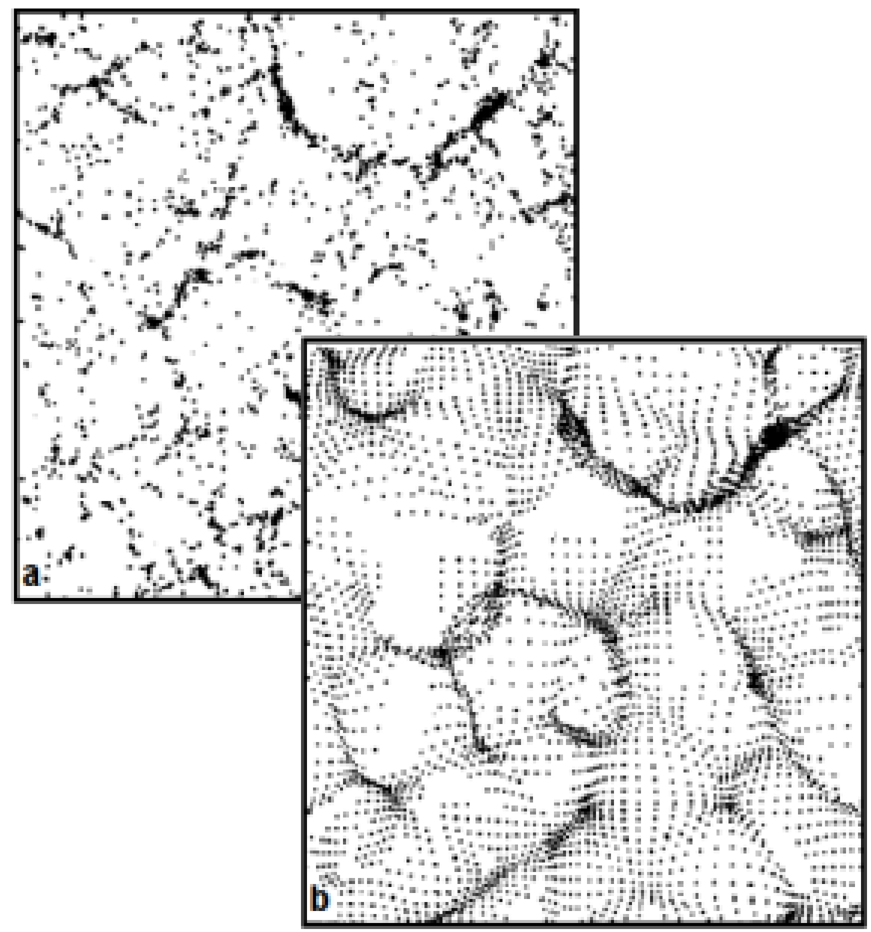
Obtaining the analytical solution for the lineal regime:

$$\delta_{\boldsymbol{k}}(t) = \delta_{\boldsymbol{k}}(0)D(t)$$

$$D(a) = \frac{5}{2}\Omega_m \left[\Omega_m^{4/7} - \Omega_\Lambda + \left(1 + \frac{\Omega_m}{2}\right)\left(1 + \frac{\Omega_\Lambda}{70}\right)\right]^{-1}$$

The cosmological setup

Even for the quasi-linear regime ($\delta \sim 1$), it is possible to obtain analytical solutions through the Zel'dovich approximation:

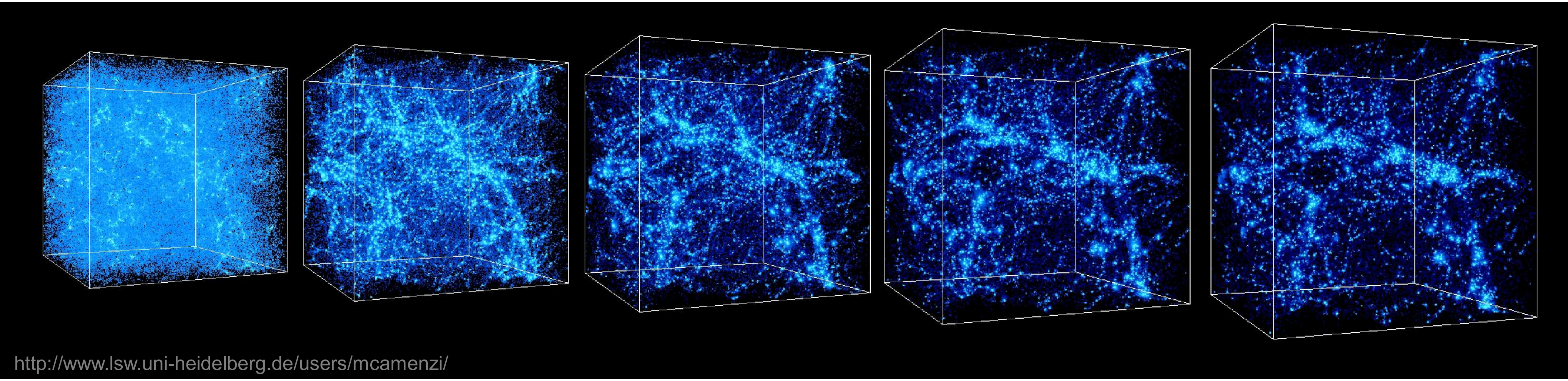


$$\boldsymbol{r}(t, \boldsymbol{q}) = a(t)\left[\boldsymbol{q} + \boldsymbol{\Psi}(\boldsymbol{q}, t)\right]$$

$$\boldsymbol{\Psi}(\boldsymbol{q}) = \int \frac{d^3k}{(2\pi)^3} e^{i\boldsymbol{k}\cdot\boldsymbol{q}} \frac{i\boldsymbol{k}}{k^2} \delta_{\boldsymbol{k}}(0)$$

The cosmological setup

For the non-linear regime, the Fourier modes of the density are not longer coupled. Numerical evolution is necessary henceforth. However, quasi-linear solutions provide initial conditions for numerical schemes.



Hydrodynamic solvers

Once clarified the importance of the hydrodynamics in Galaxy formation (for evolving the non-linear regime), two families of numerical approaches has been adopted throughout the literature:

- **Lagrangian schemes:** the fluid is evolved following the dynamics of fluid parcels.
- **Eulerian schemes:** the evolution performed over a fixed coordinate system, namely a mesh.

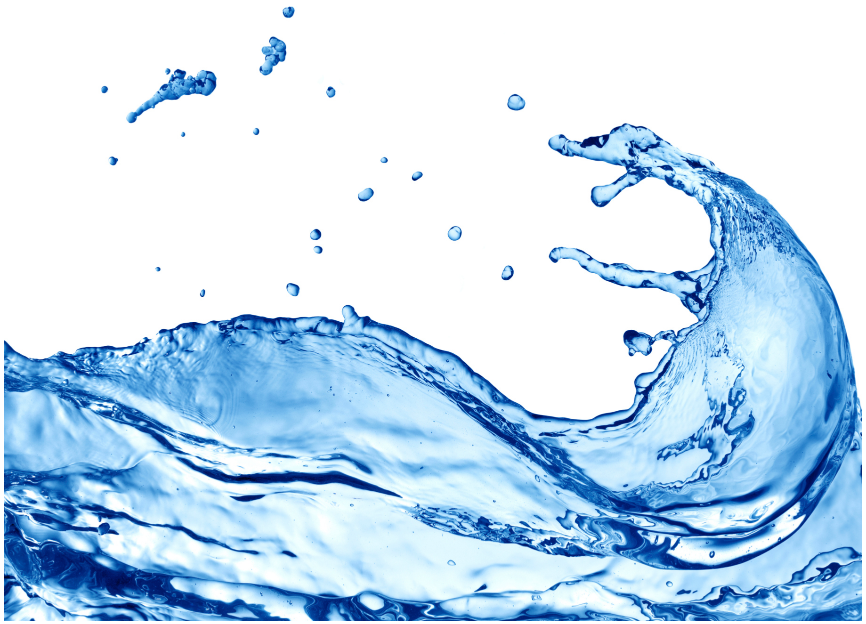
Lagrangian schemes

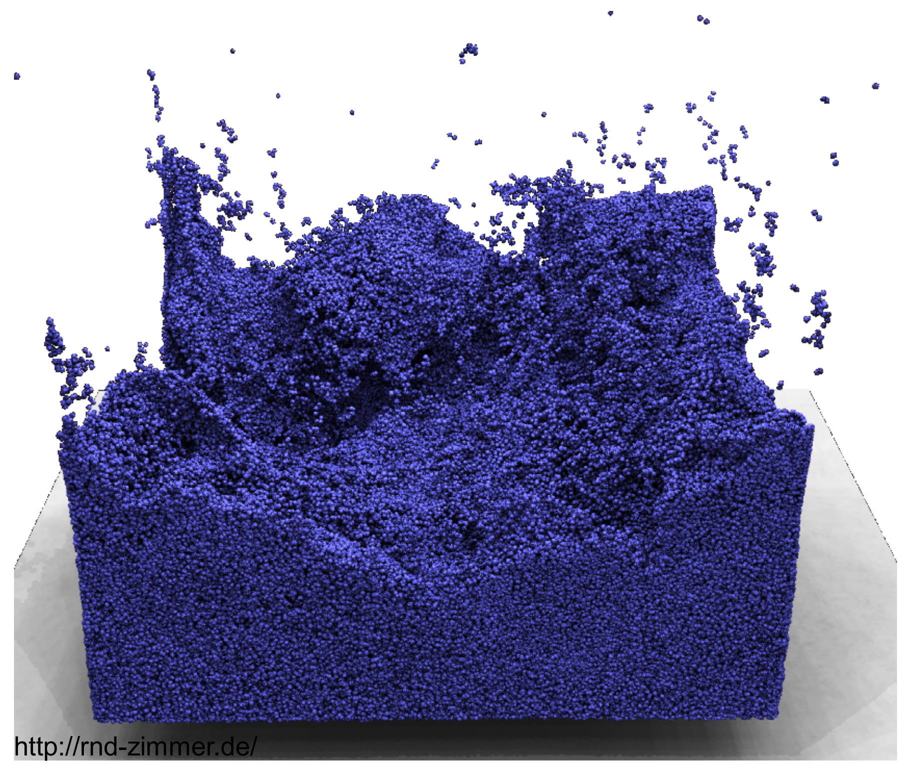
- The most representative Lagrangian scheme is **SPH** (Smoothed-Particle Hydrodynamic).
- Other schemes, normally modifications of the original **SPH**, have been also implemented.
- Among the most interesting proposals is the **VPH** (Voronoi-Particle Hydrodynamic) scheme. This scheme greatly improves the accuracy of **SPH**.

Smoothed-Particle Hydrodynamic

- Although a fluid is a continuous medium, numerical solutions always requires some kind of discretization.
- SPH reaches this by sampling the fluid through particles that represent a parcel.
- The equations of motions are then integrated over each of moving particles.
- Continuous properties are then recovered through interpolation techniques.

Smoothed-Particle Hydrodynamic





Smoothed-Particle Hydrodynamic

The heart of **SPH** is the kernel interpolant formalism:

$$F_s(\boldsymbol{r}) = \int_V \boldsymbol{F}(\boldsymbol{r}')W(\boldsymbol{r} - \boldsymbol{r}'; h)d\boldsymbol{r}'$$

Discretization can be reached approximating $\Delta\boldsymbol{r}_i = m_i/\rho_i$, yielding:

$$F_i' \equiv F_s(\boldsymbol{r}_i) \approx \sum_j^{N_{tot}} \frac{m_j}{\rho_j} F_j W(\boldsymbol{r}_i - \boldsymbol{r}_j; h_i)$$

Where h_j is the smoothing length and is associated to the volume of the fluid parcel.

Smoothed-Particle Hydrodynamic

The equations of motions may be derived from the Lagrangian

$$\mathcal{L} = \int \rho \left(\frac{v^2}{2} - u \right) dV$$

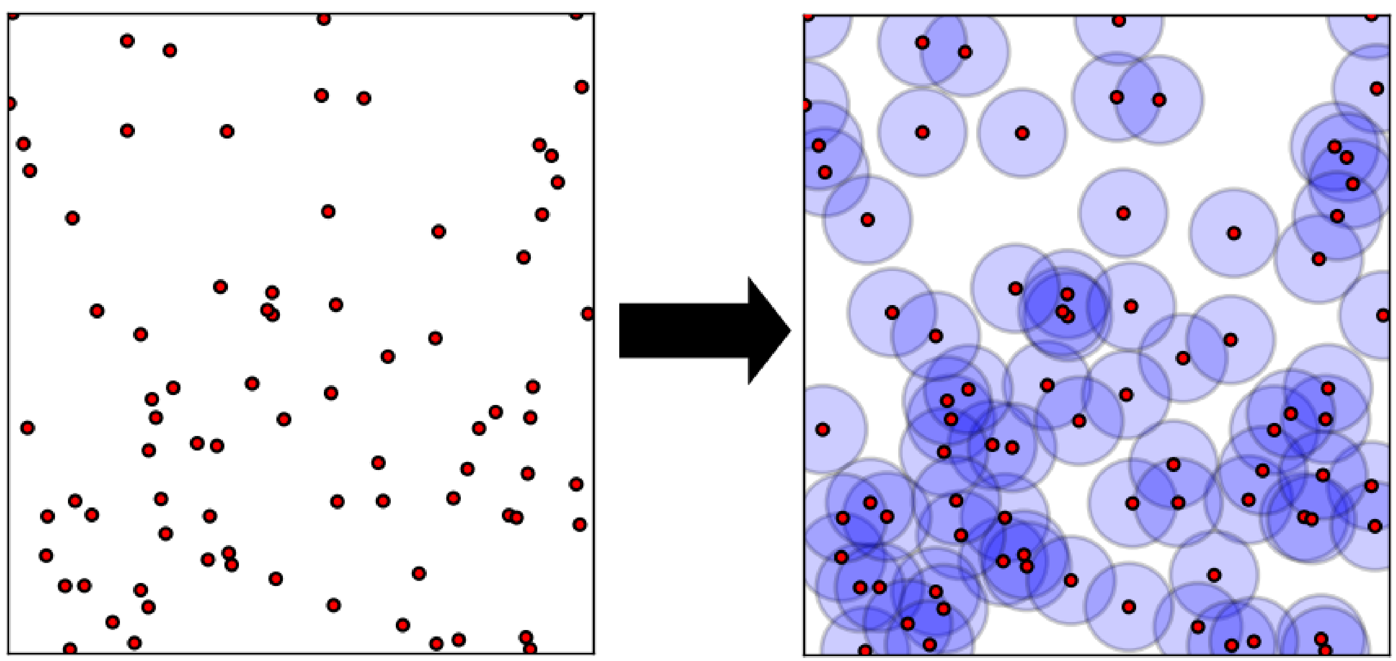
Using the equation of state of inviscid gas and discretizing, we obtain

$$\frac{d\boldsymbol{v}_i}{dt} = -\sum_{j=1}^{N} m_j \left[f_i \frac{P_i}{\rho_i^2} \nabla_i W_{ij}(h_i) + f_j \frac{P_j}{\rho_j^2} \nabla_i W_{ij}(h_j) \right]$$

Smoothed-Particle Hydrodynamic

Computational performance is greatly improved if we use a finite-range kernel:

$$W\left(\frac{r}{2h}\right) = W(q) = \frac{8}{\pi}\begin{cases} 1 - 6q^2 + 6q^3, & 0 \le q \le 1/2, \\ 2(1-q)^3, & 1/2 < q \le 1, \\ 0, & q > 1 \end{cases}$$

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