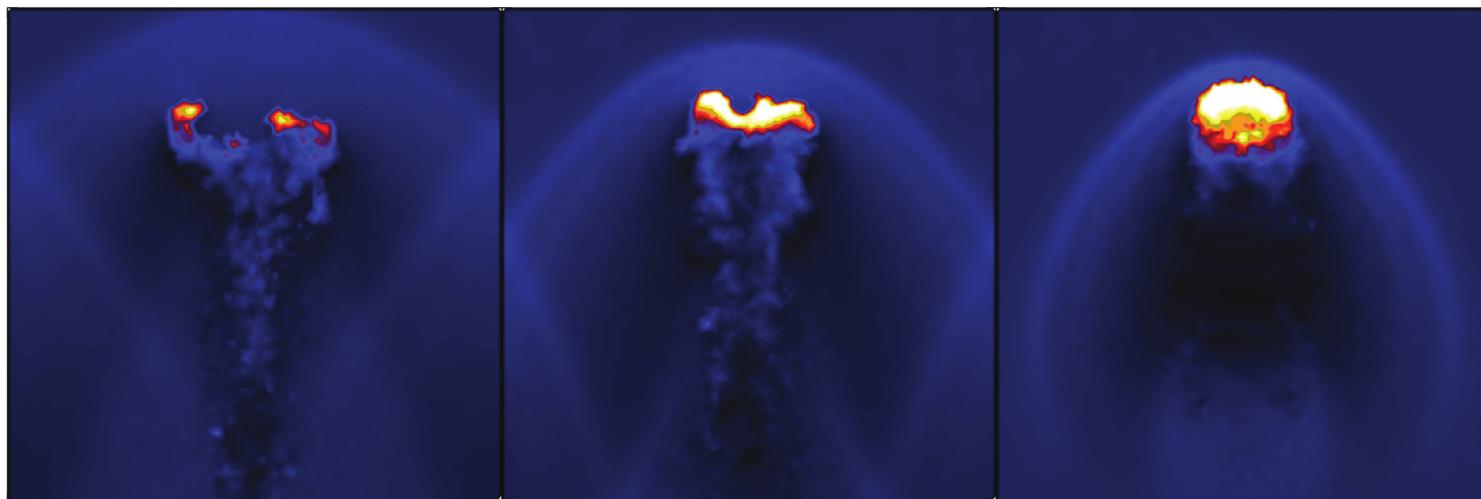


Verifying the Voronoi-Particle Hydrodynamics (VPH) scheme in Galaxy Formation

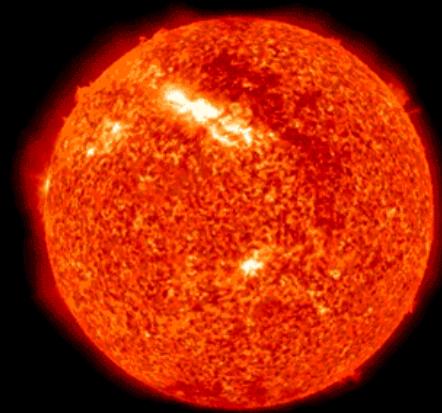
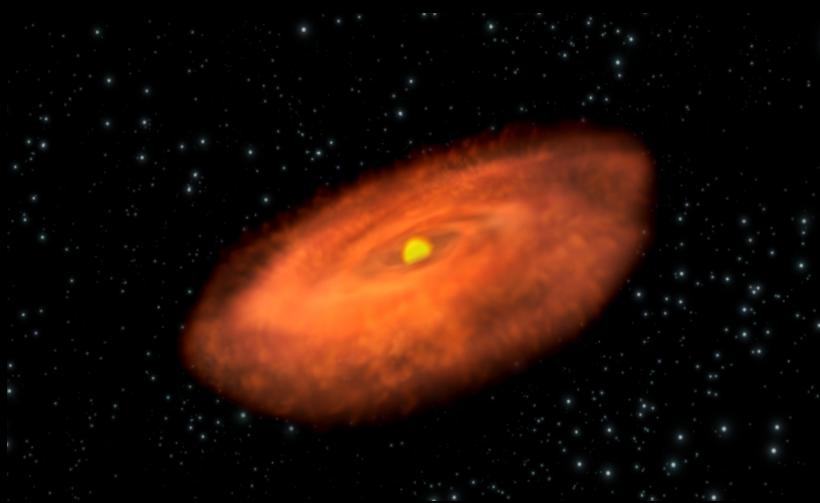


Sebastian Bustamante
Universidad de Antioquia
Medellín, Colombia

Motivation

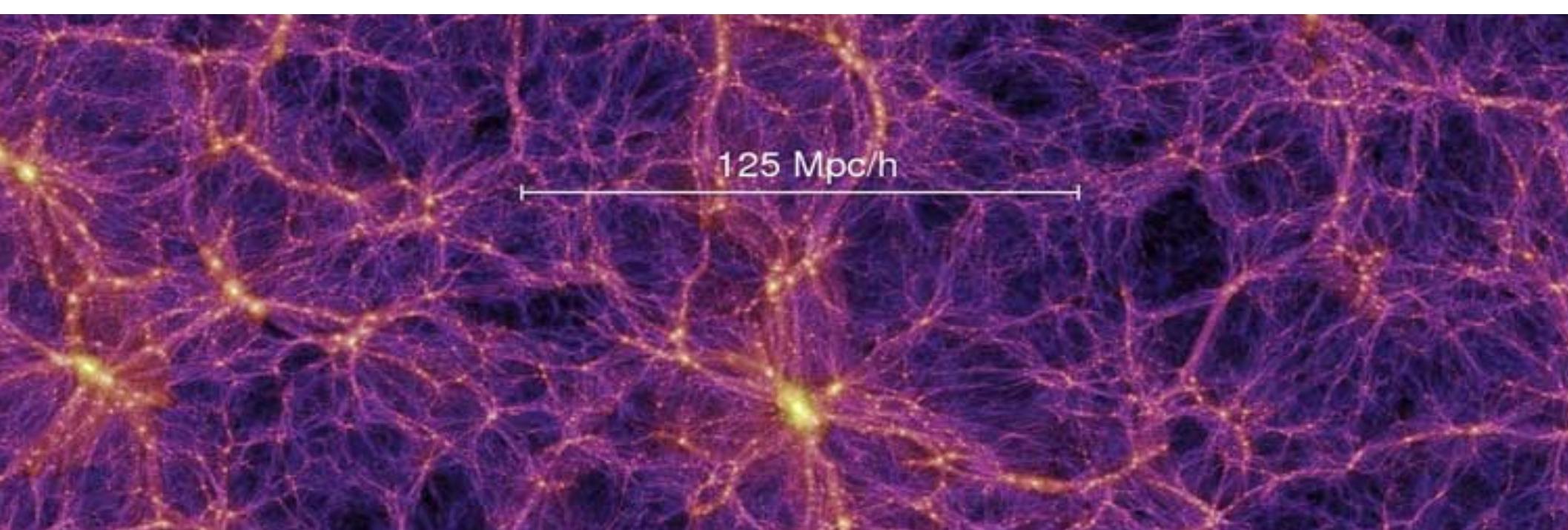
- Understanding and simulating hydrodynamics processes with good accuracy in astrophysics is a very important task as many phenomena are modeled as fluid systems.
- Provided the complexity of the hydrodynamic equations, numerical solutions are usually the only feasible approach.

Motivation



Motivation

The large-scale Universe is a very rich scenario where a plethora of hydrodynamical processes occur. In this fashion, cosmological setups are excellent systems for comparing hydrodynamic solvers.

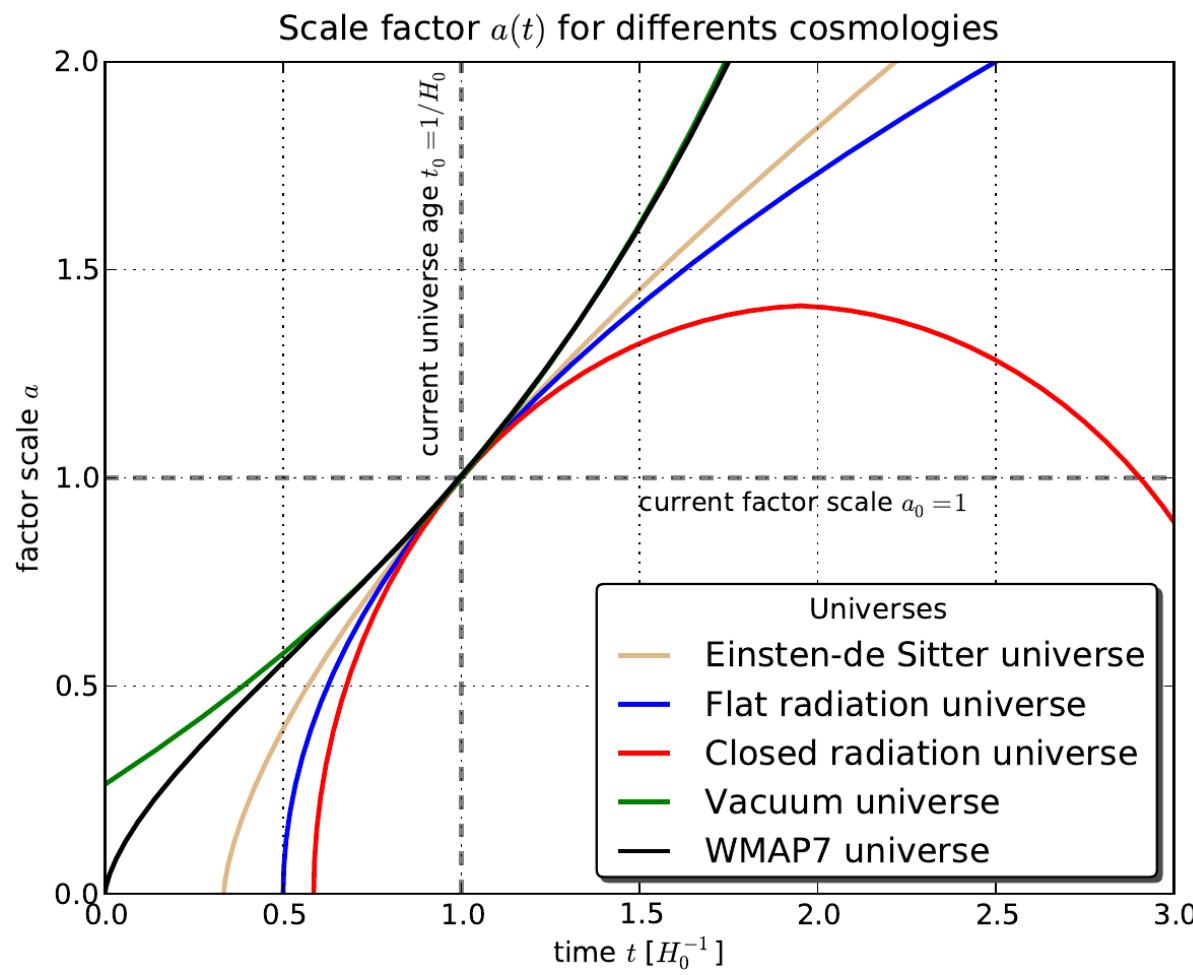


Objectives

- Quantifying the computational performance of the **VPH** technique in terms of its physical accuracy for a cosmological setup.
- Compare the improved accuracy of **VPH** with the Lagrangian scheme **SPH**.

The cosmological setup

The Universe at very large scales can be modeled as a homogeneous and isotropic fluid.



The cosmological setup

However, a homogeneous and isotropic Universe can not account for complex structures like galaxies, stars, planets and us!

Continuity equation

$$\frac{\partial \delta}{\partial t} = -\frac{1}{a} \nabla_r \cdot [(1 + \delta) \mathbf{v}]$$

Euler's equation

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{\dot{a}}{a} \mathbf{v} + \frac{1}{a} (\mathbf{v} \cdot \nabla_r) \mathbf{v} = -\frac{\nabla_r P}{a \bar{\rho}(1 + \delta)} - \frac{1}{a} \nabla_r \Phi$$

Poisson's equation

$$\nabla_r^2 \Phi = 4\pi G \bar{\rho} a^2 \delta$$

$$\rho = \bar{\rho} + \delta \rho = \bar{\rho}(1 + \delta)$$

The cosmological setup

When modes of the density field are small enough ($\delta \ll 1$), it is possible to decouple them, obtaining the next equation:

$$\frac{d^2\delta_{\mathbf{k}}}{dt^2} + 2\frac{\dot{a}}{a}\frac{d\delta_{\mathbf{k}}}{dt} = \left[4\pi G\bar{\rho} - \frac{c_s^2}{a^2}k^2\right]\delta_{\mathbf{k}}$$

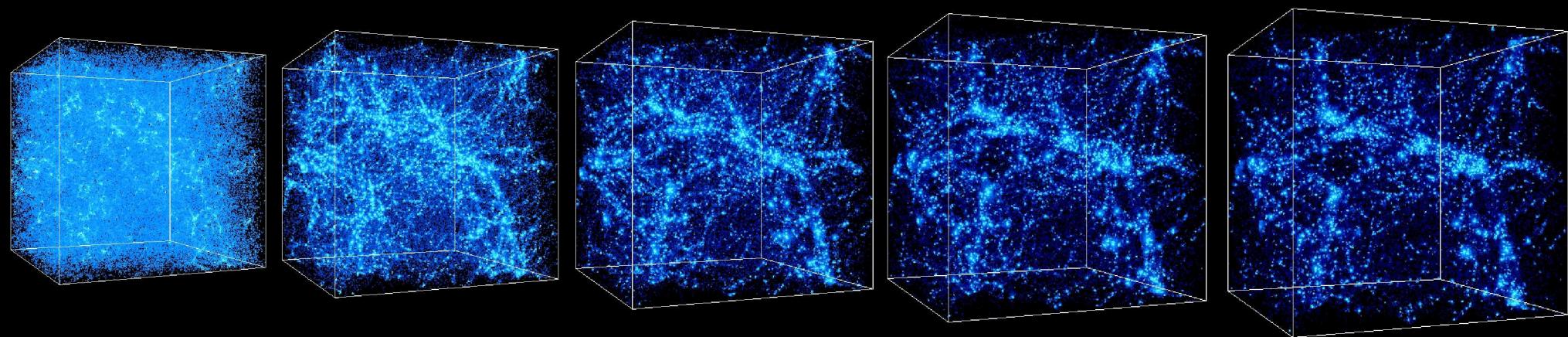
Obtaining the analytical solution for the lineal regime:

$$\delta_{\mathbf{k}}(t) = \delta_{\mathbf{k}}(0)D(t)$$

$$D(a) = \frac{5}{2}\Omega_m \left[\Omega_m^{4/7} - \Omega_{\Lambda} + \left(1 + \frac{\Omega_m}{2}\right) \left(1 + \frac{\Omega_{\Lambda}}{70}\right) \right]^{-1}$$

The cosmological setup

For the non-linear regime, the Fourier modes of the density are not longer coupled. Numerical evolution is necessary henceforth. However, quasi-linear solutions provide initial conditions for numerical schemes.



Hydrodynamic solvers

Once clarified the importance of the hydrodynamics in Galaxy formation (for evolving the non-linear regime), two families of numerical approaches has been adopted throughout the literature:

- **Lagrangian schemes:** the fluid is evolved following the dynamics of fluid parcels.
- **Eulerian schemes:** the evolution performed over a fixed coordinate system, namely a mesh.

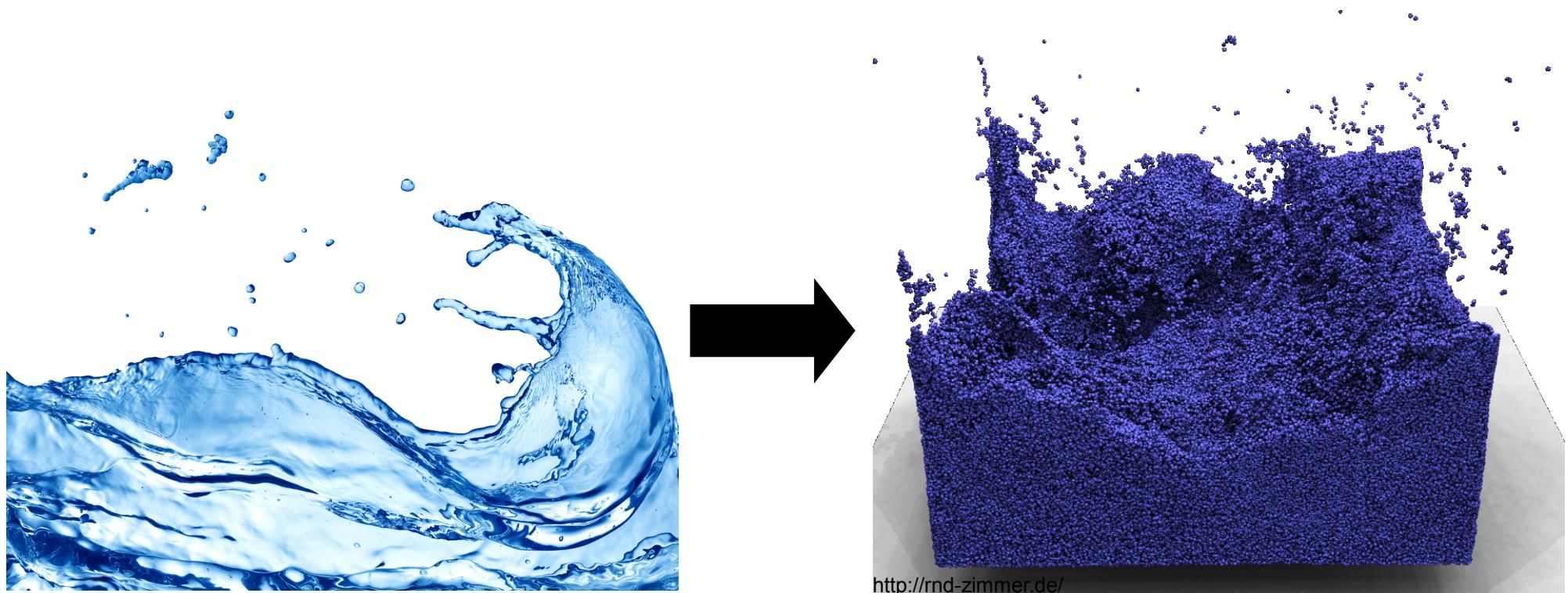
Lagrangian schemes

- The most representative Lagrangian scheme is **SPH** (Smoothed-Particle Hydrodynamic).
- Other schemes, normally modifications of the original **SPH**, have been also implemented.
- Among the most interesting proposals is the **VPH** (Voronoi-Particle Hydrodynamic) scheme. This scheme greatly improves the accuracy of **SPH**.

Smoothed-Particle Hydrodynamic

- Although a fluid is a continuous medium, numerical solutions always requires some kind of discretization.
- **SPH** reaches this by sampling the fluid through particles that represent a parcel.
- The equations of motion are then integrated over each of moving particles.
- Continuous properties are then recovered through interpolation techniques.

Smoothed-Particle Hydrodynamic



<http://rnd-zimmer.de/>

Smoothed-Particle Hydrodynamic

The heart of **SPH** is the kernel interpolant formalism:

$$F_s(\mathbf{r}) = \int_V \mathbf{F}(\mathbf{r}') W(\mathbf{r} - \mathbf{r}'; h) d\mathbf{r}'$$

Discretization can be reached approximating
 $\Delta\mathbf{r}_i = m_i/\rho_i$, yielding:

$$F'_i \equiv F_s(\mathbf{r}_i) \approx \sum_j^{N_{tot}} \frac{m_j}{\rho_j} F_j W(\mathbf{r}_i - \mathbf{r}_j; h_i)$$

Where h_j is the smoothing length and is associated to the volume of the fluid parcel.

Smoothed-Particle Hydrodynamic

The equations of motions may be derived from the Lagrangian

$$\mathcal{L} = \int \rho \left(\frac{v^2}{2} - u \right) dV$$

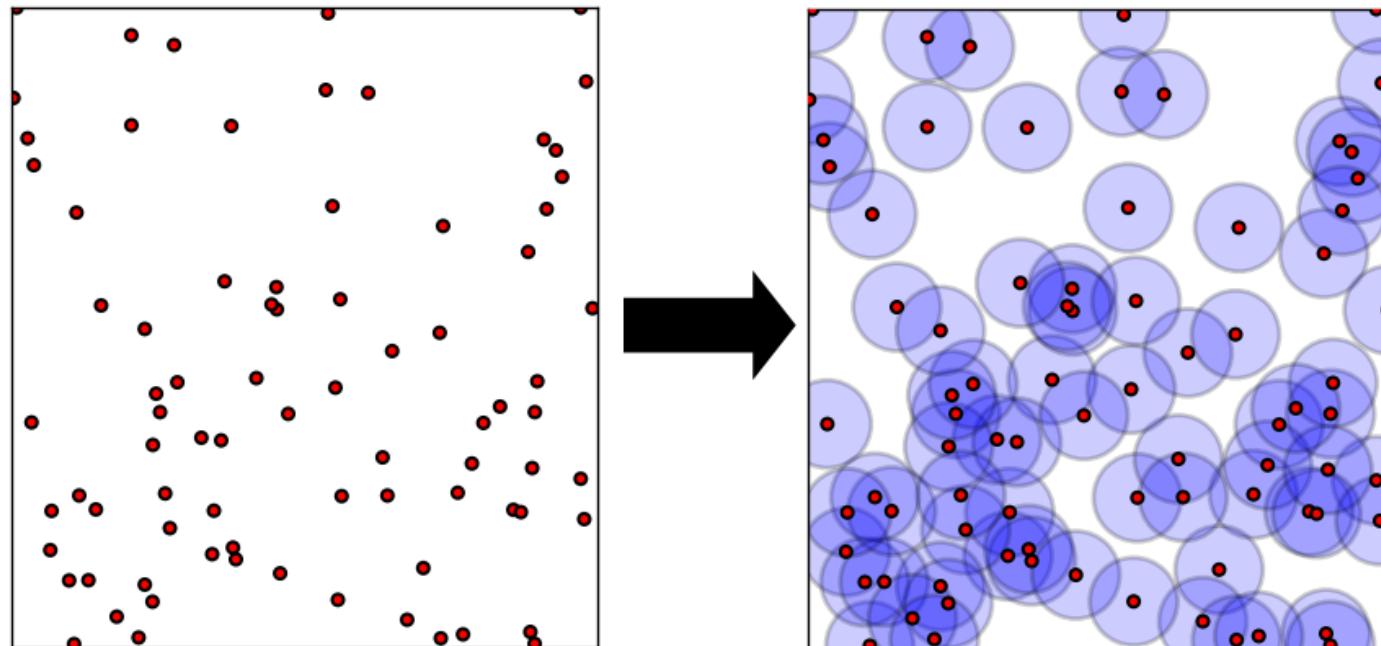
Using the equation of state of inviscid gas and discretizing, we obtain

$$\frac{d\mathbf{v}_i}{dt} = - \sum_{j=1}^N m_j \left[f_i \frac{P_i}{\rho_i^2} \nabla_i W_{ij}(h_i) + f_j \frac{P_j}{\rho_j^2} \nabla_i W_{ij}(h_j) \right]$$

Smoothed-Particle Hydrodynamic

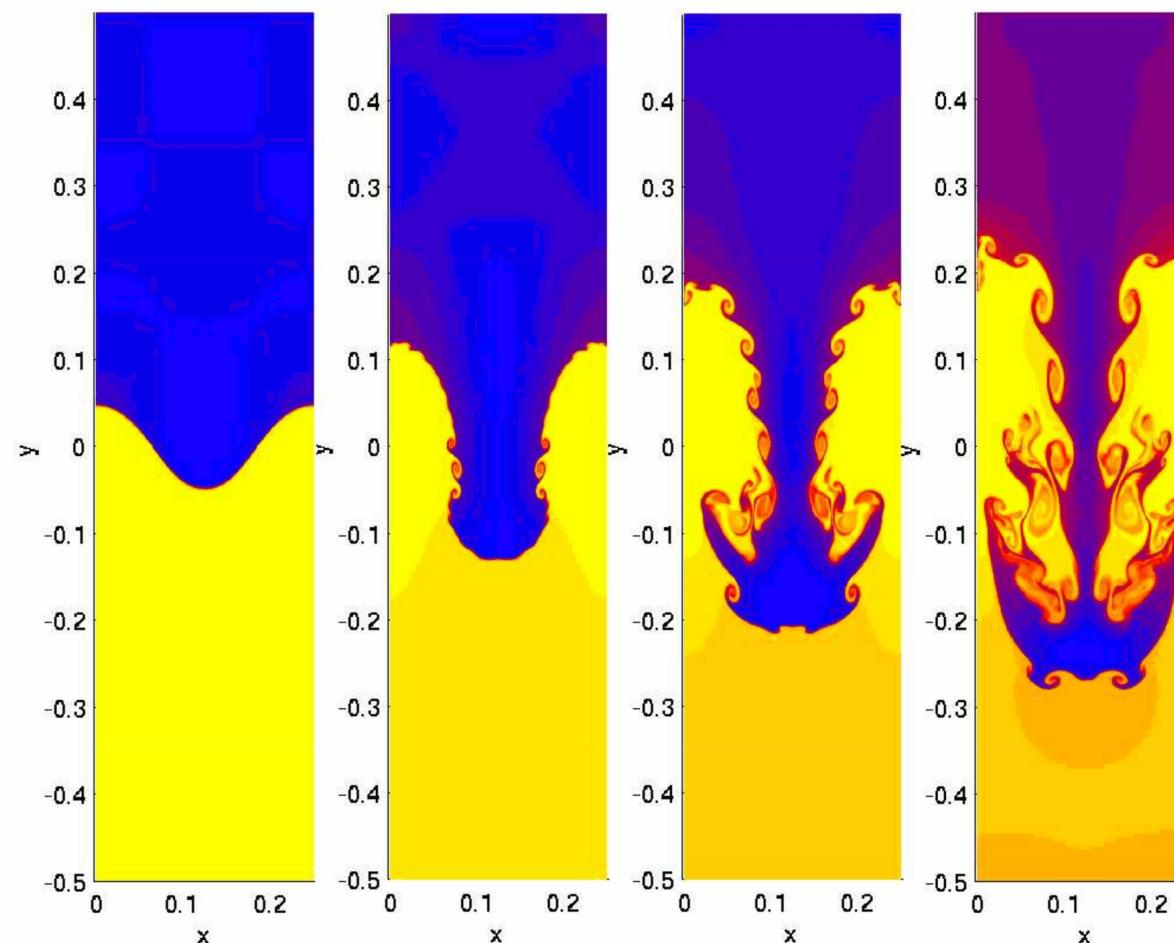
Computational performance is greatly improved if we use a compact kernel:

$$W\left(\frac{r}{2h}\right) = W(q) = \frac{8}{\pi} \begin{cases} 1 - 6q^2 + 6q^3, & 0 \leq q \leq 1/2, \\ 2(1 - q)^3, & 1/2 < q \leq 1, \\ 0, & q > 1 \end{cases}$$



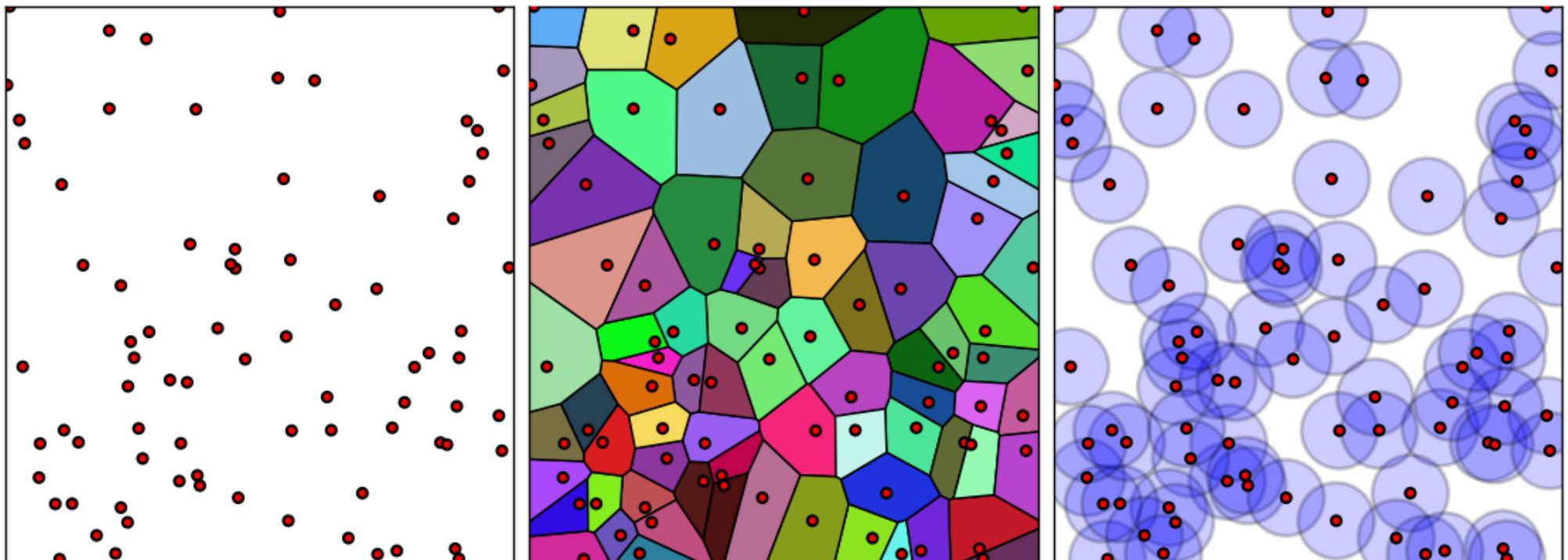
Smoothed-Particle Hydrodynamic

Due to the interpolant formalism, **SPH** produces suppressions in fluid instabilities.



Voronoi-Particle Hydrodynamic

VPH is a modification of **SPH**, where the density estimation is made by means of a Voronoi Tesselation.



Voronoi-Particle Hydrodynamic

Using the same fluid Lagrangian

$$\mathcal{L} = \int \rho \left(\frac{v^2}{2} - u \right) dV$$

discretizing and replacing the new density estimation

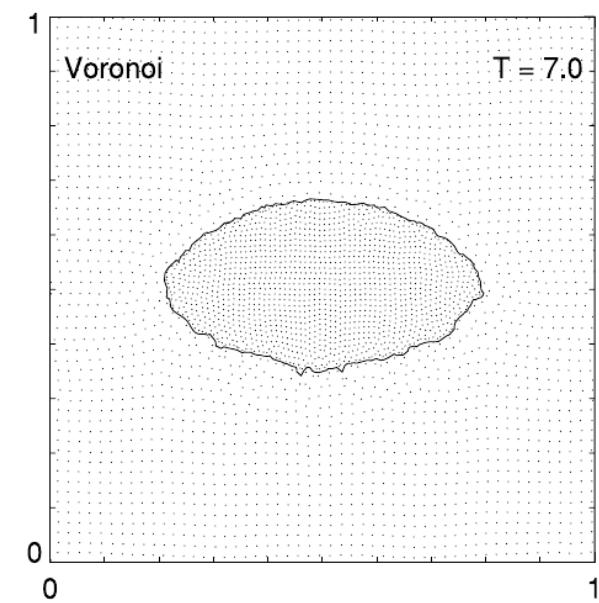
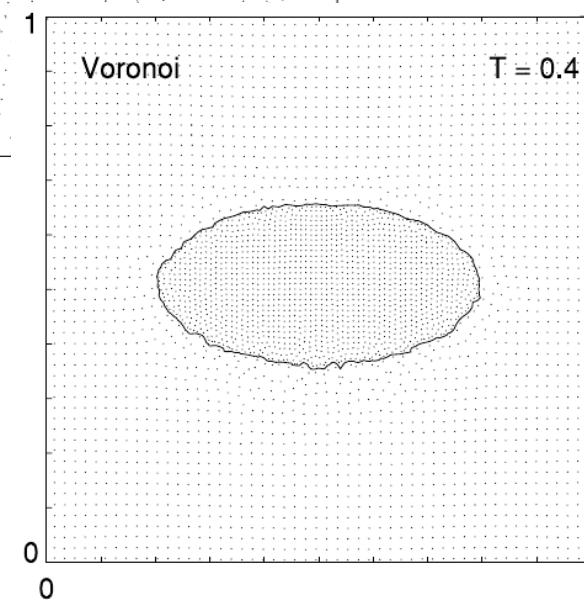
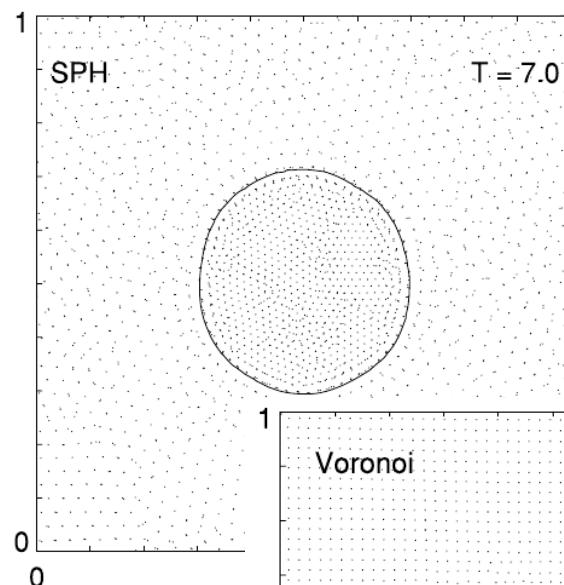
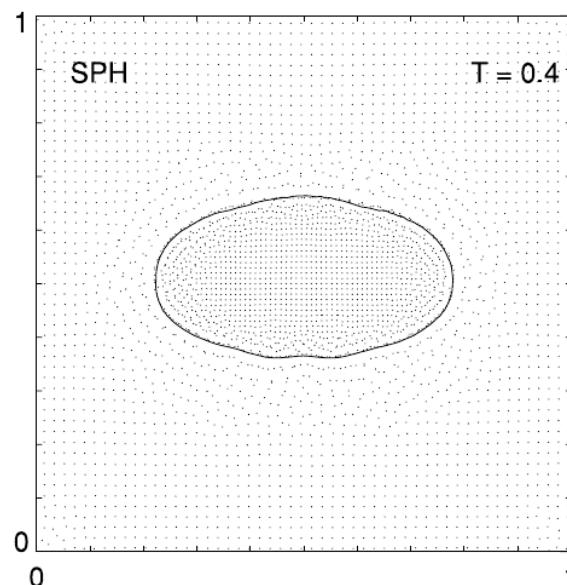
$$\rho_i = \frac{m_i}{V_i}$$

It is obtained the next equation of motion

$$m_i \frac{d\mathbf{v}_i}{dt} = - \sum_{i \neq j}^N A_{ij} \left[(P_i + P_j) \frac{\mathbf{e}_{ij}}{2} + (P_j - P_i) \frac{\mathbf{c}_{ij}}{R_{ij}} \right]$$

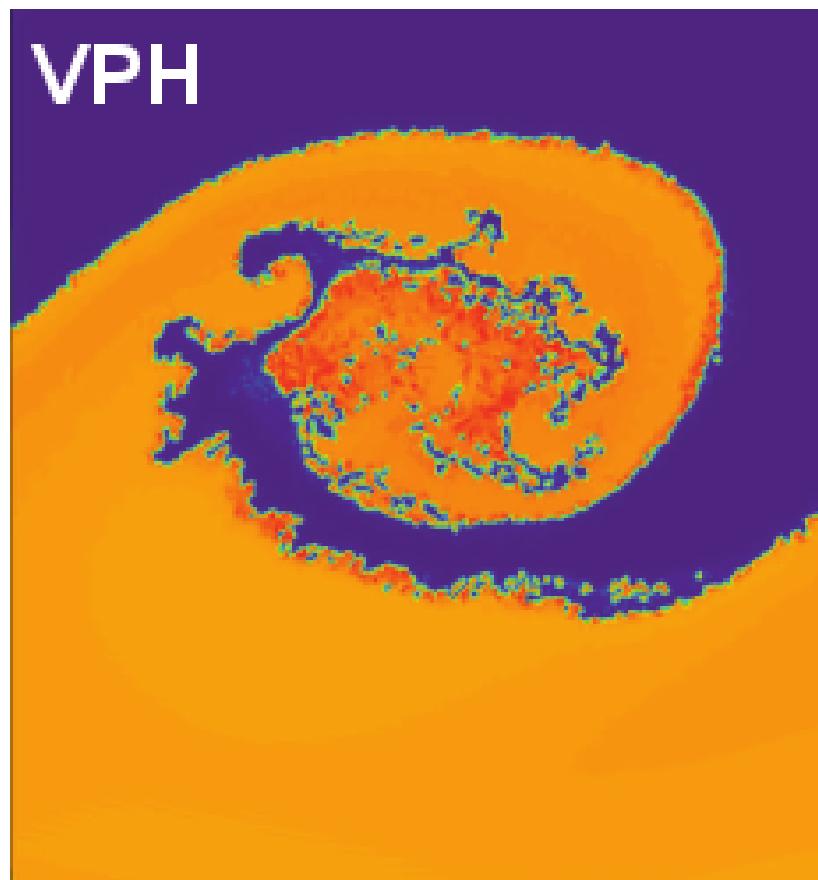
Previous results

Surface tension effects [Hess and Springel 2010]



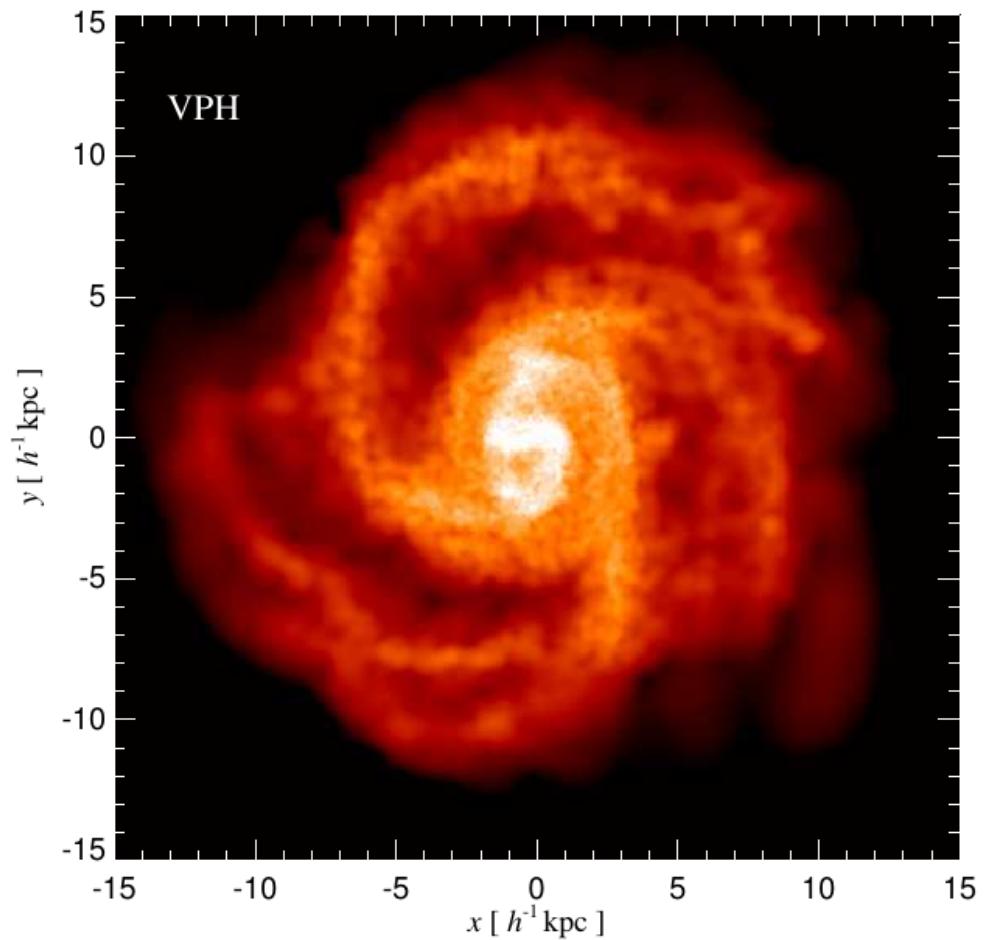
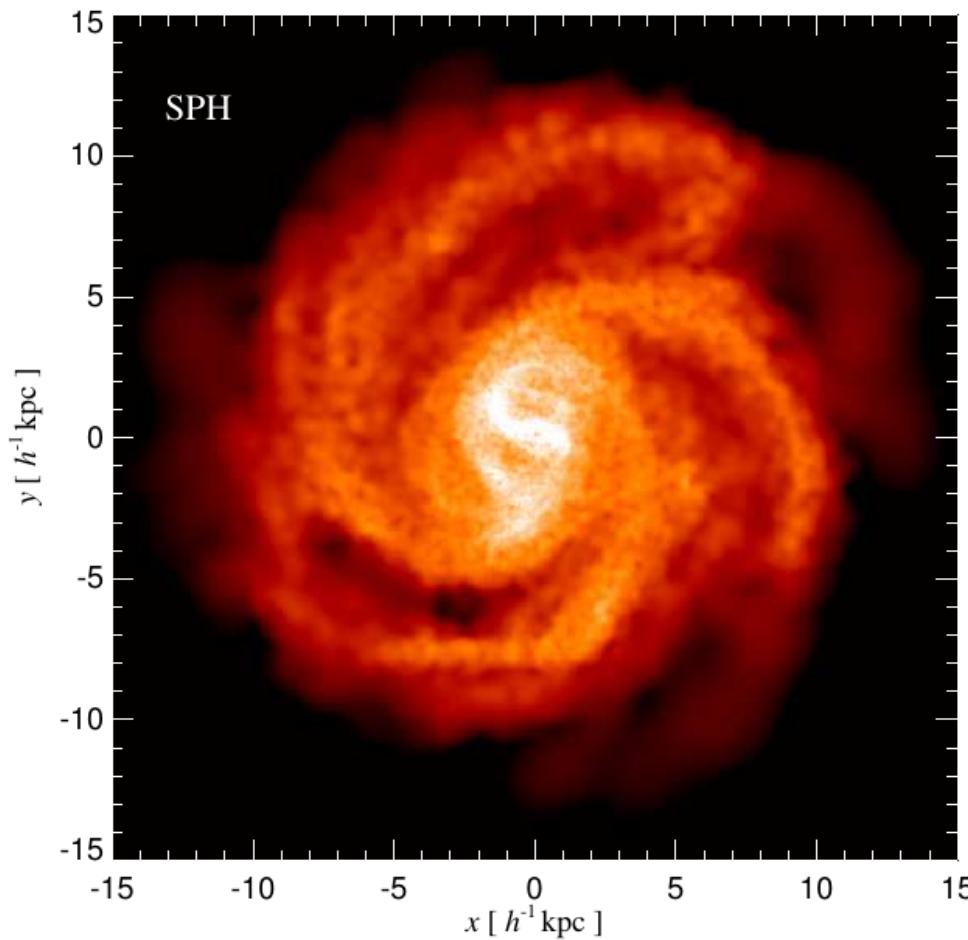
Previous results

Kelvin–Helmholtz instability for both solvers:
[Hess and Springel 2010]



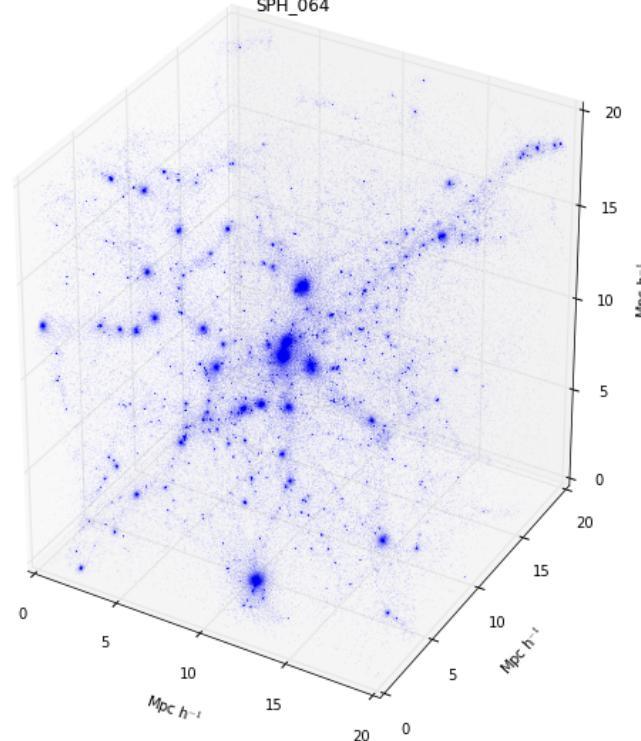
Previous results

Stars forming galaxy simulations [Hess and Springel 2012]

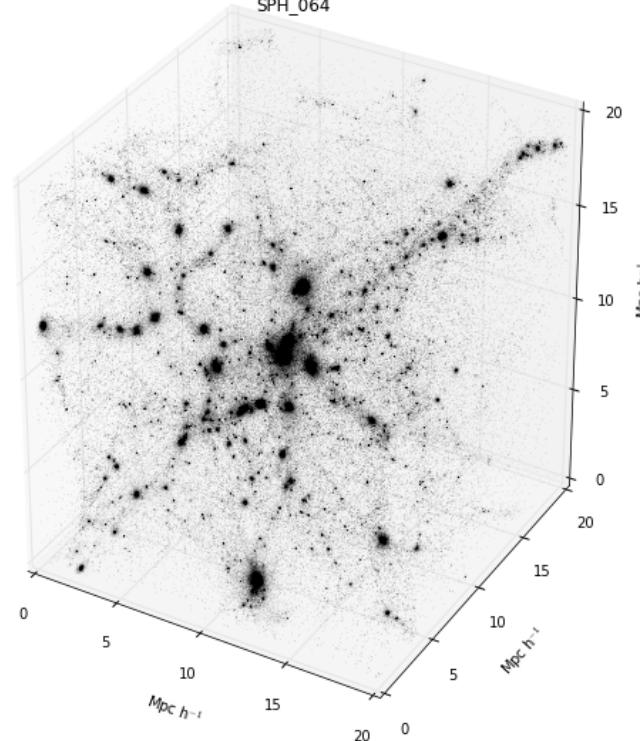


Our (few) results!

SPH_064



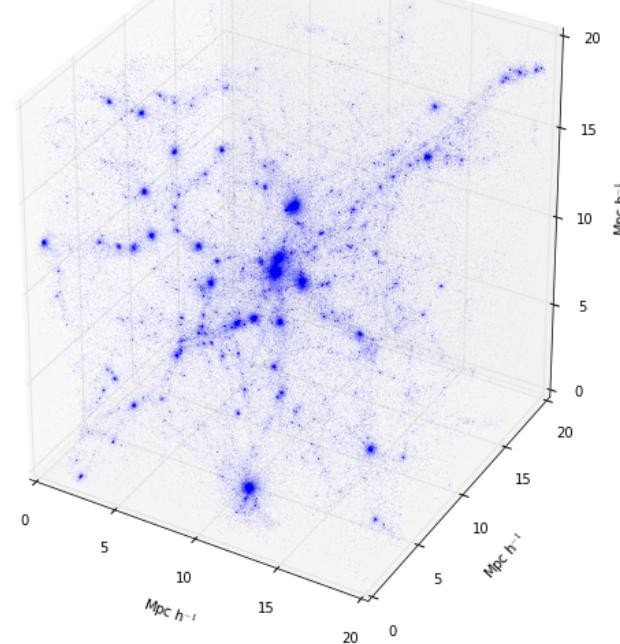
SPH_064



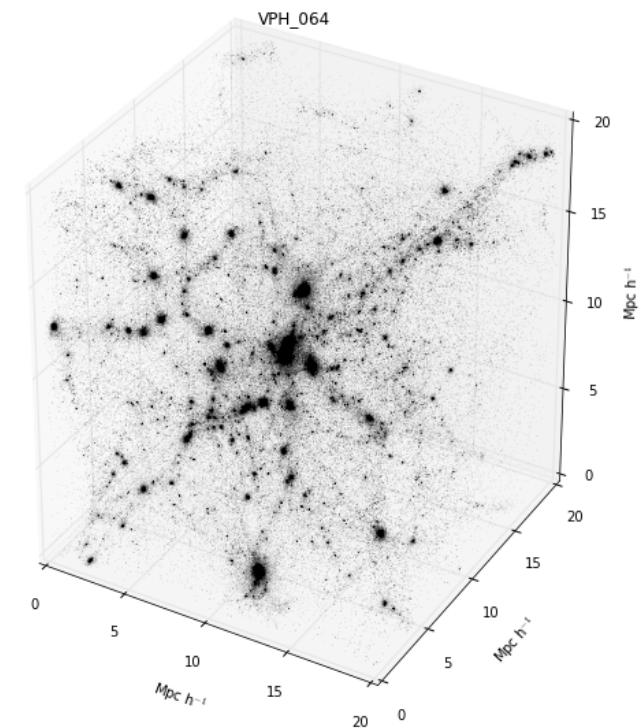
SPH

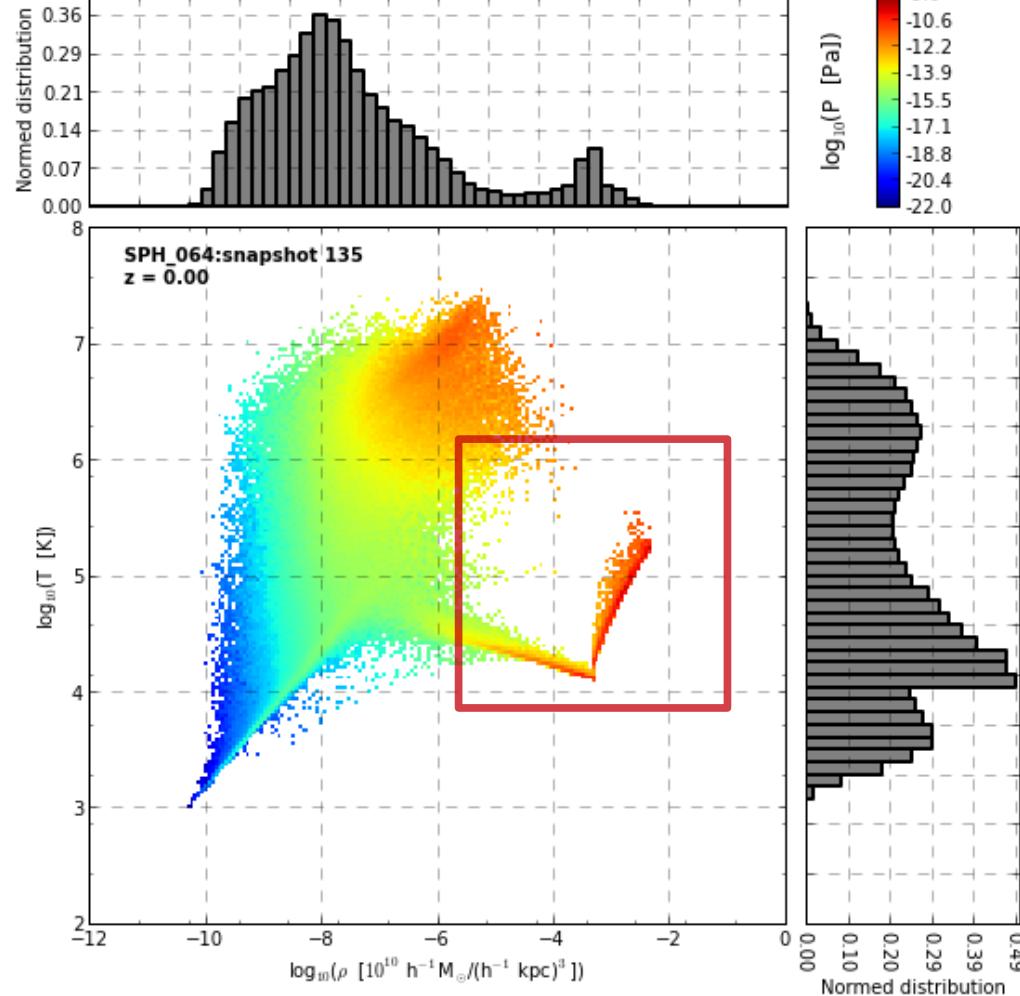
VPH

VPH_064

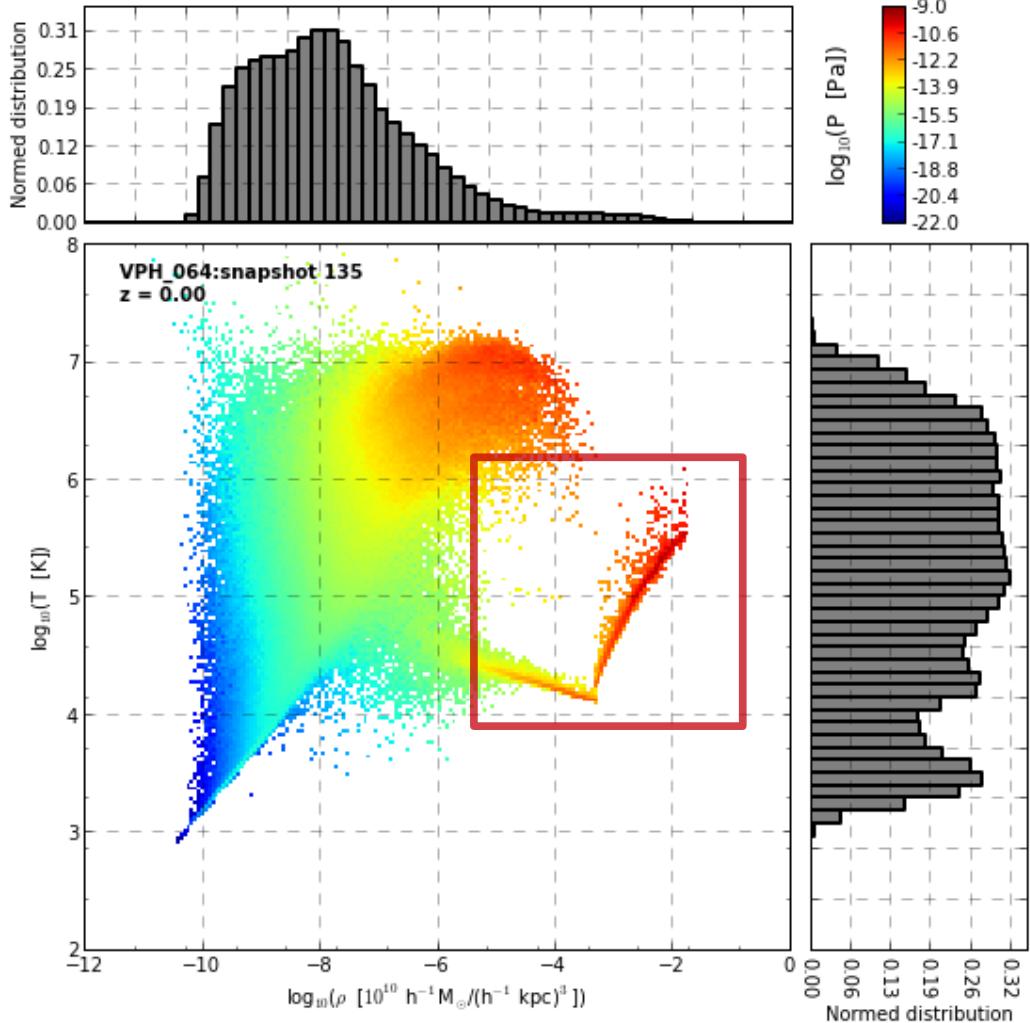


VPH_064





SPH



VPH

Conclusions

- **VPH** reduces spurious suppressions of fluid instabilities exhibited by **SPH**.
- Numerical effects like surface tensions are also diminished by **VPH**.
- For our results, **VPH** seems to be more efficient for radiative cooling than **SPH**, what is an improvement as Galaxy formation is more likely to occur in such a scenario. However further analysis are required for more robust conclusions!

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Arigato
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