A1

z.z. $\forall n, m \in \mathbb{N}_0$: $f(n,m)$ ist definiert

I.A: $n = 0$

$f(0,m) = m+1$ ✓.

I.S. Gilt auch für $n+1$

$f(n+1, m) = f(n, f(n+1, m-1))$

↳ terminiert gemäß I.V.

$f(n+1, m-1) = f(n, f(n+1, m-2))$

↳ terminiert gemäß I.V.

$f(n+1, m-2) = f(f(n), f(n+1, m-3))$

↳ terminiert gemäß I.V.

In jedem Schritt wird m um eins verringert bis $m=0$

$f(n+1, 0) = f(n, 1)$

↳ m wird wieder verringert bis $m=0$

↳ terminiert gemäß I.V.

~~$f(n-2, 0) = f(n-2, 0)$~~

~~↳ n wird verringert bis n = 0~~

□

AD - Blatt 4

A2
(1) $f(n) = \sum_{i=0}^{n} (2i+1) = 2 \sum_{i=0}^{n} i + n = 2 \frac{n(n+1)}{2} + n$

$= n^2 + n + n = n^2 + 2n$

$g(n) = 3n^2 + 17$

$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{n^2+2n}{3n^2+17} = \lim_{n\to\infty} \frac{1 + \frac{2}{n}}{3 + \frac{17}{n^2}} = \frac{1}{3} = const.$

$\Rightarrow f(n) \in O(g(n))$

$\lim_{n\to\infty} \frac{g(n)}{f(n)} = \lim_{n\to\infty} \frac{3n^2+17}{n^2+2n} = 3 = const$

$\Rightarrow g(n) \in O(f(n))$

$\Rightarrow \Theta(g(n)) = \Theta(f(n))$

(2)

$f(n) = \log(n!) = \sum_{k=1}^{n} \log(k) \leq n \cdot \log(n)$

$g(n) = 3n \log n^{5n} = 15n^2 \log n$

1. A $n = 1$

$1 \cdot f(1) = \log(1) < 15 \cdot \log 1 = g(1)$

1. S. z.z. $\forall n \in \mathbb{N}: 1 \cdot f(n) < g(n) \Rightarrow 1 \cdot f(n+1) < g(n+1)$

$1 \cdot f(n+1) = \log((n+1)!) = \sum_{k=1}^{n+1} \log(k)$ ~~$< (n+1)\log(n) < (n+1)^2 \log n < 15(n+1)^2 \log$~~ $< \sum_{k=1}^{n+1} \log(n+1) = (n+1) \log(n+1)$

$< (n+1)^2 \log(n+1) < 15 \cdot (n+1)^2 \log(n+1) = g(n+1)$

$\Rightarrow f(n) \in O(g(n))$

(3) $f(n) = 2^{n+5} + n^2$

$g(n) = \sum_{i=0}^{n} 2^i \leq 2^{n+1}$

I.A. $n = 1$

$\sum_{i=0}^{1} 2^i = 1 + 2 = 3$

$2^{n+1} = 2^2 = 4 \geq 3$ ✓

I.S. Inv. $\forall n \in \mathbb{N}$: $\sum_{i=0}^{n} 2^i \leq 2^{n+1} \Rightarrow \sum_{i=0}^{n+1} 2^i \leq 2^{n+2}$

$\sum_{i=0}^{n+1} 2^i = \sum_{i=0}^{n} 2^i + 2^{n+1} \overset{I.V.}{\leq} 2^{n+1} + 2^{n+1} = 2 \cdot 2^{n+1} = 2^{n+2}$ □

$\lim_{n\to\infty} \frac{g(n)}{f(n)} \leq \lim_{n\to\infty} \frac{2^{n+1}}{2^{n+5} + n^2} =$

$= \lim_{n\to\infty} \frac{\frac{2^{n+1}}{2^{n+5}}}{\underbrace{\frac{2^{n+5}}{2^{n+5}}}_{\to 1} + \frac{n^2}{2^{n+5}}} = \lim_{n\to\infty} \frac{\frac{1}{2^4}}{1} = \frac{1}{16}$

$\Rightarrow g(n) \in O(f(n))$

A2

(4) $f(n) = 3n^2$

$g(n) = 9^{\log_3(n)} = 3^{2\log_3 n} = 3^{(\log_3 n)^2} = n^2$

$\Rightarrow \quad f(n) \in \Theta(g(n))$

A3

$$c_{ij} := \sum_{k=1}^{r} a_{i,k} b_{k,j}$$

Standard-Methode

$$M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \qquad N = \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix}$$

$$O = \cancel{\ldots} = M \cdot N = \begin{pmatrix} o_{1,1} & o_{1,2} \\ o_{2,1} & o_{2,2} \end{pmatrix}$$

$$o_{1,1} = \sum_{k=1}^{2} m_{1,k} n_{k,1} = m_{11} \cdot n_{11} + m_{12} \cdot n_{2,1}$$

$$o_{1,2} = \sum_{k=1}^{2} m_{1,k} \cdot n_{k,2} = m_{1,1} \cdot n_{1,2} + m_{1,2} \cdot n_{2,2}$$

$$o_{2,1} = \sum_{k=1}^{2} m_{2,k} n_{k,1} = m_{2,1} \cdot n_{1,1} + m_{2,2} \cdot n_{2,1}$$

$$o_{2,2} = \sum_{k=1}^{2} m_{2,k} \cdot n_{k,2} = m_{2,1} \cdot n_{1,2} + m_{2,2} \cdot n_{2,2}$$

$\Rightarrow$ Variante 1 = Standardmethode

Variante 2:

$$\begin{aligned} o_{1,1} &:= H_1 + H_4 - H_5 + H_7 \\ &= (m_{11} + m_{22}) \cdot (n_{11} + n_{22}) + m_{22} \cdot (n_{2,1} - n_{11}) - n_{22}(m_{11} + m_{12}) \\ &\quad + (m_{1,2} - m_{22}) \cdot (n_{21} + n_{22}) \\ &= m_{11} \cdot n_{11} + \cancel{m_{22} \cdot n_{11}} + \cancel{m_{11} \cdot n_{22}} + \cancel{m_{22} \cdot n_{22}} + \cancel{m_{22} \cdot n_{21}} - \cancel{m_{22} \cdot n_{11}} \\ &\quad - \cancel{m_{11} \cdot n_{22}} - \cancel{m_{12} \cdot n_{22}} + m_{1,2} \cdot n_{2,1} - \cancel{m_{22} \cdot n_{21}} + \cancel{m_{12} \cdot n_{22}} \\ &\quad - \cancel{m_{22} \cdot n_{22}} = m_{11} n_{11} + m_{12} \cdot n_{2,1} = o_{11} \text{ (von Variante 1)} \end{aligned}$$

$$\begin{aligned} o_{1,2} &:= H_3 + H_5 = m_{11}(n_{1,2} - n_{22}) + n_{22} \cdot (m_{11} + m_{12}) \\ &= m_{11} n_{1,2} - \cancel{m_{11} \cdot n_{22}} + \cancel{m_{11} \cdot n_{22}} + m_{1,2} \cdot n_{22} \\ &= m_{11} \cdot n_{1,2} + m_{1,2} \cdot n_{22} = o_{1,2} \text{ (Variante 1)} \end{aligned}$$

$$\sigma_{2,1} := H_2 + H_4 = (m_{2,1} + m_{22}) \cdot n_{11} + m_{22}(n_{21} - n_{11})$$

$$= m_{2,1} \cdot n_{11} + \cancel{m_{22} n_{11}} + m_{22} \cdot n_{21} - \cancel{m_{22} n_{11}}$$

$$= m_{2,1} \cdot n_{11} + m_{22} \cdot n_{2,1} = \sigma_{2,1} \text{ (Variante 1)}$$

$$\sigma_{2,2} := H_1 - H_2 + H_3 + H_6$$

$$= (m_{11} + m_{22}) \cdot (n_{11} + n_{22}) - n_{11}(m_{21} + m_{22})$$

$$+ m_{11}(n_{1,2} - n_{2,2}) + (m_{21} - m_{11}) \cdot (n_{11} + n_{1,2})$$

$$= \cancel{m_{11} n_{11}} + \cancel{m_{22} n_{11}} + m_{22} \cdot n_{22} - \cancel{m_{21} n_{11}} - \cancel{m_{22} n_{11}}$$

$$+ \cancel{m_{11} n_{1,2}} - \cancel{m_{11}} \cdot n_{22} + \cancel{m_{21} n_{11}} - \cancel{m_{11} n_{11}}$$

$$+ m_{2,1} \cdot n_{1,2} - \cancel{m_{11} \cdot n_{1,2}}$$

$$= m_{2,1} \cdot n_{1,2} + m_{22} \cdot n_{22} = \sigma_{2,2} \text{ (Variante 1)}$$

~~Variante 2 ist Variante 1~~

=> Variante 2 ist Variante 1 mit mehr Hilfsmatrizen

=> Variante 2 berechnet das gleiche wie Variante 1 und damit wie die Standardmethode

~~Variante 1:~~

~~$T(n) = 4T(\frac{n}{2}) + 8n$~~

Master ~~Methode:~~ ~~$a = 4, b = 2, f(n) = \ldots$~~

A 3 : asymptotische Laufzeit

Variante 1:

$T(n) = 8 \cdot T(\frac{n}{2}) + 4n^2$

Master-Methode: $a = 8$, $b = 2$, $f(n) = (n^2)$

$\log_b a = \log_2 8 = 3 > 2 \Rightarrow T(n) \in \Theta(n^3)$

gleich wie Standard-Methode

Variante 2:

$T(n) = 7 \cdot T(\frac{n}{2}) + 18n^2$

Master Methode: $a = 7$, $b = 2$, $f(n) = n^2$

$\log_b a = \log_2 7 > 2$

$\Rightarrow T(n) \in \Theta(n^{2,81})$

besser als Standard-Methode,

erst für große Matrizen, da $18n^2$