OE5500 Term Paper

Added Mass and Damping Coefficient of a typical ship (Barge) section in Heave, Sway & Roll motion

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1 Theory

1.1 Introduction

Box barge is a symmetrical square box-barge. Due to symmetry, FEM is applied only to half part of the region, and replicate to the other side. The aim of this term paper is to find the Added Mass and Damping coefficient of a typical 2D ship section in different modes of motion, mainly in heave, sway and roll motion, using Finite Element analysis of a Boundary value problem.

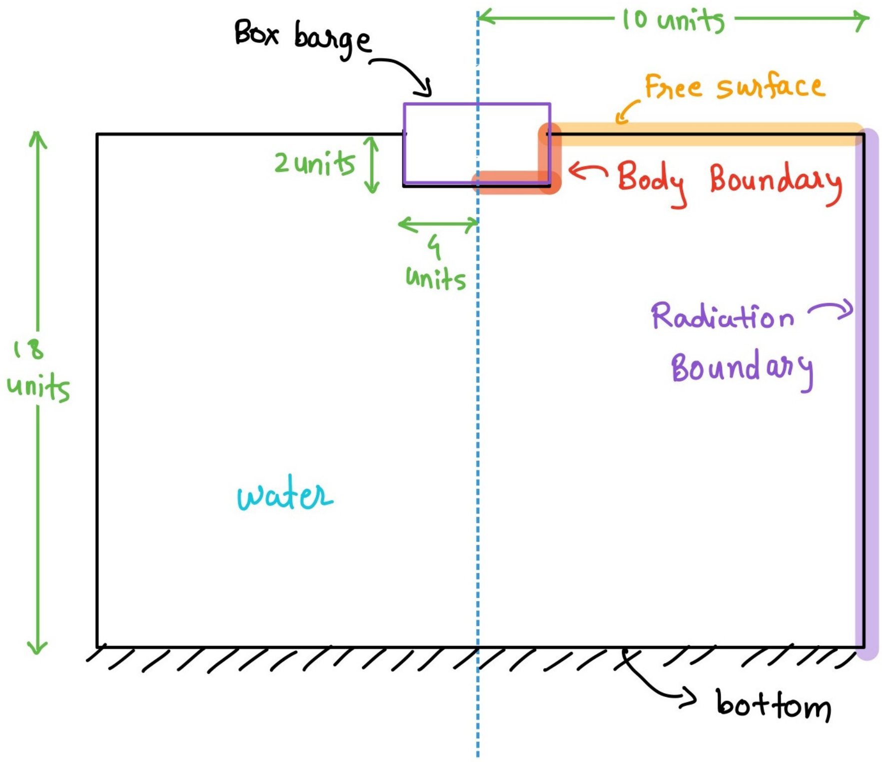

Figure 1: Box barge of the problem statement

1.2 Assumptions

1. Linear Wave theory is considered for this problem and the body is considered as rigid.
2. The fluid is assumed to be ideal and the flow is assumed to be irrotational.
3. There is no flow of energy through either bottom/free surface of the body.
4. The energy is gained or lost by the system only through waves arriving or departing at infinity or the external forces acting on the body.
5. The induced motions are small so that the body boundary conditions are satisfied very close to the equilibrium position of the body.

2 Formulation of problem

2.1 Problem Statement

The problem statement is to calculate the Hydrodynamic characteristics, namely added mass, added mass coefficient, and damping coefficient, in three different modes of motion: Heave, Sway, and Roll. This is achieved through the finite element method involving radiation and diffraction phenomena.

2.2 Governing Equation

The diffraction-radiation problem for this two-dimensional (2D) body in oblique incident waves is sinusoidal in the direction of the body axis so that the 3D Laplace equation can be reduced to a 2D Helmholtz equation. The boundary value problem for the diffracted potential can be defined by the governing Helmholtz equation as given below,

$$\nabla^2\phi = 0 \tag{1}$$

2.3 Boundary Conditions

Boundary conditions are both practically necessary for specifying a problem and of critical relevance. Boundary condition are also shown in the diagram above Figure 1. Below are the boundary condition used to solve the problem statement :

1. Free Surface Boundary Condition:
$$\frac{\partial\phi}{\partial z} = \frac{\omega^2}{g}\phi \tag{2}$$
2. Bottom Boundary Conditions :
$$\frac{\partial\phi}{\partial z} = 0 \tag{3}$$
3. Radiation Boundary Conditions :
$$\frac{\partial\phi}{\partial x} \pm ik\phi = 0 \tag{4}$$
4. Body Boundary Conditions :
$$\frac{\partial\phi}{\partial n} = n_z \tag{5}$$
6. Symmetric Boundary Conditions :
$$\frac{\partial\phi}{\partial x} = 0 \tag{6}$$
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2.4 Applying Boundary Conditions

Galerkin's form of governing equation is given as

$$\iint_\Omega N_i\phi d\Omega = 0 \tag{7}$$

Now applying green-Gauss theorem

$$-\iint_\Omega \left(\frac{\partial N_i}{\partial x}\cdot\frac{\partial\phi}{\partial x}\right) + \left(\frac{\partial N_i}{\partial z}\cdot\frac{\partial\phi}{\partial z}\right) d\Omega + \int_\tau N_i\frac{\partial\phi}{\partial n}d\tau = 0 \tag{8}$$

Applying boundary conditions

$$\int_\tau (0)d\tau + \int_\tau N_i(\pm ik\phi)d\tau + \int_\tau N_i\frac{-\partial\phi}{\partial n}d\tau + \int_\tau N_i\left(\frac{\omega^2\phi}{g}\right)d\tau - \iint_\Omega \left(\frac{\partial N_i}{\partial x}\cdot\frac{\partial\phi}{\partial x}\right) + \left(\frac{\partial N_i}{\partial z}\cdot\frac{\partial\phi}{\partial z}\right) d\Omega = 0 \tag{9}$$

$$\underbrace{\pm ik\int_\tau N_iN_jd\tau\{\phi_j\}}_{\text{Radiation Boundary condition}} + \underbrace{\int_\tau N_i\left(-\frac{\partial\phi}{\partial ns}\right)d\tau}_{\text{Body Boundary condition}} + \underbrace{\frac{\omega^2}{g}\int_\tau N_iN_jd\tau\{\phi_j\}}_{\text{Free Surface condition}} - \iint \left(\frac{\partial N_i}{\partial x}\cdot\frac{\partial\phi}{\partial x}\right) + \left(\frac{\partial N_i}{\partial z}\cdot\frac{\partial\phi}{\partial z}\right) d\Omega = 0 \tag{10}$$

3 FEM Solution

3.1 Discretization

Divide the Region **S** into **E** finite rectangular elements of 4 nodes each to discretized the domain surface. Division of region into rectangular nodes is done as per the diagram shown below :

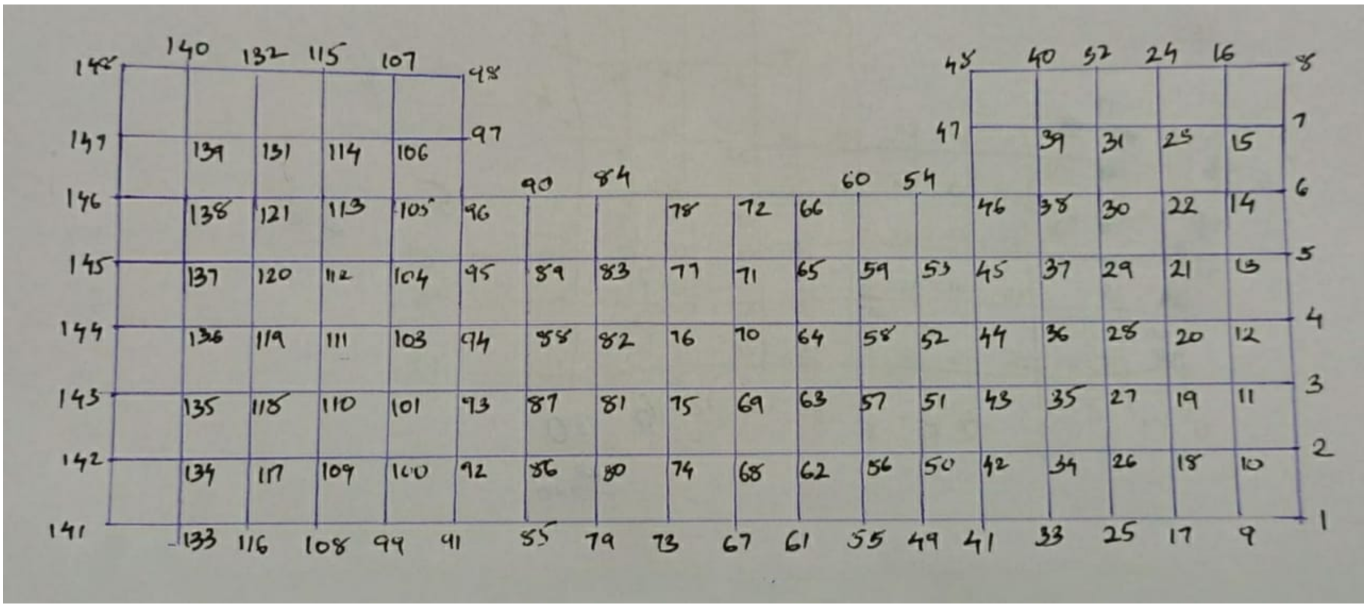

Figure 2: Finite Element Discretization of barge

3.2 K-Matrix

Assuming x and z as

$$x = N_1x_1 + N_2x_2 + N_3x_3 + N_4x_4 \tag{11}$$
$$z = N_1z_1 + N_2z_2 + N_3z_3 + N_4z_4 \tag{12}$$

Now, rewriting above equations in the matrix form

$$\begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix} \begin{bmatrix} x_1 \\ z_1 \\ x_2 \\ z_2 \\ x_3 \\ z_3 \\ x_4 \\ z_4 \end{bmatrix} = [N]\{\delta\} \tag{13}$$

Where N_1, N_2, N_3 and N_4 are shape functions for each node, given as

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta) \tag{14}$$
$$N_2 = \frac{1}{4}(1+\xi)(1-\eta) \tag{15}$$
$$N_3 = \frac{1}{4}(1+\xi)(1+\eta) \tag{16}$$
$$N_4 = \frac{1}{4}(1-\xi)(1+\eta) \tag{17}$$

To relate natural coordinates with global co-ordinates, using chain rule i.e.

$$\frac{\partial N}{\partial \xi} = \frac{\partial N}{\partial x} \cdot \frac{\partial x}{\partial \xi} + \frac{\partial N}{\partial z} \cdot \frac{\partial z}{\partial \xi} \tag{18}$$

$$\frac{\partial N}{\partial \eta} = \frac{\partial N}{\partial x} \cdot \frac{\partial x}{\partial \eta} + \frac{\partial N}{\partial z} \cdot \frac{\partial N}{\partial \eta} \tag{19}$$

Now, rewriting above equations in the matrix form,

$$\begin{Bmatrix} \dfrac{\partial N}{\partial \xi} \\ \\ \dfrac{\partial N}{\partial \eta} \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial x}{\partial \xi} & \dfrac{\partial z}{\partial \eta} \\ \\ \dfrac{\partial x}{\partial \eta} & \dfrac{\partial z}{\partial \eta} \end{bmatrix} \cdot \begin{Bmatrix} \dfrac{\partial N}{\partial x} \\ \\ \dfrac{\partial N}{\partial x} \end{Bmatrix} = [J] \cdot \begin{Bmatrix} \dfrac{\partial N}{\partial x} \\ \\ \dfrac{\partial N}{\partial x} \end{Bmatrix} \tag{20}$$

Where [**J**] is the Jacobian matrix for transformation which is given by

$$[\boldsymbol{J}] = \begin{bmatrix} \dfrac{\partial x}{\partial \xi} & \dfrac{\partial x}{\partial \eta} \\ \\ \dfrac{\partial z}{\partial \xi} & \dfrac{\partial z}{\partial \eta} \end{bmatrix} \tag{21}$$

Now,

$$\frac{\partial x}{\partial \xi} = \frac{\partial N_1}{\partial \xi} x_1 + \frac{\partial N_2}{\partial \xi} x_2 + \frac{\partial N_3}{\partial \xi} x_3 + \frac{\partial N_4}{\partial \xi} x_4 \tag{22}$$

$$\tag{23}$$

$$\frac{\partial z}{\partial \xi} = \frac{\partial N_1}{\partial \xi} z_1 + \frac{\partial N_2}{\partial \xi} z_2 + \frac{\partial N_3}{\partial \xi} z_3 + \frac{\partial N_4}{\partial \xi} z_4 \tag{24}$$

$$\tag{25}$$

$$\frac{\partial x}{\partial \eta} = \frac{\partial N_1}{\partial \eta} x_1 + \frac{\partial N_2}{\partial \eta} x_2 + \frac{\partial N_3}{\partial \eta} x_3 + \frac{\partial N_4}{\partial \eta} x_4 \tag{26}$$

$$\tag{27}$$

$$\frac{\partial z}{\partial \eta} = \frac{\partial N_1}{\partial \eta} z_1 + \frac{\partial N_2}{\partial \eta} z_2 + \frac{\partial N_3}{\partial \eta} z_3 + \frac{\partial N_4}{\partial \eta} z_4 \tag{28}$$

$$\tag{29}$$

$$\tag{30}$$

$$\frac{\partial N_1}{\partial \xi} = -\frac{1}{4}(1-\eta) \tag{31}$$

$$\frac{\partial N_2}{\partial \xi} = \frac{1}{4}(1-\eta) \tag{32}$$

$$\frac{\partial N_3}{\partial \xi} = \frac{1}{4}(1+\eta) \tag{33}$$

$$\frac{\partial N_4}{\partial \xi} = -\frac{1}{4}(1+\eta) \tag{34}$$

$$\frac{\partial N_1}{\partial \eta} = -\frac{1}{4}(1-\xi) \tag{35}$$

$$\frac{\partial N_2}{\partial \eta} = -\frac{1}{4}(1+\xi) \tag{36}$$

$$\frac{\partial N_3}{\partial \eta} = \frac{1}{4}(1+\xi) \tag{37}$$

$$\frac{\partial N_4}{\partial \xi} = -\frac{1}{4}(1-\xi) \tag{38}$$

$$[B] = \begin{bmatrix} \dfrac{\partial N_1}{\partial x} & \dfrac{\partial N_2}{\partial x} & \dfrac{\partial N_3}{\partial x} & \dfrac{\partial N_4}{\partial x} \\ \\ \dfrac{\partial N_1}{\partial z} & \dfrac{\partial N_2}{\partial z} & \dfrac{\partial N_3}{\partial z} & \dfrac{\partial N_4}{\partial z} \end{bmatrix} = [\boldsymbol{J}]^{-1} \begin{bmatrix} \dfrac{\partial N_1}{\partial \xi} & \dfrac{\partial N_2}{\partial \xi} & \dfrac{\partial N_3}{\partial \xi} & \dfrac{\partial N_4}{\partial \xi} \\ \\ \dfrac{\partial N_1}{\partial \eta} & \dfrac{\partial N_2}{\partial \eta} & \dfrac{\partial N_3}{\partial \eta} & \dfrac{\partial N_4}{\partial \eta} \end{bmatrix} \tag{39}$$

$$\boldsymbol{D} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{40}$$

$$[K]^e = \int_{-1}^{1}\int_{-1}^{1} B^T\, D\, B\, |\boldsymbol{J}|\, d\xi\, d\eta \tag{41}$$

4 Calculations of Boundary conditions for Heave, Sway and Roll

4.1 Radiation Boundary condition

First term in the equation (10) defines radiation boundary condition, given by $\pm i\, k \int_\tau N_i N_j d\tau\{\phi_j\}$. Now, assuming unit value for k equation for element stiffness matrix can be written as

$$K^{(e)} = \pm i \int_{-1}^{1} N^T\, N\, |J| d\eta \tag{42}$$

Thus, element stiffness matrix is given by

$$K^{(I)} = -\iota \int_{-1}^{1} \begin{bmatrix} \frac{1}{2}(1-\eta) \\ \frac{1}{2}(1+\eta) \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2}(1-\eta) & \frac{1}{2}(1+\eta) \end{bmatrix} \times 0.5\, d\eta \tag{43}$$

Using Gauss Quadrature rule for each element, the element stiffness matrix can be calculated. But we have used the numerical indefinite integration using the *int* command in MATLAB to compute the integration. This boundary condition calculation is same for all modes i.e. for Heave, Sway and Roll because Radiation boundary is same.

4.2 Free Surface Boundary Condition

The third term in the equation (10) represents the Free Surface Boundary condition, which is

$$\frac{\omega^2}{g} \int_\tau N_i N_j d\tau\{\phi_j\} \tag{44}$$

Now, assuming unit value for $\dfrac{\omega^2}{g}$, element stiffness matrix ($K^{(e)}$) is given by

$$K^{(e)} = \int_{-1}^{1} N^T\, N\, |J|\, d\eta \tag{45}$$

Here, determinant of Jacobian Matrix is half of the length of the element i.e. $|J| = \dfrac{L}{2}$. Thus, element stiffness matrix is given by

$$K^{(I)} = \int_{-1}^{1} \begin{bmatrix} \frac{1}{2}(1-\xi) \\ \frac{1}{2}(1+\xi) \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2}(1-\xi) & \frac{1}{2}(1+\xi) \end{bmatrix} \times d\eta \tag{46}$$

Using Gauss quadrant rule (1 point method) , stiffness matrix for each element can be calculated. This Boundary condition calculation is same for all i.e. for Heave, Sway and Roll because Free surface is same for all modes of motion.

4.3 Body Boundary Condition

Second term in the equation (10) represents the Body Boundary Condition, which is

$$\int_\tau N_i \left(-\frac{\partial \phi}{\partial ns}\right) d\tau \tag{47}$$

Now, assuming unit heave velocity

$$K^{(e)} = \int_{-1}^{1} N^T \cos\theta |J| \, d\eta \tag{48}$$

Body Boundary calculation for each mode of motion is not same, because elements comprising body boundary condition will change for each mode of motion resulting in the different matrix with different dimensions. This matrix comprises of two matrix i.e. vertical body boundary and horizontal body boundary.

All the above steps are programmed in MATLAB ver.R2022b

5 Added mass and Damping coefficient

From the stiffness element matrix, the radiation solution matrix is derived. From radiation solution matrix, hydrodynamic force, pressure, added mass, added mass coefficient and damping coefficient are determined using the empirical relations shown below,

5.1 Hydrodynamic force:

Major force in Heave will be vertical and in Sway it will be horizontal. Likewise, for roll motion both will constitute for the radiation solution matrix.

5.1.1 Equation for Vertical Force :

$$F_y = 2 \times \int_{-1}^{1} p \times \hat{n} \, dS = 2 \times \int_{-1}^{1} p \times 1 \times |J| \times d\xi \tag{49}$$

5.1.2 Equation for Horizontal Force :

$$F_x = 2 \times \int_{-1}^{1} p \times \hat{n} \, dS = 2 \times \int_{-1}^{1} p \times 1 \times |J| \times d\eta \tag{50}$$

5.1.3 Force for roll motion :

$$F = F_x + F_y \tag{51}$$

Here, for all modes of motion $|J| = L$ where, L is length of the element. For Heave F is taken as F_y and for Sway F will be F_x. Similarly, for roll $F = F_x + F_y$. Hence, the equations for added mass, added mass coefficient and radiation damping will change for each mode of motion, accordingly. Equations for all the parameters are mentioned below.

5.2 Added Mass (m_a) :

$$m_a = -\frac{\text{Im}(F)}{\omega} \times v_0 = -\frac{\text{Im}(F)}{\omega^2} \times a \tag{52}$$

where a is the principal dimension of the element, which here is taken as the length of the element as we are taking beam elements.

5.3 Added Mass Coefficient (C_m) :

$$C_m = \frac{m_a}{\rho A} = \frac{\text{Re}(F)}{\rho \times A \times a \times \omega^2} \tag{53}$$

5.4 Damping Coefficient (C_d) :

$$C_d = -\frac{\text{Im}(F)}{\rho \times A \times a \times \omega^2} \tag{54}$$

6 Conclusion

Thus, the Added mass, Added mass coefficient and Damping coefficient were found out of a typical 2D ship section (barge) by Finite element analysis of a Boundary value problem using MATLAB software for computation.