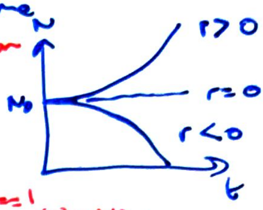
$\frac{dN}{dt} = rN \overset{\text{integration}}{\Longrightarrow} N(t) = N_0 e^{rt}$ $\boxed{\text{exponential growth}}$

$\approx \frac{\Delta N}{\Delta t}$

$\lim_{t \to \infty} N(t)$ depends on r = growth rate or doubling time

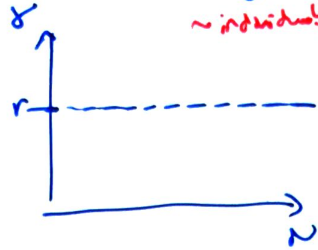
unit is time^{-1}

~ individuals/generation



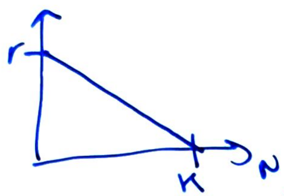
$r = \frac{1}{N}\frac{dN}{dt}$

per capita growth rate




```
if i==1
   population(i) = N0;
else
   population(i) = population(i-1)+dNdt(i-1)
```

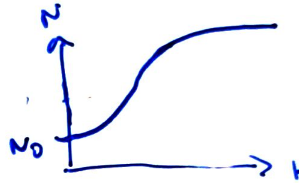
$\boxed{\text{logistic}}$



= carrying effect

$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$

```
dNdt = r*population(i)
```

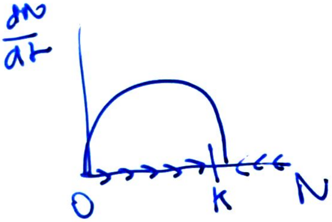


r reaches 0 : N becomes constant

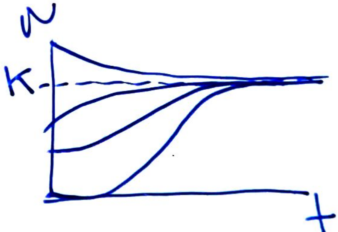
r decreases, $\frac{dN}{dt}$ decreases

when $N=K$, $r=0 \rightarrow \frac{dN}{dt} = 0 \rightarrow N$ is constant over time

when $N=0$, $\frac{dN}{dt} = 0 \rightarrow N$ is constant over time

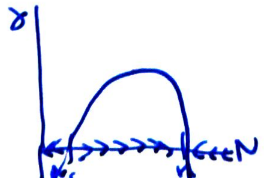


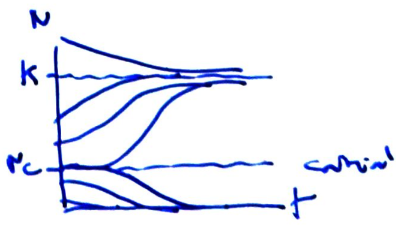
unstable stable




```
dNdt(i) = r*population(i)*(1-population(i)/K)
```

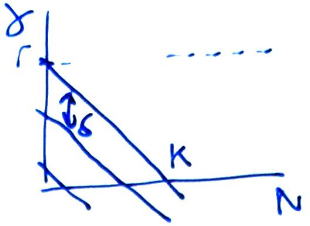
$\boxed{\text{cooperation}}$ = Allee effect



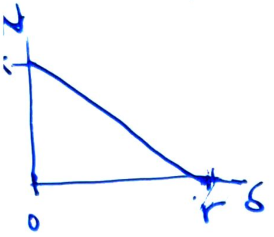


deterioration e.g. fishing

$$\frac{dN}{dt} = rN\left(1-\frac{N}{K}\right) - \delta N$$



$$\gamma = r\left(1-\frac{N}{K}\right) - \delta = 0$$

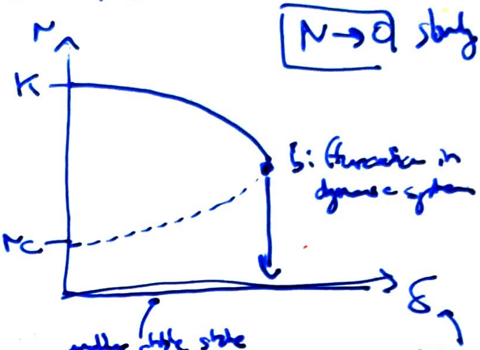


extinction $N = 0$ (or N_c ?)

$\delta = r$

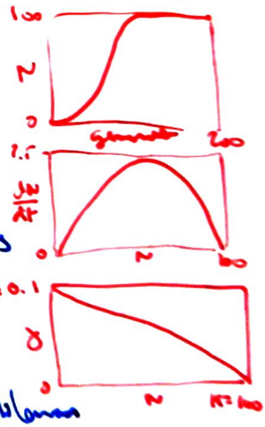
$$N = \left(1-\frac{\delta}{r}\right)K$$

$\boxed{N \to 0 \text{ stable}}$



a stable state · small changes δ → catastrophic collapse

Cooperation (Allee effect) + ↑δ → transition in dynamic systems = sudden collapse



Game theory in biology

- mutations supply strategies
- payoff replaced by fitness/growth rate
- selection on fitness → game solution

Two person game

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} a & b \\ c & d \end{pmatrix} \end{array}$$

← opponent strategy

← payoffs

own strategy

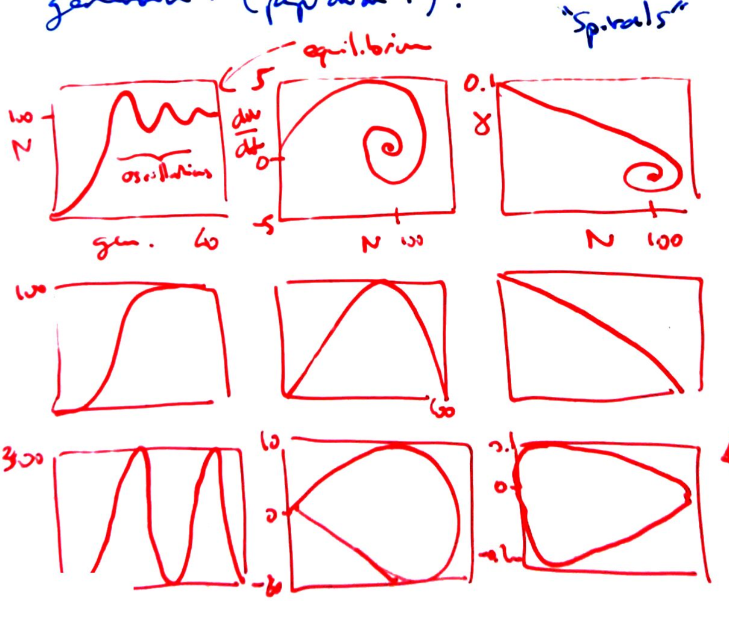
$$\begin{pmatrix} 3 & 0 \\ 5 & 1 \end{pmatrix}$$

Nash equilibrium ≠ prisoner's dilemma

population time lag τ : $\frac{dN}{dt}$ depends both on it's current size and on its size at some past time point

```
for i =1:size(generations 2)
  if i==1
      population(i)=N0;
      dNdt(i) = r*population(i)*(1-population(i)/K);
  else
    if i<=tau
        population(i)=population(i-1)+dNdt(i-1);
        dNdt(i) = r*population(i)*(1-population(i)/K);
    else
        population(i)=population(i-1)+dNdt(i-1);
        dNdt(i) = r*population(i)*(1-population(i-tau)/K);
    end
  end
end
```

for generations 2 through τ (where $1 < i \leq \tau$), the population growth rate of generation i-1 is used to calculate the population of generation i (population i).



N0=1
r=0.1
K=100
tau=10

tau=3

stable oscillations

tau=20

Harvest rate H

$$\frac{dN}{dt} = rN\left(1-\frac{N}{K}\right) - H$$

$\frac{dN}{dt} = 0 \leftarrow$ stable/sustained

$$rN\left(1-\frac{N}{K}\right) = H$$

with $N = N_0 = \frac{K}{2}$ $\rightarrow$ $H = \frac{rK}{2}\left(1 - \frac{\cancel{K}}{2\cancel{K}}\right)$

$\longleftrightarrow$ $\boxed{H = \frac{rK}{4}}$