save

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1 Paper Functions

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These are Python functions which produce the plots for the SC2021 LANL/CCU statistics paper on acceptance testing.

[1]:
```
from scipy.stats import poisson
from scipy.stats import gamma
import numpy as np
import matplotlib.pyplot as plt
import math
from scipy.optimize import fmin
from numpy import array
import pandas as pd
import seaborn as sns
from scipy.optimize import minimize
```

[2]:
```
def pff_calculator(x, F, pff):
    # in R
    # abs(1-ppois(q=q,lambda=x)-pff)
    return abs(1 - poisson.cdf(F, x) - pff)
def one_minus_poisson_cfd(x, F):
    return abs(1 - poisson.cdf(F, x))
def pfp_calculator(x, F, pfp):
    # in R
    # abs(ppois(q=q,lambda=x)-pfp)
    return abs(poisson.cdf(F, x) - pfp)
def poisson_cfd(x, F):
    return abs(poisson.cdf(F, x))
def two_party_optimization(x, F, factor, pfp, pff):
    # in R
    # abs(1-ppois(q=quse,lambda=x)-pff) + abs(ppois(q=quse,lambda=fac*x)-pfp)
    return ( abs(1 - poisson.cdf(F, x) - pff) + abs(poisson.cdf(F, x * factor)␣
 ↪- pfp) )
```

2 Plotting Parameters

Here we define some parameters for plotting.

3 Parameters of Interest

Here we set our initial constraints.

[3]:
```
pff = 0.1 # probability of false failure
pfp = 0.1 # probability of false pass
F = 1 # failures
mu = 24 # hours
factor = 2
```

4 Probability of False Fail

This is related to the vendor (producer) failing a test that they should have passed. They want to minimize that happening.

4.1 Find the Min Theta

First, we need to find the min theta for these above constraints.

[4]:
```
func = pff_calculator
ret = fmin(func, F/2, args=(F, pff), xtol = 0.00001, full_output=True)
the_min_x = ret[0][0]
the_y_val = ret[1]
print(f"The min of function '{func.__name__}' is at {the_min_x} with result␣
 ↪{the_y_val}.")
```

```
Optimization terminated successfully.
         Current function value: 0.000000
         Iterations: 13
         Function evaluations: 26
The min of function 'pff_calculator' is at 0.5318115234374999 with result
2.6544168779674138e-08.
```

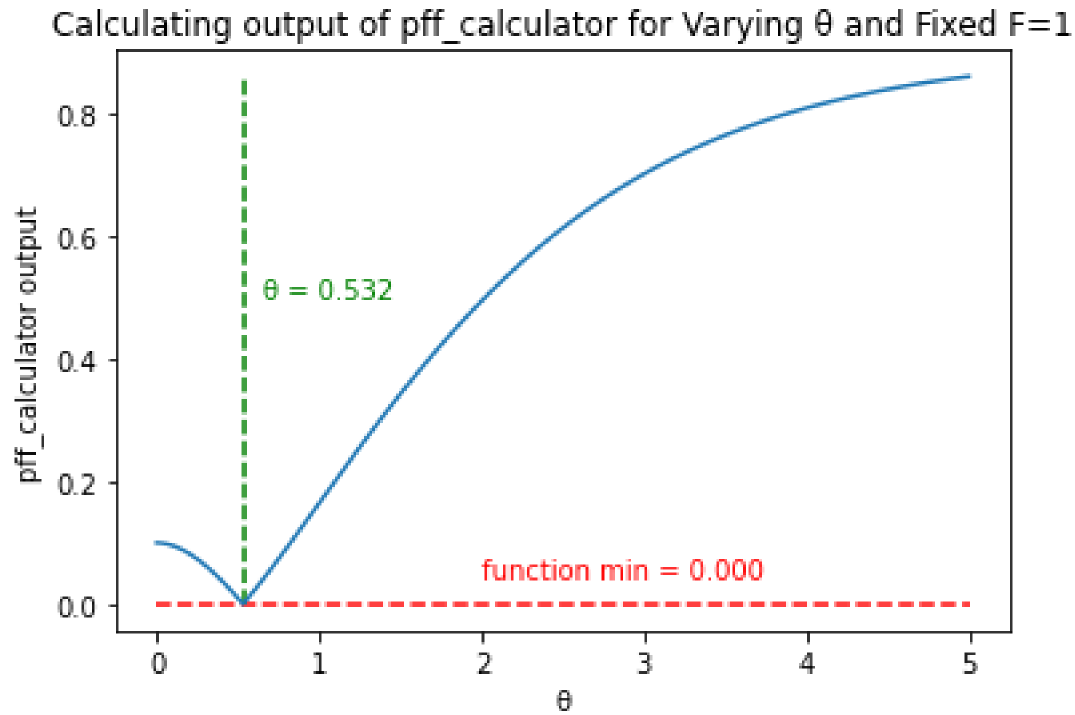
Set up our range of x values, based on what we know the min is - this is just for pretty plotting.

[5]:
```
xmin = 0
xmax = math.floor(the_min_x) * 2
if xmax < 5:
    xmax = 5
points = 10000
# generate x points
xlist = np.linspace(xmin, xmax, points)
```

4.1.1 Plot the Min

This plots the function and annotates the min

[6]:
```python
func = pff_calculator

# map the x points to the values from the above function
ylist = list(map(lambda x: func(x, F=F, pff=pff), xlist))

# plot the results, w/ annotations
plt.plot(xlist, ylist)

# # draw horizontal line at min of function
plt.hlines(y=the_y_val, xmin=min(xlist), xmax=max(xlist), color='red', linestyles="--")
plt.annotate(f"function min = {the_y_val:.3f}", xy=(2, the_y_val + max(ylist) * 0.05), color='red')

# draw vertical line at min
plt.vlines(x=the_min_x, ymin=min(ylist), ymax=max(ylist), color='green', linestyles='--')
plt.annotate(f"  = {the_min_x:.3f}", xy=(the_min_x + max(xlist) * 0.025, 0.5), color='green')

# labels and title
plt.xlabel(" ")
plt.ylabel(f"{func.__name__} output")
plt.title(f"Calculating output of {func.__name__} for Varying   and Fixed F={F}")
plt.savefig(f"NOT_USED_IN_PAPER_pff_calculator_f{F}_pff{pff}.png")
plt.savefig(f"NOT_USED_IN_PAPER_pff_calculator_f{F}_pff{pff}.pdf")
```



4.1.2 Show the Min With Constraints

Going back to our initial distribution, show this point crossing the constraint

[7]:
```
func = one_minus_poisson_cfd

# map the x points to the values from the above function
ylist = list(map(lambda x: func(x, F), xlist))
```

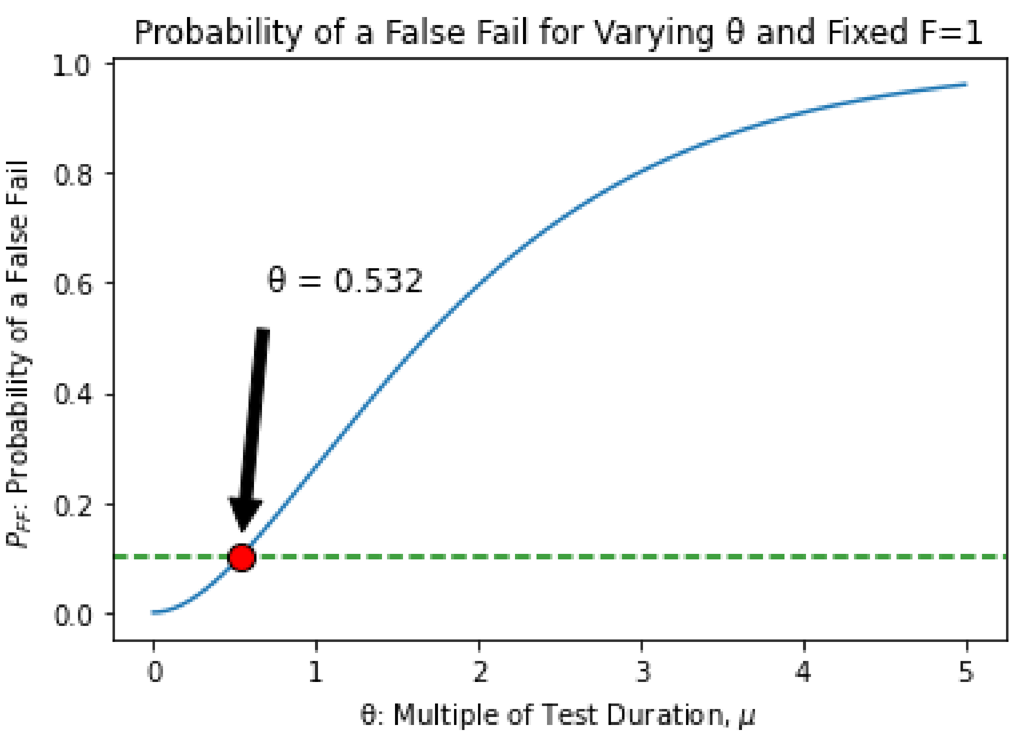
[8]:
```
df = pd.DataFrame()
df['x'] = xlist
df['pff'] = ylist
df.columns = ['theta', '$P_{FF}$']
df.head()
```

[8]:

	theta	P_{FF}
0	0.0000	0.000000e+00
1	0.0005	1.249833e-07
2	0.0010	4.997667e-07
3	0.0015	1.124100e-06
4	0.0020	1.997735e-06

[9]:
```
fig, ax = plt.subplots(figsize=(6, 4))
ax = sns.lineplot(data=df, x='theta', y='$P_{FF}$')
ax.axhline(pfp, ls='--', color='green')
ax.annotate(text=f"  = {the_min_x:.3f}",
            xy=(the_min_x, the_y_val + pfp),
            xycoords='data',
            xytext=(10, 100),
            fontsize=12,
            textcoords='offset points',
            arrowprops=dict(facecolor='black', shrink=0.1)
           )
ax.plot([the_min_x],[the_y_val + pfp],'ro', mec='black', color='red', ms=10,␣
 ↪linewidth=5)
plt.xlabel(" : Multiple of Test Duration, $\mu$") # matplotlib doesn't like␣
 ↪$\theta$ for some reason, wtf?
plt.ylabel("$P_{FF}$: Probability of a False Fail")
plt.title("Probability of a False Fail for Varying   and Fixed F=1")
fn = f"FIGURE_2_prob_false_fail_{F}_pff{pff}"
plt.savefig(f"{fn}.pdf")
plt.savefig(f"{fn}.png")
print(f"{fn}.png and .pdf created")
```

```
FIGURE_2_prob_false_fail_1_pff0.1.png and .pdf created
```



4.2 Result

These are the end results for this section:

```
[10]: print(f"The vendor / producer:\n" \
          f"  To minimize false failure, with acceptable probability of false␣
 ↪failure = {pff}\n" \
          f"  With mu (MTBF) of {mu} (hours)\n" \
          f"  And testing until <= {F} failures are observed\n" \
          f"  Wants to test for {the_min_x:.3f} * {mu} or {(the_min_x * mu):.3f}␣
 ↪hours")
```

```
The vendor / producer:
  To minimize false failure, with acceptable probability of false failure = 0.1
  With mu (MTBF) of 24 (hours)
  And testing until <= 1 failures are observed
  Wants to test for 0.532 * 24 or 12.763 hours
```

5 Probability of False Pass

This is related to the buyer (consumer) failing to reject a system that is actually bad. They want to minimize that happening.

5.1 Find the Min Theta

First, we need to find the min theta for these above constraints.

```
[11]: func = pfp_calculator
ret = fmin(func, F/2, args=(F, pfp), xtol = 0.00001, full_output=True)
the_min_x = ret[0][0]
the_y_val = ret[1]
print(f"The min of function '{func.__name__}' is at {the_min_x} with result␣
 ↪{the_y_val}.")
```

```
Optimization terminated successfully.
         Current function value: 0.000000
         Iterations: 25
         Function evaluations: 50
The min of function 'pfp_calculator' is at 3.889721679687507 with result
1.2010450728405786e-07.
```

Set up our range of x values, based on what we know the min is - this is just for pretty plotting.

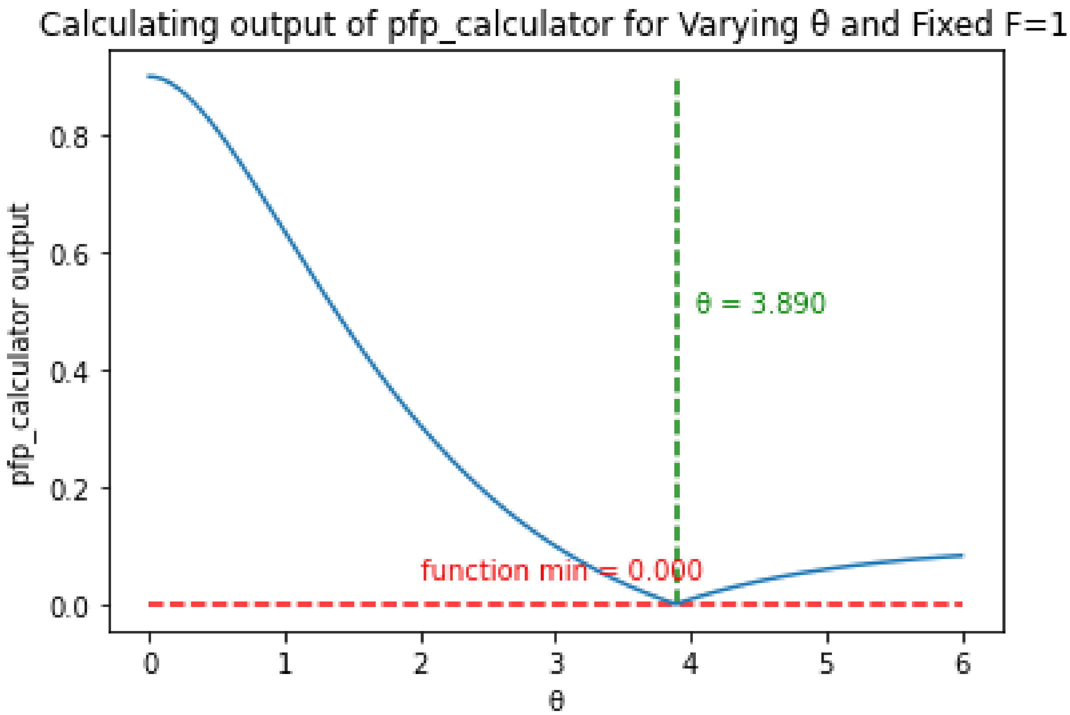
```
[12]: xmin = 0
xmax = math.floor(the_min_x) * 2
if xmax < 5:
    xmax = 5
points = 10000
# generate x points
```

```
xlist = np.linspace(xmin, xmax, points)
```

5.1.1 Plot the Min

This plots the function and annotates the min

[13]:
```
func = pfp_calculator

# map the x points to the values from the above function
ylist = list(map(lambda x: func(x, F=F, pfp=pfp), xlist))

# plot the results, w/ annotations
plt.plot(xlist, ylist)

# # draw horizontal line at min of function
plt.hlines(y=the_y_val, xmin=min(xlist), xmax=max(xlist), color='red',␣
 ↪linestyles="--")
plt.annotate(f"function min = {the_y_val:.3f}", xy=(2, the_y_val + max(ylist) *␣
 ↪0.05), color='red')

# draw vertical line at min
plt.vlines(x=the_min_x, ymin=min(ylist), ymax=max(ylist), color='green',␣
 ↪linestyles='--')
plt.annotate(f"  = {the_min_x:.3f}", xy=(the_min_x + max(xlist) * 0.025, 0.5),␣
 ↪color='green')

# labels and title
plt.xlabel(" ")
plt.ylabel(f"{func.__name__} output")
plt.title(f"Calculating output of {func.__name__} for Varying   and Fixed␣
 ↪F={F}")
plt.savefig(f"NOT_USED_IN_PAPER_pfp_calculator_f{F}_pfp{pfp}.png")
plt.savefig(f"NOT_USED_IN_PAPER_pfp_calculator_f{F}_pfp{pfp}.pdf")
```



5.1.2 Show the Min With Constraints

Going back to our initial distribution, show this point crossing the constraint

[14]:
```
func = poisson_cfd

# map the x points to the values from the above function
ylist = list(map(lambda x: func(x, F), xlist))
```

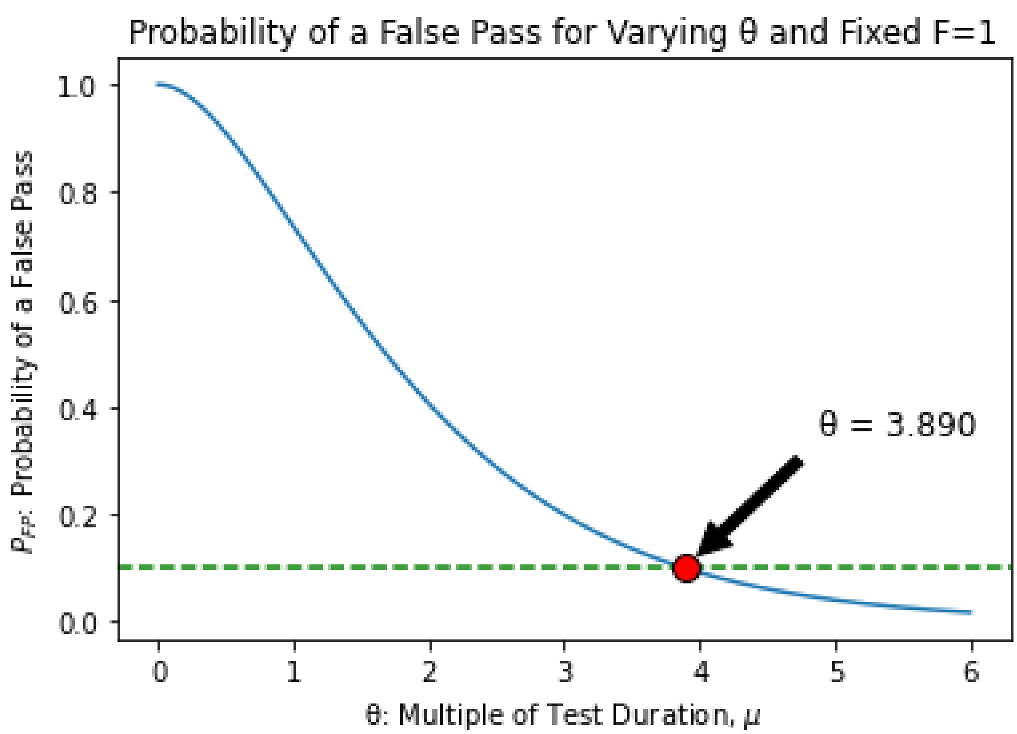
[15]:
```
df = pd.DataFrame()
df['x'] = xlist
df['pff'] = ylist
df.columns = ['theta', '$P_{FP}$']
df.head()
```

[15]:

	theta	\$P_{FP}\$
0	0.0000	1.000000
1	0.0006	1.000000
2	0.0012	0.999999
3	0.0018	0.999998
4	0.0024	0.999997

[16]:
```
fig, ax = plt.subplots(figsize=(6, 4))
ax = sns.lineplot(data=df, x='theta', y='$P_{FP}$')
ax.axhline(pfp, ls='--', color='green')
ax.annotate(text=f"  = {the_min_x:.3f}",
            xy=(the_min_x, the_y_val + pfp),
            xycoords='data',
            xytext=(50, 50),
            fontsize=12,
            textcoords='offset points',
            arrowprops=dict(facecolor='black', shrink=0.1)
           )
ax.plot([the_min_x],[the_y_val + pfp],'ro', mec='black', color='red', ms=10,␣
 ↪linewidth=5)
plt.xlabel(" : Multiple of Test Duration, $\mu$") # matplotlib doesn't like␣
 ↪$\theta$ for some reason, wtf?
plt.ylabel("$P_{FP}$: Probability of a False Pass")
plt.title("Probability of a False Pass for Varying   and Fixed F=1")
fn = f"FIGURE_1_prob_false_pass_{F}_pfp{pfp}"
plt.savefig(f"{fn}.pdf")
plt.savefig(f"{fn}.png")
print(f"{fn}.png and .pdf created")
```

```
FIGURE_1_prob_false_pass_1_pfp0.1.png and .pdf created
```



5.2 Result

These are the end results for this section:

[17]:
```
print(f"The buyer / consumer:\n" \
      f"  To minimize false pass, with acceptable probability of false pass =␣
↪{pfp}\n" \
      f"  With mu (MTBF) of {mu} (hours)\n" \
      f"  And testing until <= {F} failures are observed\n" \
      f"  Wants to test for {the_min_x:.3f} * {mu} or {(the_min_x * mu):.3f}␣
↪hours")
```

```
The buyer / consumer:
  To minimize false pass, with acceptable probability of false pass = 0.1
  With mu (MTBF) of 24 (hours)
  And testing until <= 1 failures are observed
  Wants to test for 3.890 * 24 or 93.353 hours
```

6 Constrain PFF and Match PFP with factor Tolerance

This code locks PFF at the input value and then plots PFP and the intersection of these curves

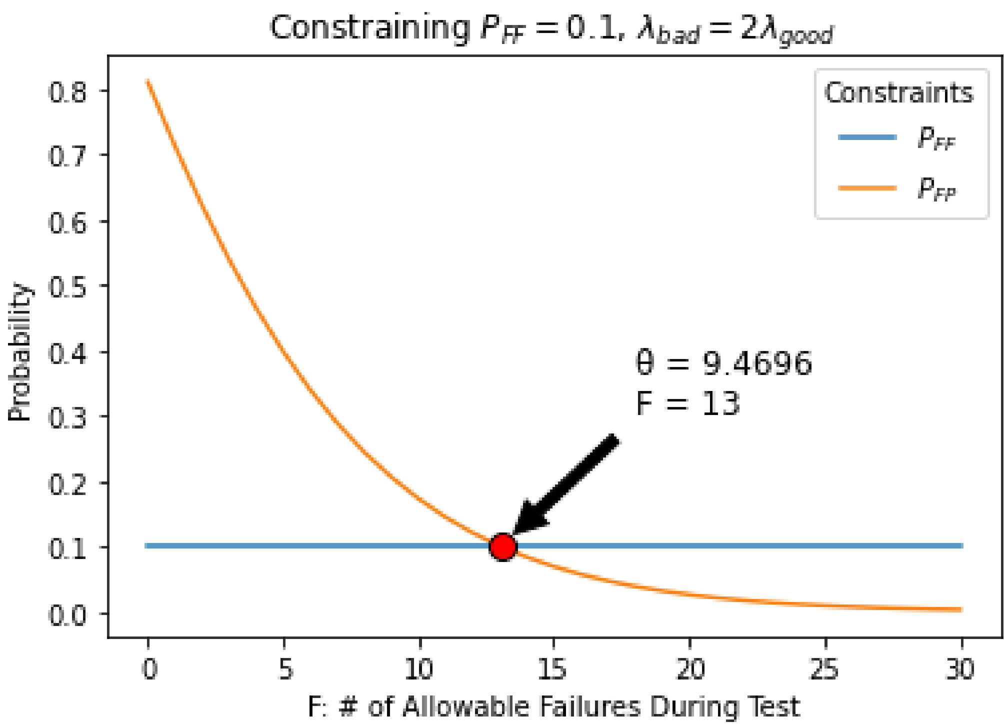
[18]:
```
# some helper code to find intersection of columns from the dataframe
def find_intersection_of_two_curves(x, y1, y2, deg):
    y1_fit = np.polyfit(x, y1, deg)
    y1_func = np.poly1d(y1_fit)

    y2_fit = np.polyfit(x, y2, deg)
    y2_func = np.poly1d(y2_fit)

    from scipy.optimize import fsolve
    def findIntersection(fun1,fun2,x0):
        return fsolve(lambda x : fun1(x) - fun2(x), x0)

    result = findIntersection(y1_func, y2_func, 0.0)
    return (result[0],y1_func(result[0]))
```

[19]:
```
def constrain_pff_find_pfp(pff, factor, title_prepend):
    # generate the dataframe with F, theta, and values of PFF, PFP (where PFP␣
↪has a `factor` tolerance)
    d = dict()
    for Ftemp in range(0, 31):
        d[Ftemp] = list()
        d[Ftemp].append(Ftemp)
        res = minimize(fun=pff_calculator, args=(Ftemp, pff), x0=Ftemp/2, tol=0.
↪0000001, method='Nelder-Mead')
        d[Ftemp].append(res['x'][0])
        d[Ftemp].append(one_minus_poisson_cfd(res['x'][0], Ftemp))
```

```
        d[Ftemp].append(poisson_cfd(res['x'] * factor, Ftemp)[0])

    df = pd.DataFrame.from_dict(d, orient='index', columns=['F','theta','$P_{FF}$','$P_{FP}$'])
    intersection_x,intersection_y = find_intersection_of_two_curves(df['F'].to_numpy(), df['$P_{FF}$'].to_numpy(), df['$P_{FP}$'].to_numpy(), 10)
    print(f"These curves intersect at x={intersection_x}, y={intersection_y}")
    idx = df['F'].sub(intersection_x).abs().idxmin()
    theta_at_intersection = df.iloc[idx]['theta']
    print(f"Theta value at this intersection is {theta_at_intersection}")

    # melt (transform) the dataframe for plotting
    df2 = df.melt(id_vars=['F','theta'], var_name='Constraints', value_name='Probability')

    # and plot it
    fig, ax = plt.subplots(figsize=(6,4))
    ax = sns.lineplot(data=df2, x='F', y='Probability', hue='Constraints')
    ax.annotate(text=f"  = {theta_at_intersection:.4f}\nF = {df.iloc[idx]['F']:.0f}",
                xy=(intersection_x, intersection_y),
                xycoords='data',
                xytext=(50, 50),
                fontsize=12,
                textcoords='offset points',
                arrowprops=dict(facecolor='black', shrink=0.1)
               )
    ax.plot([intersection_x],[intersection_y],'ro', mec='black', color='red', ms=10, linewidth=5)
    plt.xlabel("F: # of Allowable Failures During Test")
    plt.title("Constraining $P_{FF} = 0.1$, $\lambda_{bad} = %s\lambda_{good}$" % factor)
    fn = f"{title_prepend}constrained_pff{pff}_W{factor}"
    plt.savefig(f"{fn}.png");
    plt.savefig(f"{fn}.pdf");
    print(f"{fn}.png and .pdf created")
```

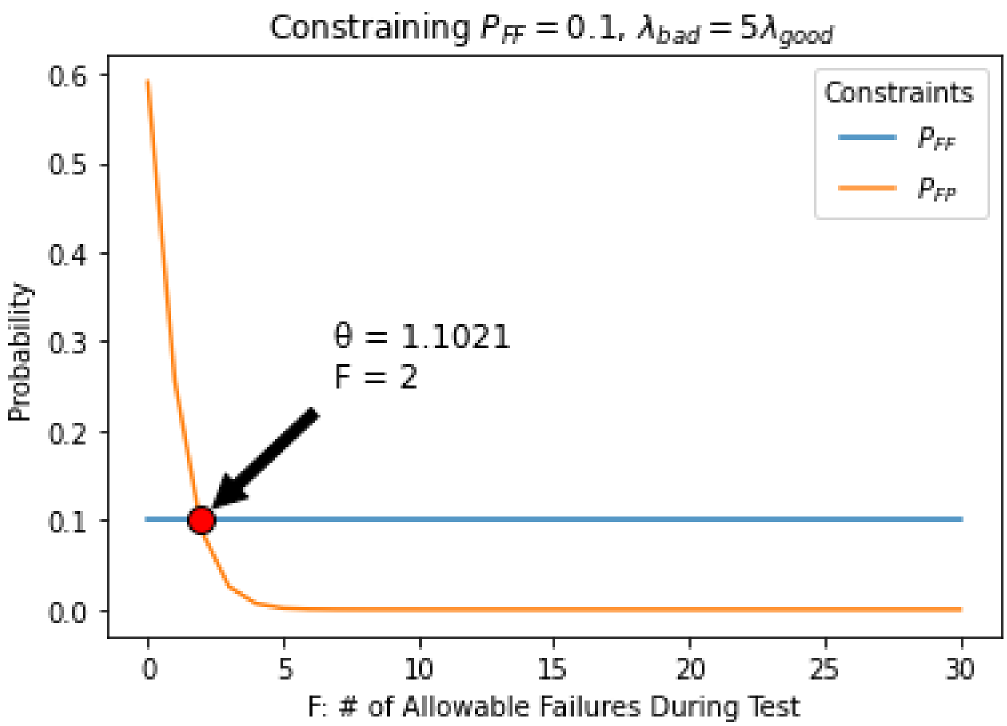
[20]:
```
constrain_pff_find_pfp(pff=0.1, factor=2, title_prepend="FIGURE_3_")
```

```
These curves intersect at x=13.038587393880091, y=0.0999999998808488
Theta value at this intersection is 9.469621187448507
FIGURE_3_constrained_pff0.1_W2.png and .pdf created
```




```
[21]: constrain_pff_find_pfp(pff=0.1, factor=5, title_prepend="NOT_USED_IN_PAPER_")
```

```
These curves intersect at x=1.9284141502834966, y=0.10000000532803094
Theta value at this intersection is 1.1020653724670408
NOT_USED_IN_PAPER_constrained_pff0.1_W5.png and .pdf created
```



7 Optimize to Both Parties' Goals

The previous section assumed that PFP would be interested in the same goal as PFF, here we allow them to vary

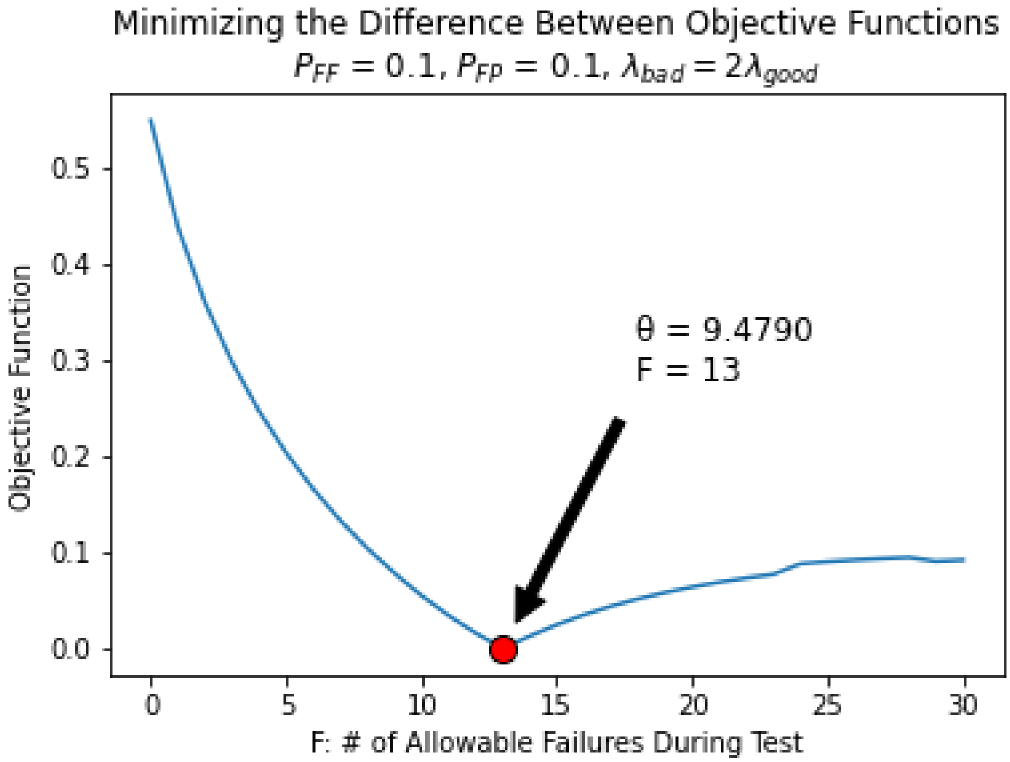
[22]:
```
def constraint_pff_and_pfp_optimize(pff, pfp, factor, Fmax, degree_poly,␣
↪title_prepend):
    # generate the dataframe with F, theta, and values of PFF, PFP (where PFP␣
↪has a `factor` tolerance)
    d = dict()
    for Ftemp in range(0, Fmax):
        d[Ftemp] = list()
        d[Ftemp].append(Ftemp)
        res = minimize(fun=two_party_optimization, args=(Ftemp, factor, pfp,␣
↪pff), x0=Ftemp/2, tol=0.0000001, method='Nelder-Mead')
        d[Ftemp].append(res['x'][0])
        d[Ftemp].append(res['fun'])

    df = pd.DataFrame.from_dict(d, orient='index',␣
↪columns=['F','theta','objective_function'])
    #display(df)
```

```python
    obj_fit = np.polyfit(df['F'].to_numpy(), df['objective_function'].
↪to_numpy(), degree_poly)
    obj_func = np.poly1d(obj_fit)
    min_res = minimize(obj_func, x0=0, method='Nelder-Mead')
    min_value = min_res['x'][0]
    print(min_value)
    idx = df['F'].sub(min_value).abs().idxmin()
    theta_at_min = df.iloc[idx]['theta']
    f_at_min = df.iloc[idx]['F']
    obj_at_min = df.iloc[idx]['objective_function']
    print(f"F value at this min is {f_at_min}")
    print(f"Theta value at this min is {theta_at_min}")
    print(f"Objective value at this min is {obj_at_min}")

    fig, ax = plt.subplots(figsize=(6,4))
    ax = sns.lineplot(data=df, x='F', y='objective_function')

    ax.annotate(text=f"  = {theta_at_min:.4f}\nF = {f_at_min:.0f}",
                xy=(f_at_min, obj_at_min),
                xycoords='data',
                xytext=(50, 100),
                fontsize=12,
                textcoords='offset points',
                arrowprops=dict(facecolor='black', shrink=0.1)
               )
    ax.plot([f_at_min],[obj_at_min],'ro', mec='black', color='red', ms=10,␣
↪linewidth=5)

    #plt.xticks(range(0, Fmax))
    plt.ylabel("Objective Function")
    plt.xlabel("F: # of Allowable Failures During Test")

    plt.title("Minimizing the Difference Between Objective Functions\n$P_{FF}$␣
↪= %s, $P_{FP}$ = %s, $\lambda_{bad} = %s\lambda_{good}$" % (pff, pfp,␣
↪factor));
    fn = f"{title_prepend}two_party_pff{pff}_pfp{pfp}_W{factor}"
    plt.savefig(f"{fn}.png");
    plt.savefig(f"{fn}.pdf");
    print(f"{fn}.png and .pdf created")
```

[23]:
```python
constraint_pff_and_pfp_optimize(pff=0.1, pfp=0.1, factor=2, Fmax=31,␣
↪degree_poly=10, title_prepend="FIGURE_4_")
```

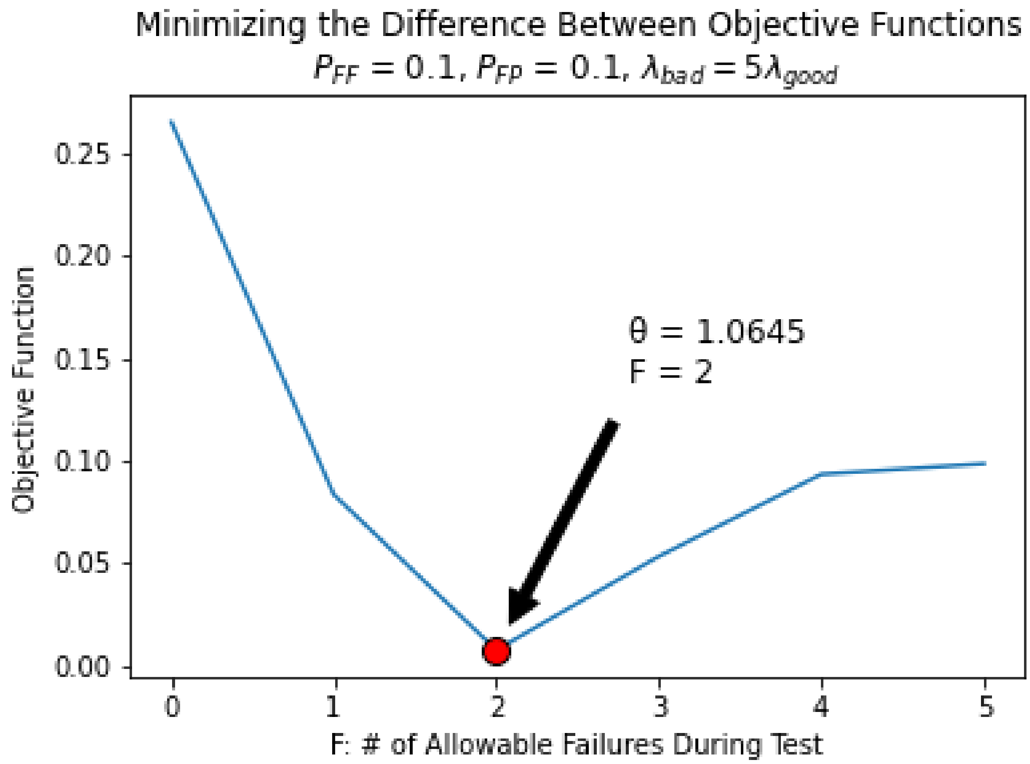
```
13.316687500000013
F value at this min is 13.0
Theta value at this min is 9.478980571031574
Objective value at this min is 0.0005720677095657073
```

```
FIGURE_4_two_party_pff0.1_pfp0.1_W2.png and .pdf created
```



[24]:
```
constraint_pff_and_pfp_optimize(pff=0.1, pfp=0.1, factor=5, Fmax=6,␣
↪degree_poly=4, title_prepend="FIGURE_7_")
```

```
2.1060000000000025
F value at this min is 2.0
Theta value at this min is 1.0644640922546391
Objective value at this min is 0.007467249196865769
FIGURE_7_two_party_pff0.1_pfp0.1_W5.png and .pdf created
```



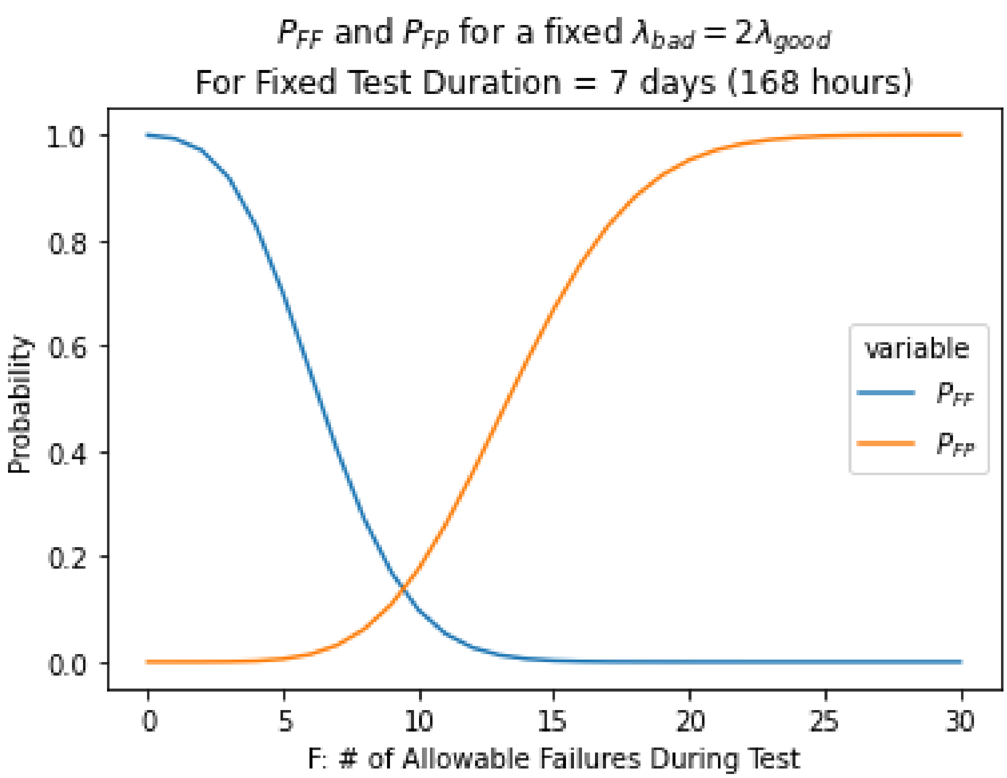
8 Constraining T

Try and fix T and see if we can optimize

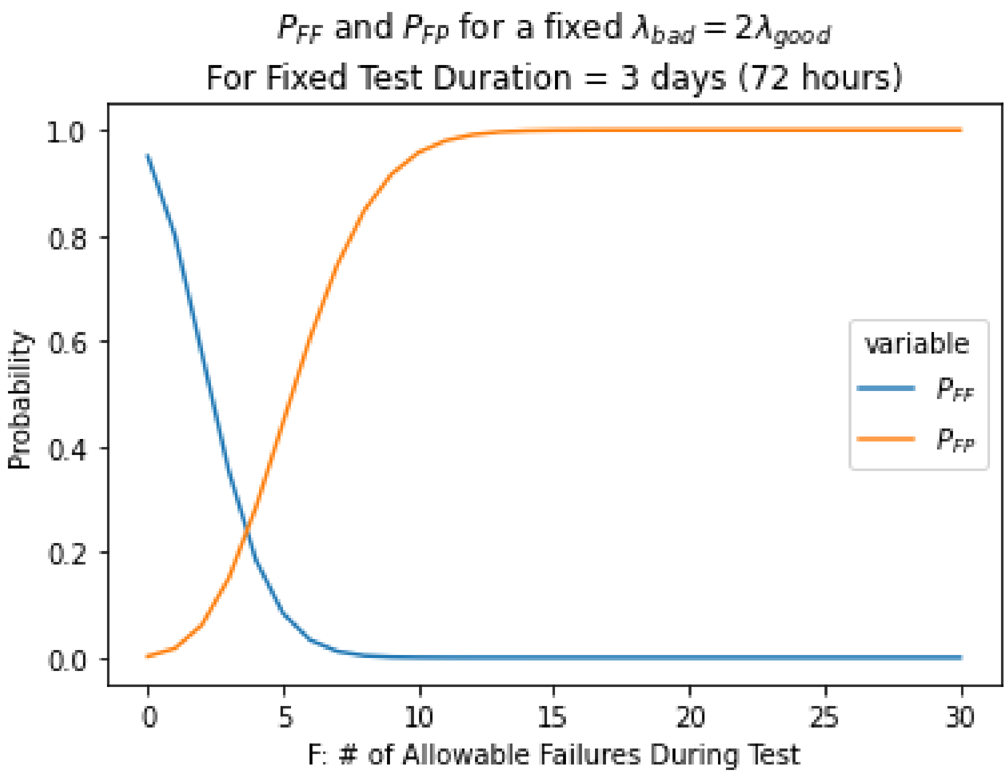
[25]:
```
def constrained_time(T, W, title_prepend):
    T_hours = T * 24
    # generate the dataframe with F, theta, and values of PFF, PFP (where PFP␣
↪has a `factor` tolerance)
    d = dict()
    for Ftemp in range(0, 31):
        d[Ftemp] = list()
        d[Ftemp].append(Ftemp)
        pff = 1 - poisson.cdf(Ftemp, T_hours / 24)
        pfp = poisson.cdf(Ftemp, W * T_hours / 24)
        d[Ftemp].append(pff)
        d[Ftemp].append(pfp)

    df = pd.DataFrame.from_dict(d, orient='index',␣
↪columns=['F','$P_{FF}$','$P_{FP}$'])
    df.head()
    df_melted = df.melt(id_vars=['F'])
    df_melted.head()
```

```
    fig, ax = plt.subplots(figsize=(6,4))
    ax = sns.lineplot(data=df_melted, x='F', y='value', hue='variable')
    plt.ylabel("Probability")
    plt.xlabel("F: # of Allowable Failures During Test")
    plt.title("$P_{FF}$ and $P_{FP}$ for a fixed $\lambda_{bad} =␣
↪%s\lambda_{good}$\n" \
              f"For Fixed Test Duration = {T} days ({T_hours} hours)" % factor)

    plt.
↪savefig(f"{title_prepend}fixed_time_interval_THours{T_hours}_F{F}_W{factor}.
↪png");
    plt.
↪savefig(f"{title_prepend}fixed_time_interval_THours{T_hours}_F{F}_W{factor}.
↪pdf");
```

[26]:
```
constrained_time(T=7, W=2, title_prepend="FIGURE_5_")
```



[27]:
```
constrained_time(T=3, W=2, title_prepend="FIGURE_6_")
```



9 Bayesian

9.1 Confidence Intervals On Determinging MTBF

10 THIS IS TABLE 2

[28]:
```python
def get_gamma_pi_95(alpha_prior, beta_prior, mu, X, T_Days):
    alpha_post = alpha_prior + X
    beta_post = beta_prior + 1
    T_Hours = T_Days * 24
    dist = gamma(alpha_post, scale=1.0/beta_post)
    # can do this if you want to sample it, without analytical results
    #random_variables = T_Hours / (dist.rvs(size=pow(10,7))) # pull 10^7 random␣
 ↪variables
    #np.count_nonzero(random_variables > mu) / len(random_variables))))

    # PI and posterior prob that measured mu >= mu
    return (tuple((T_Hours / dist.ppf(0.975),
                   T_Hours / dist.ppf(0.025),
                   dist.cdf(T_Hours / mu))))
```

[29]:
```
mu = 24
T_Days = 7
alpha_prior = pow(10, -6)
beta_prior = pow(10, -4)
```

[30]:
```
get_gamma_pi_95(alpha_prior, beta_prior, mu, 1, 7)
```

[30]: (45.54681341170713, 6636.281526794217, 0.9990887537137954)

[31]:
```
get_gamma_pi_95(alpha_prior, beta_prior, mu, 2, 7)
```

[31]: (30.155689426283086, 693.6835160314615, 0.9927093990251339)

[32]:
```
get_gamma_pi_95(alpha_prior, beta_prior, mu, 3, 7)
```

[32]: (23.255920643711523, 271.5762949453369, 0.9703794360056592)

[33]:
```
get_gamma_pi_95(alpha_prior, beta_prior, mu, 4, 7)
```

[33]: (19.164085553491788, 154.1628223358991, 0.9182709960049941)

[34]:
```
get_gamma_pi_95(alpha_prior, beta_prior, mu, 5, 7)
```

[34]: (16.405343113241006, 103.49131551158374, 0.8270721298265464)

[35]:
```
get_gamma_pi_95(alpha_prior, beta_prior, mu, 6, 7)
```

[35]: (14.399382756009041, 76.30555109109329, 0.6993809748736854)

[36]:
```
get_gamma_pi_95(alpha_prior, beta_prior, mu, 7, 7)
```

[36]: (12.865509131958868, 59.69974715291811, 0.5503930882919177)

[37]:
```
get_gamma_pi_95(alpha_prior, beta_prior, mu, 8, 7)
```

[37]: (11.64948804895228, 48.646477223965505, 0.40139032420760085)

[38]:
```
get_gamma_pi_95(alpha_prior, beta_prior, mu, 9, 7)
```

[38]: (10.658806633048547, 40.82661975420877, 0.27099988419077115)

[39]:
```
get_gamma_pi_95(alpha_prior, beta_prior, mu, 10, 7)
```

[39]: (9.83428270413837, 35.03715476074176, 0.1695749673017822)

[ ]:
```

```