

1. 예측 모델 생성 및 모델 검증

```
ad = read.csv("ad.csv")
set.seed(100)

lmMod1 = lm(sales~TV,data = ad)
lmMod2 = lm(sales~radio,data=ad)
lmMod3 = lm(sales~newspaper,data = ad)
lmMod4 = lm(sales~newspaper * TV * radio,data = ad)
lmMod5 = lm(sales~TV * radio, data = ad)
lmMod6 = lm(sales~TV * newspaper, data = ad)
lmMod7 = lm(sales~newspaper*radio,data=ad)
lmMod8 = lm(sales~(TV+radio+newspaper)^2,data=ad)
lmMod9 = step(lmMod8,direction="backward")
```

<그림 1>

위 <그림 1>의 코드처럼 lm과 다중 회귀분석의 formula를 이용해 총 9개의 모델을 생성하였다. 그리고, 다음에서 summary와 anova를 이용해 9개 모델을 검증하고, 이후 그래프로 가시화 하였다.

해당 내용은 다음과 같다.

1) lmMod1

> summary(lmMod1)#0.6099

Call:
lm(formula = sales ~ TV, data = ad)

Residuals:

Min	1Q	Median	3Q	Max
-8.3860	-1.9545	-0.1913	2.0671	7.2124

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	7.032594	0.457843	15.36	<2e-16 ***
TV	0.047537	0.002691	17.67	<2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

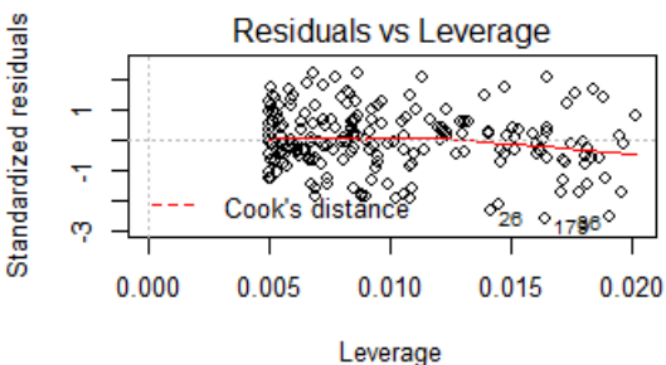
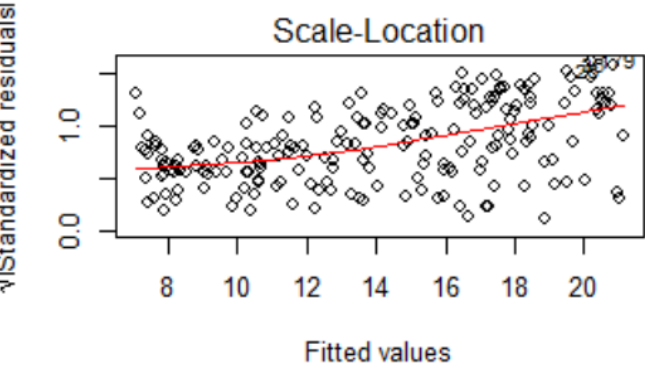
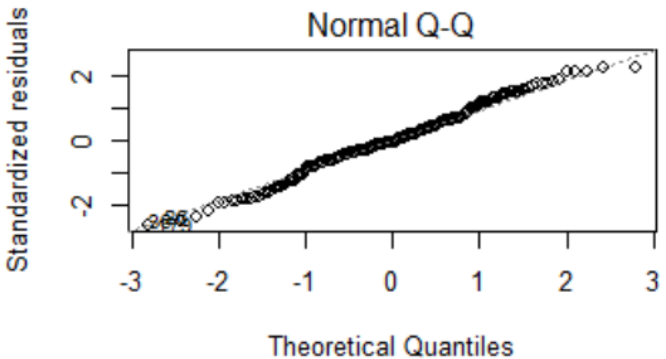
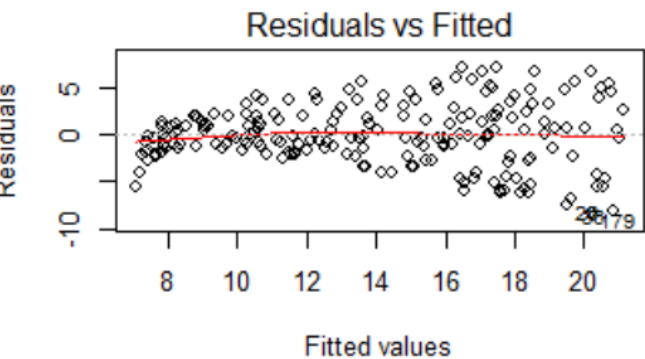
Residual standard error: 3.259 on 198 degrees of freedom
Multiple R-squared: 0.6119, Adjusted R-squared: 0.6099
F-statistic: 312.1 on 1 and 198 DF, p-value: < 2.2e-16

> anova(lmMod1)
Analysis of Variance Table

Response: sales

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
TV	1	3314.6	3314.6	312.14	< 2.2e-16 ***
Residuals	198	2102.5	10.6		

Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1



2) lmMod2

```
> summary(lmMod2)#0.3287
```

```
Call:
lm(formula = sales ~ radio, data = ad)
```

Residuals:

Min	1Q	Median	3Q	Max
-15.7305	-2.1324	0.7707	2.7775	8.1810

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	9.31164	0.56290	16.542	<2e-16 ***
radio	0.20250	0.02041	9.921	<2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.275 on 198 degrees of freedom
Multiple R-squared: 0.332, Adjusted R-squared: 0.3287
F-statistic: 98.42 on 1 and 198 DF, p-value: < 2.2e-16

```
> anova(lmMod2)
```

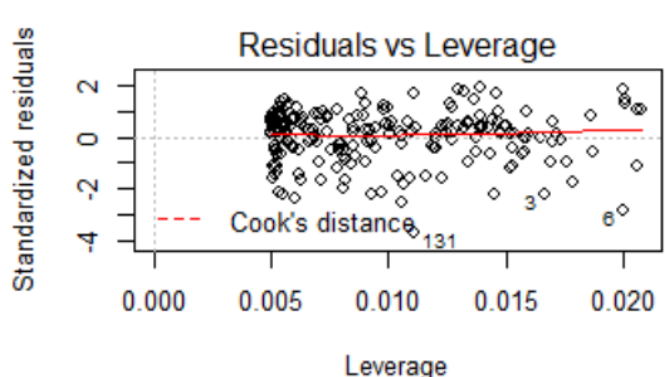
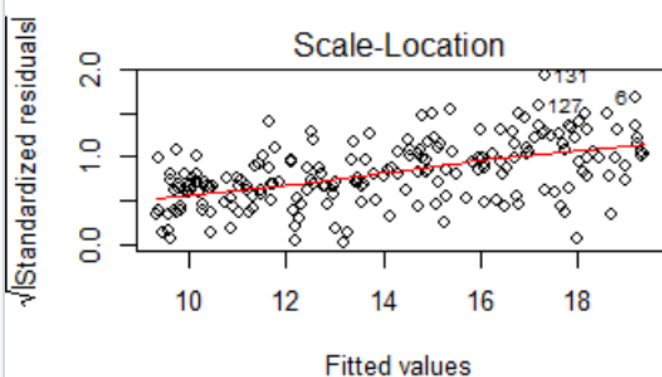
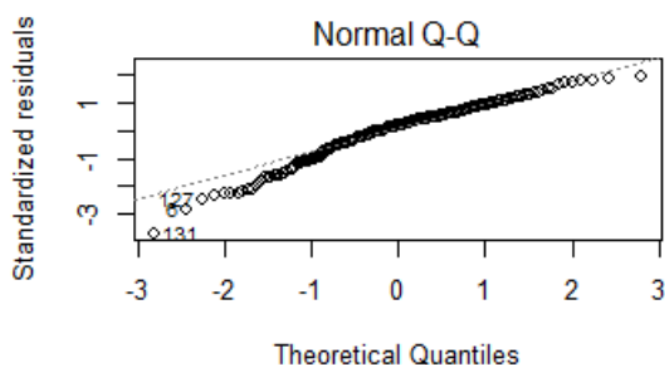
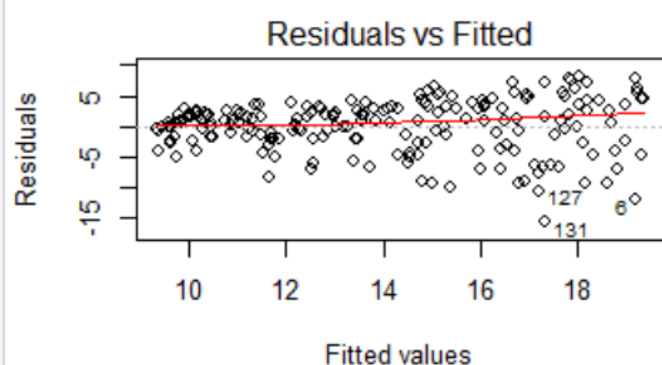
Analysis of Variance Table

Response: sales

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
radio	1	1798.7	1798.67	98.422	< 2.2e-16 ***
Residuals	198	3618.5	18.28		

Signif. codes:

0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1



3) lmMod3

```
> summary(lmMod3)#0.04733
```

```
call:
lm(formula = sales ~ newspaper, data = ad)
```

Residuals:

Min	1Q	Median	3Q	Max
-11.2272	-3.3873	-0.8392	3.5059	12.7751

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	12.35141	0.62142	19.88	< 2e-16 ***
newspaper	0.05469	0.01658	3.30	0.00115 **

signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

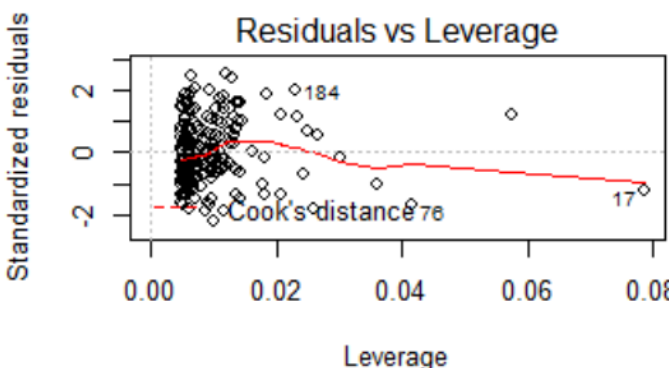
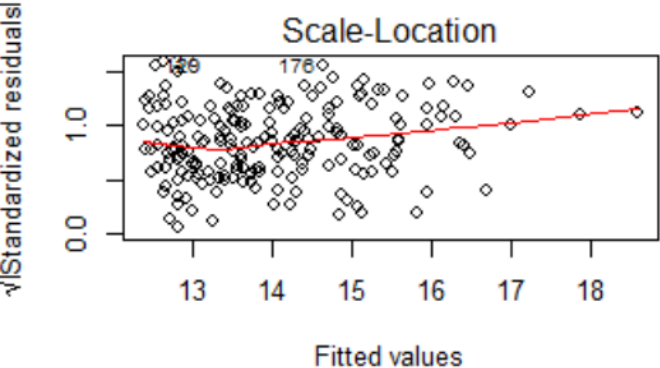
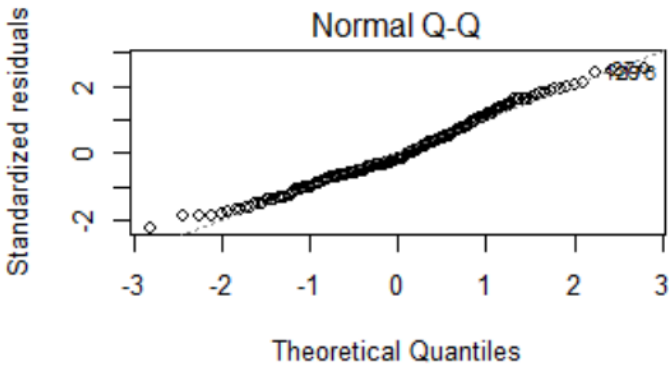
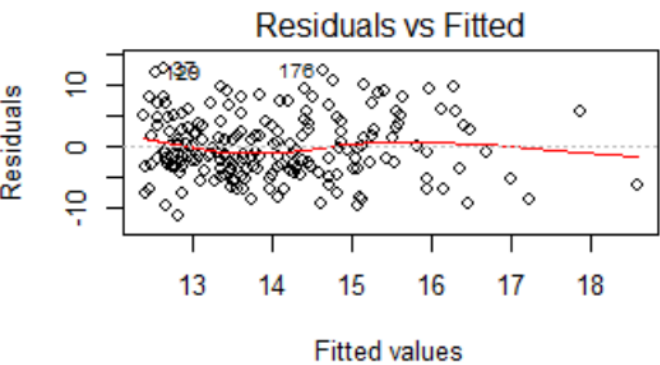
Residual standard error: 5.092 on 198 degrees of freedom
Multiple R-squared: 0.05212, Adjusted R-squared: 0.04733
F-statistic: 10.89 on 1 and 198 DF, p-value: 0.001148

```
> anova(lmMod3)
Analysis of Variance Table
```

Response: sales

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
newspaper	1	282.3	282.344	10.887	0.001148 **
Residuals	198	5134.8	25.933		

signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1



4) lmMod4

```
> summary(lmMod4)#0.9675
```

Call:
lm(formula = sales ~ newspaper * TV * radio, data = ad)

Residuals:

Min	1Q	Median	3Q	Max
-5.8955	-0.3883	0.1938	0.5865	1.5240

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	6.556e+00	4.655e-01	14.083	< 2e-16	***
newspaper	1.311e-02	1.721e-02	0.761	0.447	
TV	1.971e-02	2.719e-03	7.250	9.95e-12	***
radio	1.962e-02	1.639e-02	1.197	0.233	
newspaper:TV	-5.545e-05	9.326e-05	-0.595	0.553	
newspaper:radio	9.063e-06	4.831e-04	0.019	0.985	
TV:radio	1.162e-03	9.753e-05	11.909	< 2e-16	***
newspaper:TV:radio	-7.610e-07	2.700e-06	-0.282	0.778	

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

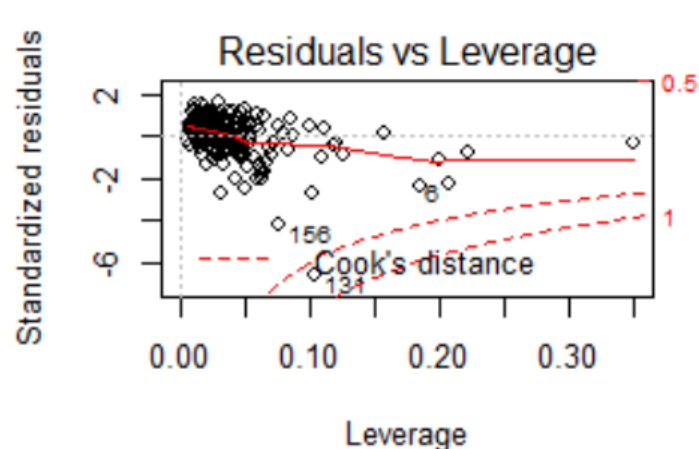
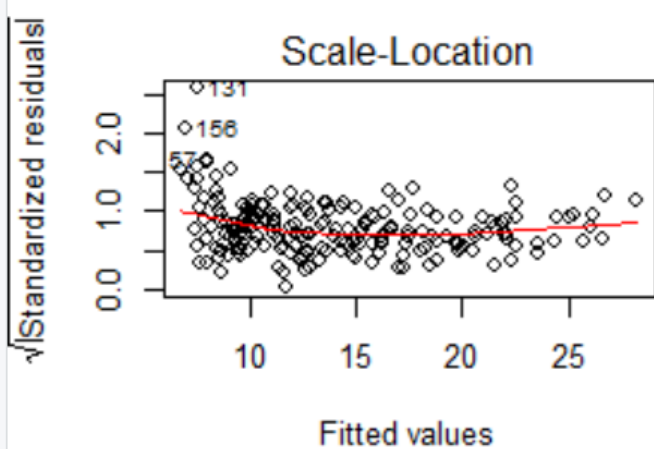
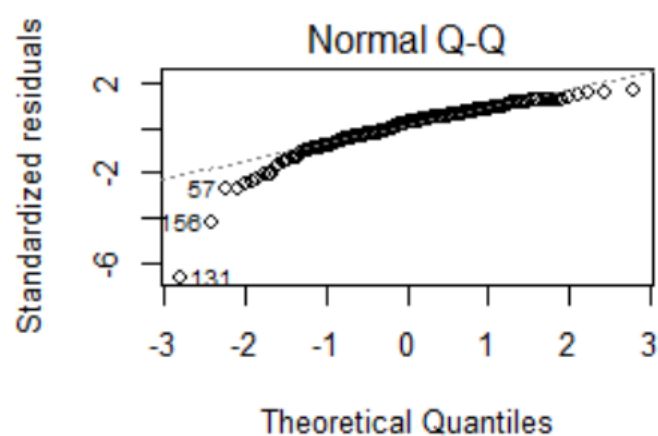
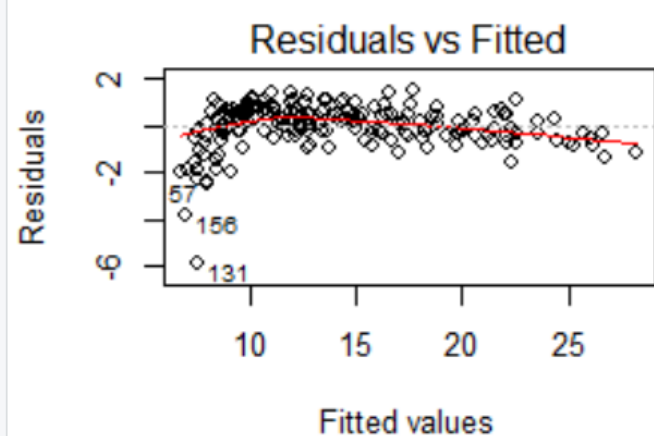
Residual standard error: 0.9406 on 192 degrees of freedom
Multiple R-squared: 0.9686, Adjusted R-squared: 0.9675
F-statistic: 847.3 on 7 and 192 DF, p-value: < 2.2e-16

```
> anova(lmMod4)  
Analysis of Variance Table
```

Response: sales

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
newspaper	1	282.3	282.3	319.1463	< 2.2e-16	***
TV	1	3216.2	3216.2	3635.4627	< 2.2e-16	***
radio	1	1361.7	1361.7	1539.2316	< 2.2e-16	***
newspaper:TV	1	33.9	33.9	38.2886	3.61e-09	***
newspaper:radio	1	3.3	3.3	3.7389	0.05463	.
TV:radio	1	349.7	349.7	395.2975	< 2.2e-16	***
newspaper:TV:radio	1	0.1	0.1	0.0794	0.77839	
Residuals	192	169.9	0.9			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1



5) lmMod5

```
> summary(lmMod5)#0.9673
```

```
Call:
lm(formula = sales ~ TV * radio, data = ad)
```

Residuals:

Min	1Q	Median	3Q	Max
-6.3366	-0.4028	0.1831	0.5948	1.5246

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	6.750e+00	2.479e-01	27.233	<2e-16	***
TV	1.910e-02	1.504e-03	12.699	<2e-16	***
radio	2.886e-02	8.905e-03	3.241	0.0014	**
TV:radio	1.086e-03	5.242e-05	20.727	<2e-16	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.9435 on 196 degrees of freedom
Multiple R-squared: 0.9678, Adjusted R-squared: 0.9673
F-statistic: 1963 on 3 and 196 DF, p-value: < 2.2e-16

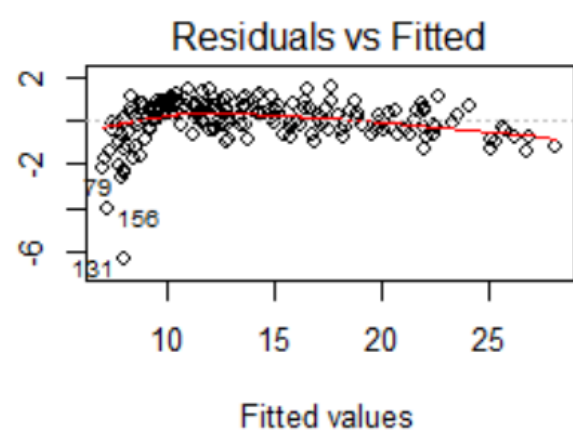
```
> anova(lmMod5)
Analysis of Variance Table
```

Response: sales

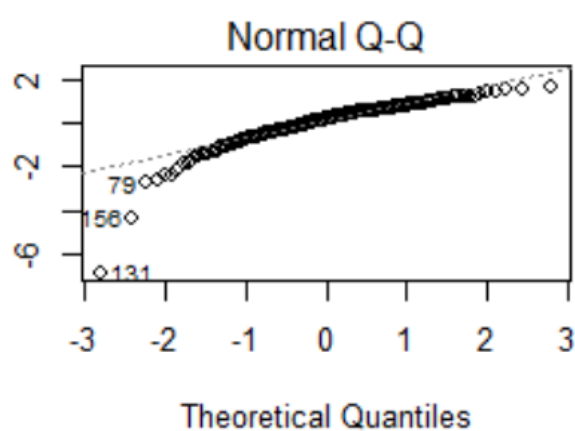
	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
TV	1	3314.6	3314.6	3723.36	< 2.2e-16	***
radio	1	1545.6	1545.6	1736.22	< 2.2e-16	***
TV:radio	1	382.4	382.4	429.59	< 2.2e-16	***
Residuals	196	174.5	0.9			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

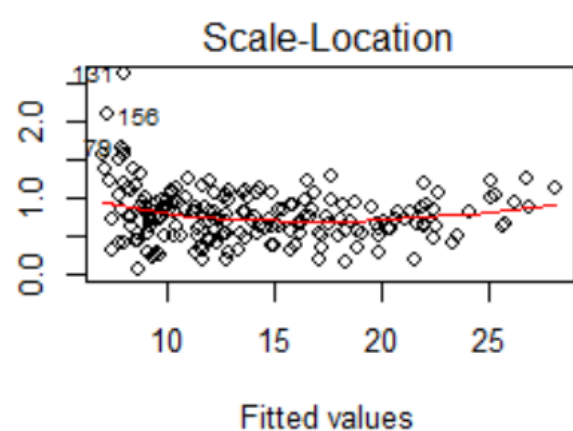
Residuals



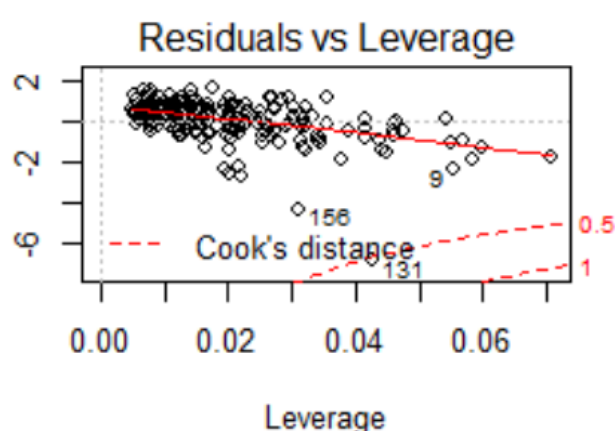
Standardized residuals



$\sqrt{|\text{Standardized residuals}|}$



Standardized residuals



6) lmMod6

```
> summary(lmMod6)#0.6432

Call:
lm(formula = sales ~ TV * newspaper, data = ad)

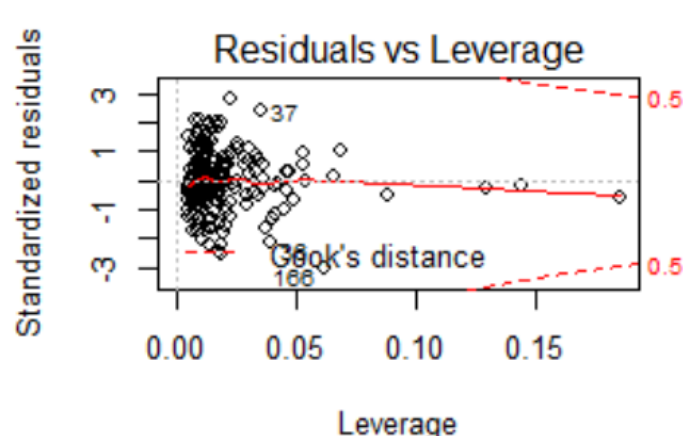
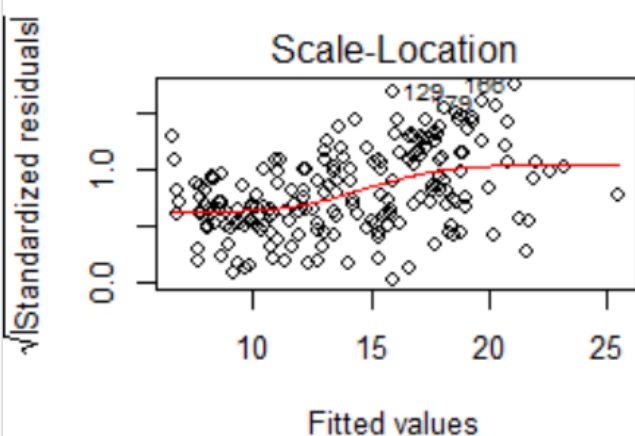
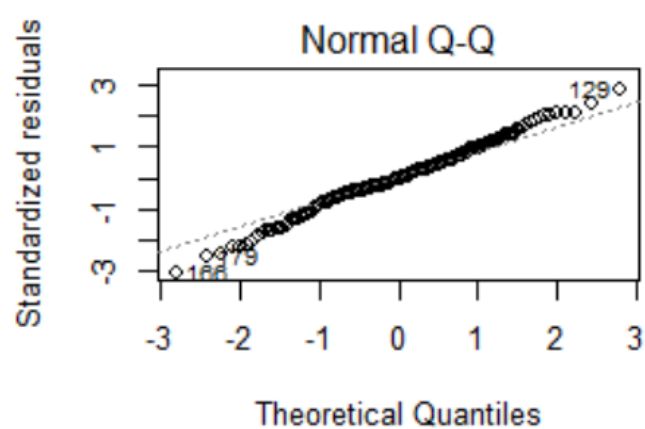
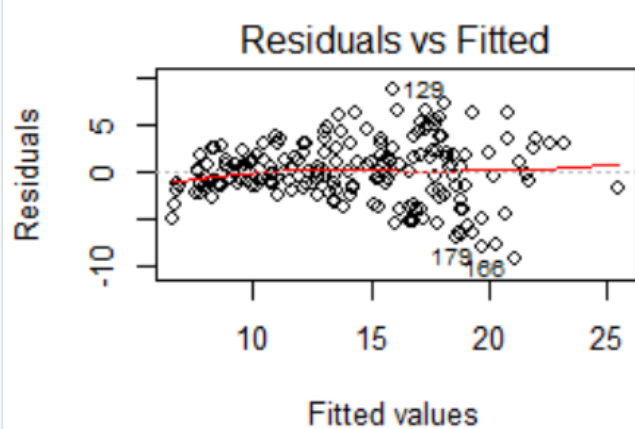
Residuals:
    Min       1Q   Median       3Q      Max
-9.1860 -1.5521 -0.0648  1.8062  8.7276

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  6.4042175   0.7333818   8.732  1.1e-15 ***
TV            0.0426585   0.0043105   9.896  < 2e-16 ***
newspaper     0.0241103   0.0192716   1.251   0.212
TV:newspaper  0.0001324   0.0001079   1.228   0.221
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.117 on 196 degrees of freedom
Multiple R-squared:  0.6485,    Adjusted R-squared:  0.6432
F-statistic: 120.6 on 3 and 196 DF,  p-value: < 2.2e-16

> anova(lmMod6)
Analysis of Variance Table

Response: sales
      Df Sum Sq Mean Sq F value    Pr(>F)
TV      1 3314.6   3314.6  341.226 < 2.2e-16 ***
newspaper  1  184.0    184.0   18.939 2.171e-05 ***
TV:newspaper  1   14.6     14.6    1.508  0.2209
Residuals 196 1903.9      9.7
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> |
```



7) lmMod7

```
> summary(lmMod7)#0.3233
```

```
Call:
lm(formula = sales ~ newspaper * radio, data = ad)

Residuals:
    Min       1Q   Median       3Q      Max
-15.6981  -2.1955   0.7567   2.7191   8.2228

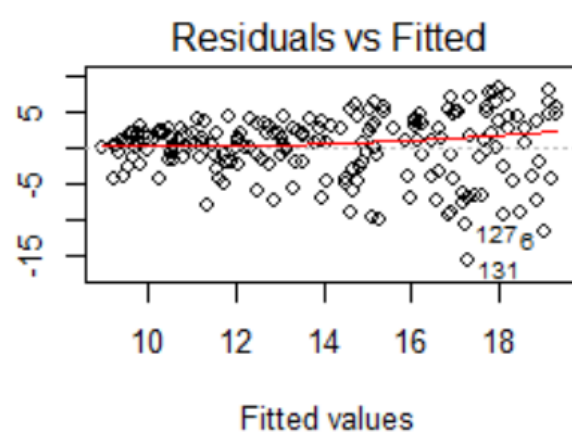
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)    8.7904734    1.0224848     8.597 2.58e-15 ***
newspaper       0.0220611    0.0345866     0.638  0.524
radio          0.2145684    0.0382985     5.603 7.08e-08 ***
newspaper:radio -0.0005259    0.0010642    -0.494  0.622
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.292 on 196 degrees of freedom
Multiple R-squared:  0.3335,    Adjusted R-squared:  0.3233
F-statistic: 32.7 on 3 and 196 DF,  p-value: < 2.2e-16

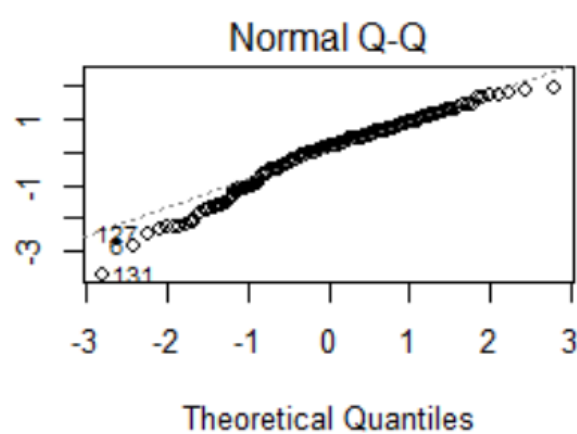
> anova(lmMod7)
Analysis of Variance Table

Response: sales
              Df Sum Sq Mean Sq F value    Pr(>F)
newspaper      1  282.3   282.34  15.3281 0.0001247 ***
radio          1 1520.0  1519.97  82.5170 < 2.2e-16 ***
newspaper:radio 1    4.5    4.50   0.2443 0.6217057
Residuals     196 3610.3   18.42
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> |
```

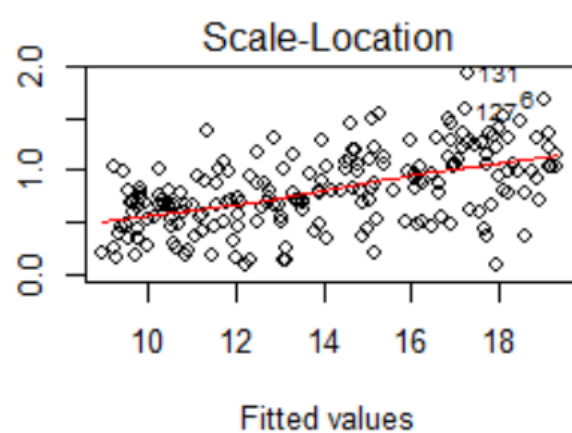
Residuals



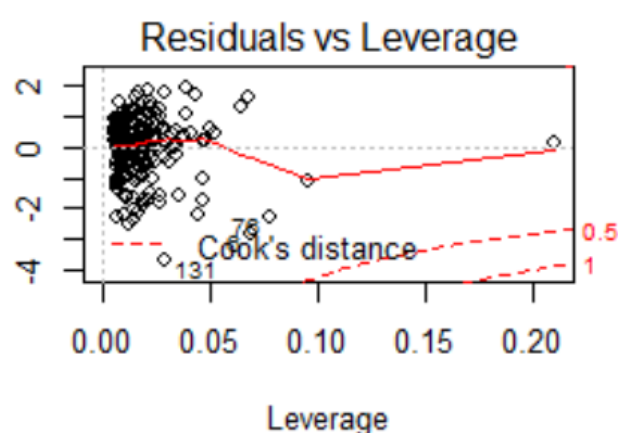
Standardized residuals



$\sqrt{|\text{Standardized residuals}|}$



Standardized residuals



8) lmMod8

```
> summary(lmMod8)#0.9677

Call:
lm(formula = sales ~ (TV + radio + newspaper)^2, data = ad)

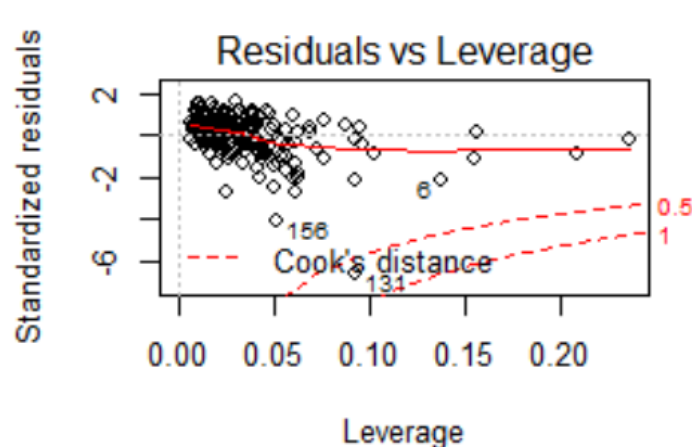
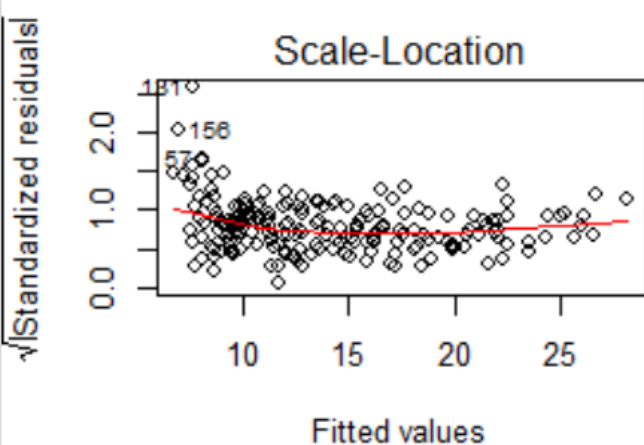
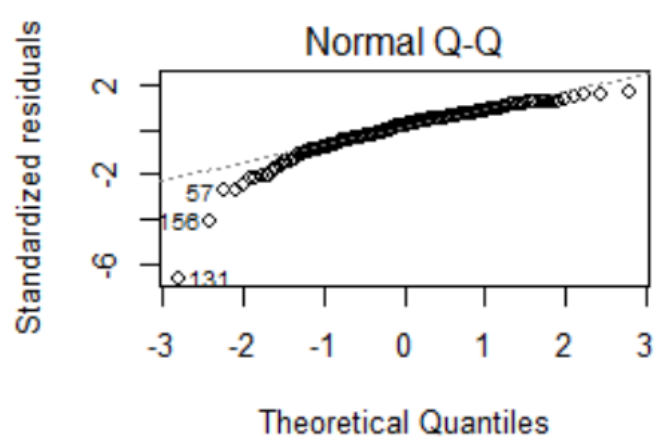
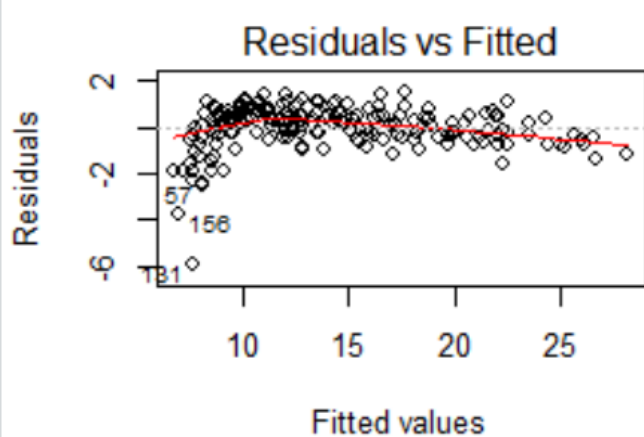
Residuals:
    Min       1Q   Median       3Q      Max
-5.9239 -0.3954  0.1873  0.5976  1.5267

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  6.460e+00  3.176e-01  20.342  <2e-16 ***
TV           2.033e-02  1.609e-03  12.633  <2e-16 ***
radio        2.293e-02  1.141e-02   2.009   0.0460 *
newspaper    1.703e-02  1.007e-02   1.691   0.0924 .
TV:radio      1.139e-03  5.716e-05  19.930  <2e-16 ***
TV:newspaper -7.971e-05  3.579e-05  -2.227   0.0271 *
radio:newspaper -1.096e-04  2.363e-04  -0.464   0.6433
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.9383 on 193 degrees of freedom
Multiple R-squared:  0.9686,    Adjusted R-squared:  0.9677
F-statistic: 993.3 on 6 and 193 DF,  p-value: < 2.2e-16
```

```
> anova(lmMod8)
Analysis of Variance Table

Response: sales
      Df Sum Sq Mean Sq  F value    Pr(>F)
TV      1 3314.6   3314.6  3764.6175 < 2e-16 ***
radio   1 1545.6   1545.6  1755.4527 < 2e-16 ***
newspaper 1    0.1     0.1    0.1008 0.75126
TV:radio 1  382.5   382.5   434.4444 < 2e-16 ***
TV:newspaper 1    4.2     4.2    4.7615 0.03031 *
radio:newspaper 1    0.2     0.2    0.2151 0.64329
Residuals 193 169.9     0.9
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> |
```



9) lmMod9

```
> summary(lmMod9)#0.9678

Call:
lm(formula = sales ~ TV + radio + newspaper + TV:radio + TV:newspaper,
    data = ad)

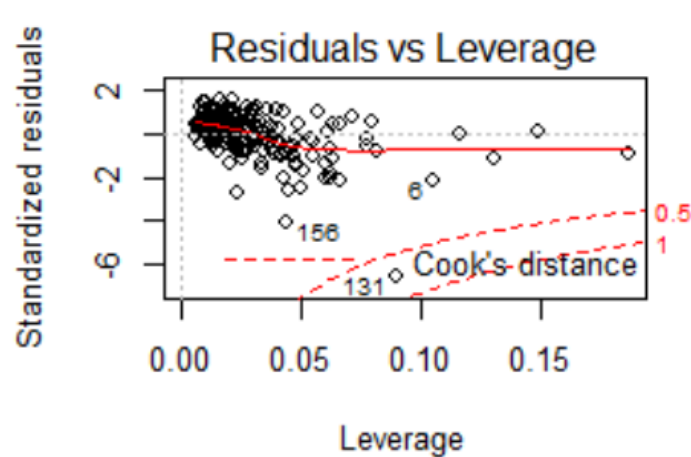
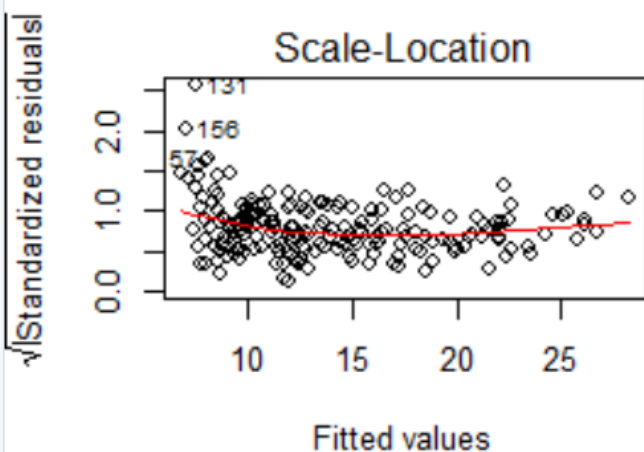
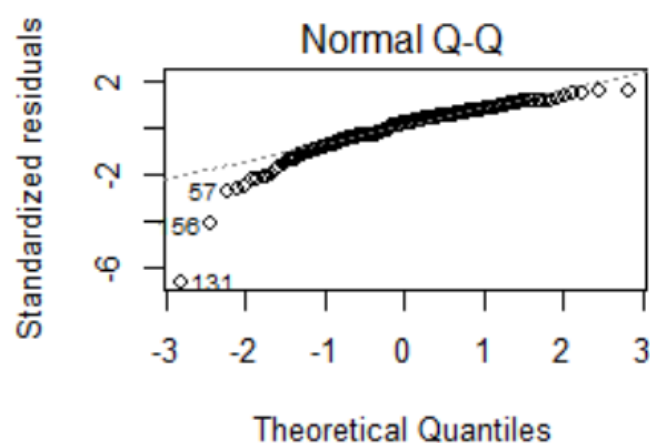
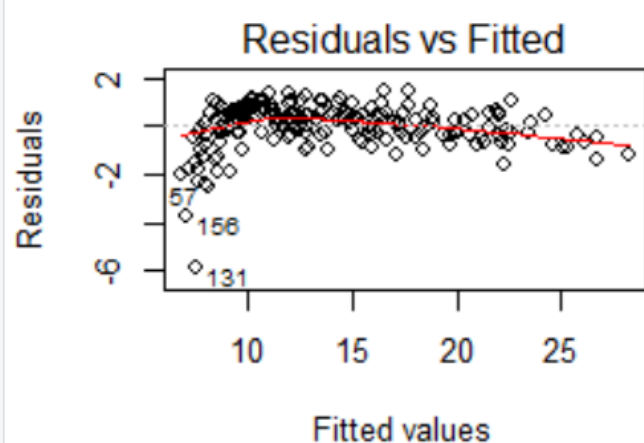
Residuals:
    Min       1Q   Median       3Q      Max
-5.9019 -0.3818  0.1937  0.5741  1.4839

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  6.541e+00  2.652e-01  24.668  <2e-16 ***
TV            2.035e-02  1.605e-03  12.675  <2e-16 ***
radio        2.018e-02  9.734e-03   2.073   0.0395 *
newspaper    1.342e-02  6.377e-03   2.105   0.0366 *
TV:radio     1.136e-03  5.664e-05  20.059  <2e-16 ***
TV:newspaper -7.719e-05  3.531e-05  -2.187   0.0300 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.9364 on 194 degrees of freedom
Multiple R-squared:  0.9686,    Adjusted R-squared:  0.9678
F-statistic: 1197 on 5 and 194 DF,  p-value: < 2.2e-16
```

```
> anova(lmMod9)
Analysis of Variance Table

Response: sales
      Df Sum Sq Mean Sq  F value    Pr(>F)
TV      1 3314.6  3314.6 3779.9098 < 2e-16 ***
radio   1 1545.6  1545.6 1762.5835 < 2e-16 ***
newspaper 1    0.1    0.1   0.1012 0.75077
TV:radio 1  382.5   382.5  436.2092 < 2e-16 ***
TV:newspaper 1    4.2    4.2   4.7808 0.02998 *
Residuals 194 170.1    0.9
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```



2. 예측 및 검정 수행

위에서 구한 내용을 이용해 가장 정확도가 큰 예측 모델을 구한다. Adjusted R-squared 값으로 추적 회귀선이 관측값을 가장 잘 설명하는 모델을 찾아야 하므로, 각 모델 별 Adjusted R-squared 값을 이용한다. 수행 결과, lmMod9 모델이 가장 Adjusted R-Squared 값이 높음을 알 수 있다.

```
trainingRowIndex = sample(1:nrow(ad),0.6*nrow(ad))
trainingData = ad[trainingRowIndex,]
testData = ad[-trainingRowIndex,]

lmMod = lmMod9
distPred = predict(lmMod, testData)
actuals_preds=data.frame(cbind(actuals = testData$sales,predict = distPred))
cor(actuals_preds)
```

<그림 2>

```
> trainingRowIndex = sample(1:nrow(ad),0.6*nrow(ad))
> trainingData = ad[trainingRowIndex,]
> testData = ad[-trainingRowIndex,]
> lmMod = lmMod9
> distPred = predict(lmMod, testData)
> actuals_preds=data.frame(cbind(actuals = testData$sales,predict = distPred))
> cor(actuals_preds)
```

	actuals	predict
actuals	1.0000000	0.9840184
predict	0.9840184	1.0000000

<그림 3>

이후, 그림 2의 코드 처럼 6:4로 training data, testing data를 나누어 testing data를 이용해 예측을 수행, 예측 된 값에 대한 검정을 수행한다. 결과는 그림 3과 같다.