Engineering and Applied Science Programs for Professionals
Whiting School of Engineering
Johns Hopkins University
605.621 Foundations of Algorithms
Homework 4
Due at the end of Module 8

Total Points 100/100

Collaboration groups will be set up in Blackboard by the end of the week. Make sure your group starts an individual thread for each collaborative problem and subproblem. You are required to participate in each of the collaborative problem and subproblem.

Self-Study Problems

All of the following problems come from the textbook and have solutions posted on the web at http://mitpress.mit.edu/algorithms.

You are permitted to use this site to examine solutions for these problems as a means of self-checking your solutions. These problems will not be graded.

Problems: 6.1-3, 6.2-6, 7.2-5, 8.2-3, 8.3-3, 9.3-1, 9.3-5, 14.1-7, 14.2-2, 14.3-7.

Problems for Grading

1. **Problem 1 Chapter 6 Parts a, b, c, d, e**
   25 Points Total 5 Points Each

   (a) CLRS 6.2-1 Using Figure 6.2 as a model, illustrate the operation of MAX-HEAPIFY$(A, 3)$ on the array $A =< 27, 17, 3, 16, 13, 10, 1, 5, 7, 12, 4, 8, 9, 0 >$.

   (b) CLRS 6.3-1 Using Figure 6.3 as a model, illustrate the operation of BUILD-MAX-HEAP on the array $A =< 5, 3, 17, 10, 84, 19, 6, 22, 9 >$.

   (c) CLRS 6.4-1 Using Figure 6.4 as a model, illustrate the operation of HEAPSORT on the array $A =< 5, 13, 2, 25, 7, 17, 20, 8, 4 >$.

   (d) CLRS 6.5-1 Illustrate the operation of HEAP-EXTRACT-MAX on the heap $A =< 15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2, 1 >$.

   (e) CLRS 6.5-2 Illustrate the operation of MAX-HEAP-INSERT on the heap $A =< 15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2, 1 >$.

2. **Problem 2 Chapter 6**
   15 Points Total

   The following problem is known as the *Dutch Flag Problem.* In this problem, the task is to rearrange an array of characters R, W, and B (for red, white, and blue, which are the colors of the Dutch national flag) so that all the R's come first, all the W's come next, and all the B's come last. Design a linear, in-place algorithm that solves this problem.

3. **Problem 3 Chapter 7** ***Note this is a Collaborative Problem***
20 Points Total 5 Points Each

CLRS 7-2 ***Quicksort with equal element values***
The analysis of the expected running time of randomized quicksort in Section 7.4.2 assumes that all element values are distinct. In this problem, we examine what happens when they are not.

(a) Suppose that all element values are equal. What would be randomized quicksort's running time in this case?

(b) The PARTITION procedure returns an index q such that each element of $A[p...q-1]$ is less than or equal to $A[q]$ and each element of $A[q+1...r]$ is greater than $A[q]$. Modify the PARTITION procedure to produce a procedure PARTITION'(A, P, R), which permutes the element of $A[p...r]$ and returns two indices q and t, where $p \leq q \leq t \leq r$, such that

- all elements of $A[q...t]$ are equal,
- each element of $A[p...q-1]$ is less than $A[q]$, and
- each element of $A[t+1...r]$ is greater than $A[q]$.

(c) Modify the RANDOMIZED-QUICKSORT procedure to call PARTITION' , and name the new procedure RANDOMIZED-QUICKSORT'. Then modify the QUICKSORT procedure to produce QUICKSORT'(A, P, R) that calls RANDOMIZED-QUICKSORT' and recurses only on partitions of elements not known to be equal to each other.

(d) Using QUICKSORT', how would you adjust the analysis in Section 7.4.2 to avoid the assumptions that all elements are distinct.

4. **Problem 4 Chapter 8 Parts a, b, c**
15 Points Total Part a 5 Points, Part b 5 Points and Part c 5 Points

(a) CLRS 8.3-1Using figure 8.3 as a model, illustrate the operation of RADIX-SORT on the following list of English words:
COW, DOG, SEA, RUG, ROW, MOB, BOX, TAB, BAR, EAR, TAR, DIG, BIG, TEA, NOW, FOX.

(b) CLRS 8.4-1Using figure 8.4 as a model, illustrate the operation of BUCKET-SORT on the array
$A =< .79, .13, .16, .64, .39, .20, .89, .53, .71, .42 >$.

(c) Explain why the worst-case running time for bucket sort is $\Theta(n^2)$. What simple change to the algorithm preserves its linear average-case running time and makes its worst-case running time $O(n \lg n)$.

5. **Problem 5 Chapter 9 Parts a and b** ***Note this is a Collaborative Problem***
10 Points Total 5 Points Each

(a) CLRS 9.1-1 Show that the second smallest of n elements can be found with $n + \lceil \lg n \rceil - 2$ comparisons in the worst case. (*Hint*: Also find the smallest element.)

(b) CLRS 9.3-9 Professor Olay is consulting for an oil company, which is planning a large pipeline running east to west through an oil field if n wells. The company wants to connect a spur pipeline from each well directly to the pipeline along a shortest route (either north or south), as shown in Figure 9.2. Given the $x-$ and $y-$coordinates of the wells, how should the professor pick the optimal location of the main pipeline, which would be the one that minimizes the total length of the spur? Show how to determine the optimal location in linear time.

6. **Problem 6 Chapter 14** ***Note this is a Collaborative Problem***
15 Points Total

CLRS 14-1.5 Given an element x in an n-node order-statistic tree and a natural number i, how can we determine the ith successor of x in the linear order of the tree in $O(\lg n)$ time?