Engineering and Applied Science Programs for Professionals
Whiting School of Engineering
Johns Hopkins University
605.621 Foundations of Algorithms
Due at the end of Module 6

Total Points 100/100

Collaboration groups been set up in Blackboard under Discussions. Make sure your group starts an individual thread for each collaborative problem and subproblem. You are required to participate in each of the collaborative problem and subproblem.

Problems for Grading

1. **Problem 1 Chapter 34**
   20 Points Total

   CLRS 34.3-2: Show that the $\leq_P$ relation is a transitive relation on languages. That is, show that if $L_1 \leq_P L_2$ and $L_2 \leq_P L_3$, then $L_1 \leq_P L_3$.

2. **Problem 2 Chapter 34**
   20 Points Total

   Recall the definition of a complete graph K_n is a graph with n vertices such that every vertex is connected to every other vertex. Recall also that a clique is a complete subset of some graph. The graph coloring problem consists of assigning a color to each of the vertices of a graph such that adjacent vertices have different colors and the total number of colors used is minimized. We define the chromatic number of a graph G to be this minimum number of colors required to color graph G. Prove that the chromatic number of a graph G is no less than the size of a maximal clique of G.

3. **Problem 3 Chapter 34** ***Note this is a Collaborative Problem***
   30 Points Total

   Suppose you're helping to organize a summer sports camp, and the following problem comes up. The camp is supposed to have at least one counselor who's skilled at each of the n sports covered by the camp (baseball, volleyball, and so on). They have received job applications from m potential counselors. For each of the n sports, there is some subset of the m applicants qualified in that sport. The question is "For a given number $k < m$, is it possible to hire at most k of the counselors and have at least one counselor qualified in each of the n sports?" We'll call this the Efficient Recruiting Problem. Prove that Efficient Recruiting is NP-complete.

4. **Problem 4 Chapter 34 *Note this is a Collaborative Problem***
30 Points Total

We start by defining the Independent Set problem. Given a graph $G = (V, E)$, we say a set of nodes $S \subseteq V$ is independent if no two nodes in S are joined by an edge. The Independent Set problem, which we denote as IS, is the following. Given G, find an independent set that is as large as possible. Stated as a decision problem, IS answers the question, does there exist a set $S \in G$ such that $|S| \leq k$? Then set k as large as possible. For this problem, you may take as given that IS is NP-complete.

A store trying to analyze the behavior of its customers will often maintain a table A where the rows of the table correspond to the customers and the columns (or fields) corresponding to products the store sells. The entry $A[i, j]$ specifies the quantity of product j that has been purchased by customers i. For example, Table 1 shows one such table.

Table 1: Customer Tracking Table

Customer	Detergent	Beer	Diapers	Cat Litter
Raj	0	6	0	3
Alanis	2	3	0	0
Chelsea	0	0	0	7

One thing that a store might want to do with this data is the following. Let's say that a subset S of the customers is diverse if no two of the customers in S have ever bought the same product (i.e., for each product, at most one of the customers S has ever bought it). A diverse set of customers can be useful, for example, s a target pool for market research.

We can now define the Diverse Subset problem (DS) as follows: Given an $m \times n$ array A as defined above and a number $k \leq m$, is there a subset of at least k customers that is divers?

Prove that DS is NP-complete.

References

[1] Thomas H. Cormen et al., Introduction to Algorithms, Third Edition, MIT Press and McGraw-Hill, 2009, ISBN-13: 978-0-262-03384-8.

[2] Stephen Cook, The Complexity of Theorem Proving Procedures, Proceedings of the third annual ACM symposium on Theory of computing, pp. 151-158, 1971

[3] Richard M. Karp, Reducibility Among Combinatorial Problems, In R. E. Miller and J. W. Thatcher (editors), Complexity of Computer Computations, New York: Plenum, pp. 85-103, 1972

[4] David Zuckerman, NP-Complete Problems Have a Version That's Hard to Approximate, IEEE, Proceedings of the Eighth Annual Structure in Complexity Theory Conference, pp. 305-312, 1993

[5] David Zuckerman, On Unapproximable Versions of NP-Complete Problems, SIAM Journal on Computing, Volume 25, Issue 6, pp. 1293-1304, 1996