

3.2.6 An **order  $n$  matrix** is an  $n \times n$  array of numbers. For example,

$$(3)$$

is a  $1 \times 1$  matrix,

$$\begin{array}{cc} 1 & 3 \\ -2 & 8 \end{array}$$

is a  $2 \times 2$  matrix and

$$\begin{array}{cccc} 1 & 3 & 4 & 6 \\ 2 & -5 & 0 & 8 \\ 3 & 7 & 6 & 4 \\ 2 & 0 & 9 & -1 \end{array}$$

is a  $4 \times 4$  matrix. Define the **minor** of an element  $x$  in a matrix as the submatrix formed by deleting the row and column containing  $x$ . In the above example of a  $4 \times 4$  matrix, the minor of the element 7 is the  $3 \times 3$  matrix

$$\begin{array}{ccc} 1 & 4 & 6 \\ 2 & 0 & 8 \\ 2 & 9 & -1 \end{array}$$

Clearly, the order of a minor of any element is 1 less than the order of the original matrix. Denote the minor of an element  $a[i, j]$  by  $\text{minor}(a[i, j])$ .

Define the **determinant** of a matrix  $a$  (written  $\text{det}(a)$ ) recursively as follows:

1. If  $a$  is a  $1 \times 1$  matrix ( $x$ ), then  $\text{det}(a) = x$ .
2. If  $a$  is of an order greater than 1, compute the determinant of  $a$  as follows:
  - a. Choose any row or column. For each element  $a[i, j]$  in this row or column, form the product:

$$\text{power}(-1, i + j) * a[i, j] * \text{det}(\text{minor}(a[i, j]))$$

where  $i$  and  $j$  are the row and column positions of the element chosen,  $a[i, j]$  is the element chosen,  $\text{det}(\text{minor}(a[i, j]))$  is the determinant of the minor of  $a[i, j]$ , and  $\text{power}(m, n)$  is the value of  $m$  raised to the  $n$ th power.

b.  $\det(a)$  = sum of all these products.

(More concisely, if  $n$  is the order of  $a$ ,

$$\det(a) = \sum_i \text{power}(-1, i + j) * a[i, j] * \det(\text{minor}(a[i, j])), \text{ for any } j$$

or

$$\det(a) = \sum_j \text{power}(-1, i + j) * a[i, j] * \det(\text{minor}(a[i, j])), \text{ for any } i).$$

Write a Java program that reads  $a$ , prints  $a$  in matrix form, and prints the value of  $\det(a)$ , where  $\det$  is a method that computes the determinant of a matrix.