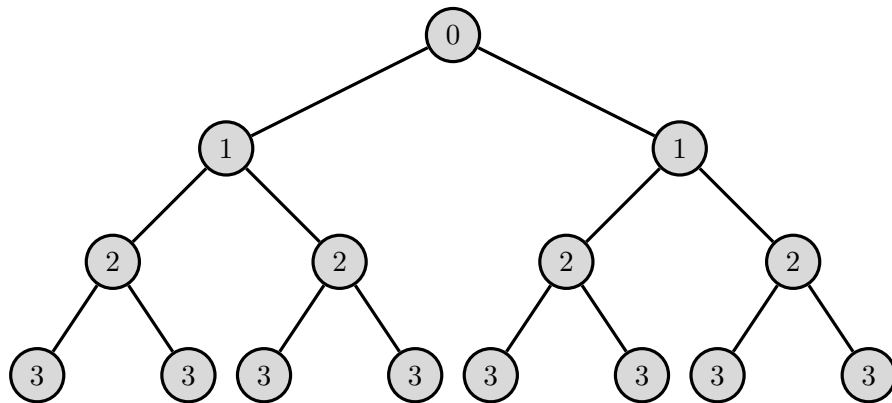

Q1

A node at level n has exactly n ancestors, assuming root node is level zero. This is because each node (except for the root node) has one parent node. So a node at level one has a single parent, the root node. A node at level two has a parent at level one, who has a parent at level zero, and so on.



Q2

Assume that the number of nodes is given by $2n - 1$ where n is the number of leaves of a binary tree. If there is a binary tree of level zero, there will be a single leaf. By the previous equation, $2(1) - 1 = 1$, and so the equation is satisfied.

If a binary tree T has two children, subtrees A and B , then the total number of leaves will be $leaves_A + leaves_B = leaves_T$, or $a + b = t$. By the previous equation, A and B each have $2a - 1$ and $2b - 1$ nodes, respectively. Thus, the total number of nodes in T is $1 + (2a - 1) + (2b - 1)$.

$$\begin{aligned}
 1 + (2a - 1) + (2b - 1) &= \\
 1 + 2a + 2b - 2 &= \\
 2a + 2b - 1 &= \\
 2(a + b) - 1 &= \\
 2t - 1 &= \\
 \boxed{2n - 1}
 \end{aligned}$$

Q3

The total number of pointer fields will be equal to the number of nodes times the number of pointer fields per node. For an m -ary tree, this will be equal to $n \times m$. All nodes except for the primary root node have a pointer from their parent node. Thus the number of “allocated” pointers is $n - 1$. The number of remaining null pointers is thus equal to $(n \times m) - (n - 1)$.

Simplifying, we see:

$$\begin{aligned}
 (n \times m) - (n - 1) &= \\
 (n \times m) - n + 1 &= \\
 nm - n + 1 &= \\
 \boxed{n(m - 1) + 1}
 \end{aligned}$$

Q4

```

/** Java/pseudocode for right in-thread binary tree with
** sequential array representation
**/

// class to represent a node in the tree
class Node{
    DataType data;
    int left; // int value representing index of left node
    int right; // int value representing index of right node
    boolean rThread;
}

// binary tree class (sequential array implementation)
public class TreeClass{
    private Node[] treeArray;

    // constructor
    public void TreeClass(int size, DataType item){
        treeArray = new Node[size];
        treeArray[0] = makeTree(item);
    }

    // returns true if tree is empty, false otherwise
    public boolean isEmpty(){
        if (treeArray[0] == null){
            return true;
        } else{
            return false;
        }
    }

    // creates a new node with data, null left and right pointers, and rThread = false
    public void makeTree(DataType item){
        Node temp = new Node;
        temp.data = item;
        temp.left = null;
        temp.right = null;
        temp.rThread = false;
        return temp;
    }
}

```

```
// creates a left child node with data
public void setLeft(int parentIndex, DataType item){
    Node p = treeArray[parentIndex];
    int childIndex = 2 * parentIndex; // set correct index for child
    if (p == null) {
        throw exception;
    }
    else if (p.left != null){
        throw exception;
    }
    else{
        Node temp = makeTree(item);
        p.left = childIndex;
        temp.right = parentIndex;
        temp.rThread = true;
        treeArray[childIndex] = temp; // place new node at appropriate location in array
    }
}

// creates a right child node with data
public void setRight(int parentIndex, DataType item){
    Node p = treeArray[parentIndex];
    int childIndex = 2 * parentIndex + 1; // set correct index for child
    if (p == null) {
        throw exception;
    }
    else if (!p.rThread){
        throw exception;
    }
    else{
        Node temp = makeTree(item);
        temp.right = p.right;
        temp.rThread = true;
        p.right = childIndex;
        p.rThread = false;
        treeArray[childIndex] = temp; // place new node at appropriate location in array
    }
}
}
```

Q5

The traverse can be easily accomplished using a recursive method.

```
/* Java/pseudocode for in-order traversal of sequential array binary tree */

public void traverse(int index){
    if (treeArray[index] != null){ // if index/node is null, do nothing
        int lc = 2*index; // set left child index
        int rc = lc+1; // set right child index
```

```
        traverse(lc);        // recursively call traverse() on left child
        System.out.println(treeArray[index].data);
        traverse(rc);        // recursively call traverse() on right child
    }
}
```

Q6

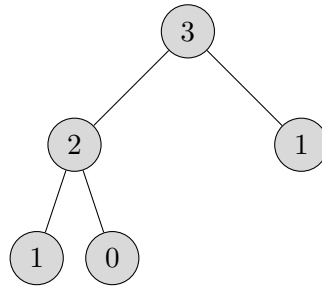
This is easily accomplished by using a recursive method to create a Fibonacci subtree at the right and left children/pointers for every order greater than or equal to two. Order one and order zero are base cases in this method.

```
/* Java/pseudocode for a Fibonacci binary tree */

// Node object used for creating Fibonacci tree
class Node{
    int data;
    Node left;
    Node right;
}

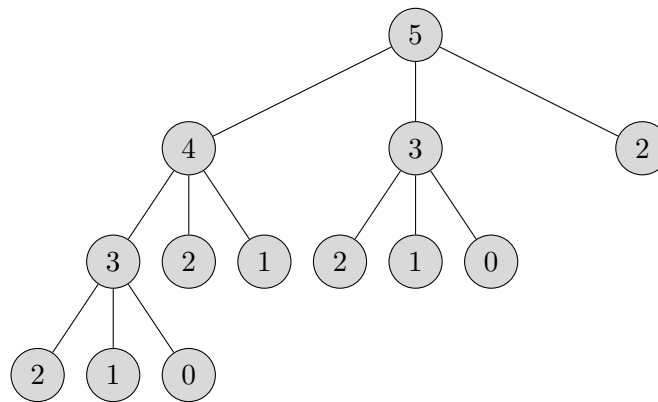
// method for creating a Fibonacci binary tree of specified order
public fibTree(int order){
    if (order == 0){
        Node temp = new Node();
        temp.data = order;
        temp.left = null;
        temp.right = null;
    }
    else if (order == 1){
        Node temp = new Node();
        temp.data = order;
        temp.left = null;
        temp.right = null;
    }
    else {
        Node temp = new Node();
        temp.data = order;
        temp.left = fibTree(order-1);
        temp.right = fibTree(order-2);
    }
}
```

As an example, if `fibTree(3)` was called, the result would be:



Q7

(a) A Fibonacci tree does not need to be binary. Any m-ary tree will work. There will be m base cases. For example, here is a ternary Fibonacci tree.



(b) The number of leaves is given by the following:

$$\#leaves_n = \#leaves_{n-1} + \#leaves_{n-2}$$

Alternatively,

$$\#leaves_n = fibonacci(n + 1)$$

(c) The depth of the tree (assuming binary, depth starts at 0) is simply $n - 1$.