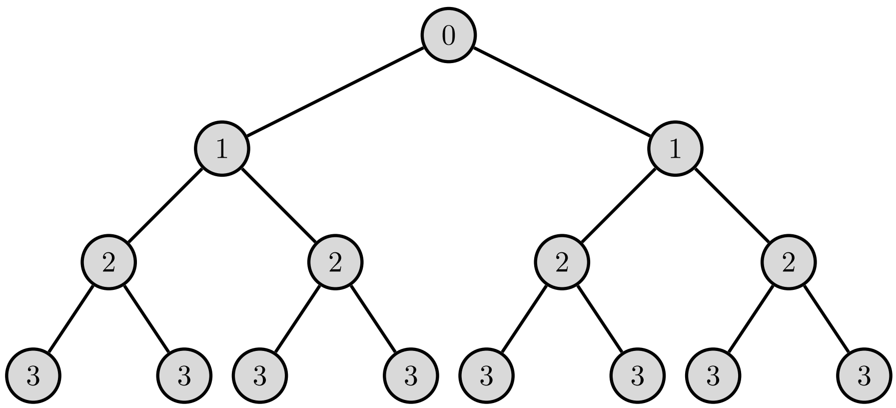
Q1

A node at level n has exactly n ancestors, assuming root node is level zero. This is because each node (except for the root node) has one parent node. So a node at level one has a single parent, the root node. A node at level two has a parent at level one, who has a parent at level zero, and so on.



Q2

Assume that the number of nodes is given by $2n-1$ where n is the number of leaves of a binary tree. If there is a binary tree of level zero, there will be a single leaf. By the previous equation, $2(1)-1=1$, and so the equation is satisfied.

If a binary tree T has two children, subtrees A and B, then the total number of leaves will be $leaves_A + leaves_B = leaves_T$, or $a+b=t$. By the previous equation, A and B each have $2a-1$ and $2b-1$ nodes, respectively. Thus, the total number of nodes in T is $1+(2a-1)+(2b-1)$.

$$\begin{aligned} 1+(2a-1)+(2b-1) &= \\ 1+2a+2b-2 &= \\ 2a+2b-1 &= \\ 2(a+b)-1 &= \\ 2t-1 &= \\ \boxed{2n-1} \end{aligned}$$

Q3

The total number of pointer fields will be equal to the number of nodes times the number of pointer fields per node. For an m-ary tree, this will be equal to $n \times m$. All nodes except for the primary root node have a pointer from their parent node. Thus the number of "allocated" pointers is $n-1$. The number of remaining null pointers is thus equal to $(n \times m)-(n-1)$.

Simplifying, we see:

$$\begin{aligned} (n \times m) - (n-1) &= \\ (n \times m) - n + 1 &= \\ nm - n + 1 &= \\ &\boxed{n(m-1)+1} \end{aligned}$$

Q4

```
/** Java/pseudocode for right in-thread binary tree with
 ** sequential array representation
 */

// class to represent a node in the tree
class Node{
   DataType data;
   int left;  // int value representing index of left node
   int right; // int value representing index of right node
   boolean rThread;
}

// binary tree class (sequential array implementation)
public class TreeClass{
   private Node[] treeArray;

   // constructor
   public void TreeClass(int size, DataType item){
      treeArray = new Node[size];
      treeArray[0] = makeTree(item);
   }

   // returns true if tree is empty, false otherwise
   public boolean isEmpty(){
      if (treeArray[0] == null){
         return true;
      } else{
         return false;
      }
   }

   //  creates a new node with data, null left and right pointers, and rThread = false
   public void makeTree(DataType item){
      Node temp = new Node;
      temp.data = item;
      temp.left = null;
      temp.right = null;
      temp.rThread = false;
      return temp;

   }
```

```
    // creates a left child node with data
    public void setLeft(int parentIndex, DataType item){
        Node p = treeArray[parentIndex];
        int childIndex = 2 * parentIndex; // set correct index for child
        if (p == null) {
            throw exception;
        }
        else if (p.left != null){
            throw exception;
        }
        else{
            Node temp = makeTree(item);
            p.left = childIndex;
            temp.right = parentIndex;
            temp.rThread = true;
            treeArray[childIndex] = temp; // place new node at appropriate location in array
        }
    }

    // creates a right child node with data
    public void setRight(int parentIndex, DataType item){
        Node p = treeArray[parentIndex];
        int childIndex = 2 * parentIndex + 1; // set correct index for child
        if (p == null) {
            throw exception;
        }
        else if (!p.rThread){
            throw exception;
        }
        else{
            Node temp = makeTree(item);
            temp.right = p.right;
            temp.rThread = true;
            p.right = childIndex;
            p.rThread = false;
            treeArray[childIndex] = temp; // place new node at appropriate location in array
        }
    }

}
```

Q5

The traverse can be easily accomplished using a recursive method.

```
/* Java/pseudocode for in-order traversal of sequential array binary tree */

public void traverse(int index){
    if (treeArray[index] != null){  // if index/node is null, do nothing
        int lc = 2*index;  // set left child index
        int rc = lc+1;     // set right child index
```

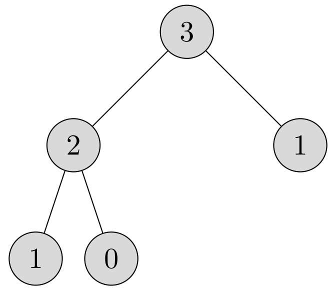
```
      traverse(lc);      // recursively call traverse() on left child
      System.out.println(treeArray[index].data);
      traverse(rc);      // recursively call traverse() on right child
   }
}
```

Q6

This is easily accomplished by using a recursive method to create a Fibonacci subtree at the right and left children/pointers for every order greater than or equal to two. Order one and order zero are base cases in this method.

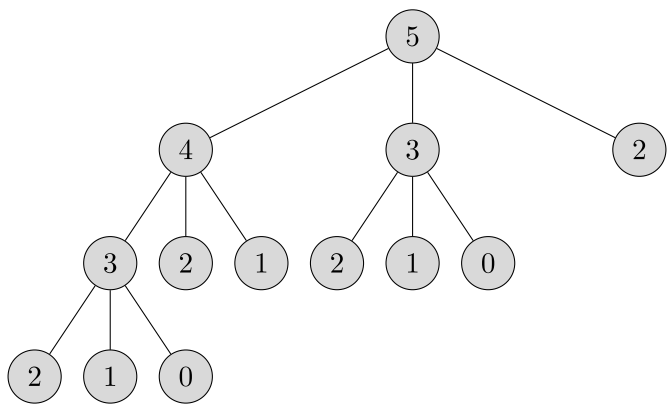
```
/* Java/pseudocode for a Fibonacci binary tree */

// Node object used for creating Fibonacci tree
class Node{
   int data;
   Node left;
   Node right;
}

// method for creating a Fibonacci binary tree of specified order
public fibTree(int order){
   if (order == 0){
      Node temp = new Node();
      temp.data = order;
      temp.left = null;
      temp.right = null;
   }
   else if (order == 1){
      Node temp = new Node();
      temp.data = order;
      temp.left = null;
      temp.right = null;
   }
   else {
      Node temp = new Node();
      temp.data = order;
      temp.left = fibTree(order-1);
      temp.right = fibTree(order-2);
   }
}
```

As an example, if fibTree(3) was called, the result would be:



Q7

(a) A Fibonacci tree does not need to be binary. Any m-ary tree will work. There will be m base cases. For example, here is a ternary Fibonacci tree.



(b) The number of leaves is given by the following:

$$\#leaves_n = \#leaves_{n-1} + \#leaves_{n-2}$$

Alternatively,

$$\#leaves_n = fibonacci(n+1)$$

(c) The depth of the tree (assuming binary, depth starts at 0) is simply $n-1$.