ME EN 534 Term Project

Apollo Spacecraft Attitude Dynamics

To be completed individually

Introduction

The objective of this project is to model the attitude dynamics of the Apollo spacecraft. The result will be a complete set of rotational equations of motion, inertial properties, and a simulator that will plot the rotational motion of the spacecraft in response to torque inputs. You will also explore the use of control moment gyroscopes (CMGs) as torque actuators on spacecraft.

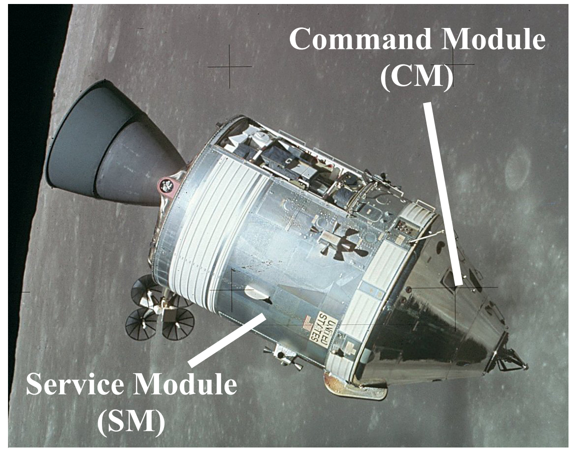

Figure 1. The Apollo command and service modules (CSM).

Apollo Modules

The Apollo spacecraft is composed of three modules. The command module (CM) is the capsule that houses the three astronauts during their time in Earth orbit, while in transit to the moon and back, and during reentry and splashdown. The service module (SM) houses life support systems, propellant, other consumables, and the main propulsion system. Together, the CM and SM are called the CSM (see Figure 1). The lunar module (LM) carries two of the astronauts to the surface of the moon and back while the CM pilot remains in the CM in lunar orbit. The SM and LM are left in orbit when the CM reenters the Earth's atmosphere at the conclusion of the mission. In this project, we will only explore the dynamics of the CSM (not the LM).

Coordinate Frames

Early in the development of the Apollo spacecraft, NASA engineers defined the body-fixed frame shown in Figure 2, with its origin at the center of mass of the CSM. We will call this frame the B frame. The x axis of the B frame is aligned with the longitudinal axis of the spacecraft; it points out of the tip of the CM and corresponds to the spacecraft's roll axis. The y axis is directed out the right side of the spacecraft and corresponds to the spacecraft's pitch axis. The z axis is orthogonal to the other two axes, points out the "belly" of the spacecraft, and corresponds to the yaw axis.

For the purpose of this project, the orientation of the B frame (and the spacecraft that it is attached to) will be specified by a standard 3-2-1 Euler angle sequence relative to some local inertial frame, with yaw angle ψ, pitch angle θ, and roll angle ϕ.

For the purposes of defining the inertial properties of the CSM and LM, NASA adopted another frame with axes parallel to the B frame, but with an origin 1000 inches in the $-x$ direction relative to the base of the CM. We will call this the A frame. The base of the CM is indicated in Figure 2.

Inertial Properties

The inertial properties of the CM, SM, and LM are listed in Figure 3. The listed moments of inertia are taken about axes through the center of mass of the individual modules. For example, for the CM, its moment of inertia about a y axis through its own center of mass is I_{yy}= 3919 slug-ft^2. The center of mass numbers are given in inches and are measured from the origin of the A frame. Note that I have covered certain values in the table because I want you to have the chance to calculate them!

Task A: Inertial Properties

Using the inertia data given in Figure 3 and the coordinate frames defined above, find the following for the CSM (CM and SM):

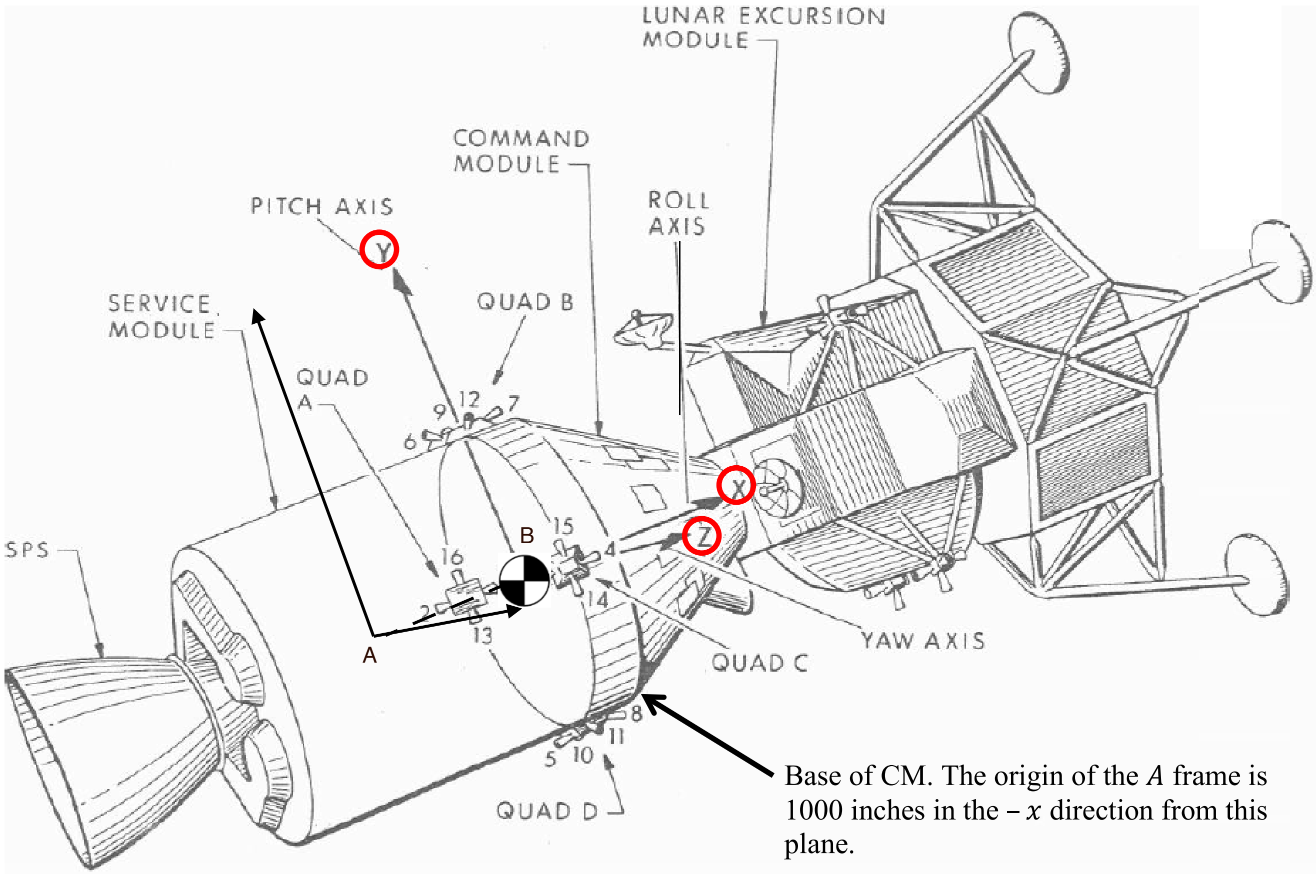

Figure 2. CSM body-fixed B frame. The frame has its origin at the center of mass. Source: *Apollo Command and Service Module Reaction Control by Digital Autopilot*, MIT Instrumentation Laboratory, 1966.

a. *General Case*, using the center of mass data given in Figure 3:
   a. The location of the center of mass of the CSM, expressed in meters, as a vector relative to the A frame. Include the propellant in your calculation.
   b. The total inertia matrix of the CSM, including propellant, about the body-fixed B axes at the center of mass of the CSM. Express your answer as a matrix with units of kg-m^2.
b. *Simplified Case*, assuming the centers of mass of all components are located on the x axis:
   a. The location of the center of mass of the CSM, expressed in meters, as a vector relative to the A frame. Include the propellant in your calculation.
   b. The total inertia matrix of the CSM, including propellant, about the body-fixed B axes at the center of mass of the CSM. Express your answer as a matrix with units of kg-m^2.

Task B: Equations of Motion

Find the rotational equations of motion that could be solved numerically to find the orientation of the spacecraft (ψ, θ, ϕ) given torque inputs about the body-fixed (B frame) xyz axes. The torques would typically be generated by thrusters, but you can leave them as general symbols (M_x, M_y, M_z) in your equations. You may present the equations as a set of six 1st-order equations in state-variable form, or as an $[M]$ matrix and $\{F\}$

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vector that could be used to put them in state-variable form. Whichever form you choose, you should (1) make it clear which form you are using, (2) reduce the equations as much as possible, and (3) collect derivative terms. Leave all the moments and products of inertia in symbolic form; do not insert numbers.

Task C: Simulations and Calculations

Using the inertial properties from Task A and the equations of motion from Task B, complete the following tasks:

a. Create a MATLAB function that simulates the rotational motion of the CSM in response to applied torques. The function parameters should be the initial body-fixed angular velocities (deg/s), initial Euler angles (deg), the (column) time vector over which the simulation should be performed (s), and column vectors containing body-fixed torques (N-m) as a function of time. The time step will always be constant. The function should return six solution vectors: $\omega_x(t)$ (deg/s), $\omega_y(t)$ (deg/s), $\omega_z(t)$ (deg/s), $\psi(t)$ (deg), $\theta(t)$ (deg), and $\phi(t)$ (deg). The function should not generate plots or any other output. The function should be named 'your_last_name,' all lower case. Everything should be contained in a single m-file so that I can take the function and compare your simulated results to my simulated results. For example, if Leonhard Euler were to submit a function, it would look like this:

```
function [wx,wy,wz,psi,theta,phi] = euler(wx0,wy0,wz0,psi0,theta0,phi0,t,Mx,My,Mz)
```

Submit a version of your function for the General Case described above (full inertia matrix). You do not need to submit your function for the Simplified Case.

b. Simulate and plot $\omega_x(t)$, $\omega_y(t)$, $\omega_z(t)$, $\psi(t)$, $\theta(t)$, and $\phi(t)$ in response to the following torque inputs for both the General Case and the Simplified Case. Simulate over t = 0 to 30 s. All initial conditions are zero. In a table, list the maximum and minimum values of ω_x, ω_y, ω_z, ψ, θ, and ϕ for both cases.

$$M_x(t) = 176\cos(0.2t) \text{ N-m},\ M_y(t) = 54 \text{ N-m},\ M_z(t) = 98\sin(0.3t) \text{ N-m}$$

c. During the flight to the moon, the CSM was placed in *passive thermal control (PTC) mode* (known as "barbeque mode"), in which the CSM rotated at a constant rate about the x axis to maintain even heating and cooling. Using the General Case described above (full inertia matrix), what are the three required body-fixed torques necessary to maintain a constant x-axis rotation of 1 deg/s without motion about other axes? Calculate the three torque values and report them in your memo. Simulate PTC mode using your function by assigning an initial condition of 1 deg/s for ω_x and zero initial conditions for the other states. Plot $\omega_x(t)$, $\omega_y(t)$, and $\omega_z(t)$ for t = 0 to t = 1000 s for two cases: (1) using the three torque values you calculated for PTC mode, and (2) using zero torques about all three axes. You will want to use a very accurate inertia matrix (preferably the same matrix you are using in your simulation) to calculate and implement the required torques.

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APOLLO LOR MISSION

WEIGHT, CENTER OF GRAVITY AND INERTIA SUMMARY

ITEM	WEIGHT POUNDS	CENTER OF GRAVITY*			MOMENTS OF INERTIA (SLUG-FT.^{2})		
		X	Y	Z	ROLL (X)	PITCH (Y)	YAW (Z)
COMMAND MODULE	9730	1043.1	-0.1	7.8	4474	3919	3684
SERVICE MODULE - Less Propellant	9690	908.2	0.7	-0.6	6222	10321	10136
TOTAL - Less Propellant							
PROPELLANT - S/M**	37295	905.9	5.6	-2.4	19162	19872	26398
TOTAL - With Propellant							
LUNAR EXCURSION MODULE	24460	623.0	0.0	0.0	13616	12776	13247
ADAPTER - LEM - C-5	3400	642.7	0.0	0.0	8372	12273	12273
TOTAL - Injected							
LAUNCH ESCAPE SYSTEM	7050	1299.1	0.0	-0.2	266	10961	10962
TOTAL - SPACECRAFT LAUNCH	91625	865.7	2.3	-0.2	52431	786479	793029

NOTES: *Centers of gravity are in the NASA reference system except that the longitudinal axis has an origin 1000 inches below the tangency point of the command module substructure mold line.

**The propellant weight of 37295 pounds provides approximately 10% $\triangle$ V margin, and is determined from an estimated time line analysis. The propellant weight is based on a specific impulse of 313.0.

Figure 3. Apollo inertial properties. Source: *Monthly Weight and Balance Report for the Apollo Spacecraft*, North American Aviation, Inc., 1963.

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Control Moment Gyroscopes

Traditionally (including in the Apollo program), spacecraft attitude control has been accomplished using reaction thrusters. More recently, engineers have used *control moment gyroscopes* (CMGs) and other momentum-exchange devices to control the attitude of satellites and other spacecraft. In fact, four large two-gimbal CMGs are used to control the attitude of the International Space Station (ISS). In many applications CMGs are preferred to thrusters because of the large control torques that can be achieved with relatively little power.

The idea behind a single-gimbal CMG is simple. To help with the discussion, consider the set of three single-gimbal CMGs shown in Figure 4. Let's focus on CMG1 for now. The CMG rotor spins at a constant angular speed Ω. The orientation of the axis of rotation of the rotor can be changed by pivoting the gimbal of CMG1 about the z axis by the angle θ_1. This results in a change in the angular momentum of CMG1, which requires an applied moment. The moment is provided by an actuator attached to the spacecraft and the bearings holding the gimbal of CMG1. In return, an equal and opposite moment is applied on the spacecraft. By rotating CMG1, we can exert an attitude-control moment on the spacecraft.

A set of three single-gimbal CMGs (as shown in Figure 4) allows us to apply moments about all three body-fixed axes of the spacecraft. With a constant Ω for all three CMG rotors, the inputs to the CMG system are the gimbal angles $(\theta_1, \theta_2, \theta_3)$ and gimbal angle rates $(\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3)$, and the output is the total moment vector exerted on the spacecraft.

Although CMGs were not used in the Apollo program, you will explore the use of a three-CMG system for attitude control of the Apollo CSM.

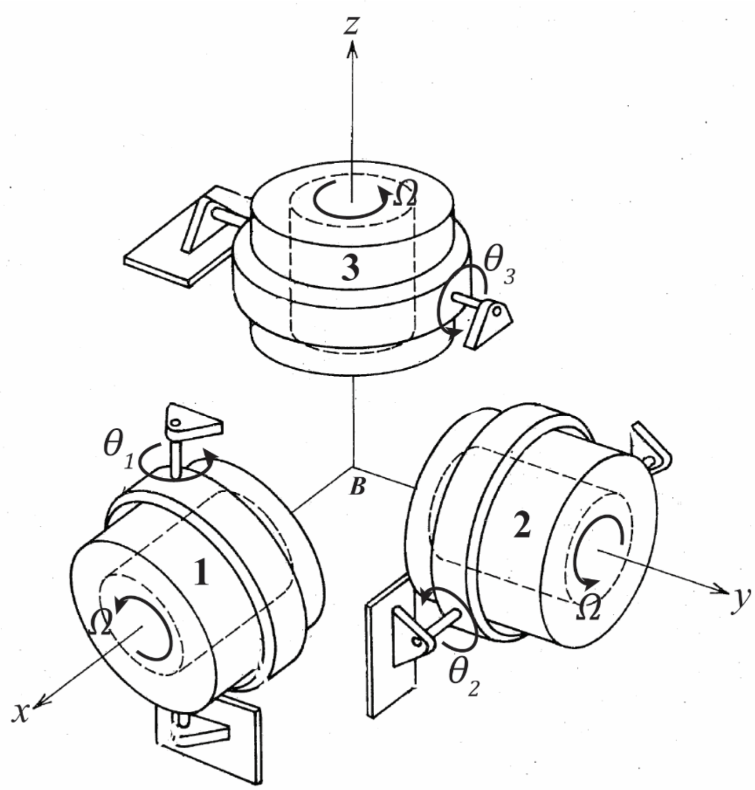

Figure 4. Three single-gimbal control moment gyroscopes (CMGs). The x,y,z axes shown are the axes of the B frame, attached to the CSM. Each CMG rotor spins at a constant speed Ω, as shown. At the instant shown, the axes of the CMG rotors are aligned with the B axes of the CSM.

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Task D: Control Moment Gyroscope Model

For the three-CMG system shown in Figure 4, which is placed at the center of mass of the CSM, do the following:

a. Develop a mathematical model that relates the gimbal angles $(\theta_1, \theta_2, \theta_3)$ and gimbal angle rates $(\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3)$ to the body-fixed moments $\left(M_x, M_y, M_z\right)$ exerted on the Apollo CSM. The inertia of each rotor about its axis of rotation is the same; call it I_R. In developing the model, I found it useful to write the angular momentum of a single rotor and then apply the fundamental moment-momentum equation $\{\dot{H}\} = \{M\}$, instead of applying Euler's equations or the expanded version of the rotational equations. For CMG1, make the assumption that $\Omega \gg \dot{\theta}_1 \gg \omega$, where ω is the angular velocity of the CSM. Repeat the process two more times, with the same assumptions, for the other two CMGs. The total moment exerted on the Apollo CSM will be the sum of moments from the three CMGs. In your memo, present three equations (or a single vector equation) representing M_x, M_y, M_z, the torques exerted on the CSM by the CMG system.

b. Consider three *identical* single-axis CMGs with a constant rotor speed of $\Omega = 6600$ rpm (the same speed as the CMGs in the International Space Station). Assuming each CMG gimbal can execute a maximum gimbal rate of $\dot{\theta}_{1,max} = \dot{\theta}_{2,max} = \dot{\theta}_{3,max} = 30$ deg/s, find the minimum rotor inertia I_R needed to achieve desired angular accelerations of the CSM of $\dot{\omega}_x = 5$ deg/s^2 and $\dot{\omega}_y = \dot{\omega}_z = 2.5$ deg/s^2. Assume the angular accelerations occur separately (only one at a time). Do your calculations for the case when the CMG gimbals are in their zero orientation and the CSM is starting from rest.

Task E: More Simulations and Calculations

Using the model, minimum rotor inertia I_R, and rotor speed $\Omega = 6600$ rpm from Task D, do the following:

a. Create a MATLAB function that simulates the rotational motion of the CSM in response to applied torques generated by three single-axis CMGs, as shown in Figure 4. The function parameters should be the initial body-fixed angular velocities (deg/s), initial Euler angles (deg), the (column) time vector over which the simulation should be performed (s), and column vectors containing CMG gimbal axes (deg) as a function of time. The time step will always be constant. The function should return six solution vectors: $\omega_x(t)$ (deg/s), $\omega_y(t)$ (deg/s), $\omega_z(t)$ (deg/s), $\psi(t)$ (deg), $\theta(t)$ (deg), and $\phi(t)$ (deg). The function should not generate plots or any other output. The function should be named 'your_last_name_cmg,' all lower case. Everything should be contained in a single m-file so that I can take the function and compare your simulated results to my simulated results. For example, if Joseph-Louis Lagrange were to submit a function, it would look like this:

```
function [wx,wy,wz,psi,theta,phi]=lagrange_cmg(wx0,wy0,wz0,psi0,theta0,phi0,t,th1,th2,th3)
```

Submit a version of your function for the General Case described above (full inertia matrix). You do not need to submit your function for the Simplified Case.

b. Simulate and plot $\omega_x(t)$, $\omega_y(t)$, $\omega_z(t)$, $\psi(t)$, $\theta(t)$, and $\phi(t)$ in response to the following CMG gimbal angle inputs for the General Case. Simulate over $t = 0$ to 30 s. All initial conditions are zero. In a table, list the maximum values of ω_x, ω_y, ω_z, ψ, θ, and ϕ.

$$\theta_1(t) = 15 \sin\frac{2\pi}{30} t \text{ deg}, \; \theta_2(t) = 0 \text{ deg}, \; \theta_3(t) = 0 \text{ deg}$$

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Use the equations of motion for the Simplified Case to explain the behavior shown in the plots. (You should have simulated the response for the General Case. However, since the products of inertia are small, the overall behavior will be similar for the Simplified Case. You can therefore use equations of motion for the Simplified Case to explain what you see.) Why do the angular velocities and Euler angles exhibit the behavior shown in the plots? Explain the initial behavior of both plots. Why do the angular velocities and Euler angles move in the directions shown in the plots?

c. Simulate and plot $\omega_x(t)$, $\omega_y(t)$, $\omega_z(t)$, $\psi(t)$, $\theta(t)$, and $\phi(t)$ in response to the following CMG gimbal angle inputs for the General Case. Simulate over $t = 0$ to 30 s. Use $\Omega = 6600$ rpm and the value for I_R that you calculated in a previous step. All initial conditions are zero. In a table, list the maximum and minimum values of ω_x, ω_y, ω_z, ψ, θ, and ϕ.

$$\theta_1(t) = 15\left(\frac{1}{1+e^{-0.3t}} - 0.5\right) \text{ deg}, \ \theta_2(t) = 5\sin\frac{2\pi}{30}t \text{ deg}, \ \theta_3(t) = -5\sin\frac{2\pi}{30}t \text{ deg}$$

Deliverables

Submit the following by email to the instructor:

1. A memo that includes all requested equations, plots, and results. Present your results clearly and succinctly. Several tasks require you to find the maximum and minimum values of ω_x, ω_y, ω_z, ψ, θ, and ϕ; present these all in a single table, with headings to indicate which results correspond to which case. Remember to include units, labels and legends on plots, etc.
2. The requested MATLAB m-files from Tasks C.b and E.a.