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Quality Ladders in the Theory of Growth

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We develop a model of repeated product improvements in a continuum of sectors. Each product follows a stochastic progression up a quality ladder. Progress is not uniform across sectors, so an equilibrium distribution of qualities evolves over time. But the rate of aggregate growth is constant. The growth rate responds to profit incentives in the R&D sector. We explore the welfare properties of our model. Then we relate our approach to an alternative one that views product innovation as a process of generating an ever-expanding range of horizontally differentiated products. Finally, we apply the model to issues of resource accumulation and international trade.

I. INTRODUCTION

Casual observation suggests the central role that quality-upgrading plays in raising our standard of living. For example, the ease and comfort of transportation have increased enormously since the horse-and-buggy gave way to a succession of automobiles of ever-higher quality. New generations of televisions provide finer detail and sharper colours. And the enjoyment of music in the home has been enhanced by the replacement of the gramophone by the phonograph, and the phonograph by the compact disc player.

Intermediate goods too have been subject to product improvement. Witness the recent revolution in desk-top computing. Or consider the advances that have taken place in integrated circuitry. These innovations have raised total factor productivity in the manufacturing of consumer goods and capital equipment, and made possible the production of entirely new final products.

The economics of quality improvement have been studied by industrial organization economists in their work on patent races. Beginning with Loury (1979), Dasgupta and Stiglitz (1980), and Lee and Wilde (1980), much effort has been devoted to understanding the incentive that firms have to bring out new and improved products. This literature typically views R&D competition as a once-and-for-all race for technological supremacy. While contributing many useful insights, the one-shot framework fails to capture an essential aspect of quality competition. This is the continual and cyclical nature of the process whereby each new product enjoys a limited run at the technological frontier, only to fade when still better products come along. Almost every product exists on a *quality ladder*, with variants below, that may already have become obsolete, and others above, that have yet to be discovered.

Recent work by Segerstrom *et al.* (1990) and Aghion and Howitt (1990) provides the beginnings of a theory of repeated quality innovations. These authors cast the patent-race paradigm in a dynamic, general-equilibrium setting. The result in each case is a model of long-run growth based on endogenous technical change.[1] Their models enable us to study the structural and institutional determinants of ongoing technological progress.

Each of these interesting efforts contains at least one unappealing element, however. In the work by Segerstrom *et al.*, patent races take place in a multitude of industries *in sequence.* That is, all research effort in the economy first is devoted to improving a single product, then another, and so on, until all products have been improved exactly once. Then the cycle repeats. In Aghion and Howitt, by contrast, the patent race takes place at an *economy-wide level.* A successful research project improves all products simultaneously. The sole innovator thereby gains monopoly power across all industries. Evidently unhappy with this implication, Aghion and Howitt make reference to antitrust laws to justify their imposition of a requirement that the monopolist must license the portfolio of patents to a continuum of arms-length competitors.

In what follows, we propose an approach that resolves these difficulties. This approach draws on the building blocks provided by Segerstrom *et al.* and Aghion and Howitt. We envision a continuum of products, each with its own quality ladder. Entrepreneurs target individual products and race to bring out the next generation. These races take place *simultaneously.* In any time interval, some of the efforts succeed while others fail. Successful ventures call forth efforts aimed at still further improvement, with each innovation building upon the last. This specification accords well with the description of the research process in, for example, Freeman (1982) and Dosi (1988).

Our model generates an equilibrium distribution of product qualities that evolves over time. Each individual product follows a stochastic progression up the quality ladder. But, in the aggregate, the innovation process we describe is a smooth one. An index of consumption grows at a constant and determinate rate in the steady state. This rate is readily calculable. It can also be compared to the socially optimal rate of growth.

We believe that our approach will prove useful in many applications. In the latter part of this paper, for example, we explore the relationship between the size of the resource base and the long-run rate of growth, and describe the long-run pattern of specialization in a two-country world economy with innovation and trade.[2] In Grossman and Helpman (1991), we study the product life-cycle with concurrent innovation by high-wage producers and imitation by low-wage producers. Grossman (1989) derives the growth effects of a variety of trade and industrial policies.

Our approach is related to an alternative one that views product innovation as a process of generating an ever-expanding range of horizontally differentiated products.[3] This latter framework has been applied to positive and normative topics in economic growth in recent work by Judd (1985), Romer (1990) and ourselves (Grossman and Helpman, 1989*a*, *b*, *c*, 1990). At first glance, the economics of the development of horizontally differentiated products seem quite distinct from those of product improvement. Yet, as we shall demonstrate below, the two approaches yield quite similar answers to many

1. Related work by Schleifer (1986) and Krugman (1990) deals with a continual process of cost innovation. Reductions in cost and improvements in quality are two different ways for firms to supply more services at a given price.

2. Grossman (1989) relates this two-country model to the evidence on Japanese success in the high-technology sectors.

3. Scherer (1980, p. 409) cites survey evidence that firms devote 59% of their research outlays to product improvement, 28% to developing new products and 13% to developing new processes. These data may understate the importance of vertical relative to horizontal differentiation in the innovation process, since many "new products" replace old products that perform similar functions.

questions. Indeed, it is possible to construct comparable variants from each class of model that share identical reduced forms. The alternative analyses do diverge, however, when it comes to normative issues.

The remainder of this paper is organized as follows. We develop our simplest one-factor model of growth due to quality improvements in the next section and explore its positive and normative properties in Section III. In Section IV we pursue the analogy between quality-based growth and variety-based growth. Section V extends the model to include an outside good and a second primary input, and investigates the implications of factor accumulation for growth. In Section VI we introduce a second country and international trade, and describe the long-run pattern of trade. Section VII concludes.

II. THE BASIC MODEL

We consider an economy with a continuum of goods indexed by ω. The (fixed) set of goods consists of the interval $[0, 1]$ whose measure is one. Each product ω can potentially be supplied in a countably-infinite number of qualities. We choose units so that the quality of every good at time $t=0$ equals one. Quality j of product ω is given by $q_j(\omega)=\lambda^j$, where $\lambda>1$ is the same for every ω. In order to attain quality j, a product must be improved j times after $t=0$. Each step up the quality ladder requires R&D. We describe this activity below.

Consumers share a common intertemporal utility function

$$U=\int_0^\infty e^{-\rho t}\log u(t)dt, \tag{1}$$

where ρ is a subjective discount rate and $\log u(t)$ represents the flow of utility at time t. Instantaneous utility is given by

$$\log u(t)=\int_0^1 \log\left[\textstyle\sum_j q_j(\omega)d_{jt}(\omega)\right]d\omega, \tag{2}$$

where $d_{jt}(\omega)$ denotes consumption of quality j of product ω at time t. Every consumer maximizes utility subject to an intertemporal budget constraint

$$\int_0^\infty e^{-R(t)}E(t)dt \leqq A(0), \tag{3}$$

where $E(t)$ represents the flow of spending at time t, $R(t)$ is the cumulative interest factor up to time t, and $A(0)$ denotes the present value of the stream of factor incomes plus the value of initial asset holdings at $t=0$. Naturally,

$$E(t)=\int_0^1 \left[\textstyle\sum_j p_{jt}(\omega)d_{jt}(\omega)\right]d\omega, \tag{4}$$

where $p_{jt}(\omega)$ is the price of quality j of product ω at time t.

The consumer maximizes utility in two stages. First, he allocates $E(t)$ to maximize $u(t)$ given prices at time t. Then he chooses the time pattern of spending to maximize U. To solve the static problem, the consumer selects for each product the single quality $j=J_t(\omega)$ that carries the lowest quality-adjusted price $p_{jt}(\omega)/q_j(\omega)$.[4] Then he allots

4. In what follows, $J_t(\omega)$ is unique.

identical expenditure shares to all products. This gives static demand functions

$$d_{jt}(\omega)=\begin{cases}E(t)/p_{jt}(\omega) & \text{for } j=J_t(\omega),\\ 0 & \text{otherwise.}\end{cases} \tag{5}$$

Substituting these demand functions into (2), and the result into (1), we can reformulate the intertemporal maximization problem as one of choosing the time pattern of spending to maximize

$$U=\int_0^\infty e^{-\rho t}\left\{\log E(t)+\int_0^1 [\log q_t(\omega)-\log p_t(\omega)]d\omega\right\}dt \tag{6}$$

subject to (3), where $q_t(\omega)$ and $p_t(\omega)$ represent the quality level and price, respectively, of $J_t(\omega)$. The solution to this problem is characterized by

$$\dot{E}/E=\dot{R}-\rho, \tag{7}$$

the budget constraint (3), and a transversality condition. Since preferences are homothetic, aggregate demands are proportional to those of the representative consumer. In what follows, we use E to denote aggregate spending and (5) to represent economy-wide demand functions.

Turning to the production side, we endow the economy with a single primary input called "labour" (but see Sections 5 and 6). One unit of labour is needed to manufacture one unit of any product, regardless of quality. Of course, better quality products cannot be produced until they have been invented in the research lab. Patent restrictions may apply as well, as we shall discuss further below.

At any point in time producers in any "industry" ω compete as price-setting oligopolists with common marginal costs equal to the wage rate $w(t)$. Then, if several firms in an industry are able to produce goods of the same quality, each sets a price equal to unit cost in the Bertrand competition and each earns zero profit. Alternatively, one producer may enjoy a quality lead over its industry rivals. Then the oligopoly equilibrium entails the leader charging a price that, adjusted for quality, falls epsilon below the unit cost of production of its nearest competitor, while that competitor sets a price just equal to its own marginal cost. At these prices the leader captures the entire industry market. The leader would not wish to deviate from this equilibrium by raising price discretely, for then it would lose all of its customers to its rival. Nor would it wish to reduce the price, because industry demand functions are unit elastic. The rival, for its part, can only make losses by lowering its price below marginal cost, while higher prices are a matter of indifference when sales are zero.

Our assumptions on the technology of product development and the nature of intellectual property rights will ensure that every industry has a unique quality leader. Also, we will show that, in each industry, the leader always stands *exactly* one step ahead of its nearest rival. Then all state-of-the-art products bear the same "limit" price,

$$p=\lambda w. \tag{8}$$

This price yields demand per product of $E/\lambda w$. The leaders each earn a flow of profits $\pi=(\lambda w-w)E/\lambda w=(1-1/\lambda)E$.

A blueprint is needed to produce any commodity. These blueprints are costly to develop. We assume that infinitely-lived patents protect the intellectual property rights of innovators, and that patent licensing is not feasible. These assumptions ensure that all manufacturing is undertaken by firms that have successfully developed new, state-of-the-art products. Alternatively, we might assume an absence of enforceable patents, but also that imitation is costly. No firm would invest in imitation knowing that it would earn zero profits in the ensuing Bertrand equilibrium.[5] Similarly, we could suppose that licensing is feasible, but that the antitrust laws prohibit no-competition clauses in licensing contracts, or that such covenants are difficult to enforce. No firm would pay a positive amount for a blueprint if the licensor were unable to commit itself to refrain from subsequent entry into Bertrand competition.[6]

An entrepreneur may target her research efforts at any of the continuum of state-of-the-art products. If she undertakes R&D at intensity $\tilde{\imath}$ for a time interval of length dt, then she will succeed in taking the next step up the quality ladder for the targeted product with probability $\tilde{\imath}dt$. This formulation mimics the one-shot, partial equilibrium, patent-race models of Lee and Wilde (1980) and others. It implies that R&D success bears a Poisson probability distribution with an arrival rate that depends only on the current level of R&D activity.

We allow free entry into the race for the next generation product. A unit of R&D activity requires a_I units of labour per unit of time for both an incumbent and for newcomers. That is, we implicitly suppose that potential entrants can, via inspection of the goods on the market, learn enough about the state of knowledge to mount their own research efforts, even if the patent laws (or the lack of complete knowledge about best production methods) prevent them from manufacturing the current generation products. This specification captures in part the often noted, public-good characteristics of technology.

Without any cost advantage, industry leaders do not invest resources to improve their own state-of-the-art products.[7] To see this, note that a research success would leave the leader with a two-step advantage over its nearest competitor, and thus enable it to increase its price to $\lambda^2 w$. This would yield a flow of *incremental* profits equal to $\Delta\pi = (1-1/\lambda^2)E-(1-1/\lambda)E=(1-1/\lambda)E/\lambda$, which, however, is strictly less than the incremental profits $(1-1/\lambda)E$ that accrue to a non-leader who achieves a research success. So leaders seeking to upgrade the quality of their own products cannot successfully compete for financing with non-leaders. Put differently, a leader would strictly prefer to devote any research funds it may raise to R&D aimed at developing a leadership position in a second market rather than to R&D aimed at widening an existing lead in its own market.

We consider now the entrepreneur's choice of industry in which to target R&D efforts, and the optimal scale of those efforts. The prize for a research success in some industry is a flow of profits that will last until the next success is achieved in the same industry. We have seen that the profit flows π are the same for all industries ω. Therefore, an entrepreneur will be indifferent as to the industry in which she devotes her R&D efforts provided that she expects her prospective leadership position to last equally long in each one. We focus hereafter on the symmetric equilibrium in which all products are targeted

5. In Grossman and Helpman (1991), we develop a two-country model in which factor prices differ across countries. Then imitation may be profitable in the low-wage country, because success in creating a clone yields strictly positive profits to the imitator in the ensuing duopoly equilibrium.

6. As a referee has pointed out to us, if licensing were feasible and contracts unrestricted, an innovator would always prefer to license her new technology to the nearest rival rather than to compete with that rival in the product market.

7. This is essentially the same result as in Reinganum (1982). She shows that a challenger has greater incentive to undertake risky R&D than an incumbent, in a one-shot patent race. Here, leaders do not undertake R&D at all, because the supply of challengers is perfectly elastic. In Grossman and Helpman (1991) we assume that leaders, by dint of their past R&D successes, are able to improve upon their own products at lower (expected) cost than outsiders. With this modification of the model, we find equilibria with positive R&D by both leaders and challengers.

to the same *aggregate* extent ι. In such an equilibrium the individual entrepreneur indeed expects profit flows of equal duration in every industry and so is indifferent as to the choice of industry.

We let v denote the present value of the uncertain profit stream that accrues to an industry leader; i.e. the stock market value of the firm. In a moment, we will relate v to the size of oligopoly profits, the expected duration of industry leadership, and the market rate of interest. But for the moment, it is enough to recognize that v is the size of the prize for a research success. An individual entrepreneur can realize v with probability $\tilde{\iota}dt$ by investing resources $a_I\tilde{\iota}$ in R&D for an interval dt at cost $wa_I\tilde{\iota}dt$. This venture is risky, because the R&D effort may fail. But the risk involved is idiosyncratic, so the market will encourage entrepreneurs to maximize the expected net benefit from R&D. Maximizing $v\tilde{\iota}dt - wa_I\tilde{\iota}dt$, we find that either the optimal scale of operation is zero or infinite, or expected benefit is zero for all $\tilde{\iota}$. In an equilibrium with positive but finite investment in R&D, we must have $v = wa_I$. As in models of perfect competition with constant returns to scale, the size of the individual R&D venture here is indeterminate.

We turn now to the stock-market valuation of the firm. The ownership shares in industry leaders pay dividends πdt over a time interval of length dt, and appreciate by $\dot{v}dt$ if no entrepreneur succeeds in supplanting the firm's leadership position. However, if the leader's product is improved during the interval dt, the shareholders will suffer a total capital loss of amount v. This happens with probability ιdt, where we recall that ι is the aggregate intensity of research by the (perhaps) many entrepreneurs who target their R&D efforts at the leader's product. All told, the shares bear an expected rate of return of $(\pi+\dot{v})/v-\iota$ per unit time. This return is risky. But once again the risk for any one leader is idiosyncratic. The stock market values the firm so that its expected rate of return just equals the safe interest rate $\dot{R}$. Using $v = wa_I$, we may write this "no-arbitrage" condition as[8]

$$\frac{\pi}{wa_I}+\frac{\dot{w}}{w}=\dot{R}+\iota.$$

We choose labour as numeraire; i.e. $w(t)=1$ for all t. Recall that $\pi=(1-1/\lambda)E$. This, together with (7) and the no-arbitrage condition, implies

$$\frac{\dot{E}}{E}=\frac{(1-1/\lambda)E}{a_I}-\rho-\iota. \tag{9}$$

This provides a differential equation for spending. The rate of growth in spending increases with the level of spending and decreases with aggregate R&D intensity.

We close the model with a market-clearing condition. Let L be the total supply of labour. Total manufacturing employment equals[9]

$$\int_0^1 [E(t)/p(t)]d\omega = E(t)/\lambda,$$

8. If we solve the differential equation implicit in the no-arbitrage condition, we find that this condition equates the cost of R&D to the expected present discounted value of profits; i.e. it requires the absence of excess profits. This requirement follows of course from our assumption of free entry into R&D. We could have specified the no-excess profits condition directly, as we did in Grossman and Helpman (1989*a*), and then derived the no-arbitrage condition by differentiating with respect to time.

9. We assume here that even at time $t=0$ each industry has a unique leader plus a competitor one rung down the quality ladder. This implies that $p=\lambda$ for all products. We will show that, with this assumption, the economy jumps immediately to the steady state. Alternatively, we might start the economy with a universally known backstop technology for quality q_0 of each good. Then the steady state that we describe is approached in the limit, after all goods have been improved for the first time.

while the R&D sector empoloys $a_I \iota$ workers. Therefore, equilibrium in the labour market requires

$$a_I \iota + E/\lambda = L. \tag{10}$$

The differential equation (9) together with the side condition (10) describe the evolution of our economy for every initial value of spending E. We depict the dynamics in Figure 1, where LL represents the resource constraint (10), and $\Pi\Pi$ describes combinations of E and ι such that $\dot{E} = 0$; i.e.

$$\frac{(1-1/\lambda)E}{a_I} = \rho + \iota. \tag{11}$$

The economy must always lie along LL, with spending rising above $\Pi\Pi$ and falling below this line. For any initial value of E below that labelled $\bar{E}$, spending eventually falls to zero, a violation of the transversality condition. For initial values of E above $\bar{E}$, ι approaches zero at a point where the level of spending implies expected profits in excess of R&D costs. This event would contradict profit maximization by entrepreneurs. We conclude that the economy must jump immediately to the steady state at point A. The equilibrium values of E and ι solve (10) and (11).

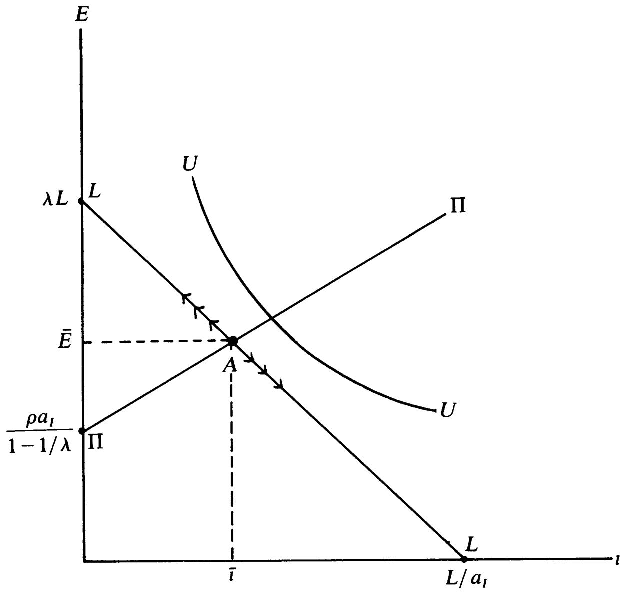

FIGURE 1

In the dynamic equilibrium, each product is improved with probability ιdt in a time interval of length dt. By the law of large numbers, a fraction ι of the products are continually being improved. These will not be the same products in every "period", nor will technological progress be uniform across sectors. In fact, our model predicts an evolving distribution of product qualities, with individual products constantly swapping relative positions within that distribution.

Before we proceed to investigate the market determinants of R&D and the welfare properties of the equilibrium, it is worthwhile to point out an alternative interpretation of our model. This interpretation treats each good ω as an intermediate input into the production of a single final consumer good. Then (2) gives the constant-returns-to-scale (Cobb–Douglas) production function for the final good, and u represents the flow of final output. The consumer's utility function remains as in (1), but now u there stands for the consumption of the single final good. This good is priced at marginal cost in a competitive equilibrium; i.e.

$$\log p_u = \int_0^1 \log [p_t(\omega)/q_t(\omega)]d\omega.$$

This provides a natural interpretation of (6) as the discounted value of the logarithm of real spending E/p_u.

From this point on the analysis proceeds as before to arrive at the differential equation (9) and the resource constraint (10). With this new interpretation, technological progress entails improvement in intermediate inputs, which serves to raise total factor productivity in the manufacture of consumer goods. It would be a simple matter to augment the model to include direct use of primary factors in the production of final goods, in which case we would have a specification similar in many respects to Romer (1990). We shall pursue this analogy further in Section IV below.

III. EQUILIBRIUM AND OPTIMAL GROWTH

We define the growth rate g to be the rate of increase of u. With the interpretation of the ω's as intermediate goods, g represents the rate of growth of final output. With our original interpretation of the ω's as consumer goods, g corresponds to the rate of increase in a quality-adjusted consumption index.

To calculate g, we substitute (5) and (8) into (2) to derive

$$\log u(t) = \log E - \log \lambda + \int_0^1 \log q_t(\omega)d\omega.$$

The last term depends upon ι and t. For any given ω, the probability of exactly m improvements in a time interval of length τ is (see Feller (1968), p. 159)

$$f(m, \tau) = \frac{(\iota\tau)^m e^{-\iota\tau}}{m!}.$$

Since in equilibrium the same intensity of R&D applies to all products, $f(m, \tau)$ represents the measure of products that are improved exactly m times over an interval of length τ. Therefore,

$$\int_0^1 \log q_t(\omega)d\omega = \textstyle\sum_{m=0}^{\infty} f(m, t) \log \lambda^m.$$

The right-hand side equals the product of $\log \lambda$ and the expected number of improvements in an interval of length t. This product equals $\iota t \log \lambda$. Hence,

$$\log u(t) = \log E - \log \lambda + \iota t \log \lambda, \tag{12}$$

and the growth rate $g = \iota \log \lambda$.

We are now prepared to study the determinants of the growth rate. We solve (10) and (11) to derive the following reduced-form expression for ι:

$$\iota = \frac{(1-1/\lambda)L}{a_I} - \frac{\rho}{\lambda}. \tag{13}$$

This equation or Figure 1 can be used to examine the comparative statics. In terms of the figure, an increase in L shifts the LL curve out and so increases equilibrium ι. Thus, as in Romer (1990), Aghion and Howitt (1990) and elsewhere, a larger resource base implies faster growth (but see Section V below). Similarly, a decline in a_I shifts LL out and also shifts $\Pi\Pi$ down. R&D effort expands due both to a resources effect and an incentive effect. The same is true about an increase in λ which not only raises ι but also spurs growth directly, because the technology jumps become larger. In short, R&D responds to profitability incentives and the economy exhibits dynamic increasing returns to scale.

We turn now to issues of welfare. Using (1), (12), and the knowledge that E and ι are constant in equilibrium, we can calculate the following exact expression for U:

$$\rho U = \log E - \log \lambda + (\iota/\rho) \log \lambda. \tag{14}$$

This representation of lifetime utility induces a preference ordering on E and ι that can be depicted by well-behaved indifference curves such as UU in Figure 1. Greater spending means higher utility early on, which may compensate for fewer quality improvements and hence lower utility later.

We find the optimal growth rate by maximizing (14) subject to (10).[10] This gives the optimal intensity of innovation

$$\iota^* = \frac{L}{a_I} - \frac{\rho}{\log \lambda}, \tag{15}$$

which we find in Figure 1 at the tangency of an indifference curve and the resource constraint LL.

We discuss the possible discrepancies between the optimal and equilibrium growth rates with the aid of Figure 2. In the figure, we have plotted the curve for $\lambda/(\lambda-1)$. This must not exceed $L/\rho a_I + 1$ if we are to have positive growth in equilibrium (compare the vertical intercepts of LL and $\Pi\Pi$ in Figure 1). Hence, $\iota = 0$ for $\lambda \leqq \lambda_0$ and $\iota > 0$ for $\lambda > \lambda_0$. Next, we compute from (13) and (15)

$$\iota^* - \iota = \frac{\rho}{\lambda}\left(\frac{L}{\rho a_I} + 1 - \frac{\lambda}{\log \lambda}\right). \tag{16}$$

Figure 2 also shows the curve for $\lambda/\log \lambda$. As is clear from the figure, $\iota > \iota^*$ for $\lambda \in (\lambda_0, \lambda_1)$ and $\lambda > \lambda_2$, whereas $\iota < \iota^*$ for $\lambda \in (\lambda_1, \lambda_2)$. In other words, the market incentives for R&D are excessive in our economy when the steps of the quality ladder are quite small or quite large, but are insufficient for steps of intermediate size.

This finding can be understood with reference to the market distortions identified by Aghion and Howitt. First, successful innovators generate a positive externality for consumers. Consumers pay the same price as before the innovation but receive a product of higher quality. This externality certainly lasts as long as the innovator maintains her monopoly position. Actually, it lasts indefinitely into the future, since all later innovations improve upon a product that is one step higher up the quality ladder than otherwise.

10. In a model with more than two activities, such as that in Section V, we would need to distinguish between the second-best growth rate which takes the oligopoly pricing of innovation products as given, and the first-best rate that sets all prices equal to marginal cost (see Grossman and Helpman, 1989*b*). To achieve the first best we would generally require two policy instruments, one to correct for externalities generated in R&D and the other to ensure optimal output of the innovative products. However, with all manufactured goods priced similarly and with labour supplied inelastically, the problem of optimal resource allocation becomes simply one of determining whether the resources devoted to R&D in the market equilibrium are too many or too few.

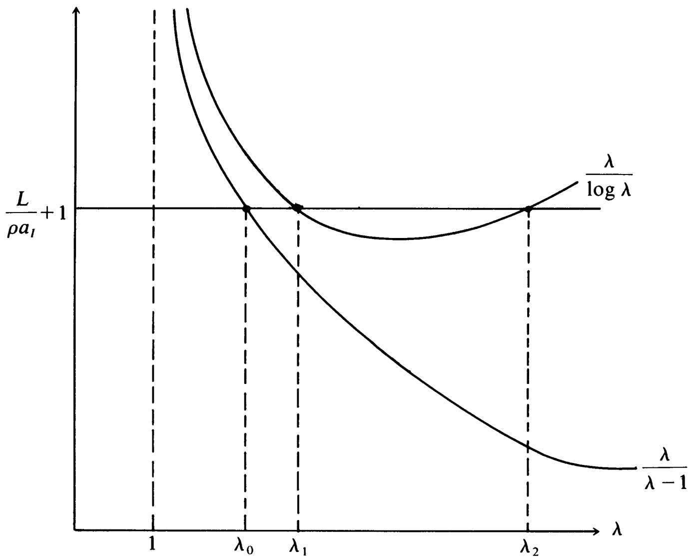

FIGURE 2

This externality, which combines what we shall call a *consumer-surplus effect* (during the life of the new product) and what Aghion and Howitt term an *intertemporal spillover effect* (extending over the lives of later products), measures $(\log \lambda)/\rho$ in terms of the utility metric in (1) and (2).

Second, a successful innovator generates a negative externality for extant industry leaders. In effect, the innovator "destroys" the producer surplus of the firm it displaces. Aghion and Howitt term this the *business-stealing effect.* This adverse effect is compounded by a multiplier, as the loss of income suffered by owners of the displaced firm means less demand and less profits for all remaining industry leaders. In all, the innovation causes instantaneous profits of others to fall by $\lambda-1$. This flow must be discounted at rate $\iota+\rho$, the effective discount rate for profits, which takes into account the expected rate of arrival of the next innovation in each industry. So the total negative externality imposed by the innovator equals $(\lambda-1)/(\iota+\rho)$. For low or high values of λ the adverse effect is larger, while the combined consumer-surplus and intertemporal-spillover effects dominate when λ takes on an intermediate value.[11]

The optimal growth rate is more likely to exceed the equilibrium rate when L/a_l is large; i.e. in large economies as measured in units of (R&D) efficiency labour. Then the optimal rate of innovation is great. Larger values of ι^* reduce the size of the business-stealing effect per unit output without changing the size of the consumer-surplus effect per unit output.

The optimum can be decentralized here by means of a tax or subsidy on R&D outlays. Let T be the multiple (or fraction) of R&D costs borne by the firm, with $T>1$ for a tax and $T<1$ for a subsidy. With such a policy in effect the no-arbitrage condition (11) is replaced by one with the left-hand side divided by T. An increase in T shifts the $\Pi\Pi$ curve upward in Figure 1 and so generates an equilibrium with greater spending and less

11. Using the expression for ι in (13), it is easy to show that the difference between the combined consumer-surplus and intertemporal spillover effects and the business-stealing effect has the same sign as the right-hand side of (16).

innovation. A decrease in T has the opposite effects on resource allocation. By varying T, the government can achieve any point along LL, including of course the optimum.

IV. QUALITY VERSUS VARIETY

We have followed Segerstrom *et al.* and Aghion and Howitt in treating endogenous product improvements as the engine of growth. As we noted in the introduction, this approach to technological progress differs from the one pursued in recent papers by Romer and ourselves (1989*a*, *b*, 1990).[12] This work treats technology-based growth as a process of generating an ever-expanding variety of horizontally differentiated products. In this section, we develop a simple variant of a variety-based growth model to demonstrate a remarkable similarity between the alternative approaches. In particular, we show below that the variety-based growth model and the quality-based growth models have identical reduced forms and we discuss the analogous roles that different parameters play in each formulation.

We continue to represent consumer preferences by (1). Now, however, the consumption index $u(t)$ exhibits love of variety over an infinite set of horizontally differentiated products, as in Dixit and Stiglitz (1977). We replace (2) by

$$u(t)=\left[\int_0^{n(t)} d_t(\omega)^{\alpha}d\omega\right]^{1/\alpha}, \qquad 0<\alpha<1, \tag{17}$$

where $n(t)$ denotes the measure of differentiated products available at time t and $d_t(\omega)$ represents consumption of brand ω at time t. In this case too the differentiated products can be interpreted either as final consumer goods or as intermediate inputs. Under the latter interpretation, (17) represents a production function and u is output of a homogeneous consumer good (see Ethier (1982)). We shall not dwell on the intermediate goods interpretation in order to save space.

This preference structure implies an intertemporal allocation of spending given by (7) and well-known static demand functions that exhibit a constant price elasticity of demand of $1/(1-\alpha)>1$ and a unitary expenditure elasticity of demand for each variety (see, for example, Grossman and Helpman (1989*c*)). We assume that a unit of any brand can be produced with one unit of labour, with labour again being the sole primary input. In this case, marginal production cost equals the wage rate w and oligopolistic price competition among suppliers of available varieties implies mark-up pricing, or

$$p=(1/\alpha)w, \tag{18}$$

for every product at each point in time. This equation is analogous to (8). It implies that profits per brand are $\pi=(1-\alpha)E/n$, where E is total spending.

Entrepreneurs must devote resources to R&D in order to bring out new products.[13] Product development requires a_D/K units of labour per increment to the set of varieties, where K is the stock of knowledge capital at a point in time. Knowledge capital is a public good; it is freely available to all potential innovators. Moreover, knowledge capital is generated as a by-product of product development.[14] For simplicity, let $K=n$. Then product development costs are $c_n=wa_D/n$. We assume that the developer of a new variety maintains indefinite monopoly power in the sub-market for her specific brand.

12. See also Judd (1985), who studies the introduction of new products via R&D, but in a dynamic framework in which growth ceases in the long run.

13. Or, more precisely, resources are spent to expand the measure of the set of available products by dn.

14. For discussion of this assumption, see Romer (1990).

Free entry by entrepreneurs ensures that, whenever innovation takes place, the present value of the infinite stream of future profits exactly matches the cost of product development. The time-derivative of this zero-profit condition gives the following no-arbitrage condition:

$$\frac{(1-\alpha)E}{wa_D}+\frac{\dot{w}}{w}-\frac{\dot{n}}{n}=\dot{R}.$$

This condition equates the interest rate to the instantaneous profit rate, $\pi/c_n = (1-\alpha)E/wa_D$, plus the rate of capital gain, $\dot{c}_n/c_n = \dot{w}/w - \dot{n}/n$.

We again choose labour as numeraire; i.e. $w(t)=1$ for all t. Let $\gamma \equiv \dot{n}/n$ be the growth rate of the number of varieties. Then, using (7), we can re-express the no-arbitrage condition as

$$\frac{\dot{E}}{E}=\frac{(1-\alpha)E}{a_D}-\rho-\gamma, \tag{19}$$

which is analogous to (9) above. Here a_D plays the role of a_I and γ plays the role of ι; the latter analogy will become clearer from what follows.

Total demand for labour is the sum of employment in R&D, $(a_D/n)\dot{n}$, and that in manufacturing, $n(E/np)=\alpha E$. Thus, labour-market clearing implies

$$a_D\gamma+\alpha E=L, \tag{20}$$

which is analogous to (10). The differential equation (19) together with the side condition (20) determine the evolution of the economy over time, given an initial value of spending. Clearly, the system can be described by means of a figure that is analogous to Figure 1, with γ replacing ι on the horizontal axis. Hence, the economy jumps to a steady state that satisfies (20) and a steady-state no-arbitrage condition,

$$(1-\alpha)E/a_D=\rho+\gamma, \tag{21}$$

that is analogous to (11).

A comparison of (10)-(11) and (20)-(21) establishes the equivalence of the two reduced-form systems. In the latter, γ replaces ι, a_D replaces a_I, and α replaces $1/\lambda$. Clearly, all comparative static calculations that apply to one system also apply to the other.

The following observations may clarify the similarities between the two approaches and the economic structures that they generate. In both models, agents invest resources to acquire the exclusive ability to manufacture a new product. Moreover, the R&D activity generates unappropriable spillovers in both cases. In the variety-based growth model, the R&D externality is quite explicit. Each completed product development project lowers the cost of later R&D efforts. In the quality-based model, the externality is implicit. When one improvement project succeeds, other researchers can quit their efforts to achieve that same innovation and begin to work on the *next* improvement. In both instances we have assumed that by observing the results from one innovative success, researchers can learn scientific and engineering facts that are useful in their own research endeavours. This seems a natural and important characteristic of research and reflects the public good nature of technology as information.

The two approaches do diverge, however, in their welfare implications. Proceeding with the analysis of the variety-based growth model, we note that (17) implies[15]

$$\log u(t)=\log E+\log\alpha+\gamma t(1/\alpha-1), \tag{22}$$

15. In writing (22) we assume without loss of generality that the initial number of differentiated products $n(0)=1$.

which is analogous to (12). Equation (22) gives the growth rate in the variety-based model as $g = \gamma(1/\alpha - 1)$. Substituting (22) into (1), we obtain the welfare function

$$\rho U = \log E + \log \alpha + (\gamma/\rho)(1/\alpha - 1), \tag{23}$$

which is comparable to (14). Now we can maximize (23) subject to (20), and compare the resulting optimal γ^* to the market equilibrium γ. We find in contrast to our earlier result, that whenever the optimal rate of innovation is positive, this rate exceeds the market determined rate (see Romer (1990) and Grossman and Helpman (1989*b*) for similar results). The reason is as follows. Each new product initially contributes $(1-\alpha)En^{1/\alpha-2}$ to consumer surplus. The marginal entrant inflicts an aggregate loss of profits of $(1-\alpha)E/\alpha n$ on the n existing firms. The marginal utility of income is $\alpha n^{1/\alpha-1}$. Thus, the static consumer-surplus effect and the static business-stealing effect just offset one another.[16] Moreover, both of these effects compound similarly over time. What remains then is the intertemporal spillover effect whereby current technological advance reduces the cost of later R&D. Therefore, the marginal innovation conveys a net positive externality in the variety-based growth model and equilibrium growth is always too slow.

V. RESOURCES AND GROWTH

We endeavour now to extend our model of quality-based innovation in order to bring out certain features of the growth process that are not evident from a one-factor, one-manufacturing-sector formulation. Specifically, we study in this section the relationship between the growth rate and the size and composition of the resource base. In the next section we examine the long-run pattern of trade in a two-country world economy. It should be apparent as we proceed, in view of our findings in the previous section, that similar results apply to an appropriately extended variant of the variety-based growth model.

We add to the model of Section II a sector that produces a homogeneous product of fixed quality. This might be a service sector, for example. We replace (1) with the augmented preferences,

$$U = \int_0^\infty e^{-\rho t}[s \log u(t) + (1-s) \log y(t)]dt, \tag{24}$$

where $y(t)$ represents consumption of the homogeneous product and $u(t)$ is as before (see (2)). Then consumers allocate at every point in time a share s of their spending to the vertically differentiated products and a share $(1-s)$ to the outside good y. The time pattern of spending follows (7).

We add as well a second primary factor of production. We shall refer to the two factors as unskilled labour (L) and skilled labour (H). Let unit manufacturing costs of the vertically differentiated products be $c_X(w_L, w_H)$ and those of the outside good be $c_Y(w_L, w_H)$, where w_i denotes the reward to input i, $i = L, H$. The cost of a unit of innovative activity is given by $c_I(w_L, w_H)$.

We assume perfect competition in the market of the outside good and Bertrand competition as before in the markets for the vertically differentiated products. Let the former good serve now as numeraire. Then we have in place of (8) the following pricing

16. This property of the CES preferences was first noted by Dixit and Stiglitz (1977). It does not extend to cases where the elasticity of substitution between differentiated products varies with the number of products.

equations:

$$p_X = \lambda c_X(w_L, w_H); \tag{25}$$

$$1 = c_Y(w_L, w_H). \tag{26}$$

Factor market clearing requires

$$a_I(w_L, w_H)\iota + a_X(w_L, w_H)X + a_Y(w_L, w_H)Y = \begin{bmatrix} H \\ L \end{bmatrix}, \tag{27}$$

where $a_i(\cdot)$ represents the cost-minimizing input vector per unit of output for $i = I, X, Y$, and X and Y denote output quantities. The input vectors are given by the gradients of the respective unit-cost functions. Static equilibrium in the commodity market entails

$$s/(1-s) = p_X X / p_Y Y. \tag{28}$$

Finally, the steady-state no-arbitrage condition reads (in place of (11)):

$$\frac{(1-1/\lambda)p_X X}{c_I(w_L, w_H)} = \rho + \iota. \tag{29}$$

As before, convergence to the steady state is immediate.

We wish to explore the relationship between the growth rate and the size of the resource base. A common feature of recent models of endogenous technological progress has been that larger economies grow faster (see Aghion and Howitt (1990), Romer (1990) and Grossman and Helpman (1990)). We shall show that this result depends critically on the general equilibrium structure of these models. In general, only larger endowments of factors that are used intensively in the growth-generating activities guarantee faster growth.

To make this point, we pursue the special case in which R&D and manufacturing of X use only skilled labour while production of the outside good uses both primary factors. With this restriction, we can combine (25)-(28) to obtain

$$a_{HI}\iota + \frac{s}{1-s}\frac{Y(w_H, L)}{\lambda w_H} + H_Y(w_H, L) = H, \tag{30}$$

where a_{HI} is the skilled labour required for a unit of R&D activity, $Y(\cdot)$ is the profit-maximizing supply of the outside good, and $H_Y(\cdot)$ represents that sector's demand for skilled labour when it employs the entire unskilled labour force. We combine (25)-(26) and (28)-(29) to obtain

$$(1-1/\lambda)\frac{s}{1-s}\frac{Y(w_H, L)}{w_H a_{HI}} = \rho + \iota. \tag{31}$$

We plot these two equations as HH and $\Pi\Pi$, respectively, in Figure 3.

Now consider an increase in H. This shifts the HH curve up and to the left, while leaving $\Pi\Pi$ unaffected. The new equilibrium involves more innovation and faster growth.[17] Here we see that accumulation of the factor used intensively in R&D indeed is conducive to growth.

When L increases, both curves shift to the right. The rate of innovation grows if and only if the rightward shift of $\Pi\Pi$ from A exceeds the rightward shift of HH. This occurs when the elasticity of substitution between skilled and unskilled labour in the

17. The growth rate of a consumption index now is $s\iota \log \lambda$.

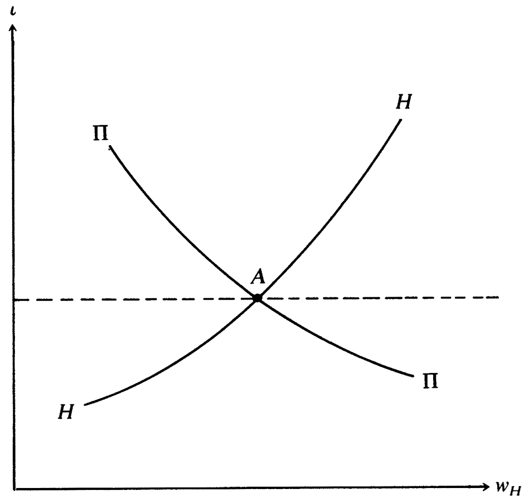

FIGURE 3

production of the outside good is larger than one, but not when it is smaller than one.[18] Intuitively, the outside good draws skilled labour away from R&D and X due to an output effect, but releases skilled labour as w_H rises due to a substitution effect. When the elasticity of substitution is large, the latter effect dominates and so more skilled labour finds itself employed in R&D in the new general equilibrium.

A more striking result emerges from a different special case. Suppose all three sectors use the primary factors in fixed proportions, and that R&D makes the most intensive use of skilled labour, while the production of the outside good makes the least intensive use of this factor. It is straightforward to show that, in these circumstances, an increase in the supply of unskilled labour *must* slow growth. The general point is that a positive monotonic relationship between resource supplies and employment of those resources in the growth-generating activities exists only for certain general equilibrium structures.

VI. INTERNATIONAL TRADE

In this section we show how quality ladders can be embedded in a model of international trade. We use the two-factor, two-sector structure of Section V to explore the nature of a trading equilibrium and to examine the long-run pattern of trade when resources in each country are devoted to improving the quality of some products.

To begin with, suppose that each vertically differentiated product must be manufactured in the country in which the most recent product improvement has taken place. That is, we rule out international licensing and multinational corporations for the moment. In

18. The rightward shift in ΠΠ is given by the change in W_{H} that leaves $Y(w_H, L)/w_H$ constant as L increases. The rightward shift in HH is larger or smaller than this as H_Y increases or decreases with L when $Y(w_H, L)/w_H$ is kept constant. Using the condition for optimal H_Y (marginal product equals wage), we find that H_Y increases with L when $Y(w_H, L)/w_H$ is constant if and only if $\theta_H+\varepsilon_H>1$, where θ_H is the share of H in the cost of producing Y and ε_H is the absolute value of the elasticity of the marginal product of skilled labour in the Y sector. Since $\theta_H+\varepsilon_H=1+\varepsilon_L(1-\sigma)$, where ε_L is defined analogously to ε_H and σ is the elasticity of substitution between skilled and unskilled labour in the production of the outside good, our claim is established.

Figure 4 we draw a rectangle whose dimensions represent the aggregate world endowments of H and L. If there were no country borders, then the equilibrium would be that described in Section V above. We ask now whether an equilibrium with free trade can reproduce the essential features of that integrated equilibrium, despite the fact that factors now are restricted to stay within their countries of origin.

The vector OQ^* in the figure represents the total employment of H and L in the activities of product improvement and manufacturing of X in the equilibrium of Section V (henceforth, the "integrated equilibrium"). This vector is given by $a_{l}\iota + a_X X$. The vector Q^*O^* similarly represents employment in the production of the outside good in the integrated equilibrium. We depict the endowments of the two countries by a point in the rectangle, with the vector of factors measured from the origin at O representing the endowment of the home country and that measured from O^* representing the endowment of the foreign country. We claim that if the endowment point, marked E, falls within the parallelogram OQO^*Q^*, then there exists a trading equilibrium with all aggregate variables identical to those of the integrated equilibrium.

At E, the home country is relatively well-endowed with skilled labour. Suppose that factor prices and interest rates in the two countries were equalized nonetheless and that their levels were the same as in the integrated equilibrium.[19] Then techniques of production would be the same. The home country achieves full employment if it employs OP in the combined activity of product improvement and manufacturing of X and OP_Y in the production of outside goods. Full employment obtains abroad when that country employs O^*P^* in the combined activity and $O^*P_Y^*$ in manufacturing of Y. These employment vectors give the same aggregate levels of activity as in the integrated equilibrium. The ratio of the line segments $\overline{OP/OQ}$ gives the number of n of vertically differentiated products that is produced at home. The home country also performs a fraction n of world R&D activity, and thereby maintains leadership in a measure n of products in all

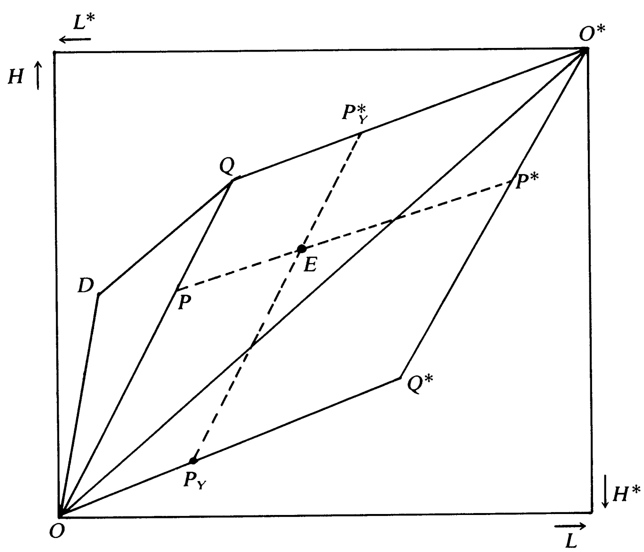

FIGURE 4

19. International trade in financial assets would of course guarantee equalization of interest rates. However, as we shall see, a steady-state equilibrium exists with identical interest rates in the two countries even when financial assets are not traded.

periods.[20] The foreign country produces the remaining $n^* = 1 - n$ vertically differentiated products and undertakes the fraction n^* of R&D effort.

It remains to be shown only that, with the proposed allocations, product markets clear and all profitability conditions are satisfied. Since we have provisionally assumed that all factor prices are the same as they were in the integrated equilibrium, all activities break even in each country, as they all did in the integrated equilibrium. Also, with interest rates as before, the no-arbitrage condition continues to be satisfied in each country. With the same costs of production, commodity prices are the same as in the integrated equilibrium. Aggregate world income is the same as well. Since preferences are homothetic, the distribution of income does not matter for aggregate demand, and so product markets clear. This completes our demonstration that commodity trade (with or without trade in financial assets) suffices to reproduce the essential features of the integrated steady-state equilibrium.

What then is the pattern of world trade? Suppose, to begin with, that financial assets are not traded; i.e. each country must finance all R&D that takes place within its borders from domestic savings. Then the trade account must balance. The homotheticity of preferences implies an identical composition of aggregate demand in the two countries. But from Figure 4 we see that the unskilled-labour-abundant (foreign) country specializes relatively in the production of the unskilled-labour-intensive (outside) good. Hence, the home country imports the outside good. With trade balanced, this country must be a net exporter of vertically differentiated products. This pattern of intersectoral trade corresponds of course to the predictions of the Heckscher–Ohlin model. Here it applies to the steady state of a dynamic world economy with continual quality-upgrading.

The two-country world economy does not converge immediately to a steady state, unless the initial ownership shares in blueprints for frontier products happen to coincide with the n and n^* of the steady-state equilibrium. In general, these shares are attained during a phase of dynamic adjustment. If international trade in financial assets takes place along the adjustment path, then typically the steady state will not be characterized by balanced trade. Although the production patterns of the steady state remain as described above, it may happen that one country will import both the outside good and (on net) vertically differentiated products. It can do so if its steady-state surplus on service account is large enough. Trade imbalance cannot reverse the pattern of trade from that predicted by the Heckscher–Ohlin theorem, however.

Finally, we relax the assumption that product improvement and manufacturing of the improved product must take place in the same location. For endowment points in OQO^*Q^* of Figure 4 firms have no incentive to separate these activities, because profit opportunities are the same in every country. We have seen that the world economy reproduces the integrated equilibrium under these circumstances. But commodity trade alone is not sufficient to reproduce the integrated equilibrium for endowments outside OQO^*Q^*. Then, if product improvement and the manufacturing of vertically differentiated products use H and L in different proportions, there may be an incentive for firms to separate geographically their research and production operations. Assume for concreteness that at common factor prices R&D employs relatively more skilled labour than does manufacturing. Let OD and DQ be the employment vectors in these two activities,

20. Entrepreneurs in the home country might, for example, be the only ones who attempt to improve a fraction n of the products, each at intensity ι. Or home entrepreneurs may attempt to improve the entire spectrum of vertically differentiated products at intensity $n\iota$. Other allocations are possible as well, so long as the aggregate innovation effort devoted to each product ω is the same and equal to ι from the integrated equilibrium.

respectively, in the integrated equilibrium. Following Grossman and Helpman (1989*c*), it is easy to show that, for endowment points inside the triangle *ODQ*, the quantities of the integrated equilibrium again can be reproduced by commodity trade when a suitable number of multinational corporations form. These multinationals conduct their R&D at home, but undertake their subsequent manufacturing in the foreign country. For some endowment points outside *ODQ* (and the symmetrically placed triangle on the opposite side of the diagonal) there will also be an incentive for multinationals to emerge. But, in these cases, the trade equilibrium with multinationals differs non-trivially from the integrated equilibrium.

VII. CONCLUDING REMARKS

We have developed a model of on-going product improvements. This model draws several building blocks from earlier work by Segerstrom *et al.* (1990) and Aghion and Howitt (1990). Entrepreneurs race to bring out the next generation of a continuum of goods. In each industry success occurs with a probability per unit time that is proportional to the total R&D resources targeted to improving that product. Each product follows a stochastic progression up the quality ladder. But the equilibrium is characterized by an aggregate rate of innovation that is determinate, and constant in the steady state.

The model captures many realistic aspects of the innovation process. Individual products become obsolete after a time. Progress is not uniform across sectors. Research responds to profit incentives. And innovators are able to benefit from observing and analysing the research successes of their rivals. These features fit the detailed historical descriptions of industrial R&D provided by Freeman (1982) and others.

We related our approach to an alternative one that treats industrial R&D as a process of creating an ever-expanding range of horizontally differentiated products. The latter framework has been applied to issues of long-run technological progress and growth by Romer (1989) and ourselves (1989*a*, *b*, 1990). We showed that the two approaches though seemingly quite distinct actually share identical reduced forms for their simplest variants. Thus, both approaches yield the same answers to many positive questions about the determinants of the long-run growth rate. However, as Aghion and Howitt pointed out, the normative analyses of the variety-based and quality-based growth models differ. In the former, the equilibrium rate of innovation always is too slow. But when growth derives from product improvements, it may be slower or faster than is optimal. We have been able to derive and contrast these results using very comparable versions of the alternative models.

In one section of this paper, we cast the innovation process in a two-country setting. We described a long-run equilibrium with product improvements taking place in each country, with intra-industry trade in vertically differentiated products, and with inter-industry trade of technologically progressive goods for homogeneous, unchanging goods. The Heckscher–Ohlin theorem predicts the long-run pattern of intersectoral trade despite the diversion of resources and to R&D and the continual technological advance that takes place.

Building on the framework developed here, we provide a richer story of international trade in a companion paper (Grossman and Helpman (1989*d*). There, all product improvements take place in a high-wage region with comparative advantage in R&D. But entrepreneurs in the low-wage region are able to produce clones of state-of-the-art products if they succeed in reverse engineering. Imitation, like innovation, requires resources and entails uncertain prospects. Unlike earlier models of the product life cycle based on horizontally differentiated products (e.g. Krugman (1979) or Grossman and Helpman (1989*a*)), this one predicts that the locus of output of a particular type of good will move back and forth between the North and the South as the former region captures market share when quality improvements take place and the latter then begins the process of imitating the new, improved product. This description of the product cycle would seem apt for many industries; e.g. personal computers and many consumer electronics.

We believe that our model is rich in its predictions, yet technically quite manageable. It might gainfully be extended to include endogenous accumulation of primary factors. We hope that it will prove useful in application to additional issues concerning innovation and long-run growth.

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