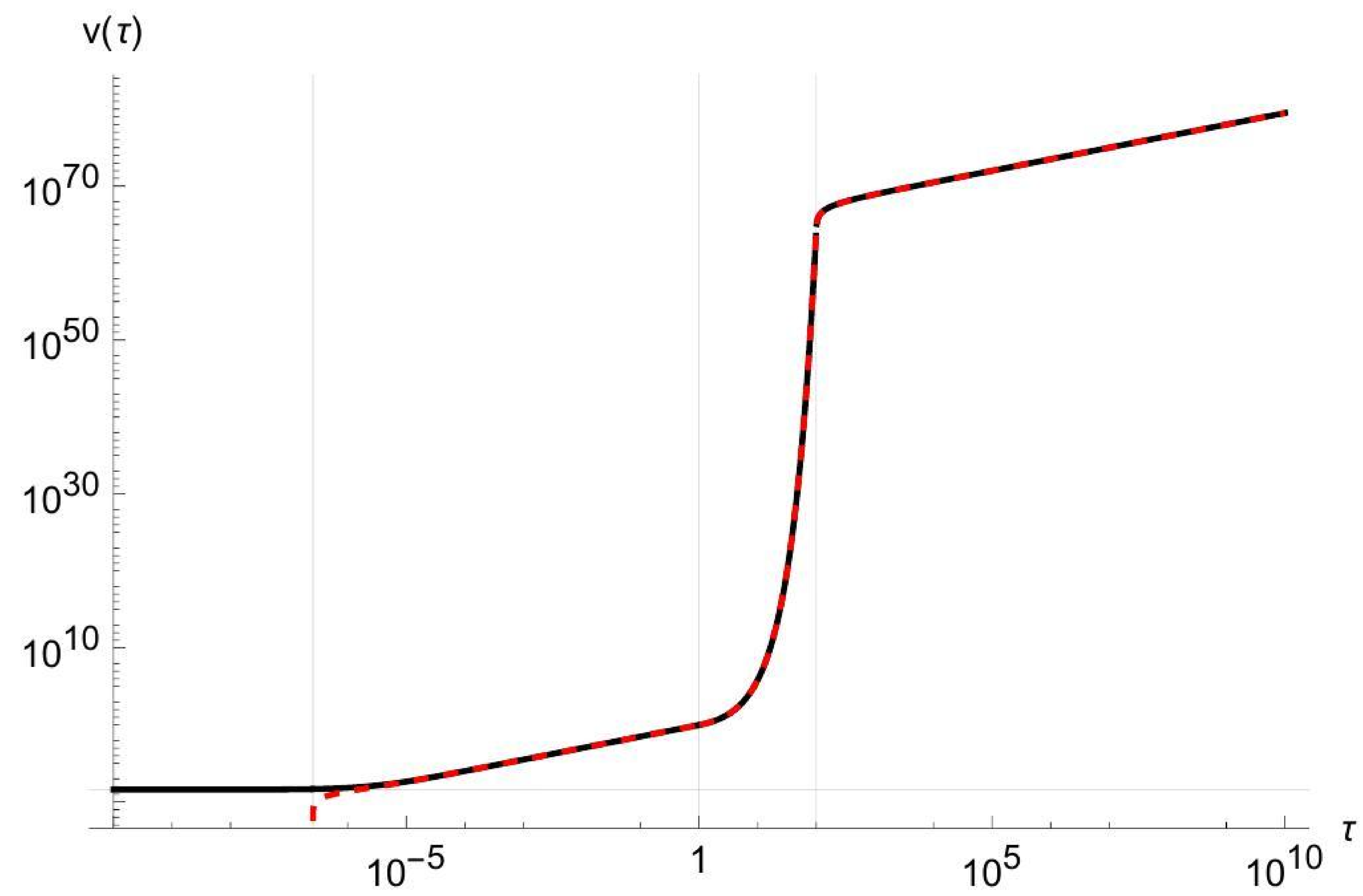

Figure 4. The evolution of $v(\tau)$ for the full model (black continuous line) compared with the standard dynamics (dashed red line). The minimum volume v_i is highlighted with a grey faded horizontal line, while the grey faded vertical lines separate the different phases (from left to right they indicate the classical Big Bang, the start of inflation and its end).

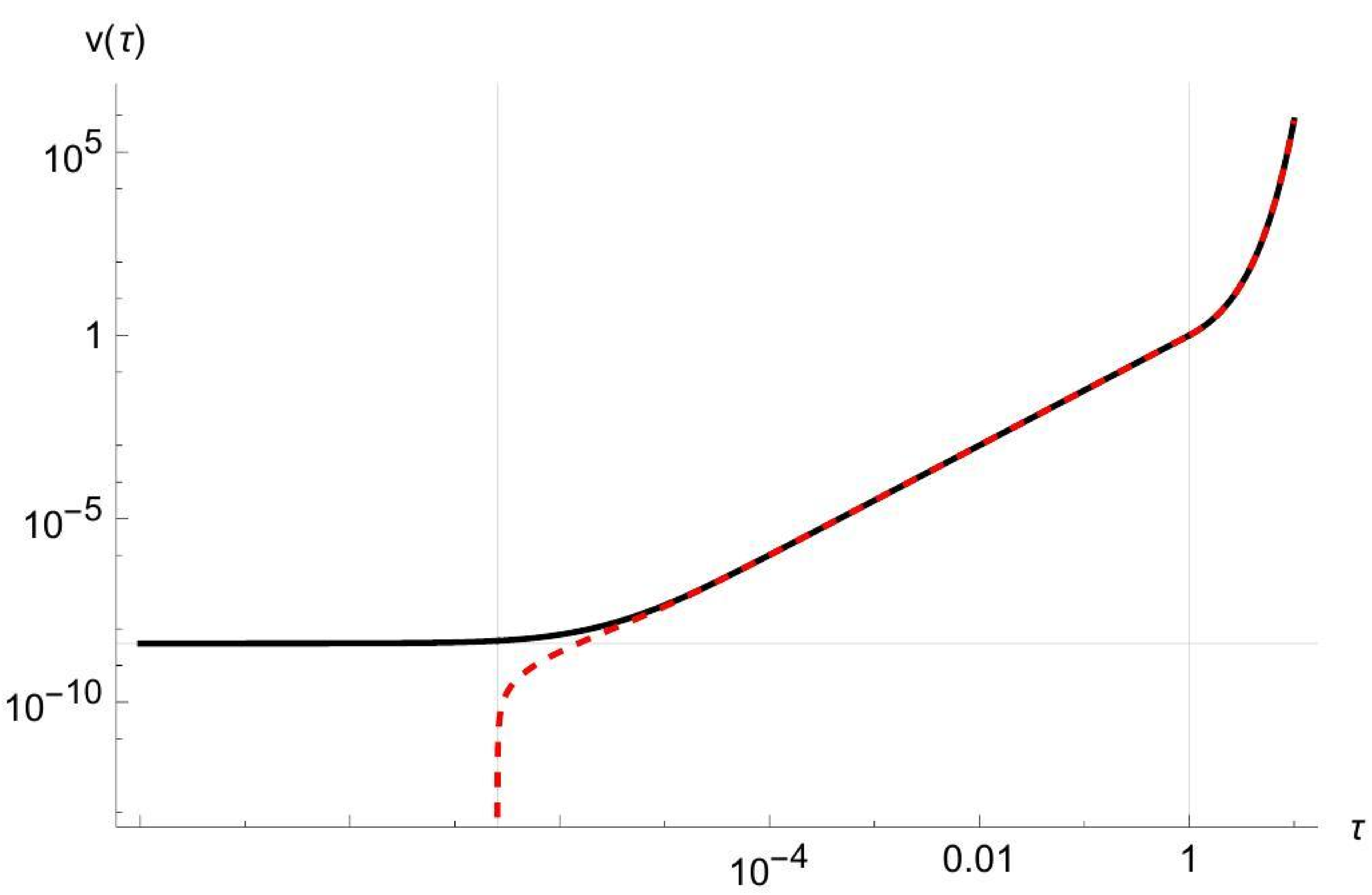

Figure 5. Zoomed-in version of Figure 4 to give a better view of the behaviour near the classical singularity.

where the constants $\overline{\rho_\gamma^{\text{pre/post}}}$ and ρ_Λ have been chosen to maintain continuity for v and $\dot{v}$.

Given the complexity of the corresponding Friedmann equations, the resolution has been performed numerically. Again, we rescaled all quantities by their corresponding value at the beginning of inflation, that is, we used as time variable $\tau = t/t_s$ and all densities have been rescaled accordingly. The result is shown in Figure 4 for the whole evolution and compared with the classical case (i.e. the one obtained with standard Poisson brackets $\{v, p_v\} = 1$); Figure 5 is the same picture zoomed near the singularity, to better highlight the asymptotic behaviour.

We see that we did not have to impose any condition such as (4) in order to obtain an asymptotic behaviour, it is implemented naturally by the modified algebra (12). Besides, the standard dynamics is recovered pretty soon and already shortly before the inflationary epoch the evolution is practically indistinguishable; this will allow us to use the classical Friedmann equation for Inflation when in the next section we will calculate the primordial Power Spectrum.

IV. MODIFIED POWER SPECTRUM OF PERTURBATIONS

In this section, as a phenomenological consequence of this model and in particular of the modified algebra (12), we aim to derive the modified Power Spectrum of primordial scalar perturbations during the inflationary epoch. We will partially follow [29] but compute the spectrum through a different method.

First of all we introduce conformal time η in the action for the scalar field:

$$d\eta = \frac{dt}{a}, \tag{22}$$

$$S_\phi = \int \frac{dt}{2} a^3 \left(\dot{\phi}^2 - 2U(\phi)\right) = \int \frac{d\eta}{2}\left(a^2(\phi')^2 - 2a^4 U(\phi)\right), \tag{23}$$

where a prime denotes a derivative with respect to η.

At this point it is useful to introduce the so-called Mukhanov-Sasaki variable ξ, a master gauge-invariant variable which is sufficient to fully describe the scalar sector of perturbations [23]:

$$\xi(x, \eta) = a\left(\delta\phi_{\text{GI}} + \frac{\phi'\,\Phi_{\text{B}}}{aH}\right), \tag{24}$$

where $\delta\phi_{\text{GI}}$ is the gauge-invariant form of the scalar field perturbations and Φ_{B} is a Bardeen potential depending on the perturbative scalar functions in the perturbed metric [29]. The action for the variable ξ is obtained as the scalar part of the second variation of the total action (that is, of both the gravitational and matter sectors) [23]:

$$\delta^2 S = \int d\eta\, d^3x \left[(\xi')^2 - \delta^{ij}\,\partial_i\xi\,\partial_j\xi + \xi^2\,\frac{z''}{z}\right], \tag{25}$$

$$z = a\sqrt{\epsilon}, \quad \epsilon = -\frac{\dot{H}}{H^2}, \tag{26}$$

where ϵ is the first slow-roll parameter.

Now, since we are working with linear perturbations where each mode evolves independently, we can perform a Fourier decomposition so that the action greatly simplifies:

$$\xi(x, \eta) = \sum_k \xi_k(\eta) e^{ikx}, \tag{27}$$

$$\delta^2 S = \int d\eta\, d^3x \sum_k \left((\xi_k')^2 - \omega_k^2(\eta)\xi_k^2\right), \tag{28}$$