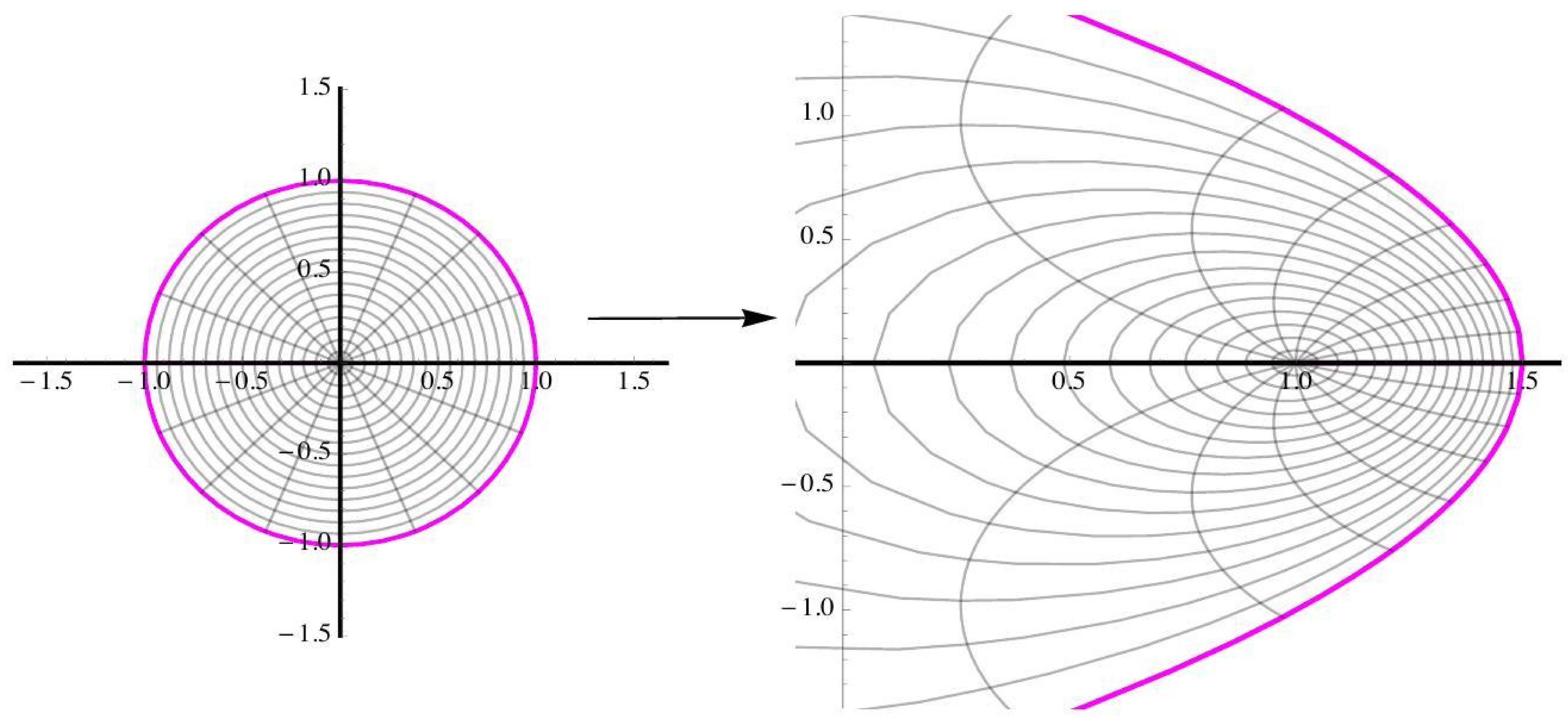

FIGURE 2. Image of open unit disk $\mathbb{D}$ under the mapping $\mathcal{LP}(z)$.

regions, which is beyond the scope of our present study and therefore it is skipped here, as it needs to be handled differently.

For brevity, let us assume $\mathcal{P}_0(z) := \mathcal{P}_{0,0}(z)$ and $\mathcal{LP}(z) := 1+\mathcal{P}_0(z)$, then the horizontal parabolic region $\mathcal{LP}(z)$ (see **Fig.** 2), is given by

$$\Omega_{\mathcal{LP}} := \{\omega \in \mathbb{C} : (\operatorname{Im}\omega)^2 < 3 - 2\operatorname{Re}\omega \text{ or } |1-\omega| < 2 - \operatorname{Re}\omega\}.$$

Since the major part of the above region lies in the left half plane, it is interesting to find the optimal radius of the domain disc for which it is fully mapped into the right half plane. Here below we define a class of analytic functions consisting of non-univalent functions.

Definition 1.1. Let the class $\mathcal{F}_{\mathcal{LP}}$ consist of functions $f \in \mathcal{A}$ satisfying the subordination

$$\frac{zf'(z)}{f(z)} \prec \mathcal{LP}(z) = 1 - \frac{2}{\pi^2}\left(\log\left(\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)\right)^2.$$

Note that if $f_0 \in \mathcal{F}_{\mathcal{LP}}$, then it can be expressed as

$$f_0(z) = z\left(\exp\int_0^z \frac{\mathcal{P}_0(t)}{t}dt\right), \tag{1.2}$$

which acts as an extremal function for many radius results. Evidently, the subclasses of starlike functions $\mathcal{S}^*_{SG}$, $\mathcal{S}^*_{\mathcal{RL}}$ and $\mathcal{SL}^*(\alpha)$ $(0 \le \alpha < 1)$ (see Table 1) are contained in $\mathcal{F}_{\mathcal{LP}}$, however $\mathcal{F}_{\mathcal{LP}} \nsubseteq \mathcal{S}$. In section 2, we examine some geometrical properties of the function $\mathcal{LP}(z)$.

2. Main Results

2.1. **Geometric Properties of the function** $\mathcal{LP}(z)$**.** In Lemma 2.1, we establish the maximum and minimum bounds of real part of the function $\mathcal{P}_0(z)$.

Lemma 2.1. *Let $z \in \mathbb{D}_r = \{z : |z| = r\}$, then for each $0 \le r < 1$ and $\alpha \in (-\pi, \pi]$, we have*

$$\mathcal{P}_0(r) \le \operatorname{Re}\mathcal{P}_0(re^{i\alpha}) \le \mathcal{P}_0(-r).$$