where

$$A := \sum_{j \geq 0} \binom{2n+1}{j} \deg s_j(C_{\Delta(X)}(X \times X)) \tag{3}$$

and

$$B := \sum_{k \geq 0} \binom{3n+2}{k} \deg s_k(C_{\Delta(X)}(X \times \sigma_2(X))). \tag{4}$$

by substituting (1) into [7, Theorem 8.2.8]. The equation (3) is just a linear combination of Segre classes of tangent bundle of X. Therefore, it is sufficient to calculate the term $s(C_{\Delta(X)}(X \times \sigma_2(X)))$ in order to determine the degree of the 3-secant variety. However, the normal cone $C_{\Delta(X)}(X \times \sigma_2(X))$ has singularities along the diagonal $\Delta(X)$, which must be expressed in simpler forms for the calculation to proceed.

3 The secant bundle

In this section, we start with introducing the concepts of higher very ampleness and secant bundles to relate information of double point formula with the information of higher secant varieties. It is often necessary to consider higher very ampleness of the projective embedding in order to ensure the desirable properties of the secant bundle and the inverse image of X under the birational morphism from the secant bundle to the secant variety. (cf. [13] and [15])

Definition 3. (cf. [1] and [4])
A line bundle $\mathcal{L}$ on a complete algebraic variety X over an algebraically closed field k is *d-very ample* if, for every zero-dimensional subscheme Z of X with length less than or equal to $d+1$, the restriction map

$$r_Z : H^0(X, \mathcal{L}) \to H^0(X, \mathcal{L} \otimes \mathcal{O}_Z)$$

is surjective.

Remark. Note that for two integers $d_1 \geq d_2$, if a line bundle $\mathcal{L}$ is d_1-very ample, $\mathcal{L}$ is also d_2-very ample.

A line bundle $\mathcal{L}$ is 0-very ample if and only if it is spanned by global sections, and it is 1-very ample if and only if it is very ample. For instance, the d-uple Veronese embedding of $\mathbb{P}^n$ is embedded by the d-very ample line bundle $\mathcal{O}(d)$, as shown in [2, Cor 2.1 and Prop 2.2].

Since information about higher secant varieties is often very complicated and difficult to work with, we define and use the secant bundle as a nonsingular birational model for the secant variety. We compute the required algebraic cycles for the degree formula of the 3-secant variety.

In this chapter, we consider the problem of computing the degree of the 3-secant variety of a nonsingular projective variety X that is embedded by a 5-very ample line bundle $\mathcal{L}$. The information about higher secant varieties can be difficult to work with, so we define and use the secant bundle as a nonsingular birational model for the secant variety. We compute the required algebraic cycles for the degree formula of the 3-secant variety. We let $X^{[2]}$ be the Hilbert scheme of 2 points of X and $\mathbb{P}^N$ be the projective space $\mathbb{P}H^0(X, \mathcal{L})$. It is known that $X^{[2]}$ is smooth for any dimension of X(cf. [6]). We consider the projections $\pi_1 : X \times X^{[2]} \to X$ and $\pi_2 : X \times X^{[2]} \to X^{[2]}$, and the universal family $Z_2 \subset X \times X^{[2]}$. We let I_{Z_2} be the ideal sheaf of Z_2 on $X \times X^{[2]}$ and $\mathcal{O}_{Z_2}$ be the structure sheaf of Z_2.