ON A CLASS OF NON-UNIVALENT FUNCTIONS ASSOCIATED WITH A PARABOLIC REGION

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Abstract. In the present investigation, we introduce and study the geometric properties of a class of analytic functions, associated with a parabolic region majorly lying in the left-half plane. Further we establish radius and majorization results for the class under study with pictorial illustrations of some of its special cases. Also we derive some sufficient conditions for the class under consideration.

1. Introduction

Let $\mathcal{A}$ be the class of analytic functions $f(z)$ defined on the open unit disc $\mathbb{D} = \{z : |z|< 1\}$ with the normalization $f(0) = 0$ and $f'(0) = 1$. Assume $\mathcal{S} \subset \mathcal{A}$ to be the class of univalent functions. Let $f(z)$ and $g(z)$ be two analytic functions, then $f(z)$ is said to be subordinate to $g(z)$, symbolically $f \prec g$, if there exist a Schwarz function $w(z)$ in $\mathbb{D}$ with $w(0) = 0$, such that $f(z) = g(w(z))$. Additionally, if $g(z)$ is univalent in $\mathbb{D}$, then $f \prec g$ if and only if $f(\mathbb{D}_r) \subset g(\mathbb{D}_r)$, where $\mathbb{D}_r = \{z : |z|< r < 1\}$. Recall that a function $f \in \mathcal{A}$ is starlike if $f(\mathbb{D})$ is starlike with respect to 0. Analytically, a function $f \in \mathcal{A}$ is starlike if

$$\frac{zf'(z)}{f(z)} \prec \frac{1+z}{1-z}.$$

We denote this class by $\mathcal{S}^*$. Ma and Minda [17] gave a unified representation for various subclasses of starlike functions by replacing the superordinate function $(1 + z)/(1 - z)$ with a more general function $\phi(z)$, and the corresponding class is denoted by $\mathcal{S}^*(\phi)$. Here $\phi(z)$ is chosen such that it is univalent, starlike with respect to $\phi(0) = 1$ and $\operatorname{Re}\phi(z) > 0$ with $\phi'(0) > 0$, also $\phi(\mathbb{D})$ is symmetric about real axis. Several Ma-Minda subclasses have been studied previously (See Table 1). In contrast to $\mathcal{S}^*(\phi)$, Uralegaddi [28] introduced and studied the class

$$\mathcal{M}(\beta) = \{f \in \mathcal{A} : \operatorname{Re}(zf'(z)/f(z)) < \beta,\ \beta > 1\}.$$

Note that $\mathcal{M}(\beta) \nsubseteq \mathcal{S}^*$ and also contains non-univalent functions. In 2006, Ravichandran et al. [25] computed the radius of starlikeness for the class $\mathcal{M}(\beta)$. Motivated by the above class, Kumar et al. [15] made a systematic study of the class $\mathcal{F}(\psi)$ containing non-univalent functions, given by

$$\mathcal{F}(\psi) := \left\{ f \in \mathcal{A} : \frac{zf'(z)}{f(z)} - 1 \prec \psi(z) \right\},$$

where $\psi(z)$ is an analytic univalent function such that $\psi(\mathbb{D})$ is starlike with respect to 0 and $\psi(0) = 0$. In general $\mathcal{F}(\psi) \nsubseteq \mathcal{S}^*(\phi)$. Further if $\phi(z) = 1+\psi(z) \prec (1+z)/(1-z)$, then $\mathcal{F}(\psi)$ reduces to $\mathcal{S}^*(1 + \psi)$. Here below we give a list of functions $\psi_i(z)$, $i = 1, \dots, 4$, which are considered in

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