

A Proofs

A.1 Proof of Theorem 1

First, line 6 in Algorithm 1 is equal to

$$x^{a'|s_i} \leftarrow \begin{cases} x^{a'|s_i} + (1 - x^{a'|s_i})\eta\pi(\mathbf{e}_{i'}, \mathbf{x}, \mathbf{y}) + O(\gamma^2) & (a' = a) \\ x^{a'|s_i} - x^{a'|s_i}\eta\pi(\mathbf{e}_{i'}, \mathbf{x}, \mathbf{y}) + O(\gamma^2) & (a' \neq a) \end{cases}, \quad (\text{A1})$$

from Definition 1.

In the stationary state of the repeated games, state s_i occurs with the probability of p_i^{st} . Then, player X (resp. Y) chooses action a (resp. b) with the probability of $x^{a|s_i}$ and $y^{b|s_i}$. If we take the limit $\eta \rightarrow 0$ for updating $1/\eta$ times, Algorithm 1 is continualized as dynamics

$$\dot{x}^{a|s_i} = p_i^{\text{st}} \sum_b y^{b|s_i} \left(x^{a|s_i} (1 - x^{a|s_i}) \pi(\mathbf{e}_{i'(a,b)}, \mathbf{x}, \mathbf{y}) + \sum_{a' \neq a} x^{a'|s_i} (-x^{a|s_i}) \pi(\mathbf{e}_{i'(a',b)}, \mathbf{x}, \mathbf{y}) \right) \quad (\text{A2})$$

$$= p_i^{\text{st}} x^{a|s_i} \left\{ \underbrace{\pi\left(\sum_b y^{b|s_i} \mathbf{e}_{i'(a,b)}, \mathbf{x}, \mathbf{y}\right)}_{=\mathbf{p}^{a|s_i}} - \sum_{a'} x^{a'|s_i} \underbrace{\pi\left(\sum_b y^{b|s_i} \mathbf{e}_{i'(a',b)}, \mathbf{x}, \mathbf{y}\right)}_{=\mathbf{p}^{a'|s_i}} \right\} \quad (\text{A3})$$

$$= p_i^{\text{st}} x^{a|s_i} \left(\pi(\mathbf{p}^{a|s_i}, \mathbf{x}, \mathbf{y}) - \underbrace{\sum_{a'} x^{a'|s_i} \pi(\mathbf{p}^{a'|s_i}, \mathbf{x}, \mathbf{y})}_{=\bar{\pi}^{s_i}(\mathbf{x}, \mathbf{y})} \right). \quad (\text{A4})$$

Here, $i'(a, b)$ indicates the next state index i' such that $s_{i'} = abs_i^-$. Eq. (A4) corresponds to Eqs (8) and (9) in the main manuscript. \square

A.2 Proof of Theorem 2

Taking the limit $\gamma \rightarrow 0$, we obtain

$$\Delta^{a|s} = \frac{\partial u^{\text{st}}(\text{Norm}(\mathbf{x}), \mathbf{y})}{\partial x^{a|s}}. \quad (\text{A5})$$

Then, if we take the limit $\eta \rightarrow 0$ for updating $1/\eta$ times, Algorithm 2 is continualized as dynamics

$$\dot{x}^{a|s}(\mathbf{x}, \mathbf{y}) = x^{a|s} \frac{\partial}{\partial x^{a|s}} u^{\text{st}}(\text{Norm}(\mathbf{x}), \mathbf{y}). \quad (\text{A6})$$

Eq. (A6) corresponds to Eq. (10). \square

A.3 Proof of Theorem 3

We assume infinitesimal $dx^{a|s_i}$, and the infinitesimal change in \mathbf{x} ;

$$x^{a|s_i} \leftarrow x^{a|s_i} + dx^{a|s_i}, \quad (\text{A7})$$

$$\mathbf{x} \leftarrow \text{Norm}(\mathbf{x}), \quad (\text{A8})$$

By this change, the Markov transition matrix $\mathbf{M}(\mathbf{x}, \mathbf{y})$ changes into $\mathbf{M}(\mathbf{x}, \mathbf{y}) + d\mathbf{M}(\mathbf{x}, \mathbf{y}, dx^{a|s_i})$, described as

$$dM_{i'i''} = dx^{a|s_i} y^{b|s_i} \times \begin{cases} 1 - x^{a|s_i} & (s_{i''} = s_i, s_{i'} = abs_i^-) \\ -x^{a'|s_i} & (s_{i''} = s_i, s_{i'} = a'bs_i^-, a' \neq a) \\ 0 & (\text{otherwise}) \end{cases}. \quad (\text{A9})$$