admits any carefully synchronizing word. For a given $\mathcal{A}$ we define its *power automaton* (which is itself a PFA) as $\mathcal{P}(\mathcal{A}) = (2^Q, \Sigma, \tau)$, where 2^Q stands for the set of all subsets of Q, and Σ is the same as in $\mathcal{A}$. The transition function $\tau : 2^Q \times \Sigma \to 2^Q$ is defined as follows. Let $Q' \subseteq Q$. For every $a \in \Sigma$ we define $\tau(Q', a) = \bigcup_{q \in Q'} \delta(q, a)$ if $\delta(q, a)$ is defined for all states $q \in Q'$, otherwise $\tau(Q', a)$ is not defined. We also note $Q.w$ as an action of a word w on a set of states Q under the function δ. Let $S \subseteq Q$. Then we denote $S.w^{-1}$ as a preimage of S under the action of a word w.
We note that the above concepts can also be considered for *deterministic finite automata* (DFA), for which the transition function is total. We define an a-cluster to be a DFA $\mathcal{A} = (Q, \{a\}, \delta)$ such that the automaton is connected. In other words it means that such automaton is a cycle on letter a with paths that leads to the states of that cycle. The set of states that induce a cycle in the a-cluster is referred to as the *center* of the cluster. The *depth* of the cluster is the length of the longest path to the center of the cluster. If q belongs to the center of the a-cluster, the *branch* of the state q are the states that has a path to q that does not have any other state belonging to the center. *Destination* of the branch is a state in the center that has an in-transition from the last state of the branch. Example of the a-cluster is depicted on Figure 1.

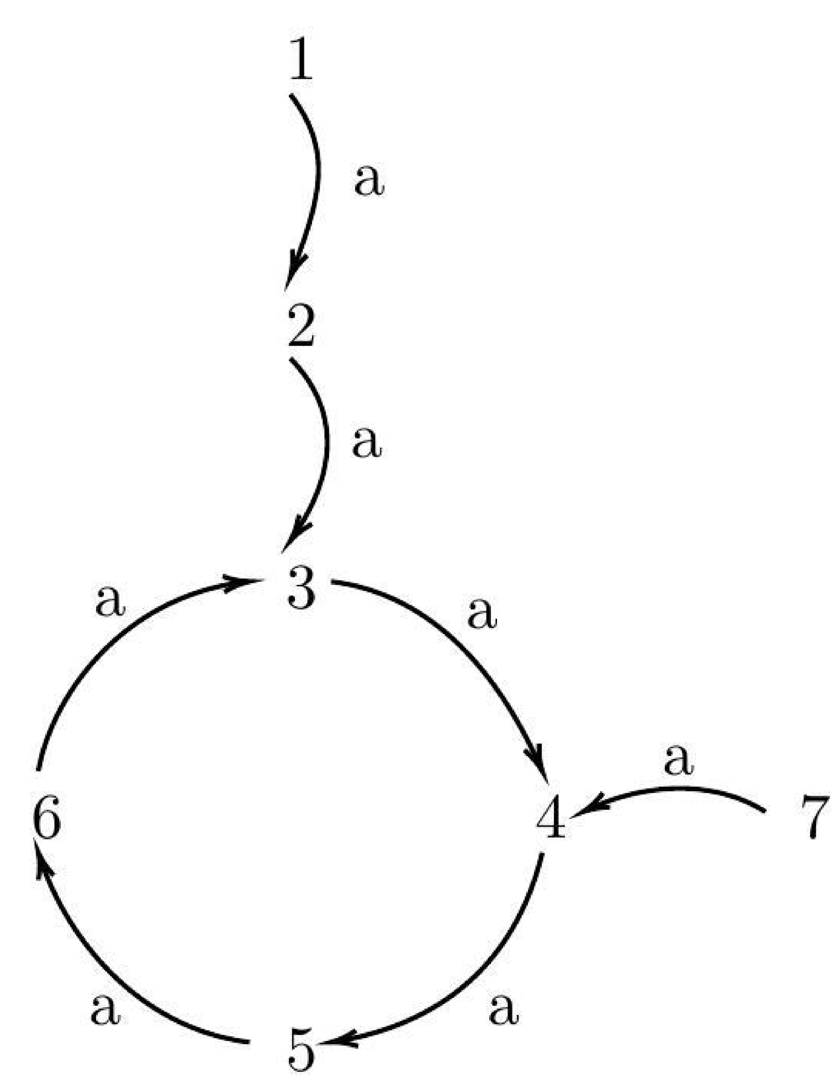

Figure 1: Example of the a-cluster

Center of that a-cluster is the set $\{3, 4, 5, 6\}$, the depth is 2 and there are two branches: $b_1 = \{1, 2\}$ and $b_2 = \{7\}$. Destination of the branch b_1 is the state 3 and of the branch b_2 is state 4.

We define the sum of two automata $\mathcal{A} = (Q_1, \Sigma_1, \delta_1)$ and $\mathcal{B} = (Q_2, \Sigma_2, \delta_2)$ as $\mathcal{A} \cup \mathcal{B} = (Q_1 \cup Q_2, \Sigma_1 \cup \Sigma_2, \delta_1 \cup \delta_2)$. We can now state the obvious fact, useful to decide whether a given PFA is carefully synchronizing.