parameterizes the scalar perturbations [23], as a small perturbation and we determine the corrections it induces on the standard states (associated to a time-dependent harmonic oscillator). As a result, we are able to determine the Spectrum corrections due to the new physics at the ground of our study, and we show under which constraints on the model parameters and initial conditions the modification is a reliably small and potentially observable feature.

We conclude by stressing that the present analysis offers an interesting new perspective on the origin of a non-singular isotropic Universe, whose underlying cut-off physics can leave a precise fingerprint on the profile of the microwave background temperature distribution.

The manuscript is organized as follows. In Section II we present the standard EU in its Hamiltonian formulation. In Section III we introduce the modified algebra and use it to derive a non-fine-tuned EU model; then, in Section IV we derive the modified Power Spectrum of perturbations using the same modified algebra implemented on the Mukhanov-Sasaki variable. In Section V we conclude the paper with a brief summary and outlook.

We use the natural units $c = \hbar = 8\pi G = 1$.

II. HAMILTONIAN FORMULATION OF THE EMERGENT UNIVERSE MODEL

Here we compactly present the standard Emergent Universe model [16, 17] starting from the Hamiltonian formulation of the FLRW homogeneous and isotropic model. We derive a non-singular, ever-expanding solution and then show the potential used to end the inflationary expansion.

A. The Standard Emergent Universe Scenario

The configurational variables that we will use for the gravitational sector are the volume $v = a^3$, where $a = a(t)$ is the cosmic scale factor, and its conjugate momentum $p_v \propto \dot{v}/v$; they have been shown to be the suitable variables to yield an universal critical energy density in Polymer Cosmology [24].

The Hamiltonian for a FLRW model with curvature filled with matter in the form of perfect fluids is

$$\mathcal{H}_g(v, p_v) = -\frac{3}{4}\, v\, p_v^2 - 3\, K\, v^{\frac{1}{3}} + \rho(v)\, v = 0, \tag{1}$$

where $K = +1$ parametrizes the spatial curvature and $\rho(v) = \sum_i \rho_i(v)$ contains all the necessary components, each obeying the following continuity equation that yields the following expression:

$$\dot{\rho}_i + \frac{\dot{v}}{v}\, \rho_i\, (1 + w_i) = 0, \quad \rho_i(v) = \overline{\rho_i}\, v^{-(1+w_i)}, \tag{2}$$

with w_i being the equation of state parameter; a simple EU model contains a radiation fluid ρ_γ with $w_\gamma = \frac{1}{3}$

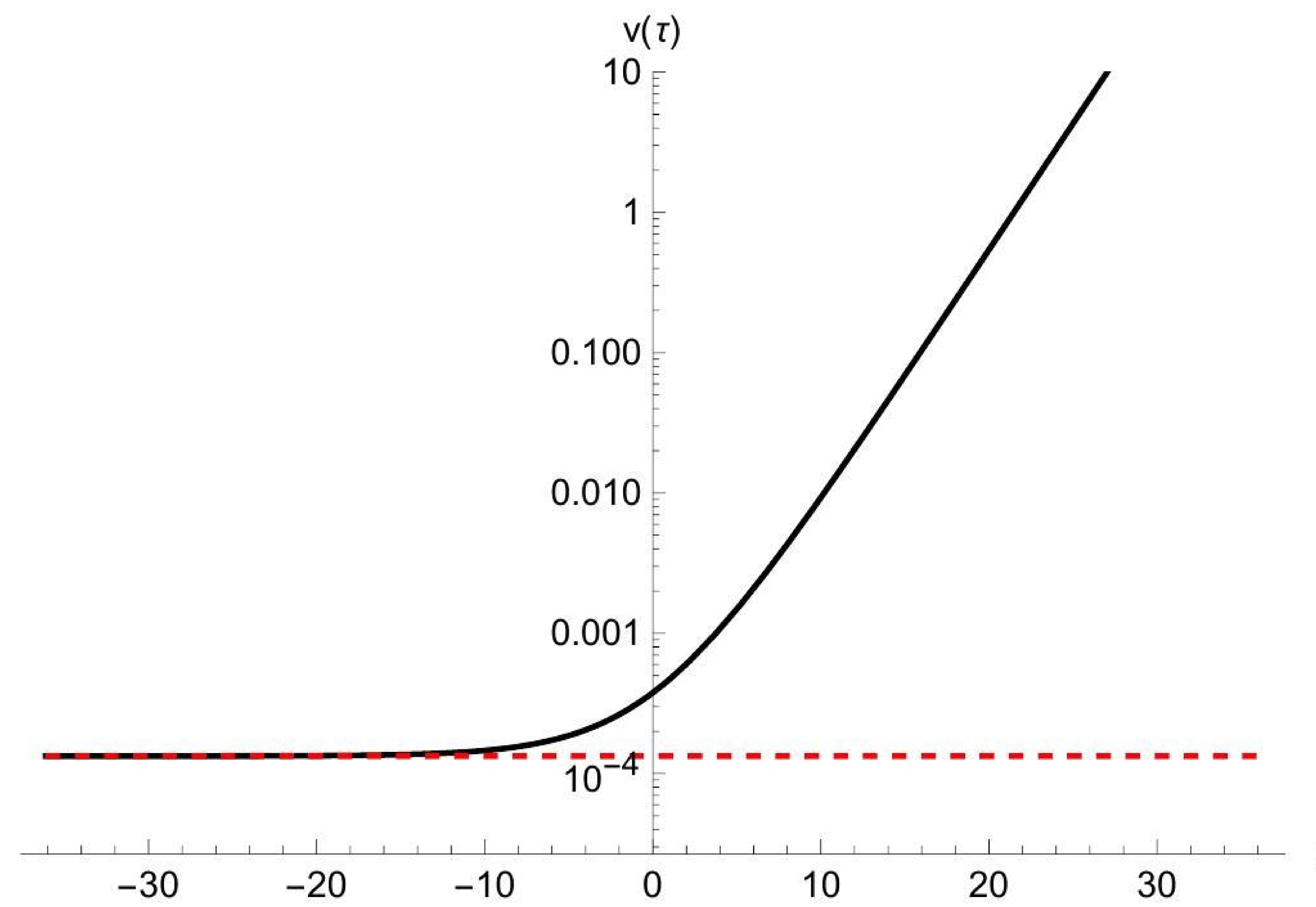

Figure 1. Evolution of the volume $v(\tau)$ in the standard EU model. The time variables are rescaled by the time t_s of the beginning of standard inflation: $\tau = \frac{t}{t_s}$. The minimum value is highlighted with a red dashed line.

and a Cosmological Constant $\rho_\Lambda = \overline{\rho_\Lambda}$ corresponding to $w_\Lambda = -1$.

From the equations of motion and the Hamiltonian constraint we obtain the Friedmann equation

$$H^2 = \left(\frac{\dot{v}}{3v}\right)^2 = \frac{\rho_\gamma + \rho_\Lambda}{3} - \frac{K}{v^{2/3}}. \tag{3}$$

By requiring the existence of a unique positive minimum v_i for the volume, we obtain the following constraint on the free parameters of the densities:

$$v_i = \left(\frac{3}{2}\,\frac{K}{\rho_\Lambda}\right)^{\frac{3}{2}}, \quad 4\,\overline{\rho_\gamma}\,\rho_\Lambda = 9\,K^2, \tag{4}$$

so that the Friedmann equation can be rewritten in terms of the minimum volume and easily solved:

$$\dot{v} = \pm 3\sqrt{\frac{K}{2}}\left(\frac{v}{v_i}\right)^{\frac{1}{3}}\left(v^{\frac{2}{3}} - v_i^{\frac{2}{3}}\right), \tag{5}$$

$$v(t) = v_i\left[1 + \exp\left(\pm\frac{\sqrt{2K}}{v_i^{1/3}}\, t\right)\right]^{\frac{3}{2}}. \tag{6}$$

We have two solutions, one expending to infinity and one contracting from infinity, depending on the sign of the exponential; of course we are interested in the expanding one with the + sign. The solution is shown in Figure 1: as expected it is asymptotically Einstein static, since $v(t \to -\infty) \to v_i > 0$, and exponentially expanding. Note that in the picture we rescaled the time variable as $\tau = t/t_s$, where $t_s \approx 10^{-36}s$ is the start of inflation in the standard cosmological model; this will be useful in later sections.

Even though inflation occurs for an infinite time in the past, at any given time $t_f \gg v_i^{1/3}$ there is a finite amount