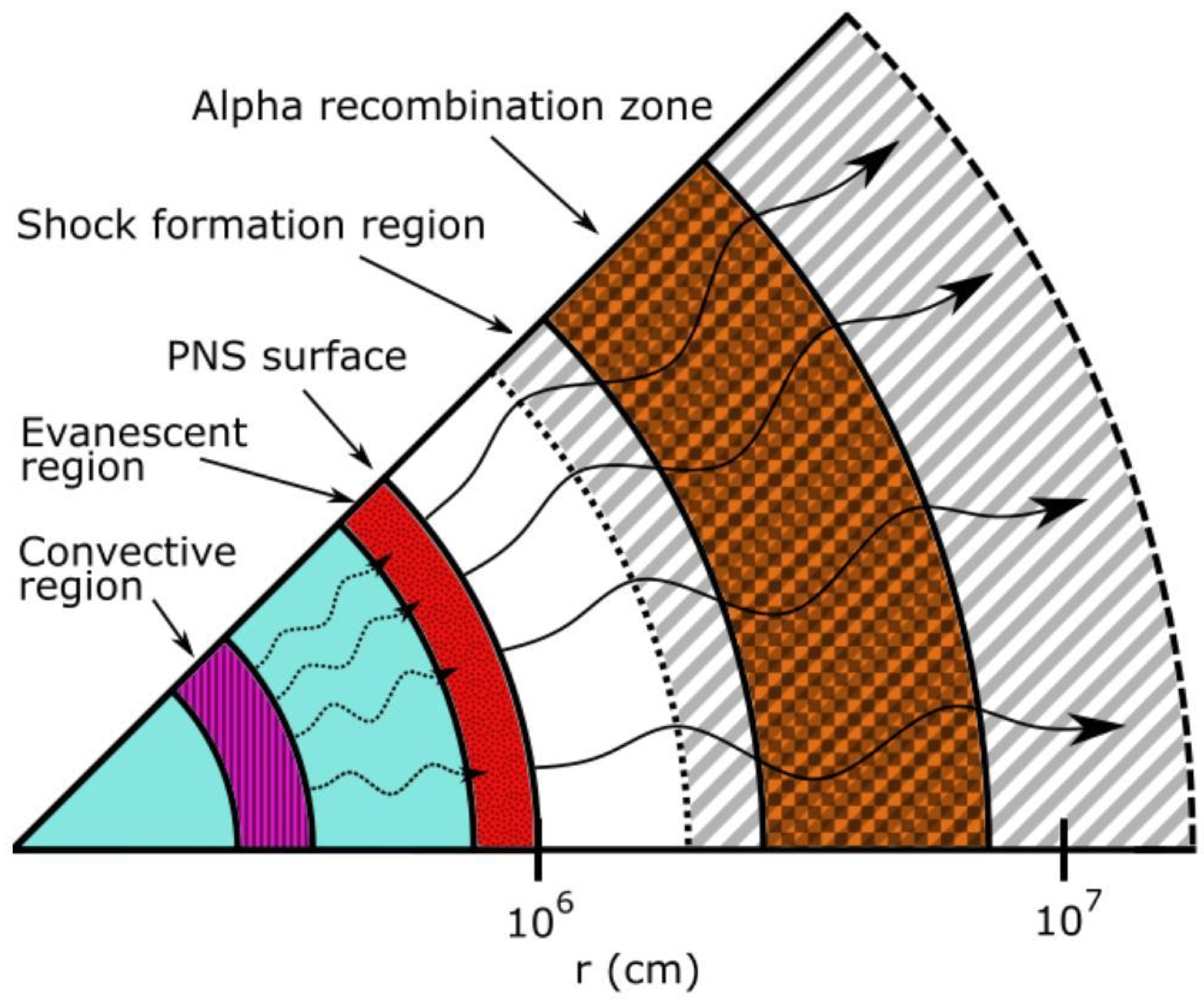

Figure 1. An approximate schematic of gravito-acoustic wave emission and propagation inside and near the PNS. Gravity waves (dashed) are generated by the convective region, attenuate in the evanescent region, and re-emerge as acoustic waves (solid) near the surface of the PNS. They then propagate outward through the wind until they form shocks and dissipate. The region of possible shock formation and heat deposition overlaps with the α-forming region. If the waves shock before or during α recombination, the additional heating will inhibit seed formation, making a strong r-process more likely.

(Pons et al. 1999). The atmosphere of the PNS is nearly isothermal due to neutrino interactions (e.g. Qian & Woosley 1996) which results in a Brunt-Väisälä frequency, ω_{BV}, that is slowly varying with radius up to the point at which the wind is launched. Based on the models described in section 5.1 and the models of Roberts & Reddy (2017), $\omega_{BV} \sim 5 \times 10^4\,\mathrm{s}^{-1}$. The Lamb frequency in this region is $\omega_L \sim c_s/r \sim 10^3\,\mathrm{s}^{-1}$. Therefore, the waves excited by convection will be evanescent through the PNS atmosphere and emerge into the acoustic branch as the density rapidly falls off in the wind region.

Using the models of the NDW described in section 5.1 with no heating and employing the WKB approximation as described in Gossan et al. (2020), we find that for a Gaussian distribution of frequencies centered at $10^3\,\mathrm{s}^{-1}$, with a standard deviation of $10^2\,\mathrm{s}^{-1}$, and angular modes ranging from $\ell = 1$ to $\ell = 6$ (assuming equal power in each mode), the average transmission efficiency is $\mathcal{T}_{\rm avg} = 0.11$. The transmission efficiency ranges between $\mathcal{T}_{\rm avg} \sim 0.01 - 0.2$ for a wide range of mean wave frequencies. Rather than try to model this wave transmission in detail, we allow for transmission efficiencies in this range and take $L_w \sim M_{\rm con}\mathcal{T} L_{\nu,\rm tot} \sim 10^{-5} - 10^{-2} L_{\nu,\rm tot}$. Although the work of Gossan et al. (2020) considered wave propagation in the pre-explosion supernova environment before a NDW had formed, their results for the transmission efficiency are similar to this range of estimates of the transmission efficiency for the post-explosion phase. We do not track the evanescent region in our models, but rather assume the waves have an acoustic character throughout the wind.

The total power of net neutrino heating in the wind is only (Qian & Woosley 1996)

$$\frac{\dot{Q}_\nu}{L_{\nu,\rm tot}} \sim 1.5 \times 10^{-4} L_{\bar{\nu}_e,51}^{2/3} R_6^{2/3} \left(\frac{1.4 M_\odot}{M_{\rm NS}}\right), \tag{2}$$

with $M_{\rm NS}$ being the PNS mass, R_6 being the PNS radius in units of 10^6 cm, $L_{\bar{\nu}_e,51}$ being the electron antineutrino luminosity in units of 10^{51} erg s^{-1}, and assuming an average neutrino energy of 12 MeV. Therefore, based on energetic arguments alone it is clear that the presence of these gravito-acoustic waves is likely to have a significant impact on the dynamics of the wind. There are two ways in which the waves can affect the wind. First, even in the linear regime, the waves will act as a source of stress in the wind (e.g. Jacques 1977) and accelerate the wind. Second, as the waves become non-linear, they will shock and dissipate their energy into heat. By changing the NDW dynamics, both of these effects can alter the nucleosynthetic yields of the wind. A faster outflow reduces the time available for carbon production to occur and will result in a more alpha-rich freeze out (Hoffman et al. 1997). More heating, if it occurs before alpha recombination, will increase the entropy of the wind and make alpha recombination occur at a lower density, also leading to a more alpha-rich freezeout. An additional source of heat or kinetic energy will also reduce the amount of neutrino capture heating required to unbind the wind material, which will in turn lower the equilibrium electron fraction of the wind. In neutron rich conditions, all of these effects will result in more favorable conditions for r-process nucleosynthesis.

3 WIND MODEL

To model the neutrino-driven wind, we solve the equations of general relativistic hydrodynamics in spherical symmetry in steady state. The background metric is assumed to be Schwarzschild sourced by the mass of the PNS $M_{\rm NS}$, i.e. we neglect self gravity. These equations are then augmented by a model equation for the evolution of the wave action and its coupling to the background flow, derived following Jacques (1977). We seek trans-sonic solutions of the wind equations, so we place the momentum equation in critical form (Thompson et al. 2001). With these assumptions, the equations of continuity, momentum conservation, entropy (s), lepton number conservation, and wave action (S) evolution give

$$\begin{aligned}
\dot{M}_{\rm NS} &= 4\pi r^2 e^\Lambda W v \rho \\
\frac{dv}{dr} &= \frac{v}{r}\frac{f_2}{f_1} \\
\frac{ds}{dr} &= \frac{\xi_s}{r} \\
\frac{dY_e}{dr} &= \frac{\xi_{Y_e}}{r} \\
\frac{dS}{dr} &= -S\left(\frac{2}{r} + \frac{1}{l_d} + \frac{1}{v_g}\frac{dv_g}{dr}\right)
\end{aligned} \tag{3}$$

where

$$\begin{aligned}
f_1 &= \left(1 - \frac{v^2}{c_s^2}\right) + \delta f_1 \\
f_2 &= -\frac{2}{W^2} + \frac{G M_{\rm NS}}{c_s^2 r}\frac{1 - \left(\frac{c_s}{c}\right)^2}{e^{2\Lambda} W^2} \\
&\quad + \frac{1}{W^2 h \rho c_s^2}\left[\xi_s \left(\frac{\partial P}{\partial s}\right)_{\rho, Y_e} + \xi_{Y_e}\left(\frac{\partial P}{\partial Y_e}\right)_{\rho, s}\right] + \delta f_2 \\
\xi_s &= \frac{r}{v}\frac{\dot{q}_{\rm tot}}{e^\Lambda W T} \\
\xi_{Y_e} &= \frac{r}{v}\frac{\dot{Y}_e}{e^\Lambda W}.
\end{aligned} \tag{4}$$

The wave action S is connected to the wave luminosity L_w via

$$S = \frac{L_w}{4\pi r^2 c_s \omega}. \tag{5}$$