The dimension i part of the Segre class $s(C)$ is denoted $s_i(C)$, and if C is equidimensional, then $s_i(C) = s^{\dim C - i}(C)$. The dimension i part of the Segre class of the normal cone of a closed immersion $X \to Y$ is denoted $s_i(C_X Y)$ or $s(X,Y)_i$.

For the better understanding of the generalized version of double point formula, we give a definition of two kind of join varieties.

Definition 2. (cf. [7, Chapter 1.3])
Let X and Y be subvarieties of $\mathbb{P}^N$. The embedded join of X and Y is the closure of the union of all lines connecting a point in X to a point in Y, denoted XY. The abstract or ruled join $J(X,Y)$ is the projective spectrum of the tensor product of the homogeneous coordinate rings of X and Y, denoted $J(X,Y)$.

For example, the ruled join of $\mathbb{P}^n$ and $\mathbb{P}^m$ is $\mathbb{P}^{n+m+1}$. The affine cone of a subvariety X is defined as the spectrum of the homogeneous coordinate ring of X. The ruled join $J(X,Y)$ consists of the closed points $[x:y]$ where x and y are points of affine cones of X and Y, respectively.

Let X and Y be two projective subvarieties of the projective space $\mathbb{P}^N$, and let x and y be closed points in X and Y, respectively, with coordinate representations $[x_0 : \cdots : x_N]$ and $[y_0 : \cdots : y_N]$. We define a rational map $J(X,Y) \dashrightarrow XY$ by

$$[x_0 : \cdots : x_N : y_0 : \cdots : y_N] \mapsto [x_0 - y_0 : \cdots : x_N - y_N].$$

The indeterminate locus of this map is defined by the equations $x_0 - y_0 = \cdots = x_N - y_N = 0$. We denote the degree of this rational map $J(X,Y) \dashrightarrow XY$ by $\deg(J/XY)$.

The intersection $X \cap Y$ is embedded into the product variety $X \times Y$ along its diagonal embedding. Let $C_{X\cap Y}(X \times Y)$ denote the normal cone of $X \cap Y$ to $X \times Y$. The following theorem decomposes the information of the embedded join into simpler pieces:

Theorem [7, Theorem 8.2.8] For subvarieties X, Y of $\mathbb{P}^N$ of degree d_X, resp. d_Y and dimension n, resp. m we have

$$\deg XY \deg(J/XY) = d_X d_Y \; - \sum_{k \geq 0} \binom{n+m+1}{k} \deg s_k(C_{X\cap Y}(X \times Y)).$$

This formula is derived from the refined Bezout's theorem, which is based on Vogel's v and β cycle construction. For further explanations, see [7, Chapter 2] and [7, Chapter 8].

If we set $Y = \sigma_2(X)$, then the embedded join XY is the 3-secant variety $\sigma_3(X)$. Thus, in order to compute $\deg \sigma_3(X)$, we need the degree of the 2-secant variety $d_Y = \deg \sigma_2(X)$ as well as the degrees of the Segre classes $s_k(C_{\Delta(X)}(X \times \sigma_2(X)))$. Assume that $\mathcal{O}_X(1)$ is a 3-very ample line bundle. The degree of the 2-secant variety d_Y can be obtained by applying [7, Theorem 8.2.8] again:

$$2 \deg \sigma_2(X) = d_X^2 - \sum_{k \geq 0} \binom{2n+1}{k} \deg s_k(C_{\Delta(X)}(X \times X)). \tag{1}$$

If $\mathcal{L}$ is 5-very ample, the degree of the rational map $J(X,Y) \dashrightarrow XY$ is 3. (cf. Lemma 4.1) Therefore, by the above theorem, the degree of the 3-secant variety of X can be computed as follows:

$$\frac{1}{3}\left(\frac{d_X(d_X^2 - A)}{2} - B\right) \tag{2}$$