A Proofs

A.1 Proof of Theorem 1

First, line 6 in Algorithm 1 is equal to

$$x^{a'|s_i} \leftarrow \begin{cases} x^{a'|s_i} + (1 - x^{a'|s_i})\eta\pi(\boldsymbol{e}_{i'}, \mathbf{x}, \mathbf{y}) + O(\gamma^2) & (a' = a) \\ x^{a'|s_i} - x^{a'|s_i}\eta\pi(\boldsymbol{e}_{i'}, \mathbf{x}, \mathbf{y}) + O(\gamma^2) & (a' \neq a) \end{cases}, \tag{A1}$$

from Definition 1.

In the stationary state of the repeated games, state s_i occurs with the probability of p_i^{st}. Then, player X (resp. Y) chooses action a (resp. b) with the probability of $x^{a|s_i}$ and $y^{b|s_i}$. If we take the limit $\eta \to 0$ for updating $1/\eta$ times, Algorithm 1 is continualized as dynamics

$$\dot{x}^{a|s_i} = p_i^{\mathrm{st}} \sum_b y^{b|s_i} \left(x^{a|s_i}(1 - x^{a|s_i})\pi(\boldsymbol{e}_{i'(a,b)}, \mathbf{x}, \mathbf{y}) + \sum_{a' \neq a} x^{a'|s_i}(-x^{a|s_i})\pi(\boldsymbol{e}_{i'(a',b)}, \mathbf{x}, \mathbf{y}) \right) \tag{A2}$$

$$= p_i^{\mathrm{st}} x^{a|s} \left\{ \pi(\underbrace{\sum_b y^{b|s_i}\boldsymbol{e}_{i'(a,b)}}_{=\boldsymbol{p}^{a|s}}, \mathbf{x}, \mathbf{y}) - \sum_{a'} x^{a'|s_i}\pi(\underbrace{\sum_b y^{b|s_i}\boldsymbol{e}_{i'(a',b)}}_{=\boldsymbol{p}^{a'|s_i}}, \mathbf{x}, \mathbf{y}) \right\} \tag{A3}$$

$$= p_i^{\mathrm{st}} x^{a|s_i} \left(\pi(\boldsymbol{p}^{a|s_i}, \mathbf{x}, \mathbf{y}) - \underbrace{\sum_{a'} x^{a'|s_i}\pi(\boldsymbol{p}^{a'|s_i}, \mathbf{x}, \mathbf{y})}_{=\bar{\pi}^{s_i}(\mathbf{x}, \mathbf{y})} \right). \tag{A4}$$

Here, $i'(a,b)$ indicates the next state index i' such that $s_{i'} = abs_i^-$. Eq. (A4) corresponds to Eqs (8) and (9) in the main manuscript. □

A.2 Proof of Theorem 2

Taking the limit $\gamma \to 0$, we obtain

$$\Delta^{a|s} = \frac{\partial u^{\mathrm{st}}(\mathrm{Norm}(\mathbf{x}), \mathbf{y})}{\partial x^{a|s}}. \tag{A5}$$

Then, if we take the limit $\eta \to 0$ for updating $1/\eta$ times, Algorithm 2 is continualized as dynamics

$$\dot{x}^{a|s}(\mathbf{x}, \mathbf{y}) = x^{a|s}\frac{\partial}{\partial x^{a|s}} u^{\mathrm{st}}(\mathrm{Norm}(\mathbf{x}), \mathbf{y}). \tag{A6}$$

Eq. (A6) corresponds to Eq. (10). □

A.3 Proof of Theorem 3

We assume infinitesimal $\mathrm{d}x^{a|s_i}$, and the infinitesimal change in $\mathbf{x}$;

$$x^{a|s_i} \leftarrow x^{a|s_i} + \mathrm{d}x^{a|s_i}, \tag{A7}$$

$$\mathbf{x} \leftarrow \mathrm{Norm}(\mathbf{x}), \tag{A8}$$

By this change, the Markov transition matrix $\boldsymbol{M}(\mathbf{x}, \mathbf{y})$ changes into $\boldsymbol{M}(\mathbf{x}, \mathbf{y}) + \mathrm{d}\boldsymbol{M}(\mathbf{x}, \mathbf{y}, \mathrm{d}x^{a|s})$, described as

$$\mathrm{d}M_{i'i''} = \mathrm{d}x^{a|s_i} y^{b|s_i} \times \begin{cases} 1 - x^{a|s_i} & (s_{i''} = s_i,\ s_{i'} = abs_i^-) \\ -x^{a'|s_i} & (s_{i''} = s_i,\ s_{i'} = a'bs_i^-, a' \neq a) \\ 0 & (\text{otherwise}) \end{cases}. \tag{A9}$$