allow an r-process to proceed. The cube of the entropy, divided by the dynamical timescale, has often been used as a criterion for determining r-process feasibility (Qian & Woosley 1996; Hoffman et al. 1997).

The NDW was initially predicted to be neutron rich, and Woosley et al. (1994) found it underwent a strong r-process that closely matched the solar r-process abundance pattern, in large part due to the high entropies found in their calculations. Subsequent work (e.g. Witti et al. 1994; Qian & Woosley 1996; Otsuki et al. 2000; Thompson et al. 2001) failed to reproduce conditions suitable for a strong r-process, finding entropies significantly lower than Woosley et al. (1994). Later work has explored the impact of other possible physics on the NDW, but has generally shown that, outside of extreme conditions – high PNS mass, unrealistically low electron fractions, or magnetar-strength magnetic fields – a wind heated purely by neutrinos does not reach high enough entropies or short enough dynamical timescales during seed formation to allow for a strong r-process (e.g. Thompson et al. 2001; Metzger et al. 2007; Wanajo 2013). The inclusion of corrections from general relativity tends to make conditions more favorable for the r-process, but a very high PNS mass is still required for a strong r-process to proceed (Cardall & Fuller 1997; Otsuki et al. 2000; Thompson et al. 2001). A number of studies have explored the effects of rotation and magnetic fields in varying dimensionality (Metzger et al. 2007; Vlasov et al. 2014; Thompson & ud Doula 2018; Desai et al. 2022), further confirming that extreme conditions - high PNS masses and magnetar-strength magnetic fields - are required for conditions to favor an r-process.

Other studies have focused on the electron fraction in the wind, as Hoffman et al. (1997) predicts that a lower Y_e will allow for strong r-processing with lower entropies. The electron fraction is set by the neutrino physics at work in the wind, which has been studied in increasing detail. Simulations by Fischer et al. (2010) and Hüdepohl et al. (2010) found that the neutrino spectrum from the PNS was likely to result in a proton-rich wind, precluding an r-process altogether. Subsequent work by Roberts et al. (2012) and Martínez-Pinedo et al. (2012) found slightly neutron-rich conditions when nuclear mean field effects were included. Later studies from Pllumbi et al. (2015) and Xiong et al. (2019) included neutrino oscillation effects, again finding that only proton-rich or slightly neutron-rich conditions were likely to occur in the wind. In short, it seems unlikely that the generally low seed-formation entropy can be compensated by an increased neutron fraction.

Rather, the most promising avenue for a strong r-process in the NDW is to invoke a secondary heating effect that takes place in the seed-forming region of the wind (Qian & Woosley 1996). Suzuki & Nagataki (2005) proposed damped Alfvén waves as a source for this heating, finding that waves generated by magnetar-strength magnetic fields could deposit sufficient energy in this region to predict a strong r-process. Metzger et al. (2007) also suggested that a small amount of additional heating from acoustic waves, deposited in the seed-forming region, could drive a strong r-process independent of magnetorotational effects. More recently, Gossan et al. (2020) suggested that gravito-acoustic waves generated by PNS convection could have an important effect on the dynamics of the NDW. Most recently, supernova simulations by Nagakura et al. (2020) and Nagakura et al. (2021) indicate that such convection is a common and significant feature across a broad range of progenitors, so convection-driven effects in the wind are likely to be important in most supernovae. They find that PNS convection is strongest in the first 1-2 seconds post-bounce, then gradually subsides. Gravito-acoustic wave heating is therefore likely to operate in the early stages of the NDW, when it is most likely to be neutron rich (e.g. Roberts et al. 2012). These effects are powered by the gravitational contraction of the PNS, which provides an energy reservoir of some 10^{53} erg during contraction and deleptonization (Gossan et al. 2020). Even a small fraction of this binding energy coupling to the wind via wave emission could have a significant impact.

In light of this, we present here a systematic parameter study of the effects of convection-driven gravito-acoustic waves on the dynamics and nucleosynthetic behavior of the NDW. These waves are excited by convective motions in the PNS as internal gravity waves, which tunnel through the PNS atmosphere and emerge as acoustic waves in the NDW itself. As they propagate through the wind, these waves provide an additional source of stress, driving a faster outflow. They can also shock, efficiently depositing their energy into the wind and acting as a secondary heat source. Our objective in this paper is to determine the conditions in which a strong r-process can take place when the effects of these waves are included. To this end, we assume a spherically symmetric and slightly neutron-rich wind and investigate the impact of varying the energy contained in the waves reaching the wind region, as well as the frequency of the waves, which impacts the radius of shock formation and their subsequent rate of energy deposition.

The paper is structured as follows: Section 2 outlines the physics behind the generation of these gravito-acoustic waves, and how they deposit energy into the wind. Section 3 describes the equations used to model the wind, and section 4 describes the computational method we use for running the simulations. In section 5 we present our results. Our results show that r-processing will take place in significant regions of the parameter space, for both fiducial and extreme PNS conditions.

2 GRAVITO-ACOUSTIC WAVES AS A SECONDARY HEATING SOURCE

Shortly after core collapse (~200 ms), a convective region develops in the outer mantle of the proto-neutron star (Dessart et al. 2006; Gossan et al. 2020). Turbulent convection will excite gravito-acoustic waves from the interface between the interior convective region and an exterior radiative region. Both gravity wave modes and acoustic modes will be excited, in addition to non-propagating modes, but due to the Mach number dependence of the wave excitation, the energy flux will be dominated by waves in the gravity wave branch (Goldreich & Kumar 1990). The emitted gravity wave luminosity is expected to be

$$L_g \approx M_{\rm con} L_{\rm con} \approx M_{\rm con} L_{\nu,\rm tot} \quad (1)$$

where $L_{\rm con}$ and $M_{\rm con}$ denote the convective luminosity and Mach number, respectively. Furthermore, the convective and total neutrino luminosities $L_{\rm con}$ and $L_{\nu,\rm tot}$ should be approximately equal, as convection is expected to be efficient in the PNS mantle and will carry the majority of the energy flux. $M_{\rm con}$ is expected to fall between 10^{-2} and 10^{-1} (Dessart et al. 2006; Gossan et al. 2020). Some fraction of the power emitted in gravity waves may propagate from the convective region, through the isothermal atmosphere where the waves will pass through an evanescent region, and into the wind where they will emerge as acoustic waves that can impact the dynamics of the NDW. A schematic of the wave propagation and dissipation in and around the PNS is shown in figure 1.

Gravity waves are emitted from this convective region with frequencies of $\omega \sim 10^2 - 10^4$ rad s^{-1} (Dessart et al. 2006; Gossan et al. 2020). During the NDW phase of PNS evolution, the convective region is expected to be fairly close to the surface of the PNS