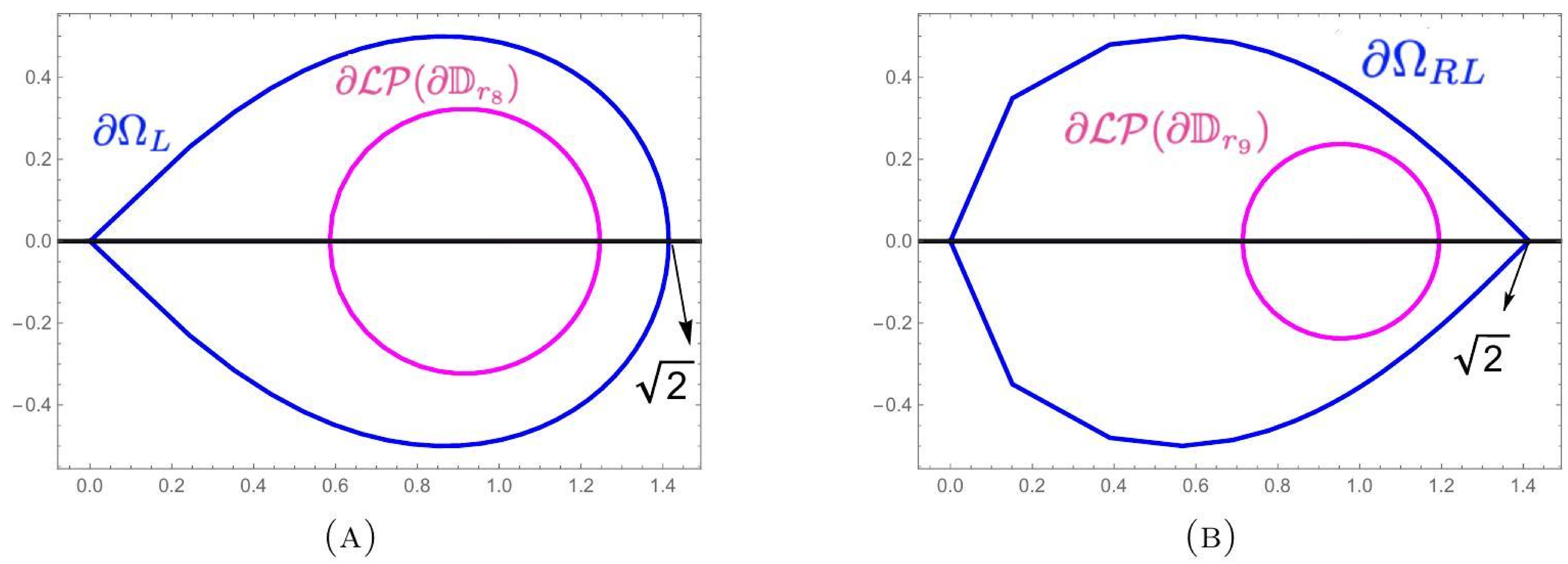
(ii) $f \in \mathcal{S}^*_{\mathcal{RL}}$ *in* $|z| < \tanh^2(\pi\sqrt[4]{\sqrt{2}\,\eta}(1-\sqrt{2\eta})/2\sqrt{2}) \approx 0.283\ldots$.

Remark 2.15. From **Fig.** 5, it is evident that the radii obtained in Corollary 2.14 can be further improved.



(A) (B)

FIGURE 5. Above figures correspond to Corollary 2.14, with radii r_8 and r_9 given by: (a) $r_8 \approx 0.376\ldots$ (b) $r_9 \approx 0.283\ldots$.

We now define the class $\mathfrak{F}$ constructed with the help of ratios of two analytic functions $f, g \in \mathcal{A}$, studied by Mundalia and Kumar [21].

$$\mathfrak{F} = \left\{ f \in \mathcal{A} : \operatorname{Re}\left(\frac{f(z)}{g(z)}\right) > 0 \ \&\ \operatorname{Re}\left(\frac{(1-z)^{1+A}g(z)}{z}\right) > 0, -1 \leq A \leq 1 \right\}.$$

In patricular, for $A = -1$ and $A = 1$, the class $\mathfrak{F}$ reduces to the following classes, respectively,

$$\mathfrak{F}_1 = \left\{ f \in \mathcal{A} : \operatorname{Re}\left(\frac{f(z)}{g(z)}\right) > 0 \ \&\ \operatorname{Re}\left(\frac{g(z)}{z}\right) > 0 \right\}$$

and

$$\mathfrak{F}_2 = \left\{ f \in \mathcal{A} : \operatorname{Re}\left(\frac{f(z)}{g(z)}\right) > 0 \ \&\ \operatorname{Re}\left(\frac{(1-z)^2 g(z)}{z}\right) > 0 \right\}.$$

For proving the next theorem, we require the following lemma of Ravichandran et al. [24].

Lemma 2.16. *If* $p \in \mathcal{P}_n[A,B]$, *then for* $|z| = r$

$$\left| p(z) - \frac{1 - ABr^{2n}}{1 - B^2 r^{2n}} \right| \leq \frac{|A-B| r^n}{1 - B^2 r^{2n}}.$$

Particularly, if $p \in \mathcal{P}_n(\alpha)$, *then*

$$\left| p(z) - \frac{1 + (1-2\alpha) r^{2n}}{1 - r^{2n}} \right| \leq \frac{2(1-\alpha) r^n}{1 - r^{2n}}.$$

Theorem 2.17. *Let* $-1 \leq A \leq 1$, *and suppose* $f \in \mathcal{F}_{\mathcal{LP}}$, *then the sharp* $\mathfrak{F}-$*radius is given by*

$$\mathcal{R}_{\mathfrak{F}}(\mathcal{F}_{\mathcal{LP}}) = \frac{1}{2A+3}\left(\sqrt{A^2 + 12A + 28} - (5 + A)\right) =: R_{\mathfrak{F}}.$$

Proof. Since $f \in \mathfrak{F}$, then by definition of the class $\mathfrak{F}$, we have $f(z) = p_1(z)g(z)$ and $g(z) = zp_2(z)(1-z)^{-(1+A)}$, where for each $i = 1, 2$, $p_i : \mathbb{D} \to \mathbb{C}$ are analytic functions such that $p_i(0) = 1$ and $\operatorname{Re} p_i(z) > 0$. This leads to $f(z) = zp_1(z)p_2(z)(1-z)^{-(1+A)}$, and as a consequence of logarithmic differentiation, we obtain

$$\frac{zf'(z)}{f(z)} = \frac{1 + Az}{1 - z} + \frac{zp_1'(z)}{p_1(z)} + \frac{zp_2'(z)}{p_2(z)}.$$

For each $-1 \leq A \leq 1$, Lemma 2.16 leads to,

$$\left| \frac{zf'(z)}{f(z)} - \frac{1 + Ar^2}{1 - r^2} \right| \leq \frac{(5 + A) r}{1 - r^2} = R. \tag{2.5}$$