Emergent Universe Model from Modified Heisenberg Algebra

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We provide an Emergent Universe picture in which the fine-tuning on the initial conditions is replaced by cut-off physics, implemented on a semiclassical level when referred to the Universe dynamics and on a purely quantum level for the quantum fluctuations of the inflaton field. The adopted cut-off physics is inspired by Polymer Quantum Mechanics but expanded in the limit of a small lattice step. On a quasi-classical level, this results in modified Poisson Brackets for the Hamiltonian Universe dynamics similar to a Generalized Uncertainty Principle algebra. The resulting Universe is indeed asymptotically Einstein-static, emerging from a finite volume configuration in the distant past and then properly reconnecting with the most relevant Universe phases. The calculation of the modifications of the primordial inflaton spectrum is then performed by treating new physics as a small correction on the standard Hamiltonian of each Fourier mode of the field.

The merit of this study is to provide a new paradigm for a non-singular Emergent Universe, which is associated with a precise fingerprint on the temperature distribution of the microwave background, in principle observable by future experiments.

I. INTRODUCTION

One of the most relevant open questions in Relativistic Cosmology concerns the existence of the initial singularity [1–6]. Indeed, as shown in well-known papers [7, 8], the existence of a singular instant in the past of our Universe where the curvature invariant diverges and the Einstein equations are no longer predictive is a general feature of the cosmological problem, which has nothing to do with the highly symmetric nature of the Robertson-Walker (RW) geometry describing the isotropic Universe [2]. For this reason, any physics possibly able to overcome the singularity of the primordial Universe acquires a particular relevance. If the canonical quantization in the Wheeler-DeWitt formulation has been unable to provide a non-singular quantum cosmology [9, 10] (see also [11] for a different perspective on this scenario), the reformulation in terms of Ashtekar variables in the so-called Loop Quantum Cosmology (LQC) [12] has determined the existence of a Big Bounce, i.e. a Universe with a non-zero minimal volume where the collapsing and the expanding branches of the dynamics are connected and the singularity is avoided (for a review of LQC and Polymer Quantum Mechanics (PQM) [13] approaches to the emergence of a bouncing cosmology, see [14]). However, a Big Bounce can appear also in classical modified gravity, as discussed for instance in [15].

Here, we will consider not a bouncing cosmology, but simply a non-singular cosmology that comes from assigning specific initial conditions on the closed RW model dynamics, known as the "Emergent Universe" (EU) [16, 17]. The possibility for a non-singular Universe is due to a precise balance between the matter components and the positive spatial curvature of the considered model, which however has to be regarded as a fine-tuning on the Cauchy problem. The interest for such an EU model was recently renewed by the analyses of the Planck data sets [18, 19], which seem to allow for a present-day positive curvature of the Universe [20, 21].

Here, after reviewing the original literature on the subject of the classical EU model, we show how it can be obtained thanks to a modified Uncertainty Principle coming from an expanded Polymer formulation for a small enough lattice parameter [22], when we consider the classical cosmological dynamics via correspondingly modified Poisson algebra. The relevance of this formulation of the EU picture relies on the generality of its non-singular behavior, without the need for a constraint on the initial condition to be required ab initio. In other words, including a quasi-classical modification of the symplectic algebra similar in its phenomenology to a modified gravity approach, we are able to get an EU with an asymptotic non-singular beginning for the synchronous time approaching negative infinity. We also properly characterize the different phases of the Universe evolution, starting with a radiation-dominated era close to the classical singularity, passing through an inflationary de Sitter period obtained including a constant energy density term, and ending again with a radiation-dominated Universe (the study of a late-time dark energy-dominated era, possible for an EU as mentioned in [16], is beyond the scope of this work).

An important part of the present analysis is dedicated to the calculation of the primordial Spectrum corrections when the inflaton field obeys the same symplectic algebra at the ground of the obtained quasi-classical dynamics, but implemented on the pure quantum sector. We treat the additional term emerging in the Fourier-decomposed Hamiltonian for the Mukhanov-Sasaki variable, which

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