

prime divisors p of n_2 . An optimal CAC are constructed in [LT05, Lev07] when $n_2 = 1$. If $n_2 \neq 1$, it is proved in [FLS14] that an optimal CAC of length n can be constructed from an optimal CAC of length n_2 . It is also proved in [FLS14] that an optimal CAC of a prime power length can be constructed if we know how to construct an optimal CAC of prime length provided that the prime p in question is a non-Wieferich prime. For other odd lengths or *tight/equi-difference* CACs, we refer to [Mom07, WF13, LMSJ14, MM17, HLS] for the constructions. It turns out that CACs of prime lengths are the fundamental cases needed to be constructed. This naturally leads us to study CACs of *prime lengths* and weight 3.

Let p be an odd prime and denote by $\mathbb{F}_p = \mathbb{Z}/p\mathbb{Z}$ a finite field of p elements. Recall that a codeword of the form $\{0, a, 2a\}$ is said to be *equi-difference*. In the paper [LT05], the authors show that there exists an optimal CAC consisting of $\frac{p-1}{4}$ equi-difference codewords in the case where $4 \mid o_p(2)$. In contrast, if $4 \nmid o_p(2)$ then a CAC consisting of equi-difference codewords only is usually not optimal. By analyzing nonequi-difference codewords, an upper bound of the size of optimal CAC of odd length is given in [FLS14]. Let us recall their results for the case of CAC with prime lengths. Put

$$\mathcal{O}(p) = \begin{cases} \frac{p-1}{2o_p(2)} & \text{if } o_p(2) \text{ is odd,} \\ \frac{p-1}{o_p(2)} & \text{if } o_p(2) \equiv 2 \pmod{4}, \\ 0 & \text{if } 4 \mid o_p(2) \end{cases}$$

and $M(p)$ to be the size of optimal CAC of length p . Then, by [FLS14, Lemma 3] one has

$$(1) \quad \frac{p-1-2\mathcal{O}(p)}{4} \leq M(p) \leq \frac{p-1-2\mathcal{O}(p)}{4} + \left\lfloor \frac{\mathcal{O}(p)}{3} \right\rfloor.$$

Note that in the case where $\mathcal{O}(p) \leq 2$, inequality (1) already gives that $M(p) = \frac{p-1-2\mathcal{O}(p)}{4}$. For $\mathcal{O}(p) \geq 3$ the authors provide an algorithm for constructing nonequi-difference CAC and conjectured that the algorithm produces a CAC consists of $\frac{p-1-2\mathcal{O}(p)}{4}$ equi-difference codewords and $\left\lfloor \frac{\mathcal{O}(p)}{3} \right\rfloor$ nonequi-difference codewords. In other words, the upper bound in (1) can be attained and hence the CAC obtained by their algorithm is actually an optimal CAC. The key property needed for their algorithm to work is given as Conjecture A below. For our purpose, we rephrase their conjecture in terms of cosets of the subgroup generated by -1 and 2 in the multiplicative group $\mathbb{F}_p^\times = \mathbb{F}_p \setminus \{0\}$ of \mathbb{F}_p .

Conjecture A ([FLS14, Conjecture 1]). Let p be a non-Wieferich prime. Then there are $3 \left\lfloor \frac{\mathcal{O}(p)}{3} \right\rfloor$ cosets $A_1, B_1, C_1, \dots, A_{\left\lfloor \frac{\mathcal{O}(p)}{3} \right\rfloor}, B_{\left\lfloor \frac{\mathcal{O}(p)}{3} \right\rfloor}, C_{\left\lfloor \frac{\mathcal{O}(p)}{3} \right\rfloor}$ of the subgroup generated by -1 and 2 in \mathbb{F}_p^\times such that for each $i = 1, \dots, \left\lfloor \frac{\mathcal{O}(p)}{3} \right\rfloor$ there exists a triple $(a_i, b_i, c_i) \in A_i \times B_i \times C_i$ satisfying

$$a_i + b_i + c_i = 0 \quad \text{in } \mathbb{F}_p.$$