of lines 4 and 5 between these algorithms are proved as

$$\begin{cases} x'^{a_1|s_i} \leftarrow x'^{a_1|s_i} + \gamma \\ \mathbf{x}' \leftarrow \mathrm{Norm}(\mathbf{x}') \\ \Delta^{a_1|s_i} \leftarrow \dfrac{u^{\mathrm{st}}(\mathbf{x}',\mathbf{y}) - u^{\mathrm{st}}(\mathbf{x},\mathbf{y})}{\gamma} \end{cases} \tag{A30}$$

$$\Leftrightarrow \begin{cases} x'^{a_1|s_i} \leftarrow x'^{a_1|s_i} + (1 - x'^{a_1|s_i})\gamma + O(\gamma^2) \\ x'^{a_2|s_i} \leftarrow x'^{a_2|s_i} - x'^{a_2|s_i}\gamma + O(\gamma^2) \\ \Delta^{a_1|s_i} \leftarrow \dfrac{u^{\mathrm{st}}(\mathbf{x}',\mathbf{y}) - u^{\mathrm{st}}(\mathbf{x},\mathbf{y})}{\gamma} \end{cases} \tag{A31}$$

$$\Leftrightarrow \begin{cases} x'_i \leftarrow x'_i + (1 - x'_i)\gamma \\ \Delta_i \leftarrow \dfrac{u^{\mathrm{st}}(\boldsymbol{x}',\boldsymbol{y}) - u^{\mathrm{st}}(\boldsymbol{x},\boldsymbol{y})}{\gamma} \end{cases} \tag{A32}$$

$$\Leftrightarrow \begin{cases} x'_i \leftarrow x'_i + \gamma \\ \Delta_i \leftarrow (1 - x'_i)\dfrac{u^{\mathrm{st}}(\boldsymbol{x}',\boldsymbol{y}) - u^{\mathrm{st}}(\boldsymbol{x},\boldsymbol{y})}{\gamma} \end{cases}. \tag{A33}$$

Here, we ignore terms of $O(\gamma^2)$ and use the definition of $\boldsymbol{x}$ (i.e., $x'_i = x'^{a_1|s_i} = 1 - x'^{a_2|s_i}$) between Eqs. (A31) and (A32). Thus, we use the continualized version of this algorithm;

$$\dot{\boldsymbol{x}} = \boldsymbol{x} \circ (\mathbf{1} - \boldsymbol{x}) \circ \frac{\partial}{\partial \boldsymbol{x}} u^{\mathrm{st}}(\boldsymbol{x},\boldsymbol{y}). \tag{A34}$$

B.2 Approximation of learning dynamics

In **Section 4.2** and **5.1**, we introduce a method to approximate the learning dynamics up to k-th order terms for deviations from the Nash equilibrium. The stationary state condition of the one-memory two-action game is given by

$$\boldsymbol{p}^{\mathrm{st}} = \boldsymbol{M}\boldsymbol{p}^{\mathrm{st}}, \tag{A35}$$

$$\boldsymbol{M} = \begin{pmatrix} x_1y_1 & x_2y_2 & x_3y_3 & x_4y_4 \\ x_1\tilde{y}_1 & x_2\tilde{y}_2 & x_3\tilde{y}_3 & x_4\tilde{y}_4 \\ \tilde{x}_1y_1 & \tilde{x}_2y_2 & \tilde{x}_3y_3 & \tilde{x}_4y_4 \\ \tilde{x}_1\tilde{y}_1 & \tilde{x}_2\tilde{y}_2 & \tilde{x}_3\tilde{y}_3 & \tilde{x}_4\tilde{y}_4 \end{pmatrix}. \tag{A36}$$

Here, for any variable $\mathcal{X}$, we define $\tilde{\mathcal{X}} := 1 - \mathcal{X}$. In addition, let us denote $O(\delta^k)$ term in any variable $\mathcal{X}$ as $\mathcal{X}^{(k)}$. The neighbor of the Nash equilibrium, by substituting $\boldsymbol{x} = x^*\mathbf{1} + \boldsymbol{\delta}$ and $\boldsymbol{y} = y^*\mathbf{1} + \boldsymbol{\epsilon}$, we can decompose $\boldsymbol{M} = \sum_{k=0}^{2} \boldsymbol{M}^{(k)}$ as

$$\boldsymbol{M} = \underbrace{(\boldsymbol{x}^* \circ \boldsymbol{y}^*) \otimes \mathbf{1}}_{=\boldsymbol{M}^{(0)}} + \underbrace{(\boldsymbol{y}^* \circ \mathbf{1}_x) \otimes \boldsymbol{\delta} + (\boldsymbol{x}^* \circ \mathbf{1}_y) \otimes \boldsymbol{\epsilon}}_{=\boldsymbol{M}^{(1)}} + \underbrace{\mathbf{1}_z \otimes (\boldsymbol{\delta} \circ \boldsymbol{\epsilon})}_{=\boldsymbol{M}^{(2)}}, \tag{A37}$$

$$\boldsymbol{x}^* := (x^*, x^*, \tilde{x}^*, \tilde{x}^*), \qquad \boldsymbol{y}^* := (y^*, \tilde{y}^*, y^*, \tilde{y}^*), \tag{A38}$$

$$\mathbf{1}_x := (+1,+1,-1,-1)^{\mathrm{T}}, \qquad \mathbf{1}_y := (+1,-1,+1,-1)^{\mathrm{T}}, \qquad \mathbf{1}_z := \mathbf{1}_x \circ \mathbf{1}_y = (+1,-1,-1,+1)^{\mathrm{T}}. \tag{A39}$$

In the same way, we can decompose $\boldsymbol{p}^{\mathrm{st}} \simeq \sum_{k=0} \boldsymbol{p}^{\mathrm{st}(k)}$ as

$$\boldsymbol{p}^{\mathrm{st}(0)} = \boldsymbol{M}^{(0)}\boldsymbol{p}^{\mathrm{st}(0)} = \underbrace{(\boldsymbol{p}^{\mathrm{st}(0)} \cdot \mathbf{1})}_{=1} \boldsymbol{x}^* \circ \boldsymbol{y}^* =: \boldsymbol{p}^*, \tag{A40}$$

$$\boldsymbol{p}^{\mathrm{st}(1)} = \boldsymbol{M}^{(1)}\boldsymbol{p}^{\mathrm{st}(0)} = (\boldsymbol{\delta} \cdot \boldsymbol{p}^*)\boldsymbol{y}^* \circ \mathbf{1}_x + (\boldsymbol{\epsilon} \cdot \boldsymbol{p}^*)\boldsymbol{x}^* \circ \mathbf{1}_y, \tag{A41}$$

$$\boldsymbol{p}^{\mathrm{st}(2)} = \boldsymbol{M}^{(2)}\boldsymbol{p}^{\mathrm{st}(0)} + \boldsymbol{M}^{(1)}\boldsymbol{p}^{\mathrm{st}(1)} \tag{A42}$$

$$\begin{aligned} &= (\boldsymbol{\delta} \circ \boldsymbol{\epsilon} \cdot \boldsymbol{p}^*)\mathbf{1}_z + \{(\boldsymbol{\delta} \cdot \boldsymbol{p}^*)(\boldsymbol{\delta} \circ \boldsymbol{y}^* \cdot \mathbf{1}_x) + (\boldsymbol{\epsilon} \cdot \boldsymbol{p}^*)(\boldsymbol{\delta} \circ \boldsymbol{x}^* \cdot \mathbf{1}_y)\}\, \boldsymbol{y}^* \circ \mathbf{1}_x \\ &\quad + \{(\boldsymbol{\delta} \cdot \boldsymbol{p}^*)(\boldsymbol{\epsilon} \circ \boldsymbol{y}^* \cdot \mathbf{1}_x) + (\boldsymbol{\epsilon} \cdot \boldsymbol{p}^*)(\boldsymbol{\epsilon} \circ \boldsymbol{x}^* \cdot \mathbf{1}_y)\}\, \boldsymbol{x}^* \circ \mathbf{1}_y. \end{aligned} \tag{A43}$$