map. At the end (Section 5), we explain why a similar statement is not proved for the adjacency relation and invertible bounded linear or conjugate-linear operators.

2. Main result

Let H be a complex Hilbert space of dimension not less than 3. For every positive integer $k < \dim H$ we denote by $\mathcal{G}_k(H)$ and $\mathcal{G}^k(H)$ the Grassmannians formed by k-dimensional subspaces of H and closed subspaces of H whose codimension is k, respectively. Note that $\mathcal{G}^k(H) = \mathcal{G}_{n-k}(H)$ if $\dim H = n$ is finite. In the case when H is infinite-dimensional, we write $\mathcal{G}_\infty(H)$ for the Grassmannian of closed subspaces of H whose dimension and codimension both are infinite.

Let $\mathcal{G}$ be one of the Grassmannians $\mathcal{G}_k(H), \mathcal{G}^k(H)$ or $\mathcal{G}_\infty(H)$. Elements $X, Y \in \mathcal{G}$ are called *adjacent* if $X \cap Y$ is a hyperplane in both X, Y. In the case when $\mathcal{G}$ is $\mathcal{G}_1(H)$ or $\mathcal{G}^1(H)$, any two distinct elements of $\mathcal{G}$ are adjacent. Recall that two closed subspaces of H are *compatible* if there is an orthonormal basis of H such that each of these subspaces is spanned by a subset of this basis. We say that $X, Y \in \mathcal{G}$ are *ortho-adjacent* if they are adjacent and compatible.

Elements $X, Y \in \mathcal{G}$ are said to be *connected* if there is a finite sequence

$$X = X_0, X_1, \dots, X_m = Y$$

of elements of $\mathcal{G}$ such that X_{i-1}, X_i are adjacent for every $i \in \{1, \dots, m\}$. This holds if and only if

$$\dim(X/(X \cap Y)) = \dim(Y/(X \cap Y)) < \infty.$$

Consequently, any two distinct elements of $\mathcal{G}$ are connected if $\mathcal{G}$ is $\mathcal{G}_k(H)$ or $\mathcal{G}^k(H)$. If adjacent $Z, Z' \in \mathcal{G}$ are not ortho-adjacent, then there is an element of $\mathcal{G}$ ortho-adjacent to both Z, Z'. Therefore, for any connected $X, Y \in \mathcal{G}$ there is a finite sequence

$$X = X'_0, X'_1, \dots, X'_{m'} = Y$$

of elements of $\mathcal{G}$ such that X'_{i-1}, X'_i are ortho-adjacent for every $i \in \{1, \dots, m'\}$.

Let $X \in \mathcal{G}_\infty(H)$. Consider the subset of all elements of $\mathcal{G}_\infty(H)$ connected with X. Any two distinct elements of this subset are connected; furthermore, the subset is maximal with respect to this property. Every such subset will be called a *connected component* of $\mathcal{G}_\infty(H)$.

Every unitary or anti-unitary operator on H induces bijective transformations of $\mathcal{G}_k(H), \mathcal{G}^k(H)$ and $\mathcal{G}_\infty(H)$ which preserve the ortho-adjacency relation in both directions. The orthocomplementary map $X \to X^\perp$ is a bijection of $\mathcal{G}_k(H)$ to $\mathcal{G}^k(H)$ and a bijective transformation of $\mathcal{G}_\infty(H)$ preserving the ortho-adjacency in both directions.

Two 1-dimensional subspaces of H are ortho-adjacent if and only if they are orthogonal. By Uhlhorn's version of Wigner's theorem [23], every bijective transformation of $\mathcal{G}_1(H)$ preserving the orthogonality relation in both directions is induced by a unitary or anti-unitary operator. The main result of [19] states that the same holds for every bijective transformation of $\mathcal{G}_k(H)$ preserving the ortho-adjacency relation in both directions if $\dim H \neq 2k$; in the case when $\dim H = 2k \geq 6$, every such transformation is induced by a unitary or anti-unitary operator or it is the composition of the orthocomplementary map and a transformation induced by a unitary or anti-unitary operator. If $\dim H = 2k = 4$, then the latter statement fails and a descriptions of ortho-adjacency preserving transformations is an open problem, see [11, 19] for the details.