for all $i', i'' \in \{1, \ldots, |\mathcal{S}|\}$. Then, the equilibrium state $\boldsymbol{p}^{\mathrm{st}}(\mathbf{x}, \mathbf{y})$ changes into $\boldsymbol{p}^{\mathrm{st}}(\mathbf{x}, \mathbf{y}) + \mathrm{d}\boldsymbol{p}^{\mathrm{st}}(\mathbf{x}, \mathbf{y}, \mathrm{d}x^{a|s})$. Here, note that $\mathrm{d}\boldsymbol{M}$ and $\mathrm{d}\boldsymbol{p}^{\mathrm{st}}$ are a matrix and a vector of order $O(\mathrm{d}x^{a|s})$, respectively. From the stationary state condition, both $(\boldsymbol{E} - \boldsymbol{M})\boldsymbol{p}^{\mathrm{st}} = 0$ and $(\boldsymbol{E} - (\boldsymbol{M} + \mathrm{d}\boldsymbol{M}))(\boldsymbol{p}^{\mathrm{st}} + \mathrm{d}\boldsymbol{p}^{\mathrm{st}}) = 0$ hold.

$$(\boldsymbol{E} - (\boldsymbol{M} + \mathrm{d}\boldsymbol{M}))(\boldsymbol{p}^{\mathrm{st}} + \mathrm{d}\boldsymbol{p}^{\mathrm{st}}) - (\boldsymbol{E} - \boldsymbol{M})\boldsymbol{p}^{\mathrm{st}} = 0 \tag{A10}$$

$$\Leftrightarrow (\boldsymbol{E} - \boldsymbol{M})\mathrm{d}\boldsymbol{p}^{\mathrm{st}} = \mathrm{d}\boldsymbol{M}\boldsymbol{p}^{\mathrm{st}} + \mathbf{1} \times O((\mathrm{d}x^{a|s})^2). \tag{A11}$$

Here, the term of $O((\mathrm{d}x^{a|s})^2)$ is small enough to be ignored. Then, the rest term $\delta\boldsymbol{M}\boldsymbol{p}^{\mathrm{st}}$ is calculated as

$$(\mathrm{d}\boldsymbol{M}\boldsymbol{p}^{\mathrm{st}})_{i'} = \sum_{i''} \mathrm{d}M_{i'i''}p^{\mathrm{st}}_{i''} = \mathrm{d}M_{i'i}p^{\mathrm{st}}_{i} \tag{A12}$$

$$= \mathrm{d}x^{a|s_i}p^{\mathrm{st}}_{i}y^{b|s_i} \times \begin{cases} 1 - x^{a|s_i} & (s_{i'} = abs_i^-) \\ -x^{a'|s_i} & (s_{i'} = a'bs_i^-, a' \neq a) \\ 0 & (\text{otherwise}) \end{cases} . \tag{A13}$$

$$= \mathrm{d}x^{a|s_i}p^{\mathrm{st}}_{i}\left(p^{a|s_i}_{i'} - \sum_{a'} x^{a'|s_i}p^{a'|s_i}_{i'}\right). \tag{A14}$$

Thus,

$$(\boldsymbol{E} - \boldsymbol{M})\mathrm{d}\boldsymbol{p}^{\mathrm{st}} = \mathrm{d}x^{a|s_i}p^{\mathrm{st}}_{i}\left(\boldsymbol{p}^{a|s_i} - \sum_{a'} x^{a'|s_i}\boldsymbol{p}^{a'|s_i}\right) \tag{A15}$$

$$\mathrm{d}\boldsymbol{p}^{\mathrm{st}} = \mathrm{d}x^{a|s_i}p^{\mathrm{st}}_{i}(\boldsymbol{E} - \boldsymbol{M})^{-1}\left(\boldsymbol{p}^{a|s_i} - \sum_{a'} x^{a'|s_i}\boldsymbol{p}^{a'|s_i}\right) \tag{A16}$$

$$\Leftrightarrow \frac{\mathrm{d}\boldsymbol{p}^{\mathrm{st}}}{\mathrm{d}x^{a|s_i}} = \frac{\partial}{\partial x^{a|s_i}}\boldsymbol{p}^{\mathrm{st}}(\mathrm{Norm}(\mathbf{x}), \mathbf{y}) = p^{\mathrm{st}}_{s_i}(\boldsymbol{E} - \boldsymbol{M})^{-1}\left(\boldsymbol{p}^{a|s_i} - \sum_{a'} x^{a'|s_i}\boldsymbol{p}^{a'|s_i}\right) \tag{A17}$$

$$\Leftrightarrow \frac{\partial}{\partial x^{a|s_i}}\boldsymbol{p}^{\mathrm{st}}(\mathrm{Norm}(\mathbf{x}), \mathbf{y}) \cdot \boldsymbol{u} = p^{\mathrm{st}}_{i}(\boldsymbol{E} - \boldsymbol{M})^{-1}\left(\boldsymbol{p}^{a|s_i} - \sum_{a'} x^{a'|s_i}\boldsymbol{p}^{a'|s_i}\right) \cdot \boldsymbol{u} \tag{A18}$$

$$\Leftrightarrow \frac{\partial}{\partial x^{a|s_i}}u^{\mathrm{st}}(\mathrm{Norm}(\mathbf{x}), \mathbf{y}) = p^{\mathrm{st}}_{i}\left(\pi(\boldsymbol{p}^{a|s_i}, \mathbf{x}, \mathbf{y}) - \sum_{a'} x^{a'|s_i}\pi(\boldsymbol{p}^{a'|s_i}, \mathbf{x}, \mathbf{y})\right). \tag{A19}$$

The left-hand (resp. right-hand) side of Eq. (A19) corresponds to continualized MMGA (resp. MMRD). □

A.4 Proof of Theorem 4

Let us prove that X's strategy in Nash equilibrium is uniquely $\boldsymbol{x} = x^*\mathbf{1}$. First, we define u^* and v^*;

$$u^* = x^*y^*u_1 + x^*(1-y^*)u_2 + (1-x^*)y^*u_3 + (1-x^*)(1-y^*)u_4 \tag{A20}$$

$$= \frac{u_1u_4 - u_2u_3}{u_1 - u_2 - u_3 + u_4} \tag{A21}$$

$$(= -v^*) \tag{A22}$$

as X's and Y's payoffs in the Nash equilibrium in the zero-memory game. If X uses the Nash equilibrium strategy $\boldsymbol{x} = x^*\mathbf{1}$, the stationary state condition $\boldsymbol{p}^{\mathrm{st}} = \boldsymbol{M}\boldsymbol{p}^{\mathrm{st}}$ satisfies

$$\boldsymbol{p}^{\mathrm{st}} = \begin{pmatrix} x^*y_1 & x^*y_2 & x^*y_3 & x^*y_4 \\ x^*\tilde{y}_1 & x^*\tilde{y}_2 & x^*\tilde{y}_3 & x^*\tilde{y}_4 \\ \tilde{x}^*y_1 & \tilde{x}^*y_2 & \tilde{x}^*y_3 & \tilde{x}^*y_4 \\ \tilde{x}^*\tilde{y}_1 & \tilde{x}^*\tilde{y}_2 & \tilde{x}^*\tilde{y}_3 & \tilde{x}^*\tilde{y}_4 \end{pmatrix}\boldsymbol{p}^{\mathrm{st}} \tag{A23}$$

$$\Rightarrow \boldsymbol{p}^{\mathrm{st}} = (x^*\boldsymbol{p}^{\mathrm{st}} \cdot \boldsymbol{y}, x^*(1 - \boldsymbol{p}^{\mathrm{st}} \cdot \boldsymbol{y}), \tilde{x}^*\boldsymbol{p}^{\mathrm{st}} \cdot \boldsymbol{y}, \tilde{x}^*(1 - \boldsymbol{p}^{\mathrm{st}} \cdot \boldsymbol{y}))^{\mathrm{T}} \tag{A24}$$

$$\Rightarrow \boldsymbol{p}^{\mathrm{st}} \cdot \boldsymbol{u} = u^* \tag{A25}$$

$$\Leftrightarrow u^{\mathrm{st}} = u^*. \tag{A26}$$