context of the above class $\mathcal{F}(\psi)$,

$$\psi_i(z) = \begin{cases} \gamma z(1+\eta z)^{-2} & , i=1 \\ z(1-\alpha z^2)^{-1} & , i=2 \\ z(1-z)^{-1}(1+\beta z)^{-1} & , i=3 \\ (A-B)^{-1}\log((1+Az)/(1+Bz)) & , i=4, \end{cases} \tag{1.1}$$

where $A = \alpha e^{i\tau}, B = \alpha e^{-i\tau}$ with $\tau \in (0, \pi/2]$, and $\alpha, \beta, \eta \in (0,1], \gamma > 0$ (see [12, 15, 16, 19]). Kumar and Gangania in [15], introduced the class $\mathcal{S}_\gamma(\eta) = \mathcal{F}(\psi_1)$ and obtained the radius of starlikeness. Cho et al. [2] dealt with certain sharp radius problems for the class $\mathcal{BS}(\alpha) = \mathcal{F}(\psi_2)$. Infact Masih et al. [19] studied the class $\mathcal{S}_{cs}(\beta) = \mathcal{F}(\psi_3)$, where $0 \le \beta < 1$, discussed the growth theorem and established sharp estimates of logarithmic coefficients for $0 \le \beta \le 1/2$. Further for $1/2 < \beta \le 1$, the class $\mathcal{S}_{cs}(\beta)$ contains non-univalent functions, infact for $0 \le \beta \le 1/2$ the class $\mathcal{S}_{cs}(\beta) \subset \mathcal{S}^*$. In 2022, Kumar et al. [16] introduced the class $\mathcal{F}(A,B) = \mathcal{F}(\psi_4)$ and established some radii results.

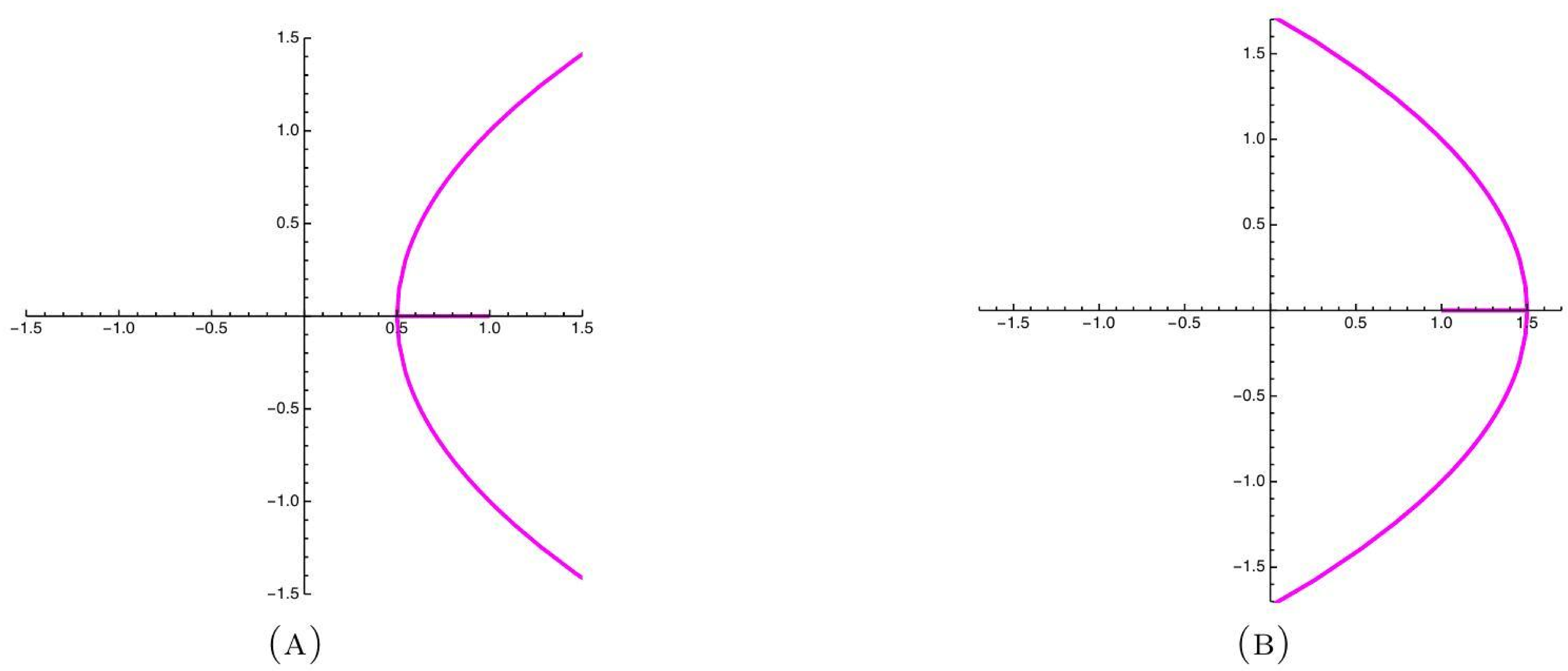

FIGURE 1. Graphs representing different parabolas with a common focus (1,0) (a) $1+\mathcal{P}_{0,\pi}(\partial\mathbb{D})$, (b) $1+\mathcal{P}_{0,0}(\partial\mathbb{D})$.

Motivated essentially by the above classes and observations, we now study a subclass of $\mathcal{A}$ containing non-univalent functions. For $\tau, \theta \in (-\pi, \pi]$, the transformation

$$\omega = ((2e^{i\theta/2}\sqrt{2}/\pi)(\tan^{-1}(e^{i(\tau-\pi/2)}\sqrt{z})))^2$$

maps the boundary of $\mathbb{D}$ onto a parabola (see **Fig.** 1), given by

$$\begin{aligned} \mathcal{P}_{\tau,\theta}(z) &:= \frac{2e^{i(\theta+\pi)}}{\pi^2}\left(\log\left(\frac{1+e^{i\tau}\sqrt{z}}{1-e^{i\tau}\sqrt{z}}\right)\right)^2 \\ &= \frac{8e^{i(\theta+\pi)}}{\pi^2}\sum_{n=1}^{\infty}\left(\frac{e^{2i\tau n}}{n}\sum_{k=0}^{n-1}\frac{1}{2k+1}\right)z^n, \end{aligned}$$

where the branch of $\sqrt{z}$ is chosen so that $\operatorname{Im}\sqrt{z} \ge 0$. Note that the function $\mathcal{P}_{\tau,\theta}(z)$ is univalent in $\mathbb{D}$ and $1+\mathcal{P}_{0,\pi}(z)$ (see **Fig.** 1(a)) is the parabolic function introduced independently by Ma-Minda [18] and Ronning [26], and used it extensively in context of parabolic starlike functions (see [10, 26]). Further, Kanas [11] discussed some differential subordination techniques and later studied admissibility results [9] involving $1+\mathcal{P}_{0,\pi}(z)$. Note that the boundary of the function $\mathcal{P}_{0,0}(z)$ is a horizontal parabola with an opening in the left-half plane (see **Fig.** 1(b)). In the present study, we consider the function $1+\mathcal{P}_{0,0}(z)$, in context of non-univalent functions, which is completely different from the way Ronning, Ma-Minda and Kanas (see [9, 11, 18, 26]) handled parabolic regions. Obviously, the other choices of τ and θ leads to oblique parabolic