

We obtain the following final decomposition of the EUR PnL into four parts:

$$\begin{aligned}
P_{(t,T]}^{(A)} &= \frac{A_t + A_T}{2} (\chi_T - \chi_t) & (= P_{(t,T]}^{(A)}(\chi)) \\
&+ \frac{\chi_t + \chi_T}{2} \left( \frac{A_T(r_T, x_T) - A_T(r_t, x_T)}{2} + \frac{A_t(r_T, x_t) - A_t(r_t, x_t)}{2} \right) & (= P_{(t,T]}^{(A)}(r)) \\
&+ \frac{\chi_t + \chi_T}{2} \left( \frac{A_T(r_T, x_T) - A_T(r_T, x_t)}{2} + \frac{A_t(r_t, x_T) - A_t(r_t, x_t)}{2} \right) & (= P_{(t,T]}^{(A)}(x)) \\
&+ \frac{\chi_t + \chi_T}{2} \left( \frac{A_T(r_T, x_t) - A_t(r_T, x_t)}{2} + \frac{A_T(r_t, x_T) - A_t(r_t, x_T)}{2} \right). & (= P_{(t,T]}^{(A)}(\text{carry}))
\end{aligned}$$

The interpretation of  $P_{(t,T]}^{(A)}(r)$  is the following: at an arbitrary time point  $u$  the difference  $A_u(r_T, x_u) - A_u(r_t, x_u)$  measures the PnL of the market price at that time that would be induced by a discounting curve change from  $r_t$  to  $r_T$ . The PnL  $P_{(t,T]}^{(A)}(r)$  is defined as the arithmetic mean of this difference for the two time points  $u = t$  and  $u = T$ . The precisely same logic applies to the interpretation of  $P_{(t,T]}^{(A)}(x)$ , only with  $r$  replaced by  $x$ . Finally, the PnL  $P_{(t,T]}^{(A)}(\text{carry})$  intuitively should measure the change between the asset values  $A_t$  and  $A_T$  that is only due to time passing, without the effects of  $r$  and  $x$ . Since the variables  $r$  and  $x$  change their values within the period  $(t, T]$ , one reasonable approach is to use an “average” of the variables  $r, x$  on the period  $(t, T]$ . Since we only have  $r, x$  available at the two time points  $t$  and  $T$ , a pragmatic idea to accomplish such average is to mix the possible pairs  $(r_u, x_s)$  for  $u, s \in \{t, T\}$  in a way that is as “neutral” as possible. This is precisely what’s done in the definition of  $P_{(t,T]}^{(A)}(\text{carry})$ .

**Remark 2.1 (On the approximative nature of our definitions)**

Similar as in Remark 1.1, we point out that our definition of  $P_{(t,T]}^{(A)}(r)$  and  $P_{(t,T]}^{(A)}(x)$  in terms of an arithmetic average of start and end time point values is only a proxy to reality. Clearly, an average that would take into account all time points  $u \in (t, T]$  would be more desirable from a theoretical perspective. For instance, based on the multivariate Itô formula, under the assumption that  $\{r_u\}_{u \in (t,T]}$  and  $\{x_u\}_{u \in (t,T]}$  are realizations of semi-martingales, take values in  $\mathbb{R}$  (i.e. are not function-valued), and under the assumption that  $r$  and  $x$  are independent, we obtain the decomposition

$$\begin{aligned}
A_T - A_t &= \left( \int_{(t,T]} \frac{\partial}{\partial t} A_u(r_u, x_u) du \right) \\
&+ \left( \int_{(t,T]} \frac{\partial}{\partial r} A_s(r_u, x_u) dr_u + \frac{1}{2} \int_{(t,T]} \frac{\partial^2}{\partial r^2} A_s(r_u, x_u) d[r, r]_u \right) \\
&+ \left( \int_{(t,T]} \frac{\partial}{\partial x} A_s(r_u, x_u) dx_u + \frac{1}{2} \int_{(t,T]} \frac{\partial^2}{\partial x^2} A_s(r_u, x_u) d[x, x]_u \right),
\end{aligned}$$

and the three terms in  $(.)$ -brackets could be interpreted as performance due to carry, changes in  $r$ , and changes in  $x$ , respectively. While already this formula is difficult to implement in practice, we point out that typically  $r_u$  is a function (an interest rate term structure), so that a generalization in this regard requires significantly more advanced