of $e-$folds given by

$$N_e = \frac{1}{3}\,\ln\!\left(\frac{v(t_f)}{v_i}\right) \approx \frac{t_f}{v_i^{1/3}}. \quad (7)$$

In the next subsection we report the mechanism to realize the EU scenario.

B. The Emergent Potential

A simple way to create a past-infinite exponential expansion and end it at a finite time t_f is to use a scalar field. Therefore we can add the scalar field term to the Hamiltonian (1):

$$\mathcal{H}(v, p_v, \phi, p_\phi) = \mathcal{H}_g(v, p_v) + \rho_\phi(v, \phi, p_\phi)\, v = 0, \quad (8)$$

$$\rho_\phi(v, \phi, p_\phi) = \frac{p_\phi^2}{2v^2} + U(\phi), \quad (9)$$

where $U(\phi)$ is a potential and $p_\phi = \dot{\phi}\, v$ is the momentum conjugate to the scalar field. From the equations of motion we find that the scalar field obeys a Klein-Gordon-like equation:

$$\ddot{\phi} + \frac{\dot{v}}{v}\,\dot{\phi} + \frac{\partial U}{\partial \phi} = 0. \quad (10)$$

The ideal potential for an EU model has a plateau (i.e. an asymptote) at $\phi \to -\infty$ and a well with an absolute minimum at $\phi = \phi_f$; it takes the form

$$U(\phi) = U_f + (U_i - U_f)\left[\exp\!\left(\frac{\phi - \phi_f}{\mathcal{E}}\right) - 1\right]^2, \quad (11)$$

where U_i is the asymptotic value in the infinite past, U_f is the minimum value and $\mathcal{E}$ is a constant energy scale parametrizing the width of the well. The form of the potential is shown in Figure 2 for generic values of the parameters.

For $t \to -\infty$ we have $\phi \to -\infty$ and the field is in the plateau of the potential; this implies $\dot{\phi}^2 \ll U$ and therefore the energy density $\rho_\phi \approx U_i$ is nearly constant, playing the role of the Cosmological Constant ρ_Λ of the previous subsection. When we approach $t = t_f$ the field falls into the well until it reaches the minimum $U_f \ll U_i$, and the exponential expansion ends. (Note that it is possible to set $U_f \neq 0$ to represent the late-time cosmological constant [16], but this is beyond the scope of this study.)

As mentioned before, the infinite time of inflation produces a finite amount of expansion; provided that v_i and t_f are respectively chosen small and large enough, a very large amount of $e-$folds can be produced as follows from eq (7), thus solving all the paradoxes of Friedmann evolution in a similar manner to the standard inflationary theory [1]. However, analogously to the latter, this model is also subject to some form of fine-tuning and criticisms.

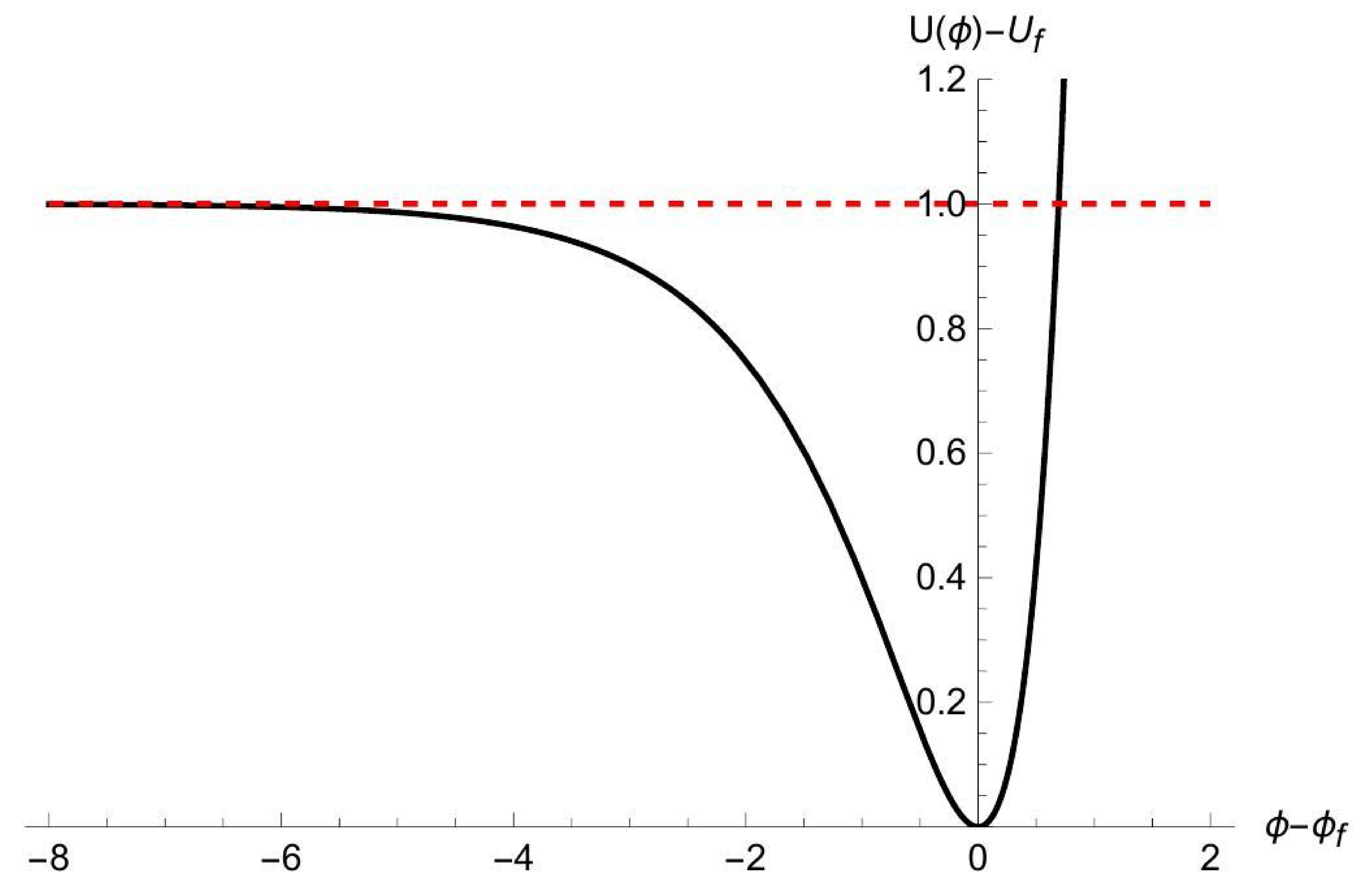

Figure 2. The potential $U(\phi) - U_f$ as function of $\phi - \phi_f$ for $U_i - U_f = 1$, $\mathcal{E} = 1$. The asymptote is highlighted with a red dashed line.

C. Fine-Tuning

As mentioned above, some fine-tuning is needed in the EU to reproduce observational parameters, such as density perturbations of the order $\mathcal{O}\big(10^{-5}\big)$ and a late-time Cosmological Constant $\Omega_\Lambda \approx 0.7$; however, all inflationary universe models need some amount of fine-tuning. The specific geometrical fine-tuning problem in the EU models is the requirement of a particular choice of the initial volume v_i and of the primordial cosmological constant ρ_Λ or U_i. This choice must then be supplemented by a further fine-tuning: a choice of initial kinetic energy such that the inequality $\dot{\phi}^2 \ll U_i$ holds. Both conditions are required to attain an asymptotically Einstein-static state.

The authors of [16, 17] acknowledge the necessity of fine-tuning in this model, but claim that the situation is not too different from any other inflationary model. Besides, they argue that the advantages of having a non-singular, highly symmetric initial state overcome the troubles of fine-tuning. However, the scope of this work is to provide a mechanism to generate an EU model with the minimum fine tuning necessary.

III. EMERGENT UNIVERSE FROM A MODIFIED ALGEBRA

In this section we present a modified Heisenberg algebra, that in the classical limit translates to modified Poisson brackets, which is able to yield an EU-like solution without the need of much fine-tuning.

The modified algebra, inspired by quantum gravity and quantum cosmological theories such as PQM [13, 25] and the Generalised Uncertainty Principle (GUP) representation [22, 26–28], takes the form

$$[\hat{q}, \hat{p}] = i\,\left(1 - \mu^2 \hat{p}^2\right), \quad (12)$$