the asymptotic behaviour:

$$\mathcal{P}^{\rm mod}(k) = \frac{k^3}{4\pi^2}\frac{f^2(\eta)}{2a^2\epsilon}\left(1-2\sqrt{2}\,\mu^2\,{\rm Re}(d_2 e^{-2i\varphi})\right)\Big|_{-k\eta\ll 1} = \\ = \frac{H_s^2}{8\pi^2\epsilon}\left(1-\frac{4\mu^2}{7k^5\eta^6}\right)\Big|_{-k\eta\ll 1}. \tag{74}$$

Now, if we performed the limit $-k\eta \to 0$ our correction would diverge; however, inflation doesn't actually go on forever but ends at some finite instant; therefore we choose to compute the Spectrum at the value $\eta = \eta_f$ that is the end of inflation. At this point we need to reintroduce some constants, until now set to one, so that our correction is dimensionless: μ^2 has the dimensions of the inverse of an energy, so we define $E_\mu = 1/\mu^2$ as the deformation energy scale; then we introduce the Planck energy $m_P c^2 = c\,\hbar/\ell_P$ where ℓ_P is the Planck length, and we set $c\,\eta_f = 2\pi/\overline{k}$, where $\overline{k}$ is a pivot scale. If we choose the standard pivotal scale $\overline{k} = 0.002 Mpc^{-1}$ used in the analysis of the CMB spectra, the spectrum can then be rewritten as

$$\mathcal{P}^{\rm mod}(k) = \frac{H_s^2}{8\pi^2\epsilon}\left(1-\frac{4}{7}\frac{\ell_P\overline{k}}{(2\pi)^6}\left(\frac{m_P c^2}{E_\mu}\right)\left(\frac{\overline{k}}{k}\right)^5\right) = \\ \approx \mathcal{P}^{\rm std}\left(1-10^{-65}\left(\frac{m_P c^2}{E_\mu}\right)\left(\frac{\overline{k}}{k}\right)^5\right); \tag{75}$$

then, by asking that at the pivot scale $k = \overline{k}$ corrections be of order lower than 10^{-3}, we obtain a constraint on the deformation energy scale:

$$E_\mu \gtrsim 10^{-62} m_P c^2. \tag{76}$$

In Figure 6 we see the modified Power Spectrum (rescaled to the standard one) for different values of the ratio $r = m_P c^2/E_\mu$; the bigger deviation would happen for small values of k corresponding to large scales.

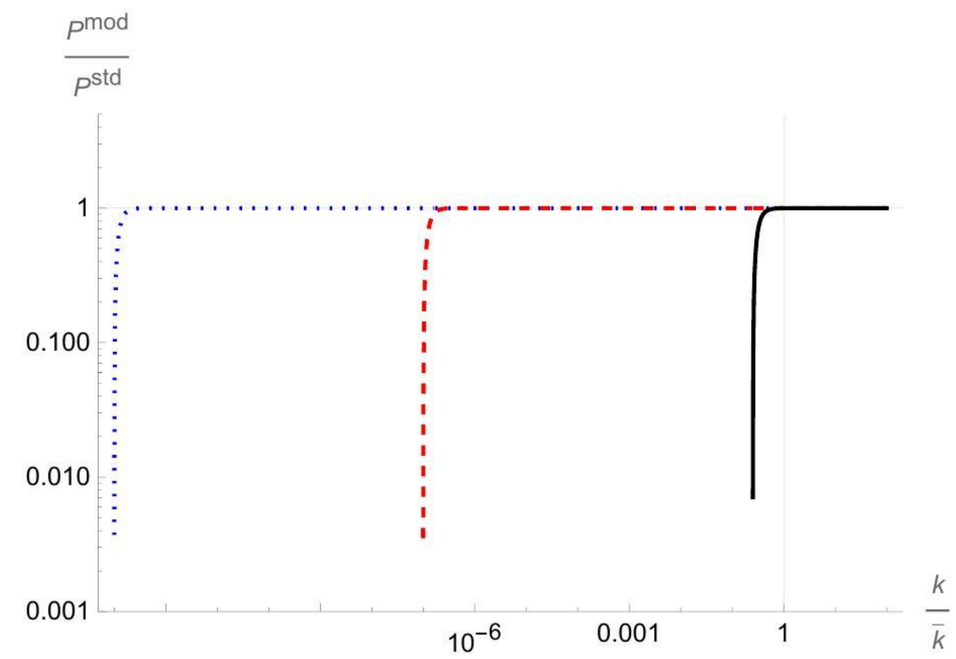

Figure 6. The modified Power Spectrum $\mathcal{P}^{\rm mod}$ rescaled to the standard one $\mathcal{P}^{\rm std}$ for $r = 10^{62}$ (black continuous line), $r = 10^{30}$ (red dashed line) and $r = 1$ (blue dotted line). The pivotal scale $k = \overline{k}$ and the standard flat spectrum are indicated by faded grey lines.

V. CONCLUDING REMARKS

We started from the so-called Emergent Universe, i.e. a non-singular standard cosmology with positive curvature on which a Cauchy problem is assigned which balances the matter contribution with the spatial curvature of the model. As a result, the initial phases of the cosmological dynamics are characterized by a a non-zero space volume, approached for the synchronous time going to negative infinity.

The main point in the analysis above was the possibility to construct a non-singular dynamics similar to that of an EU model by implementing a modified Uncertainty Principle. The restated symplectic algebra provided in the quasi-classical limit is, de facto, inspired from Polymer Quantum Mechanics when the basic commutation relation is expanded for a small lattice step. We have shown in detail how the picture of an EU properly arises from the implementation of our dynamical scheme to the positively-curved isotropic Universe. The relevance of our restated cosmological dynamics consisted in the possibility to have a finite volume limit in the distant past of our Universe without any fine-tuning of the initial conditions, but as a natural and general feature of the modified symplectic algebra, phenomenologically similar to a modified gravity approach. We discussed in detail the different Universe phases in the proposed scheme, with particular emphasis on the possibility to have an inflationary de Sitter period that is well reconnected to the subsequent radiation dominated era, where most part of the actual Universe morphology is determined via Baryogenesis, Nucleosynthesis, and structure formation [2].

A relevant part of the proposed study concerned the implementation of the modified Uncertainty Principle to the pure quantum dynamics of the inflaton field. We constructed the Hamiltonian for each Fourier mode of the quantum scalar field, which corresponded to that of a time-dependent harmonic oscillator (as in the standard spectrum case) plus a small perturbation controlled by the value of the cut-off parameter. We then performed a suitable perturbation theory procedure to calculate the modified expectation value of the squared Fourier harmonics of the Mukhanov-Sasaki variable constructed from the inflaton field, hence computing the corrections to the primordial perturbation Spectrum. Finally, we carefully analyzed the constraints and the proper initial conditions we have to impose on our model in order for the correction to the standard spectrum to live in an observational window for future experiments on the microwave background temperature distribution [2].

The present model has the merit to make the non-singular EU model a general feature of the isotropic Universe when a specific sector of cut-off physics is addressed. Furthermore, such a non purely classical feature of the Universe dynamics is expected to leave a specific trace on