

We also get $\mathbf{p}^{\text{st}(k)} = \mathbf{M}^{(2)}\mathbf{p}^{\text{st}(k-2)} + \mathbf{M}^{(1)}\mathbf{p}^{\text{st}(k-1)}$ for further orders of $k \geq 2$. Then, the equilibrium payoff is given by

$$u^{\text{st}(0)} = \mathbf{p}^{\text{st}(0)} \cdot \mathbf{u} = u^{\text{st}}, \quad (\text{A44})$$

$$u^{\text{st}(1)} = \mathbf{p}^{\text{st}(1)} \cdot \mathbf{u} = 0, \quad (\text{A45})$$

$$u^{\text{st}(2)} = \mathbf{p}^{\text{st}(2)} \cdot \mathbf{u} = (\mathbf{u} \cdot \mathbf{1}_z)(\boldsymbol{\delta} \circ \boldsymbol{\epsilon} \cdot \mathbf{p}^*), \quad (\text{A46})$$

$$u^{\text{st}(3)} = \mathbf{p}^{\text{st}(3)} \cdot \mathbf{u} = (\mathbf{M}^{(2)}\mathbf{p}^{\text{st}(1)} + \mathbf{M}^{(1)}\mathbf{p}^{\text{st}(2)}) \cdot \mathbf{u} = (\mathbf{u} \cdot \mathbf{1}_z)(\boldsymbol{\delta} \circ \boldsymbol{\epsilon} \cdot \mathbf{p}^{\text{st}(1)}) \quad (\text{A47})$$

$$= (\mathbf{u} \cdot \mathbf{1}_z) \{ (\boldsymbol{\delta} \cdot \mathbf{p}^*)(\boldsymbol{\delta} \circ \boldsymbol{\epsilon} \circ \mathbf{y}^* \cdot \mathbf{1}_x) + (\boldsymbol{\epsilon} \cdot \mathbf{p}^*)(\boldsymbol{\delta} \circ \boldsymbol{\epsilon} \circ \mathbf{x}^* \cdot \mathbf{1}_y) \}, \quad (\text{A48})$$

$$u^{\text{st}(4)} = \mathbf{p}^{\text{st}(4)} \cdot \mathbf{u} = (\mathbf{M}^{(2)}\mathbf{p}^{\text{st}(2)} + \mathbf{M}^{(1)}\mathbf{p}^{\text{st}(3)}) \cdot \mathbf{u} = (\mathbf{u} \cdot \mathbf{1}_z)(\boldsymbol{\delta} \circ \boldsymbol{\epsilon} \cdot \mathbf{p}^{\text{st}(2)}) \quad (\text{A49})$$

$$\begin{aligned} &= (\mathbf{u} \cdot \mathbf{1}_z) [(\boldsymbol{\delta} \circ \boldsymbol{\epsilon} \cdot \mathbf{p}^*)(\boldsymbol{\delta} \circ \boldsymbol{\epsilon} \cdot \mathbf{1}_z) \\ &\quad + \{ (\boldsymbol{\delta} \cdot \mathbf{p}^*)(\boldsymbol{\delta} \circ \mathbf{y}^* \cdot \mathbf{1}_x) + (\boldsymbol{\epsilon} \cdot \mathbf{p}^*)(\boldsymbol{\delta} \circ \mathbf{x}^* \cdot \mathbf{1}_y) \} (\boldsymbol{\delta} \circ \boldsymbol{\epsilon} \circ \mathbf{y}^* \cdot \mathbf{1}_x) \\ &\quad + \{ (\boldsymbol{\delta} \cdot \mathbf{p}^*)(\boldsymbol{\epsilon} \circ \mathbf{y}^* \cdot \mathbf{1}_x) + (\boldsymbol{\epsilon} \cdot \mathbf{p}^*)(\boldsymbol{\epsilon} \circ \mathbf{x}^* \cdot \mathbf{1}_y) \} (\boldsymbol{\delta} \circ \boldsymbol{\epsilon} \circ \mathbf{x}^* \cdot \mathbf{1}_y)]. \end{aligned} \quad (\text{A50})$$

Here, we used

$$\mathbf{M}^{(1)\text{T}}\mathbf{u} = \underbrace{(\mathbf{y}^* \circ \mathbf{1}_x \cdot \mathbf{u})}_{=0} \boldsymbol{\delta} + \underbrace{(\mathbf{x}^* \circ \mathbf{1}_y \cdot \mathbf{u})}_{=0} \boldsymbol{\epsilon} = \mathbf{0}, \quad (\text{A51})$$

$$\mathbf{M}^{(2)\text{T}}\mathbf{u} = (\mathbf{u} \cdot \mathbf{1}_z) \boldsymbol{\delta} \circ \boldsymbol{\epsilon}. \quad (\text{A52})$$

Then, the gradient of this payoff is given by

$$\frac{\partial u^{\text{st}(1)}}{\partial \boldsymbol{\delta}} = \mathbf{0}, \quad (\text{A53})$$

$$\frac{\partial u^{\text{st}(2)}}{\partial \boldsymbol{\delta}} = (\mathbf{u} \cdot \mathbf{1}_z) \boldsymbol{\epsilon} \circ \mathbf{p}^*, \quad (\text{A54})$$

$$\frac{\partial u^{\text{st}(3)}}{\partial \boldsymbol{\delta}} = (\mathbf{u} \cdot \mathbf{1}_z) \{ (\boldsymbol{\delta} \cdot \mathbf{p}^*) \boldsymbol{\epsilon} \circ \mathbf{y}^* \circ \mathbf{1}_x + (\boldsymbol{\epsilon} \cdot \mathbf{p}^*) \boldsymbol{\epsilon} \circ \mathbf{x}^* \circ \mathbf{1}_y + (\boldsymbol{\delta} \circ \boldsymbol{\epsilon} \circ \mathbf{y}^* \cdot \mathbf{1}_x) \mathbf{p}^* \}, \quad (\text{A55})$$

$$\begin{aligned} \frac{\partial u^{\text{st}(4)}}{\partial \boldsymbol{\delta}} &= (\mathbf{u} \cdot \mathbf{1}_z) [(\boldsymbol{\delta} \circ \boldsymbol{\epsilon} \cdot \mathbf{p}^*)(\boldsymbol{\epsilon} \circ \mathbf{1}_z) + (\boldsymbol{\delta} \circ \boldsymbol{\epsilon} \cdot \mathbf{1}_z)(\boldsymbol{\epsilon} \circ \mathbf{p}^*) \\ &\quad + \{ (\boldsymbol{\delta} \cdot \mathbf{p}^*)(\boldsymbol{\delta} \circ \mathbf{y}^* \cdot \mathbf{1}_x) + (\boldsymbol{\epsilon} \cdot \mathbf{p}^*)(\boldsymbol{\delta} \circ \mathbf{x}^* \cdot \mathbf{1}_y) \} (\boldsymbol{\epsilon} \circ \mathbf{y}^* \circ \mathbf{1}_x) \\ &\quad + \{ (\boldsymbol{\delta} \cdot \mathbf{p}^*)(\boldsymbol{\epsilon} \circ \mathbf{y}^* \cdot \mathbf{1}_x) + (\boldsymbol{\epsilon} \cdot \mathbf{p}^*)(\boldsymbol{\epsilon} \circ \mathbf{x}^* \cdot \mathbf{1}_y) \} (\boldsymbol{\epsilon} \circ \mathbf{x}^* \circ \mathbf{1}_y) \\ &\quad + (\boldsymbol{\delta} \circ \boldsymbol{\epsilon} \circ \mathbf{y}^* \cdot \mathbf{1}_x) \{ (\boldsymbol{\delta} \cdot \mathbf{p}^*) \mathbf{y}^* \circ \mathbf{1}_x + (\boldsymbol{\epsilon} \cdot \mathbf{p}^*) \mathbf{x}^* \circ \mathbf{1}_y + (\boldsymbol{\delta} \circ \mathbf{y}^* \cdot \mathbf{1}_x) \mathbf{p}^* \} \\ &\quad + (\boldsymbol{\delta} \circ \boldsymbol{\epsilon} \circ \mathbf{x}^* \cdot \mathbf{1}_y) (\boldsymbol{\epsilon} \circ \mathbf{y}^* \cdot \mathbf{1}_x) \mathbf{p}^*]. \end{aligned} \quad (\text{A56})$$

The learning dynamics (of continualized MMGA) in two-action one-memory games are given by

$$\dot{\boldsymbol{\delta}} = + (\mathbf{x}^* \mathbf{1} + \boldsymbol{\delta}) \circ (\tilde{\mathbf{x}}^* \mathbf{1} - \boldsymbol{\delta}) \circ \frac{\partial u^{\text{st}}}{\partial \boldsymbol{\delta}}, \quad (\text{A57})$$

$$\dot{\boldsymbol{\epsilon}} = - (\mathbf{y}^* \mathbf{1} + \boldsymbol{\epsilon}) \circ (\tilde{\mathbf{y}}^* \mathbf{1} - \boldsymbol{\epsilon}) \circ \frac{\partial u^{\text{st}}}{\partial \boldsymbol{\epsilon}}. \quad (\text{A58})$$