We can decompose $\dot{\boldsymbol{\delta}} \simeq \sum_{k=0} \dot{\boldsymbol{\delta}}^{(k)}$ and $\dot{\boldsymbol{\epsilon}} \simeq \sum_{k=0} \dot{\boldsymbol{\epsilon}}^{(k)}$ as

$$\dot{\boldsymbol{\delta}}^{(0)} = +x^* \tilde{x}^* \frac{\partial u^{\mathrm{st}(1)}}{\partial \boldsymbol{\delta}}, \tag{A59}$$

$$\dot{\boldsymbol{\delta}}^{(1)} = +x^* \tilde{x}^* \frac{\partial u^{\mathrm{st}(2)}}{\partial \boldsymbol{\delta}} - (x^* - \tilde{x}^*)\boldsymbol{\delta} \circ \frac{\partial u^{\mathrm{st}(1)}}{\partial \boldsymbol{\delta}}, \tag{A60}$$

$$\dot{\boldsymbol{\delta}}^{(2)} = +x^* \tilde{x}^* \frac{\partial u^{\mathrm{st}(3)}}{\partial \boldsymbol{\delta}} - (x^* - \tilde{x}^*)\boldsymbol{\delta} \circ \frac{\partial u^{\mathrm{st}(2)}}{\partial \boldsymbol{\delta}} + \boldsymbol{\delta} \circ \boldsymbol{\delta} \circ \frac{\partial u^{\mathrm{st}(1)}}{\partial \boldsymbol{\delta}}, \tag{A61}$$

$$\dot{\boldsymbol{\delta}}^{(3)} = +x^* \tilde{x}^* \frac{\partial u^{\mathrm{st}(4)}}{\partial \boldsymbol{\delta}} - (x^* - \tilde{x}^*)\boldsymbol{\delta} \circ \frac{\partial u^{\mathrm{st}(3)}}{\partial \boldsymbol{\delta}} + \boldsymbol{\delta} \circ \boldsymbol{\delta} \circ \frac{\partial u^{\mathrm{st}(2)}}{\partial \boldsymbol{\delta}}, \tag{A62}$$

$$\dot{\boldsymbol{\epsilon}}^{(0)} = -y^* \tilde{y}^* \frac{\partial u^{\mathrm{st}(1)}}{\partial \boldsymbol{\epsilon}}, \tag{A63}$$

$$\dot{\boldsymbol{\epsilon}}^{(1)} = -y^* \tilde{y}^* \frac{\partial u^{\mathrm{st}(2)}}{\partial \boldsymbol{\epsilon}} + (y^* - \tilde{y}^*)\boldsymbol{\epsilon} \circ \frac{\partial u^{\mathrm{st}(1)}}{\partial \boldsymbol{\epsilon}}, \tag{A64}$$

$$\dot{\boldsymbol{\epsilon}}^{(2)} = -y^* \tilde{y}^* \frac{\partial u^{\mathrm{st}(3)}}{\partial \boldsymbol{\epsilon}} + (y^* - \tilde{y}^*)\boldsymbol{\epsilon} \circ \frac{\partial u^{\mathrm{st}(2)}}{\partial \boldsymbol{\epsilon}} - \boldsymbol{\epsilon} \circ \boldsymbol{\epsilon} \circ \frac{\partial u^{\mathrm{st}(1)}}{\partial \boldsymbol{\epsilon}}, \tag{A65}$$

$$\dot{\boldsymbol{\epsilon}}^{(3)} = -y^* \tilde{y}^* \frac{\partial u^{\mathrm{st}(4)}}{\partial \boldsymbol{\epsilon}} + (y^* - \tilde{y}^*)\boldsymbol{\epsilon} \circ \frac{\partial u^{\mathrm{st}(3)}}{\partial \boldsymbol{\epsilon}} - \boldsymbol{\epsilon} \circ \boldsymbol{\epsilon} \circ \frac{\partial u^{\mathrm{st}(2)}}{\partial \boldsymbol{\epsilon}}. \tag{A66}$$

In cases of one-memory penny-matching games, the solution is obtained if we substitute

$$\boldsymbol{x}^* = \frac{1}{2}\mathbf{1}, \quad \boldsymbol{y}^* = \frac{1}{2}\mathbf{1}, \quad \boldsymbol{p}^* = \frac{1}{4}\mathbf{1}. \tag{A67}$$

B.3 Method to Calculate the Stationary State

Regarding **Section 5.1**, we use an analytical solution of the stationary state, which is known only in the case of two-action one-memory games as

$$p_1^{\mathrm{st}} = k\{(x_4 + (x_3 - x_4)y_3)(y_4 + (y_2 - y_4)x_2) - x_3 y_2 (x_2 - x_4)(y_3 - y_4)\}, \tag{A68}$$

$$p_2^{\mathrm{st}} = k\{(x_4 + (x_3 - x_4)y_4)(\tilde{y}_3 - (y_1 - y_3)x_1) - x_4 \tilde{y}_1 (x_1 - x_3)(y_3 - y_4)\}, \tag{A69}$$

$$p_3^{\mathrm{st}} = k\{(\tilde{x}_2 - (x_1 - x_2)y_1)(y_4 + (y_2 - y_4)x_4) - \tilde{x}_1 y_4 (x_2 - x_4)(y_1 - y_2)\}, \tag{A70}$$

$$p_4^{\mathrm{st}} = k\{(\tilde{x}_2 - (x_1 - x_2)y_2)(\tilde{y}_3 - (y_1 - y_3)x_3) - \tilde{x}_2 \tilde{y}_3 (x_1 - x_3)(y_1 - y_2)\}, \tag{A71}$$

$$k = \frac{1}{p_1^{\mathrm{st}} + p_2^{\mathrm{st}} + p_3^{\mathrm{st}} + p_4^{\mathrm{st}}}, \tag{A72}$$

under the notation in Assumption 1.

In other parts (**Sections 5.2** and **5.3**), we calculate the stationary state of a Markov transition matrix $\boldsymbol{M}$ by the power iteration method. Compared to the analytical solution $\boldsymbol{p}^{\mathrm{st}}$, the computational solution $\hat{\boldsymbol{p}}^{\mathrm{st}}$ is accurate except for 10^{-9} error in L^2 norm (i.e., $\|\hat{\boldsymbol{p}}^{\mathrm{st}} - \boldsymbol{p}^{\mathrm{st}}\|_2 \leq 10^{-9}$).