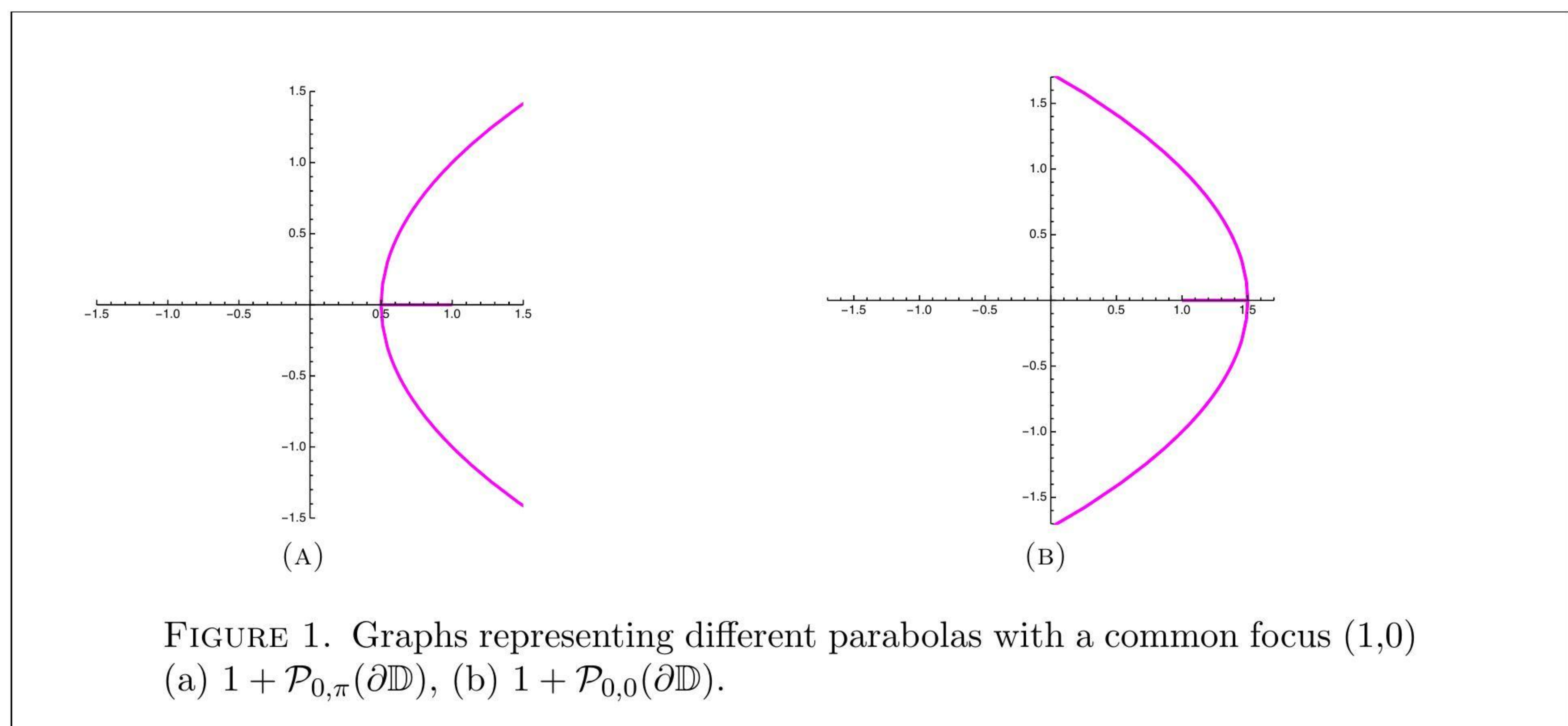


context of the above class $\mathcal{F}(\psi)$,

$$\psi_i(z) = \begin{cases} \gamma z(1 + \eta z)^{-2} & , i = 1 \\ z(1 - \alpha z^2)^{-1} & , i = 2 \\ z(1 - z)^{-1}(1 + \beta z)^{-1} & , i = 3 \\ (A - B)^{-1} \log((1 + Az)/(1 + Bz)) & , i = 4, \end{cases} \quad (1.1)$$

where $A = \alpha e^{i\tau}$, $B = \alpha e^{-i\tau}$ with $\tau \in (0, \pi/2]$, and $\alpha, \beta, \eta \in (0, 1]$, $\gamma > 0$ (see [12, 15, 16, 19]). Kumar and Gangania in [15], introduced the class $\mathcal{S}_\gamma(\eta) = \mathcal{F}(\psi_1)$ and obtained the radius of starlikeness. Cho et al. [2] dealt with certain sharp radius problems for the class $\mathcal{BS}(\alpha) = \mathcal{F}(\psi_2)$. Infact Masih et al. [19] studied the class $\mathcal{S}_{cs}(\beta) = \mathcal{F}(\psi_3)$, where $0 \leq \beta < 1$, discussed the growth theorem and established sharp estimates of logarithmic coefficients for $0 \leq \beta \leq 1/2$. Further for $1/2 < \beta \leq 1$, the class $\mathcal{S}_{cs}(\beta)$ contains non-univalent functions, infact for $0 \leq \beta \leq 1/2$ the class $\mathcal{S}_{cs}(\beta) \subset \mathcal{S}^*$. In 2022, Kumar et al. [16] introduced the class $\mathcal{F}(A, B) = \mathcal{F}(\psi_4)$ and established some radii results.



Motivated essentially by the above classes and observations, we now study a subclass of \mathcal{A} containing non-univalent functions. For $\tau, \theta \in (-\pi, \pi]$, the transformation

$$\omega = ((2e^{i\theta/2}\sqrt{2}/\pi)(\tan^{-1}(e^{i(\tau-\pi/2)}\sqrt{z})))^2$$

maps the boundary of \mathbb{D} onto a parabola (see **Fig. 1**), given by

$$\begin{aligned} \mathcal{P}_{\tau,\theta}(z) &:= \frac{2e^{i(\theta+\pi)}}{\pi^2} \left(\log \left(\frac{1 + e^{i\tau}\sqrt{z}}{1 - e^{i\tau}\sqrt{z}} \right) \right)^2 \\ &= \frac{8e^{i(\theta+\pi)}}{\pi^2} \sum_{n=1}^{\infty} \left(\frac{e^{2i\tau n}}{n} \sum_{k=0}^{n-1} \frac{1}{2k+1} \right) z^n, \end{aligned}$$

where the branch of \sqrt{z} is chosen so that $\text{Im} \sqrt{z} \geq 0$. Note that the function $\mathcal{P}_{\tau,\theta}(z)$ is univalent in \mathbb{D} and $1 + \mathcal{P}_{0,\pi}(z)$ (see **Fig. 1(a)**) is the parabolic function introduced independently by Ma-Minda [18] and Ronning [26], and used it extensively in context of parabolic starlike functions (see [10, 26]). Further, Kanas [11] discussed some differential subordination techniques and later studied admissibility results [9] involving $1 + \mathcal{P}_{0,\pi}(z)$. Note that the boundary of the function $\mathcal{P}_{0,0}(z)$ is a horizontal parabola with an opening in the left-half plane (see **Fig. 1(b)**). In the present study, we consider the function $1 + \mathcal{P}_{0,0}(z)$, in context of non-univalent functions, which is completely different from the way Ronning, Ma-Minda and Kanas (see [9, 11, 18, 26]) handled parabolic regions. Obviously, the other choices of τ and θ leads to oblique parabolic