

admits any carefully synchronizing word. For a given  $\mathcal{A}$  we define its *power automaton* (which is itself a PFA) as  $\mathcal{P}(\mathcal{A}) = (2^Q, \Sigma, \tau)$ , where  $2^Q$  stands for the set of all subsets of  $Q$ , and  $\Sigma$  is the same as in  $\mathcal{A}$ . The transition function  $\tau : 2^Q \times \Sigma \rightarrow 2^Q$  is defined as follows. Let  $Q' \subseteq Q$ . For every  $a \in \Sigma$  we define  $\tau(Q', a) = \bigcup_{q \in Q'} \delta(q, a)$  if  $\delta(q, a)$  is defined for all states  $q \in Q'$ , otherwise  $\tau(Q', a)$  is not defined. We also note  $Q.w$  as an action of a word  $w$  on a set of states  $Q$  under the function  $\delta$ . Let  $S \subseteq Q$ . Then we denote  $S.w^{-1}$  as a preimage of  $S$  under the action of a word  $w$ .

We note that the above concepts can also be considered for *deterministic finite automata* (DFA), for which the transition function is total. We define an *a-cluster* to be a DFA  $\mathcal{A} = (Q, \{a\}, \delta)$  such that the automaton is connected. In other words it means that such automaton is a cycle on letter  $a$  with paths that leads to the states of that cycle. The set of states that induce a cycle in the  $a$ -cluster is referred to as the *center* of the cluster. The *depth* of the cluster is the length of the longest path to the center of the cluster. If  $q$  belongs to the center of the  $a$ -cluster, the *branch* of the state  $q$  are the states that has a path to  $q$  that does not have any other state belonging to the center. *Destination* of the branch is a state in the center that has an in-transition from the last state of the branch. Example of the  $a$ -cluster is depicted on Figure 1.

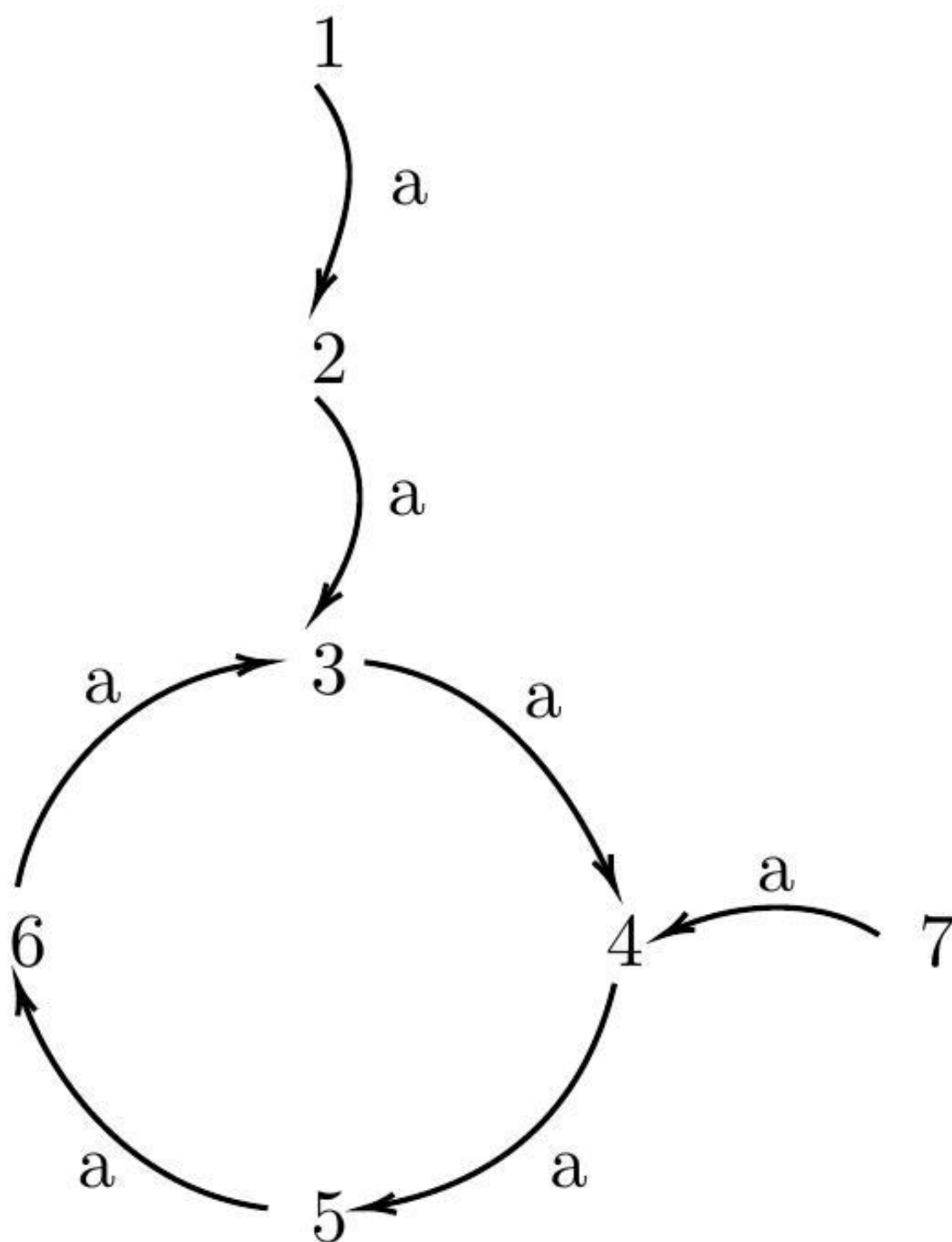


Figure 1: Example of the  $a$ -cluster

Center of that  $a$ -cluster is the set  $\{3, 4, 5, 6\}$ , the depth is 2 and there are two branches:  $b_1 = \{1, 2\}$  and  $b_2 = \{7\}$ . Destination of the branch  $b_1$  is the state 3 and of the branch  $b_2$  is state 4.

We define the sum of two automata  $\mathcal{A} = (Q_1, \Sigma_1, \delta_1)$  and  $\mathcal{B} = (Q_2, \Sigma_2, \delta_2)$  as  $\mathcal{A} \cup \mathcal{B} = (Q_1 \cup Q_2, \Sigma_1 \cup \Sigma_2, \delta_1 \cup \delta_2)$ . We can now state the obvious fact, useful to decide whether a given PFA is carefully synchronizing.