points on a smooth surface X. The motivation for this calculation was suggested by Donaldson with the computation of theoretical physics. The problem is as follows : let n be a positive integer and $|H|$ be a linear system that induces a map $X \to \mathbb{P}^{3n-2}$. Let $X^{[n]}$ be the Hilbert scheme of n points of X. For each zero-dimensional subscheme $\xi \in X^{[n]}$ if the map

$$H^0(\mathbb{P}^{3n-2}, \mathcal{O}_{\mathbb{P}}(1)) \to H^0(\xi, \mathcal{O}_{\xi}(1))$$

fails to be surjective, then ξ does not impose independent conditions on the linear system $|H|$. The authors computation on top Segre classes and N_n would give formuli for the degree of all higher secant varieties of a surface under sufficient higher very ample embedding conditions.

As a generalization of above results on the degree of the 3-secant variety $\sigma_3(X)$ for arbitrary dimension of X, the main theorem is presented as follows :

Theorem 1.1 (Main theorem). *Let X be a nonsingular projective variety embedded by a 5-very ample line bundle. Let n and d be the dimension and the degree of X. Then the degree of the 3-secant variety $\sigma_3(X)$ is given by the following formula :*

$$\frac{1}{3!}\{d^3 - \sum_{k=0}^{n} d\ a_{n,k}\ \deg s_k(T_X) + \sum_{k=0}^{n}\sum_{a=0}^{k} 2^{k-a+n+1}\binom{3n+2}{n-k}\ \deg(s_a(T_X)\cdot s_{k-a}(T_X))\}.$$

where $a_{n,k} = \binom{2n+1}{n-k} + 2\sum_{i=k}^{n}(-1)^{i-k}\binom{3n+2}{n-i}\binom{i-k+n}{n}$.

When attempting to compute the degree of the 3-secant variety for arbitrary dimension of X, it is not possible to imitate the approach in [10] by Lehn. It is because the universal family $Z_3 \subset X \times X^{[3]}$ may be singular(cf. [6]). Additionally the refined Bezout's theorem cannot be applied directly. One possible approach is to set V as the triple join $J(X, X, X)$ of X and σ as the addition map defined by $[x_0, \cdots x_N, y_0, \cdots, y_N, z_0, \cdots z_N] \mapsto [x_0 + y_0 + z_0, \cdots, x_N + y_N + z_N]$. The image $\sigma(V)$ is the 3-secant variety $\sigma_3(X)$. However the indeterminacy locus of σ can be non-reduced, making computation of $v^i(\underline{\sigma}, V)$ much more difficult. Therefore it is not advisable to imitate the approach in [7].

To circumvent these issues, we propose to use the secant bundle, which was introduced by R.L.E. Schwarzenberger in [11] for the purpose of giving fiber bundle structure of secant lines. While the original construction was based on symmetric product of a given variety, rather than the Hilbert scheme of points, in this paper we adopt the convention presented in [3] and [15] where the secant bundle is a projective bundle over a Hilbert scheme of points. This approach has been used in previous works such as [3], [13], [14] and [15], where the secant bundles was used as a tool for describing the singularity and the normality of the 2-secant variety. In this paper, we utilize the biraional morphism from the secant bundle to the 2-secant variety as a resolution of singularities.

In Section 2, we define the total Segre class of a cone and introduce a generalized version of double point formula. This formula allows us to compute the total Segre class $s(X, \sigma_2(X))$ in order to determine the degree of the 3-secant variety. However, the singularity of the 2-secant variety may impede this computation. To overcome this issue, we introduce the notion of higher very ampleness of a line bundle $\mathcal{L}$ and the secant bundle in Section 3. We take the secant bundle as a nonsingular birational model for the 2-secant variety when the line bundle $\mathcal{L}$ satisfies the higher very ampleness condition. As shown in [13], the inverse image of X under this birational morphism is isomorphic to the universal family $Z_2 \subset X \times X^{[2]}$ when $\mathcal{L}$ is 3-very ample. This allows us to regard