2.3. **Sufficient Conditions for the class $\mathcal{F}_{\mathcal{LP}}$.** In this section, we establish some sufficient conditions for the class $\mathcal{F}_{\mathcal{LP}}$. Below, we state a Lemma given by Jack [7], which is utilised in context of the class under consideration.

Lemma 2.22. [7, Lemma 1, p.470] *Let $\nu(z)$ be a non-constant analytic function in $\mathbb{D}$, such that $\nu(0) = 0$. If $|\nu(z)|$ attains its maximum value on the circle $|z|= r$ at a point z_0, then $z_0\nu'(z_0) = k\nu(z_0)$, where k is real and $k \geq 1$.*

Theorem 2.23. *Suppose $0 \leq t \leq 1$ and let $f \in \mathcal{A}$ satisfy the following differential inequality*

$$\left|t\left(1+\frac{zf''(z)}{f'(z)}\right)+(1-t)\frac{zf'(z)}{f(z)}-1\right| < \frac{1}{6}(3+2t), \quad z \in \mathbb{D}, \tag{2.12}$$

then $f \in \mathcal{F}_{\mathcal{LP}}$.

Proof. Consider an analytic function $\nu(z)$ with $\nu(0) = 0$. Assume $f \in \mathcal{A}$ such that

$$\frac{zf'(z)}{f(z)} - 1 = \frac{1}{2}\nu(z).$$

We show that $|\nu(z)|< 1$ in $\mathbb{D}$. Suppose on the contrary $|\nu(z)|\geq 1$, then by an application of Lemma 2.22, there exists $z_0 \in \mathbb{D}$ such that for $k \geq 1$, $|\nu(z_0)|= 1$ and $z_0\nu'(z_0) = k\nu(z_0)$. Substituting $\nu(z_0) = e^{i\mu}$, $-\pi < \mu \leq \pi$ leads to

$$\begin{aligned}
&\left|t\left(1+\frac{zf''(z)}{f'(z)}\right)+(1-t)\frac{zf'(z)}{f(z)}-1\right| \\
&= \left|t\left(1+\frac{\nu(z_0)}{2}+\frac{k\nu(z_0)}{2+\nu(z_0)}\right)+(1-t)\left(1+\frac{\nu(z_0)}{2}\right)-1\right| \\
&= \left|\frac{tke^{i\mu}}{2+e^{i\mu}}+\frac{e^{i\mu}}{2}\right| \geq \frac{1}{6}(3+2t).
\end{aligned}$$

This is contradiction to the assumption given in (2.12). Thus $|\nu(z)|< 1$, which means that $zf'(z)/f(z)$ lies in the disc $|(zf'(z)/f(z)) - 1|< 1/2$. Hence in view of Lemma 2.5, (with $a = 1$) required result is achieved. ∎

For $t = 1/2$, $t = 0$ and $t = 1$ in Theorem 2.23, we obtain the following corollary,

Corollary 2.24. *Let $f \in \mathcal{A}$ satisfy the following differential inequalities*

(i) $|(zf'(z)/f(z) + zf''(z)/f'(z)) - 1| < 4/3$, *or*
(ii) $|(zf'(z)/f(z)) - 1| < 1/2$, *or*
(iii) $|zf''(z)/f'(z)| < 5/6$,

then $f \in \mathcal{F}_{\mathcal{LP}}$.

Conclusion

In the present investigation, we introduce a class of analytic functions associated with certain parabolic region. In particular, we have considered a case when parabola is lying majorly in the left half plane and symmetric about real axis. The other cases namely, oblique parabolic regions are still open, for similar investigations.

Appendix

The classes we come across in the present investigation are listed below for the ready reference of the reader.