

We obtain the following final decomposition of the EUR PnL into four parts:

$$\begin{aligned}
P_{(t,T]}^{(A)} &= \frac{A_t + A_T}{2} (\chi_T - \chi_t) & (= P_{(t,T]}^{(A)}(\chi)) \\
&+ \frac{\chi_t + \chi_T}{2} \left(\frac{A_T(r_T, x_T) - A_T(r_t, x_T)}{2} + \frac{A_t(r_T, x_t) - A_t(r_t, x_t)}{2} \right) & (= P_{(t,T]}^{(A)}(r)) \\
&+ \frac{\chi_t + \chi_T}{2} \left(\frac{A_T(r_T, x_T) - A_T(r_T, x_t)}{2} + \frac{A_t(r_t, x_T) - A_t(r_t, x_t)}{2} \right) & (= P_{(t,T]}^{(A)}(x)) \\
&+ \frac{\chi_t + \chi_T}{2} \left(\frac{A_T(r_T, x_t) - A_t(r_T, x_t)}{2} + \frac{A_T(r_t, x_T) - A_t(r_t, x_T)}{2} \right). & (= P_{(t,T]}^{(A)}(\text{carry}))
\end{aligned}$$

The interpretation of $P_{(t,T]}^{(A)}(r)$ is the following: at an arbitrary time point u the difference $A_u(r_T, x_u) - A_u(r_t, x_u)$ measures the PnL of the market price at that time that would be induced by a discounting curve change from r_t to r_T . The PnL $P_{(t,T]}^{(A)}(r)$ is defined as the arithmetic mean of this difference for the two time points $u = t$ and $u = T$. The precisely same logic applies to the interpretation of $P_{(t,T]}^{(A)}(x)$, only with r replaced by x . Finally, the PnL $P_{(t,T]}^{(A)}(\text{carry})$ intuitively should measure the change between the asset values A_t and A_T that is only due to time passing, without the effects of r and x . Since the variables r and x change their values within the period $(t, T]$, one reasonable approach is to use an “average” of the variables r, x on the period $(t, T]$. Since we only have r, x available at the two time points t and T , a pragmatic idea to accomplish such average is to mix the possible pairs (r_u, x_s) for $u, s \in \{t, T\}$ in a way that is as “neutral” as possible. This is precisely what’s done in the definition of $P_{(t,T]}^{(A)}(\text{carry})$.

Remark 2.1 (On the approximative nature of our definitions)

Similar as in Remark 1.1, we point out that our definition of $P_{(t,T]}^{(A)}(r)$ and $P_{(t,T]}^{(A)}(x)$ in terms of an arithmetic average of start and end time point values is only a proxy to reality. Clearly, an average that would take into account all time points $u \in (t, T]$ would be more desirable from a theoretical perspective. For instance, based on the multivariate Itô formula, under the assumption that $\{r_u\}_{u \in (t,T]}$ and $\{x_u\}_{u \in (t,T]}$ are realizations of semi-martingales, take values in \mathbb{R} (i.e. are not function-valued), and under the assumption that r and x are independent, we obtain the decomposition

$$\begin{aligned}
A_T - A_t &= \left(\int_{(t,T]} \frac{\partial}{\partial t} A_u(r_u, x_u) du \right) \\
&+ \left(\int_{(t,T]} \frac{\partial}{\partial r} A_s(r_u, x_u) dr_u + \frac{1}{2} \int_{(t,T]} \frac{\partial^2}{\partial r^2} A_s(r_u, x_u) d[r, r]_u \right) \\
&+ \left(\int_{(t,T]} \frac{\partial}{\partial x} A_s(r_u, x_u) dx_u + \frac{1}{2} \int_{(t,T]} \frac{\partial^2}{\partial x^2} A_s(r_u, x_u) d[x, x]_u \right),
\end{aligned}$$

and the three terms in (\cdot) -brackets could be interpreted as performance due to carry, changes in r , and changes in x , respectively. While already this formula is difficult to implement in practice, we point out that typically r_u is a function (an interest rate term structure), so that a generalization in this regard requires significantly more advanced