Remark 1. If H is infinite-dimensional and f is a bijective transformation of $\mathcal{G}^k(H)$ preserving the ortho-adjacency in both directions, then $X \to f(X^\perp)^\perp$ is a bijective transformation of $\mathcal{G}_k(H)$ which also preserves the ortho-adjacency in both directions. If the latter transformation is induced by a unitary or anti-unitary operator U, then f is also induced by U.

The following example shows that the above statement fails for bijective transformations of $\mathcal{G}_\infty(H)$ preserving the ortho-adjacency relation in both directions. Let U be a non-identity unitary operator on H which preserves a certain connected component $\mathcal{C} \subset \mathcal{G}_\infty(H)$. Consider the bijective transformation f of $\mathcal{G}_\infty(H)$ defined as follows: $f(X) = U(X)$ if $X \in \mathcal{C}$ and $f(X) = X$ if $X \notin \mathcal{C}$. It is clear that f is ortho-adjacency preserving in both directions.

Theorem 1. *Let f be a bijective transformation of $\mathcal{G}_\infty(H)$ which preserves the ortho-adjacency relation in both directions. Then the restriction of f to every connected component of $\mathcal{G}_\infty(H)$ is induced by a unitary or anti-unitary operator or it is the composition of the orthocomplementary map and a map induced by a unitary or anti-unitary operator.*

The restrictions of f to distinct connected components can be related to different operators.

3. A characterization of adjacency in terms of ortho-adjacency

If $X, Y \in \mathcal{G}_\infty(H)$ are not ortho-adjacent and there is $Z \in \mathcal{G}_\infty(H)$ ortho-adjacent to both X, Y, then one of the following possibilities is realized:

- X, Y are adjacent;
- $X \cap Y$ is of codimension 2 in both X, Y.

Indeed, $X \cap Z$ and $Y \cap Z$ are hyperplanes of Z and X, Y are adjacent if these hyperplanes coincide; the same holds if the hyperplanes are distinct and their intersection is a proper subspace of $X \cap Y$; we obtain the second possibility only when the hyperplanes are distinct and their intersection coincides with $X \cap Y$.

Lemma 1. *Let X, Y be compatible elements of $\mathcal{G}_\infty(H)$ whose intersection is of codimension 2 in both X, Y. Then there is $Z \in \mathcal{G}_\infty(H)$ ortho-adjacent to both X, Y. For every such Z there are precisely two elements of $\mathcal{G}_\infty(H)$ ortho-adjacent to each of X, Y, Z.*

Proof. The orthogonal sum of $X \cap Y$, a 1-dimensional subspace of $X \cap (X \cap Y)^\perp$ and a 1-dimensional subspace of $Y \cap (X \cap Y)^\perp$ is an element of $\mathcal{G}_\infty(H)$ ortho-adjacent to both X, Y.

Let Z be an element of $\mathcal{G}_\infty(H)$ ortho-adjacent to both X, Y. The subspaces X, Y, Z are mutually compatible and there is an orthonormal basis B of H such that each of these subspaces is spanned by a subset of B. Then $X \cap Y$ and the 2-dimensional subspaces

$$X' = X \cap (X \cap Y)^\perp, \;\; Y' = Y \cap (X \cap Y)^\perp$$

are also spanned by subsets of B. Furthermore, $X \cap Y$ is contained in Z and

$$P = Z \cap X', \;\; Q = Z \cap Y'$$

are 1-dimensional. Let P' and Q' be the 1-dimensional subspaces which are the orthogonal complements of P in X' and Q in Y', respectively. Since each of the