Proposition [8, Proposition 4.2 (a)] Let $f : Y^{'} \to Y$ be a morphism of pure-dimensional schemes, $X \subset Y$ a closed subscheme, $X^{'} = f^{-1}(X)$ the inverse image scheme, $g : X^{'} \to X$ the induced morphism
(a) If f is proper, Y irreducible, and f maps each irreducible component of $Y^{'}$ onto Y, then

$$g_*(s(X^{'}, Y^{'})) = \deg(Y^{'}/Y)s(X, Y).$$

Equation (6) can be used to calculate (4). Consider the following closed immersions:

$$\Gamma_q \subset X \times Z_2 \subset X \times \mathbb{P}(E_{\mathcal{L}})$$

Since each term is nonsingular, each of these closed immersions is a regular immersion. Therefore, we have the following exact sequence of normal bundles:

$$0 \to N_{\Gamma_q}(X \times Z_2) \to N_{\Gamma_q}(X \times \mathbb{P}(E_{\mathcal{L}})) \to N_{X\times Z_2}(X \times \mathbb{P}(E_{\mathcal{L}}))|_{\Gamma_q} \to 0 \tag{7}$$

After simplification, we obtain the following:

$$s(N_{\Gamma_q}(X \times Z_2)) = (\Gamma_q \to X)^* s(T_X) = (\mathrm{id} \times r)^* s(T_{\Delta(X)})$$
$$s(N_{X\times Z_2}(X \times \mathbb{P}(E_{\mathcal{L}}))|_{\Gamma_q}) = (\Gamma_q \to Z_2)^* s(N_{Z_2}\mathbb{P}(E_{\mathcal{L}}))$$

Using (7), we get:

$$s(N_{\Gamma_q}(X \times \mathbb{P}(E_{\mathcal{L}}))) = (\mathrm{id} \times r)^* s(T_{\Delta(X)}) \cdot (\Gamma_q \to Z_2)^* s(N_{Z_2}\mathbb{P}(E_{\mathcal{L}})). \tag{8}$$

Note that $(\mathrm{id} \times r|_{\Gamma_q})_* \circ (\Gamma_q \to Z_2)^*$ is $(\Delta(X) \to X)^* \circ r_*$ on Chow rings. From (6) and (8), we get the following :

$$s(\Delta(X), X \times \sigma_2(X)) = (\Delta(X) \to X)^*(s(T_X) \cdot r_* s(N_{Z_2}\mathbb{P}(E_{\mathcal{L}})) \cap [X]) \tag{9}$$

So, it remains to compute the total Segre class $s(\mathcal{N}_{Z_2}\mathbb{P}(E_{\mathcal{L}}))$. Since $r^{-1}(X)$ can be regarded as an effective divisor of $\mathbb{P}(E_{\mathcal{L}})$, we can compute $s(Z_2, \mathbb{P}(E_{\mathcal{L}}))$ by [8, Cor 4.2.2] as follow:

$$s(Z_2, \mathbb{P}(E_{\mathcal{L}})) = \frac{[Z_2]}{1 + [Z_2]}. \tag{10}$$

In order to proceed, it is necessary to express the term $[Z_2]$ in terms of the tautological line bundle ζ of $\mathbb{P}(E_{\mathcal{L}})$ and $\pi^*\beta$, where β is a divisor on $X^{[2]}$ (cf. Fulton 2013, Chapter 3.3). This is achieved by calculating the first Chern class of $E_{\mathcal{L}}$ in proposition 3.1.

Let h be the divisor corresponding to a line bundle $\mathcal{L}$ on X. We denote the pullback of h under the i-th projection as h_i. The morphism ρ is an involution map, so $\rho_*\eta^*h_1 = \rho_*\eta^*h_2$. We define $H = \rho_*\eta^*h_1 = \rho_*\eta^*h_2$ and $\delta = \frac{1}{2}\rho_*E$. The following proposition may be known to experts but I could not find appropriate references, so I will provide a proof.

Proposition 3.1. $c_1(E_{\mathcal{L}}) = H - \delta$

Proof. Let $\pi : \mathbb{P}(E_{\mathcal{L}}) \to X^{[2]}$ be the projection map of a projective bundle. Consider the normal sheaf $\mathcal{N} := \mathcal{N}_{Z_2/X\times X^{[2]}}$ and the closed immersion $j : Z_2 \to X \times X^{[2]}$ with the composition $q : Z_2 \to X$ of η and the first projection. The morphism $\pi_1|_{Z_2} : Z_2 \to X^{[2]}$ is a finite flat morphism, and $\pi_{2\,*}(\pi_1^*\mathcal{L} \otimes \mathcal{O}_{Z_2})$ is a locally free sheaf on $X^{[2]}$, so by Grauert's theorem, all higher direct images $R^i\pi_{2\,*}(\pi_1^*\mathcal{L} \otimes \mathcal{O}_{Z_2})$ vanish for $i \geq 1$. Let T_X be the tangent sheaf of X.