Report:- Mini Project 2
Path Planning using Artificial Potential Field
SCARA Manipulator (3RRP)

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Objective

1. Write a MATLAB code for path planning of the robot from initial to final configuration such that it does not hit the obstacles. Using potential field for path planning.
2. Consider three-point obstacles and make your program generic for any arbitrary locations of point obstacle within the workspace of the robot.

1 Kinematic Model of the Robot

The line diagram of SCARA Manipulator (3RRP) is given in the figure.1.

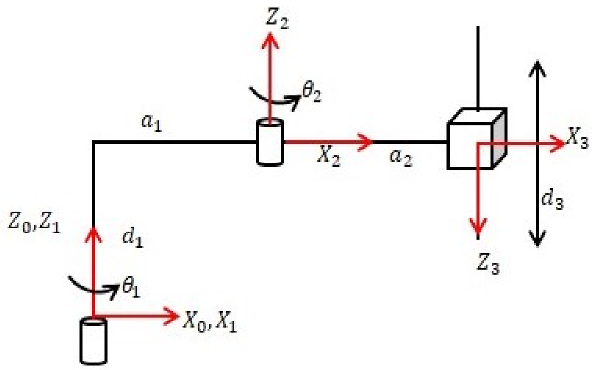

Figure 1: SCARA MANIPULATOR

The DH parameters of the manipulator is given in table 1.

Table 1: DH Parameters

S. No.	Twist Angle(θ)	Link Length(a)	Joint offset(d)	Joint Angle(θ)
1.	0	0	0	θ_1
2	0	a_1	d_1	θ_2
3	π	a_2	d_3	0

1.1 Forward Kinematics

The Homogeneous transformation matrix of every joint in base frame is given as

$$T_0^1 = \begin{bmatrix} cos\theta_1 & -sin\theta_1 & 0 & 0 \\ sin\theta_1 & cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^2 = \begin{bmatrix} cos(\theta_1+\theta_2) & -sin(\theta_1+\theta_2) & 0 & a_1cos\theta_1 \\ sin(\theta_1+\theta_2) & cos(\theta_1+\theta_2) & 0 & a_1sin\theta_1 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = \begin{bmatrix} cos(\theta_1+\theta_2) & sin(\theta_1+\theta_2) & 0 & a_1cos\theta_1 + a_2cos(\theta_1+\theta_2) \\ sin(\theta_1+\theta_2) & -cos(\theta_1+\theta_2) & 0 & a_1sin\theta_1 + a_2 + sin(\theta_1+\theta_2) \\ 0 & 0 & -1 & d_1 - d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The End effector coordinate (i.e third origin) is obtained, provided the joint variables and they are

$$X_3 = a_1cos(\theta_1) + a_2cos(\theta_1+\theta_2)$$

$$Y_3 = a_1sin(\theta_1) + a_2sin(\theta_1+\theta_2)$$

$$Z_3 = d_1 - d_3$$

The coordinate of the second origin is given as

$$X_2 = a_1cos(\theta_1)$$

$$Y_2 = a_1sin(\theta_1)$$

$$Z_2 = d_1$$

The forward kinematics plot of the manipulator using cycloidal trajectory is given in the fig2. The parameters of the manipulator are provided table2 and the initial and final values of the joint variables are given in the table3

Table 2: Parameters of the Manipulator

Parameter	Value
a_1	0.5
a_2	0.5
d_1	0.3

Table 3: Initial and Final values of the Joint variables

Joint Variable	Initial Value	Final Value
θ_1	0 rad	pi/2
θ_2	0.0873 rad	2pi/3
d_3	0.01	0.3

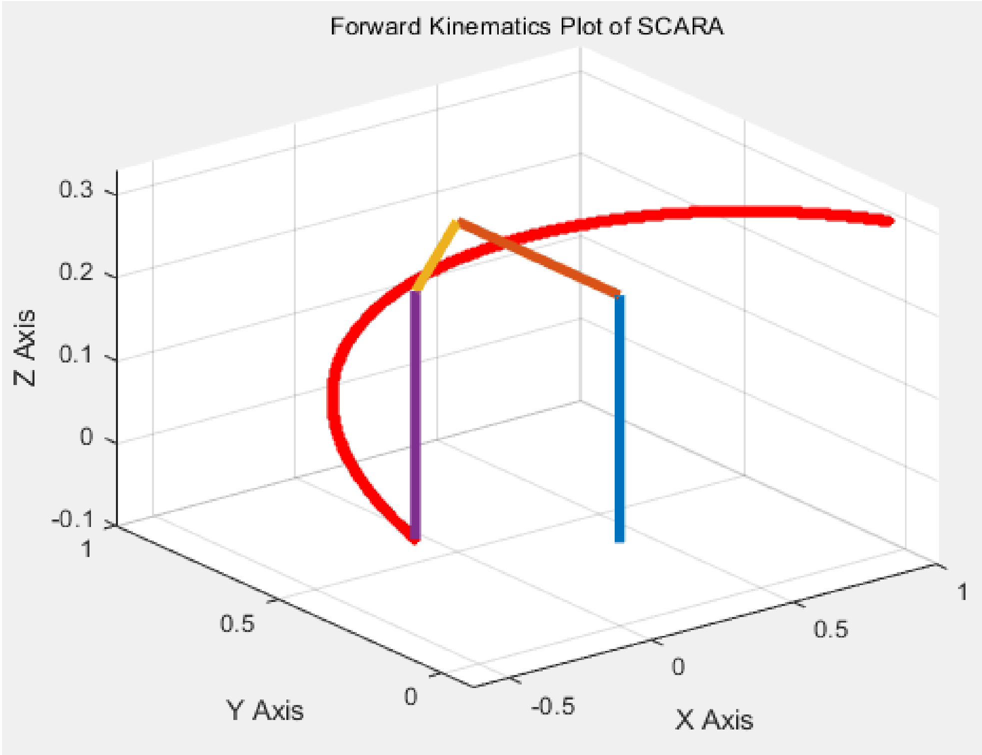

Figure 2: Forward Kinematics Plot of SCARA Manipulator

1.2 Inverse Kinematics

The joint variables are obtained, provided the end-effector coordinate and they are obtained as

$$cos\theta_2 = \frac{X^2 + Y^2 - a_1^2 - a_2^2}{2a_1a_2}$$

$$sin\theta_2 = \sqrt{1 - (cos\theta_2)^2}$$

$$\theta_2 = tan^{-1}\left(\frac{sin\theta_2}{cos\theta_2}\right)$$

$$\theta_1 = tan^{-1}\left(\frac{Y}{X}\right) - tan^{-1}\left(\frac{a_2sin\theta_2}{a_1 + a_2cos\theta_2}\right)$$

The inverse kinematics of the manipulator is calculated by using the calculated end effector coordinate and after the forward kinematics is calculated and the forward kinematics plot is given in the fig.3.

1.3 Jacobian

The Jacobian of joint 2 and joint 3 is

$$J^{02} = \begin{bmatrix} -a_1sin\theta_1 & 0 & 0 \\ a_1cos\theta_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$J^{03} = \begin{bmatrix} -a_2sin(\theta_1+\theta_2) - a_1sin\theta_1 & -a_2sin(\theta_1+\theta_2) & 0 \\ a_2cos(\theta_1+\theta_2) + a_1cos\theta_1 & a_2cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

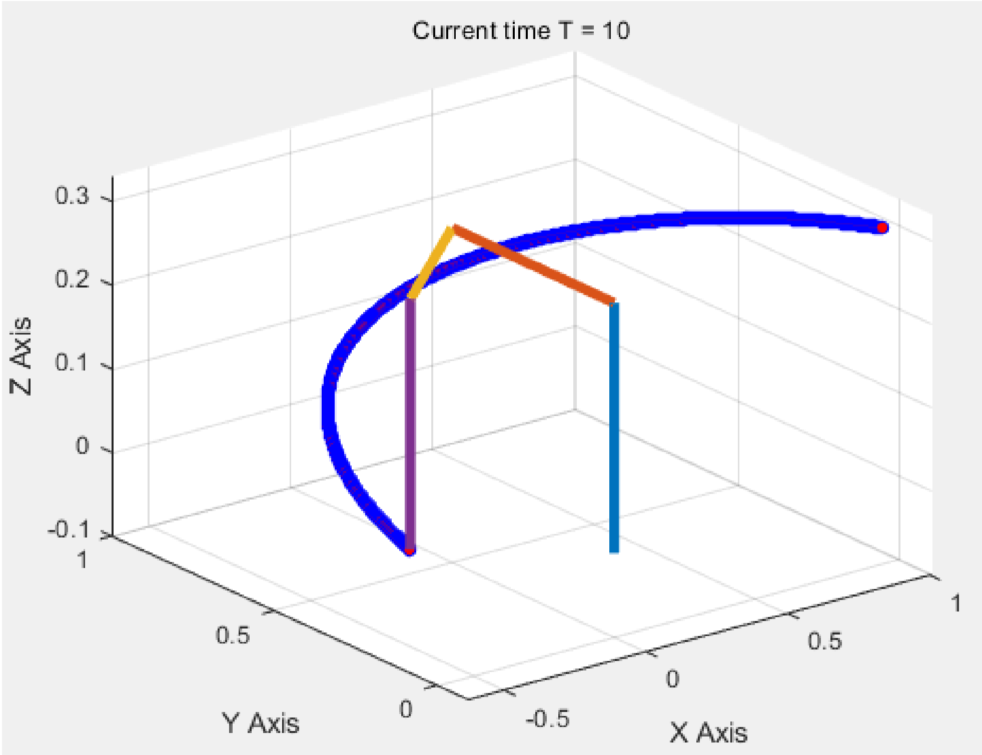

Figure 3: Forward Kinematics Plot calculated from the Inverse Kinematics

2 Path Planning

Path planning is, computing a collision-free path for a rigid or articulated moving robot among static obstacles. The path is a sequence of robot configuration in a particular order without regards to the timing of these configurations. The path planning problem is to find a path from an initial configuration q_i to a final configuration q_f, such that the robot does not collide with any obstacle as it traverses the path. In other words, a collision-free path from q_i to q_f is a continuous path, $gamma$:[0,1]$\rightarrow Q_{free}$, with $gamma(0) = q_i$ and $\gamma(1) = Q_f$.
An artificial potential field algorithm is used to compute a sequence of discrete configurations in the configuration space. This algorithm incrementally explores Q_{free} while searching for a path. The idea behind the potential field approach is to treat the robot as a point particle in the configuration space under the influence of an artificial potential field. The robot is attracted to the goal configuration while being repelled from the static obstacles. In potential field path planning, the potential fields are directly defined on the workspace of the robot. For an n-link manipulator, the potential field is defined for each of the origins of the n DH frames(excluding fixed). These potential workspace fields attract the origins of the DH frames to their goal location while repelling them from obstacles. These fields used to define motions in the configuration space using the Jacobian matrix.

2.1 The Attractive Potential Field

To attract the robot to its goal configuration, we define an attractive potential filed $U_{att,i}$ for the i^{th} origin of DH Frame. When all n origins reach their goal positions, the arm has reached its goal configuration. In this mini-project, we have used parabolic well potential field. This field grows quadratically with distance.

$$U_{att,i}(q) = \frac{1}{2}\zeta_i||O_i(q_i) - O_i(q_f)||^2$$

Where ζ_i is influence of attractive potential for O_i. The workspace attractive force forO_i is equal to the negative gradient of $U_{att,i}$, which is given by

$$F_{att,i}(q) = -\nabla U_{att,i}(q) = -\zeta_i(O_i(q_i) - O_i(q_f))$$

The manipulator shown in figure 1, have 4 origins i.e O_0, O_1, O_2, O_3 in which O_0 and O_1 are at same position. So the potential field is defined only for origins O_2, O_3. The workspace forces at these origins are given by

$$F_{att,2} = \zeta_2(O_2(q_i) - O_2(q_f))$$
$$F_{att,3} = \zeta_3(O_3(q_i) - O_3(q_f))$$

q_i and q_f are the initial and goal configurations.

2.2 The Repulsive Potential Field

To prevent the collision between the robot and obstacles, we define a workspace repulsive potential field for the origin of each frame (excluding frame 0 and 1). The repulsive potential filed should repel the robot from obstacles, never allowing the robot to collide with an obstacle and when the robot is far away from the obstacle, that obstacle should exert little or no influence on the motion of the robot. In the repulsive potential field, we define a radius of influence ρ_0 of an obstacle. The little or no influence on the motion means that an obstacle does not repel O_i if the distance from O_i to the obstacle is more significant than ρ_0. The potential field that meets the criteria described above is given as

$$U_{rep,i}(q) = \begin{cases} \frac{1}{2}\eta_i\left(\frac{1}{\rho(O_i(q_i))} - \frac{1}{\rho_0}\right)^2 & ; \rho(O_i(q_i)) \leq \rho_0 \\ 0 & ; \rho(O_i(q_i)) > \rho_0 \end{cases}$$

Where $\rho(O_i(q_i))$ is the shortest distance between O_i and any workspace obstacle. The workspace repulsive force is equal to the negative gradient of $U_{rep,i}(q)$. For $\rho(O_i(q_i)) \leq \rho_0$ this repulsive force is given by

$$F_{rep,i}(q) = \begin{cases} \eta_i\left(\frac{1}{\rho(O_i(q_i))} - \frac{1}{\rho_0}\right)^2 \frac{1}{\rho(O_i(q_i))^2}\nabla\rho(O_i(q_i)) & ; \rho(O_i(q_i)) \leq \rho_0 \\ 0 & ; \rho(O_i(q_i)) > \rho_0 \end{cases}$$

Where the notation $\nabla\rho(O_i(q_i))$ indicates the gradient $\nabla\rho(x)$ evaluated at $x = O_i(q_i)$. If b is the point on the obstacle then the $\rho(O_i(q_i)) = ||O_i(q_i) - b||$, and its gradient is

$$\nabla\rho(x) = \frac{O_i(q_i) - b}{||O_i(q_i) - b||}$$

and

$$\rho(O_i(q_i)) = \left(||O_i(q_i) - b||\right)$$

this is, the unit vector directed from b toward $O_i(q_i)$. In this project we have to consider three point obstacles and those are b_1, b_2, b_3 and the radius of influence is ρ_0. The repulsive forces on the origin $O2$ and O_3 (because the 0 and 1 origin are at same place) is given by

1. For the first obstacle b_1

$$F_{rep,2_1}(q) = \begin{cases} \eta_2\left(\frac{1}{\rho(O_2(q_i))} - \frac{1}{\rho_0}\right)^2 \frac{1}{\rho(O_2(q_i))^2}\left(\frac{O_2(q_i) - b_1}{||O_2(q_i) - b_1||}\right) & ; \rho(O_2(q_i)) \leq \rho_0 \\ 0 & ; \rho(O_2(q_i)) > \rho_0 \end{cases}$$

Where

$$\rho(O_2(q_i)) = \big(||O_2(q_i) - b_1||\big)$$

$$F_{rep,3_1}(q) = \begin{cases} \eta_3\left(\dfrac{1}{\rho(O_3(q_i))} - \dfrac{1}{\rho_0}\right)^2 \dfrac{1}{\rho(O_3(q_i))^2}\left(\dfrac{O_3(q_i) - b_1}{||O_3(q_i) - b_1||}\right) & ; \rho(O_3(q_i)) \leq \rho_0 \\ 0 & ; \rho(O_3(q_i)) > \rho_0 \end{cases}$$

Where

$$\rho(O_3(q_i)) = \big(||O_3(q_i) - b_1||\big)$$

2. For the second obstacle b_2

$$F_{rep,2_2}(q) = \begin{cases} \eta_2\left(\dfrac{1}{\rho(O_2(q_i))} - \dfrac{1}{\rho_0}\right)^2 \dfrac{1}{\rho(O_2(q_i))^2}\left(\dfrac{O_2(q_i) - b_2}{||O_2(q_i) - b_2||}\right) & ; \rho(O_2(q_i)) \leq \rho_0 \\ 0 & ; \rho(O_2(q_i)) > \rho_0 \end{cases}$$

Where

$$\rho(O_2(q_i)) = \big(||O_2(q_i) - b_2||\big)$$

$$F_{rep,3_2}(q) = \begin{cases} \eta_3\left(\dfrac{1}{\rho(O_3(q_i))} - \dfrac{1}{\rho_0}\right)^2 \dfrac{1}{\rho(O_3(q_i))^2}\left(\dfrac{O_3(q_i) - b_2}{||O_3(q_i) - b_2||}\right) & ; \rho(O_3(q_i)) \leq \rho_0 \\ 0 & ; \rho(O_3(q_i)) > \rho_0 \end{cases}$$

Where

$$\rho(O_3(q_i)) = \big(||O_3(q_i) - b_2||\big)$$

3. For the third obstacle b_3

$$F_{rep,2_3}(q) = \begin{cases} \eta_2\left(\dfrac{1}{\rho(O_2(q_i))} - \dfrac{1}{\rho_0}\right)^2 \dfrac{1}{\rho(O_2(q_i))^2}\left(\dfrac{O_2(q_i) - b_3}{||O_2(q_i) - b_3||}\right) & ; \rho(O_2(q_i)) \leq \rho_0 \\ 0 & ; \rho(O_2(q_i)) > \rho_0 \end{cases}$$

Where

$$\rho(O_2(q_i)) = \big(||O_2(q_i) - b_3||\big)$$

$$F_{rep,3_3}(q) = \begin{cases} \eta_3\left(\dfrac{1}{\rho(O_3(q_i))} - \dfrac{1}{\rho_0}\right)^2 \dfrac{1}{\rho(O_3(q_i))^2}\left(\dfrac{O_3(q_i) - b_3}{||O_3(q_i) - b_3||}\right) & ; \rho(O_3(q_i)) \leq \rho_0 \\ 0 & ; \rho(O_3(q_i)) > \rho_0 \end{cases}$$

Where

$$\rho(O_3(q_i)) = \big(||O_3(q_i) - b_3||\big)$$

2.3 Mapping Workspace Forces to Joint Torques

The workspace forces defined in the previous subsection need to be mapped into the joint torques. The joint torques due to the workspace forces are calculated by

$$\tau = J_v^T F$$

Where J_v includes the top three rows of the manipulator Jacobian. Here for each origin $O_i$an appropriate Jacobian must be constructed. The joint torques for the manipulator shown in the figure 1 is

$$\tau_{att} = (J^{02})^T F_{att,2} + (J^{03})^T F_{att,3}$$

$$\tau_{rep} = (J^{02})^T F_{rep,2_1} + (J^{02})^T F_{rep,2_2} + (J^{02})^T F_{rep,2_3} + (J^{03})^T F_{rep,3_1} + (J^{03})^T F_{rep,3_2} + (J^{03})^T F_{rep,3_3}$$

$$\tau = \tau_{att} + \tau_{rep}$$

2.4 Gradient Descent

Gradient descent is a well-known approach for solving optimization problems. The idea is simple. Starting at the initial configuration, take a small step in the direction of the negative gradient (which is the direction that decreases the potential as quickly as possible). The gradient descent gives a new configuration, and the process is repeated until the final configuration is reached. More formally, we can define a gradient descent algorithm as follows

Gradient Descent Algorithm
1. $i = 0, q^0 = q_i$
2. **if** $||q_i - q_f|| > \epsilon$

$$q_{i+1} = q_i + \alpha_i \frac{\tau(q_i)}{||\tau(q_i)||}$$

i++

else

return $(q_0, q_{01}, ...q_i)$

end if

3. goto 2.

3 Steps for path planning using potential field

To plan the path of the manipulator by using artificial potential field, following steps should be followed.

1. Define the initial and goal configuration.
2. For the given goal configuration create an attractive potential field.
3. Locate all the obstacles in the workspace.
4. Create the repulsive potential field for the obstacles.
5. Find the joint torques in configuration space.
6. Sum all the joint torques in configuration space.
7. Use gradient descent to reach the target configuration.

4 Simulation Results

For the simulation the robot parameters given in table.2 and the initial and final goal configuration defined in table.3 are used. The simulation has been carried out into steps, in the first step the path is planned without any workspace obstacles and in the second step three workspace obstacles are inserted in the workspace, and then the path is planned.

4.1 Path Planning without Obstacle

In the first step manipulator shown in figure.1, the path is planned without any obstacles in the workspace using parabolic artificial potential field. The simulation result is shown in the figure.4. The influence of attractive potential field is chosen as $\zeta_3 = 2.5$ and $\zeta_2 = 3$, with the step size of $\alpha = 0.0001$.

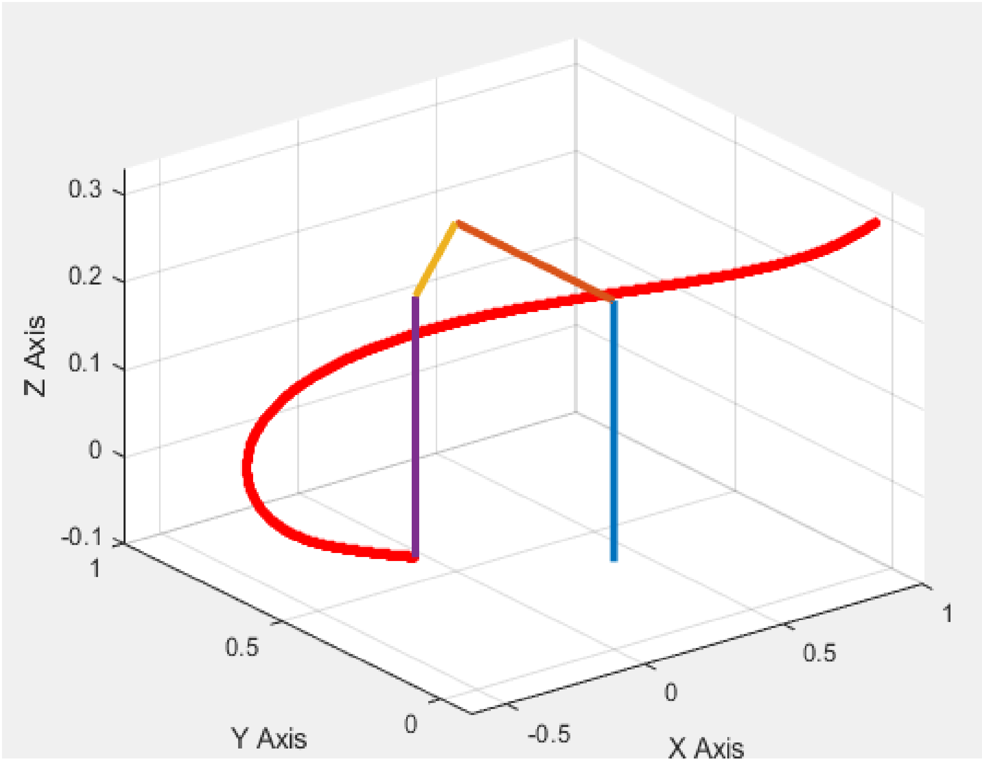

Figure 4: Path of SCARA without Obstacle

4.2 Path Planing with Obstacles

In the second step the three obstacles are placed in the workspace of the manipulator. The simulation result is shown in the figure. .The radius of influence is taken as $\rho_0 = 0.2$ and the influence of repulsive potential field is chosen as $\eta_3 = 0.5$ and $\eta_2 = 0.5$. The coordinate of the obstacles in the workspace are given in the table.4.

Table 4: Coordinates of the Obstacles

Obstacle	X-Coordinate	Y-Coordinate	Z-Coordinate
b_1	0.898	0.397	0.248
b_2	0.58	0.8	0.2
b_3	-0.58	0.4	0

5 Conclusion

The path of SCARA manipulator is planned using the artificial potential field with three static obstacles in the workspace. The simulation is carried out in two steps. In the first step, the path is planned without any obstacle in the workspace. In the second step of the simulation, three static obstacles are introduced in the workspace of the manipulator. Firstly attractive potential fields are

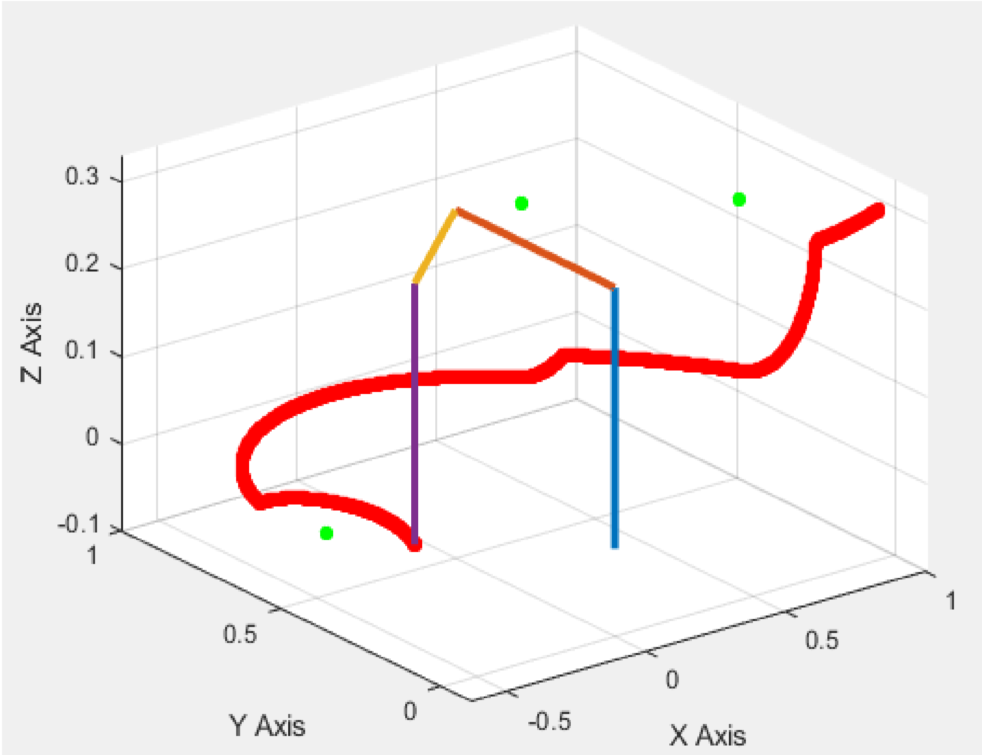

Figure 5: Path of SCARA with three obstacles

generated between the initial and goal configurations after that repulsive potential filed are generated of the obstacles which repel the origins of the manipulator away from the obstacles. The workspace forces are mapped into joint torques of every origin. At last, a gradient descent algorithm is used to guide the manipulator to its goal position.