

Problem 3: Runtime Analysis

a) void f1(int n) {

int i = 0;

while (i < n) {

// something $\theta(1)$ time

i = i * 2;

}

}

$\theta(1)$

$\theta(\log(\log n))$

$\theta(1)$

$\theta(1)$

times	n	$\log n$	$\log(\log n)$
0	1	0	0
1	2	1	1
2	4	2	2
3	8	3	3
4	16	4	4

$$\theta(1) + \sum_{i=0}^{\log(\log n)} \theta(2^i) = \theta(1) + \theta(2 \log(\log n)) = \theta(\log(\log n))$$

b) void f2(int n) {

for (int i = 1; i <= n; i++) {

if (i % (int) sqrt(n) == 0) {

for (int k = 0; k < pow(i, 3); k++) {

// something $\theta(1)$ time

}

}

}

}

$\theta(n)$

$\theta(n^4/n) = \theta(\sqrt{n})$

$\theta(i^3)$

$\theta(1)$

$$\sum_{i=1}^n (\theta(1)) + \sum_{i=0}^{\sqrt{n}} \left(\sum_{k=0}^{i^3} \theta(1) \right) = \theta(n) + \theta(n^4/n) = \theta(n^3)$$

c) for (int i = 1; i <= n; i++) {

for (int k = 1; k <= n; k++) {

if (A[k] == i) {

for (int m = 1; m <= n; m = m + m) {

// do something $\theta(1)$ time

}

}

}

}

$\theta(n)$

$\theta(n)$

runs max. n times

$\theta(\log n)$

$\theta(1)$

$$\sum_{i=1}^n \sum_{k=1}^n (\theta(1)) + \sum_{i=1}^n \left(\sum_{k=1}^n \left(\sum_{m=1}^{\log n} \theta(1) \right) \right) = \sum_{i=1}^n \theta(n) + \sum_{i=1}^n \left(\sum_{k=1}^n \theta(\log n) \right) = \theta(n^2) + \theta(n^2 \log n) = \theta(n^2 \log n)$$

d) int f(int n) {

int *a = new int[10];

int size = 10;

for (int i = 0; i < n; i++) {

if (i == size) {

int newSize = 3 * size / 2;

int *b = new int[newSize];

for (int j = 0; j < size; j++) b[j] = a[j];

delete[] a;

a = b;

size = newSize;

}

}

}

a[i] = i * i;

$\theta(n)$

$\theta(n)$

$\theta(n-1)$

$\theta(1)$

$\theta(1)$

$\theta(1)$

$\theta(1)$

$\theta(1)$

$\theta(1)$

$\theta(1)$

$\theta(1)$

$\theta(1)$

if statement executes while $n > 10 \cdot (3/2)^k$, where k is num times if statement is true. Based on n, the if statement runs $\log_{3/2}(n/10)$ times. By change of base, $\theta(\log_{3/2}(n/10)) = \theta(\frac{\log_2(n/10)}{\log_2(3/2)}) = \theta(\log n)$. The only non- $\theta(1)$ runtime line in the if statement is the for loop from j=0 to j < size, but every time that loop executes, it will simply execute $3/2$ times more than before - the first time, it will run 10 times (constant), the second it will run $3/2$ (constant) times 10 (constant) = 15 (constant) times, and so on, meaning everything in the if statement takes constant time, aka $\theta(1)$.

So:

$$\theta(1) + \theta(1) + \sum_{i=0}^{n-1} \theta(1) + \sum_{i=0}^{\log n} \theta(1) = \theta(1) + \theta(1) + \theta(n-1) + \theta(\log n) = \theta(n)$$