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Question 1

d(i) $y[n] = x[n] + 3x[n-2]$

Linearity check:

For a system to be linear, it must satisfy both additivity and homogeneity.

Additivity:

$$T\{x_1[n] + x_2[n]\} = (x_1 + x_2) + 3(x_1 + x_2)[n-2]$$

$$= x_1 + x_2 + 3x_1[n-2] + 3x_2[n-2]$$

$$= (x_1 + 3x_1[n-2]) + (x_2 + 3x_2[n-2])$$

$$= T\{x_1[n]\} + T\{x_2[n]\}$$ $\therefore$ Additivity holds.

Homogeneity:

$$T\{\alpha x[n]\} = (\alpha x) + 3(\alpha x)[n-2]$$

$$= \alpha x + 3\alpha x[n-2]$$

$$= \alpha x + \alpha 3x[n-2]$$

$$= \alpha(x + 3x[n-2])$$

$$= \alpha T\{x[n]\}$$

$\therefore$ Homogeneity holds

Both additivity ~~and~~ and homogeneity holds. Thus, this system is linear.

a(ii) $y[n] = n^2 x[n-1]$

Check Additivity:

$$T\{x_1[n] + x_2[n]\} = n^2 (x_1 + x_2)[n-1]$$

$$= n^2 (x_1[n-1] + x_2[n-1])$$

$$= n^2 x_1[n-1] + n^2 x_2[n-1]$$

$$= x_1 n^2 [n-1] + x_2 n^2 [n-1]$$

$$= T\{x_1[n]\} + T\{x_2[n]\}$$

$\therefore$ Additivity holds.

Check Homogeneity

$$T\{\alpha x[n]\} = n^2 \alpha x[n-1]$$

$$= \alpha (x n^2 [n-1])$$

$$= \alpha (x[n^3 - n^2])$$

$$= \alpha (x[n])$$

$\therefore$ Homogeneity also holds.

Both additivity and homogeneity are satisfied. Thus, this graph is also linear.

b(i) time domain signal

$$x(t) = \cos^2(4000\pi t)$$

The nyquit frequency is half the sampling rate:

$$f_s \geqslant 2f_{max}$$

$$x[n] = \cos(\omega_0 n)$$

Frequency: $T = \frac{1}{f}$

Angular frequency

$$\omega_0 = 2\pi f_0$$

$$f_0 = \frac{\omega_0}{2\pi}$$

$$\omega_0 = 4000\pi \qquad f_0 = \frac{4000\pi}{2\pi}$$

$$x[t] = \frac{1 + \cos(2t)}{2}$$

$$= \frac{1 + \cos(2 \times 4000\pi t)}{2}$$

$$= \frac{1 + \cos(8000\pi t)}{2} = \frac{1}{2} + \frac{1}{2}\cos(8000\pi t)$$

Plug into

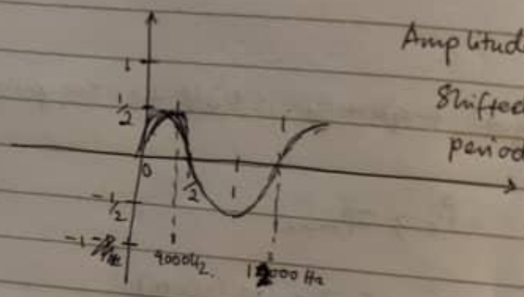
$$f_0 = \frac{\omega_0}{2\pi} = \frac{8000\pi}{2\pi} = 4000$$ ← max Frequency.

$$f_s \geqslant 2f_{max}$$

$$f_s \geqslant 2(4000)$$

$f_s \geqslant 8000$ Hz ← This is the min nyquist sample.

b(ii) $\frac{1}{2} + \frac{1}{2}\cos(8000\pi t)$



Amplitude is 1/2

Shifted to right by 1/2 period of 8000π

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Question 2

$x[n] * h[n]$

given $h[n] = 2\delta[n] + \delta[n-1]$

and $x[n] = [5, -1, 4]$.

n	0	1	2
$x(n)$	5	-1	4

~~h[n]~~ n	0	1
$h[n]$	2	1

Convolution $y[n] = \sum_k x[k]\, h[n-k]$

$y[0] = [x_0] * h[0]$

$= 5 \times 2$

$= 10$

$y[1] = [x_0]h[1] * h[0]x[1]$

$= 5(1) + (-1)(2)$

$= 3$

$y[2] = x_1 h_1 + x_2 h_0$

$= (-1)(1) + 4(2)$

$= 7$

$y[3] = x_2 \times h_1$

$= 4(1)$

$= 4$

$y[n] = [10, 3, 7, 4]$

[Open]

Question 3

$$x(t) = \frac{2}{T}t \quad \text{for} \quad -\frac{T}{2} < t < \frac{T}{2}$$

$\frac{T}{2} = \pi$, so the range in π

$-\pi < t < \pi$.

The period : $T = 2\pi$

Check for even symmetry:

$$x(-t) = x(t)$$

$$x(-t) = \frac{2}{T}(-t) = -\frac{2}{T}t \neq x(t)$$

So not even.

$$x(-t) = -x(t)$$

$$x(-t) = -\frac{2}{T}t = -x(t)$$

So it meets the odd rule. Therefore, if signal is odd, there is no cosine.

Derivation of trigonometric Fourier:

$$x(t) = \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t) \qquad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\sin(n\omega_0 t)\, dt$$

$$= \frac{2}{2\pi}\int_{-\pi}^{\pi} x(t)\sin(nt)\, dt$$

odd function = 0

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} \sin(nt)\, dt = \frac{1}{\pi}\cdot 0 = 0.$$