Scott Clark
MPIPKS

Billiards

$$\Psi_{n,m} = \sqrt{\frac{4}{L_x L_y}} \sin\left(\frac{n\pi x}{L_x}\right) \sin\left(\frac{m\pi y}{L_y}\right)$$

$$E\Psi = \frac{-\hbar}{2m}\left(\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2}\right)$$

$$E = \frac{\hbar^2 \pi^2}{2m}\left(\frac{n^2}{L_x^2} + \frac{m^2}{L_y^2}\right)$$

$$\Delta E \Delta t = \hbar \Rightarrow \Delta E = \frac{\hbar}{\Delta t}$$

$$p = \hbar k = \frac{\hbar 2\pi}{\lambda} \Rightarrow \frac{\lambda}{p} == \Delta t$$

$$E = \frac{p^2}{2m} \Rightarrow p = \sqrt{2E}$$

$$\Delta E = \frac{\sqrt{2E}}{\lambda}$$

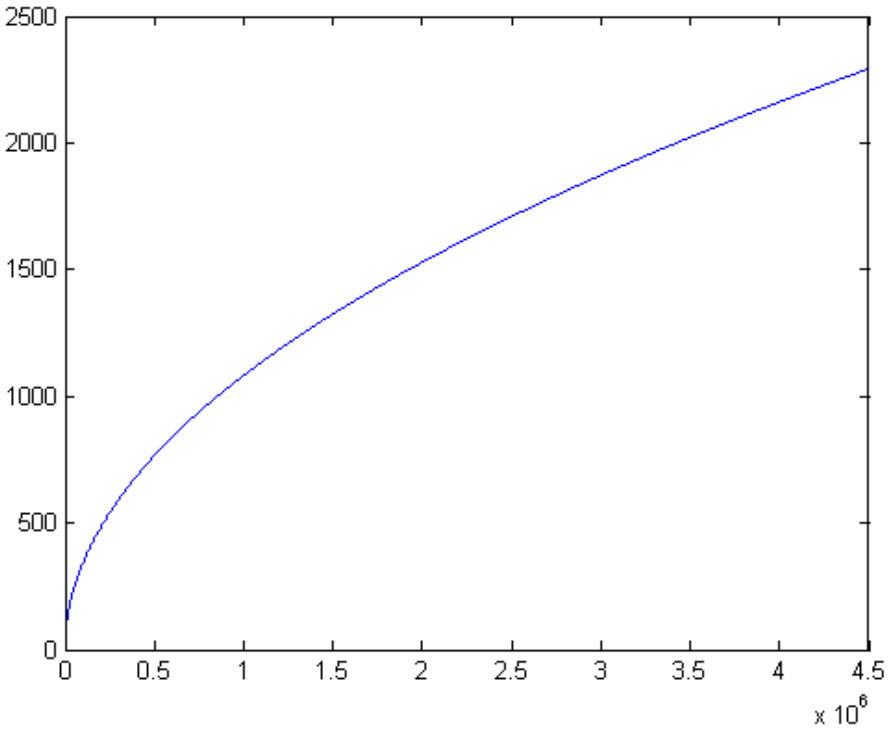


Figure 1: Energy with delta E

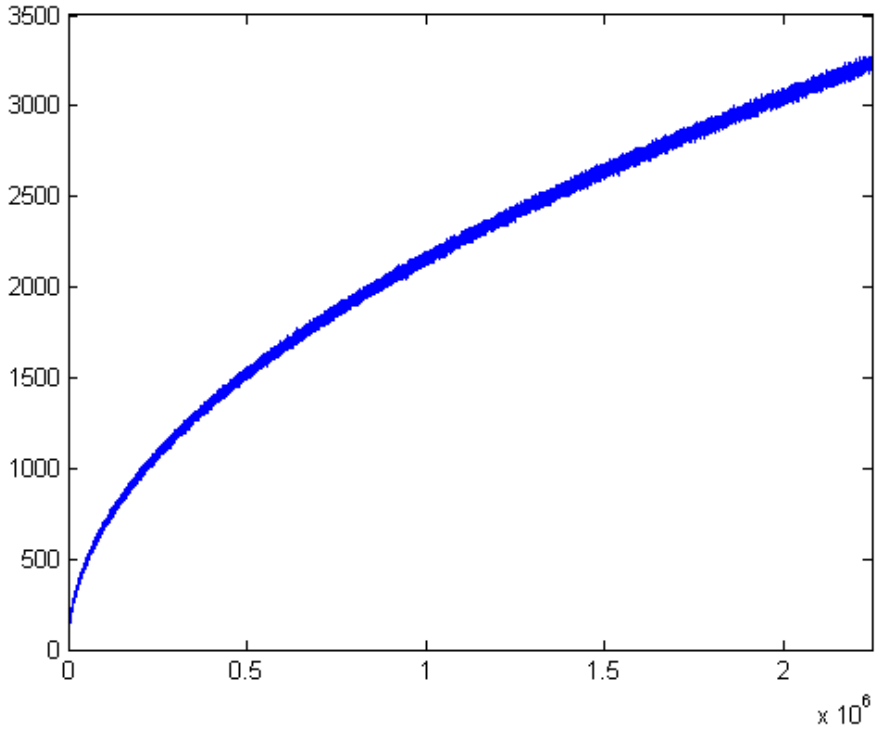


Figure 2: Neff for each energy (number of other states within $\Delta$ E of E)

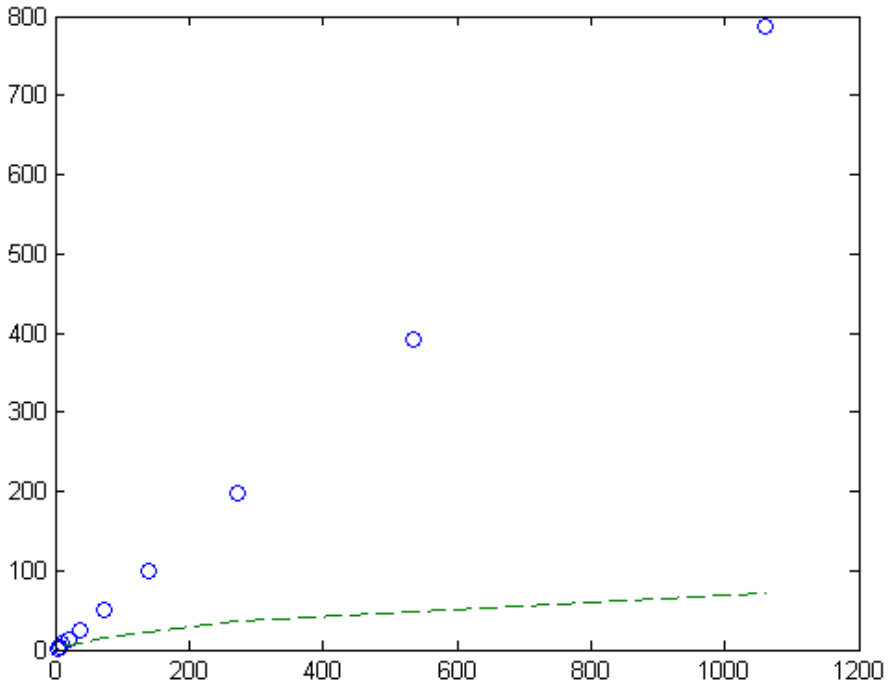


Figure 3: Nodes found 'o' and Neff '–' vs E

now we plot the distribution of the maximums (for the real case)

$$E = 1061$$

$$\Delta E = 35.1912$$

$$n_{eff} = 72$$

$$n_{maxima} = 787.6230$$

$$\bar{x} = 37.1436$$

$$\sigma = 6.4767$$

$$a = 34.2289$$

$$b = 5.0499$$

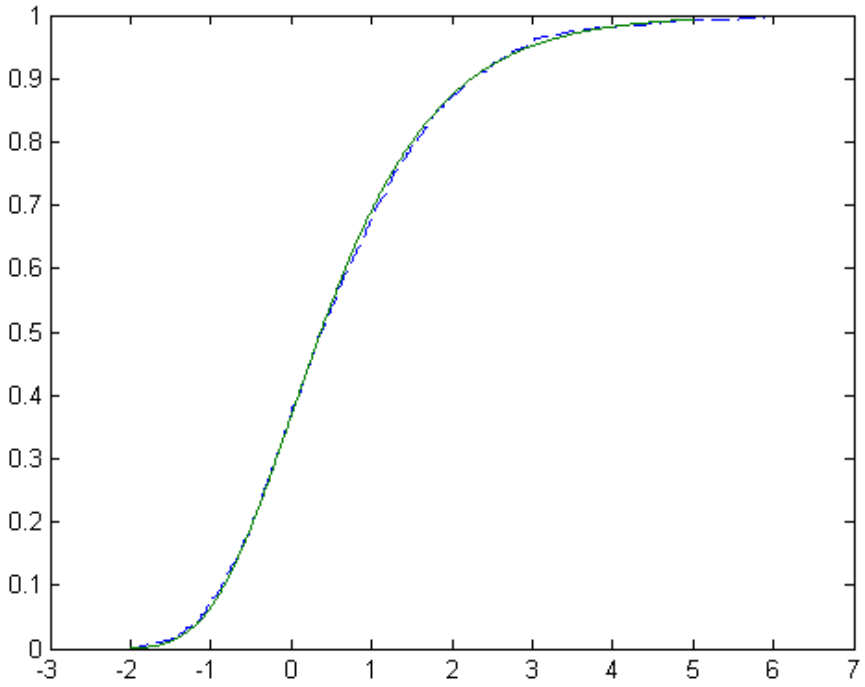


Figure 4: Gumbel and distribution of maximums

$$E = 536.2136$$

$$\Delta E = 25.0172$$

$$n_{eff} = 48$$

$$n_{maxima} = 391.6190$$

$$\bar{x} = 27.5574$$

$$\sigma = 5.1807$$

$$a = 25.2259$$

$$b = 4.0394$$

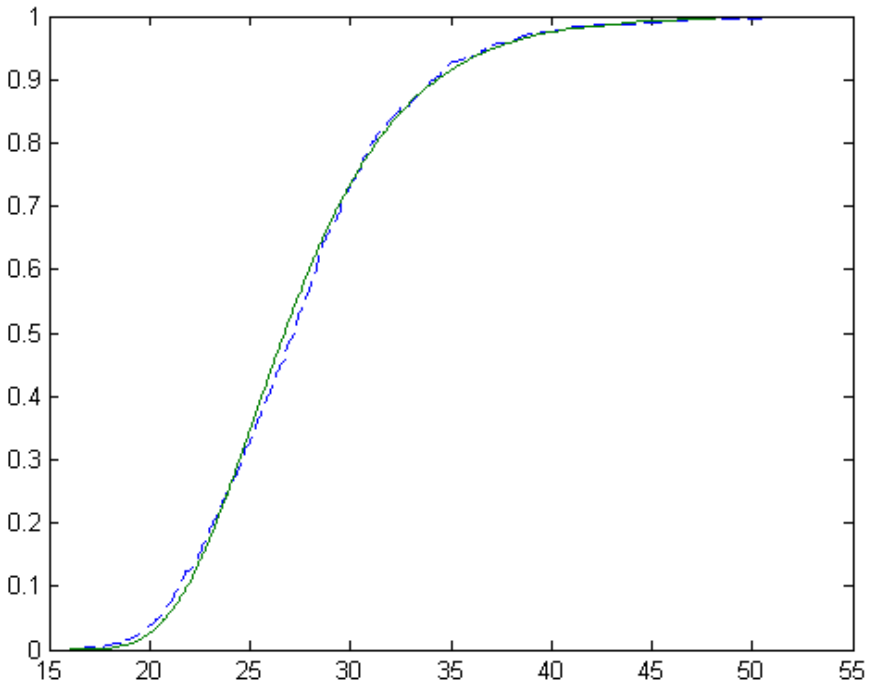


Figure 5: Gumbel and distribution of maximums

$$E = 273.5530$$

$$\Delta E = 17.8686$$

$$n_{eff} = 36.0000$$

$$n_{maxima} = 198.7900$$

$$\bar{x} = 21.8394$$

$$\sigma = 4.6864$$

$$a = 19.7303$$

$$b = 3.6540$$

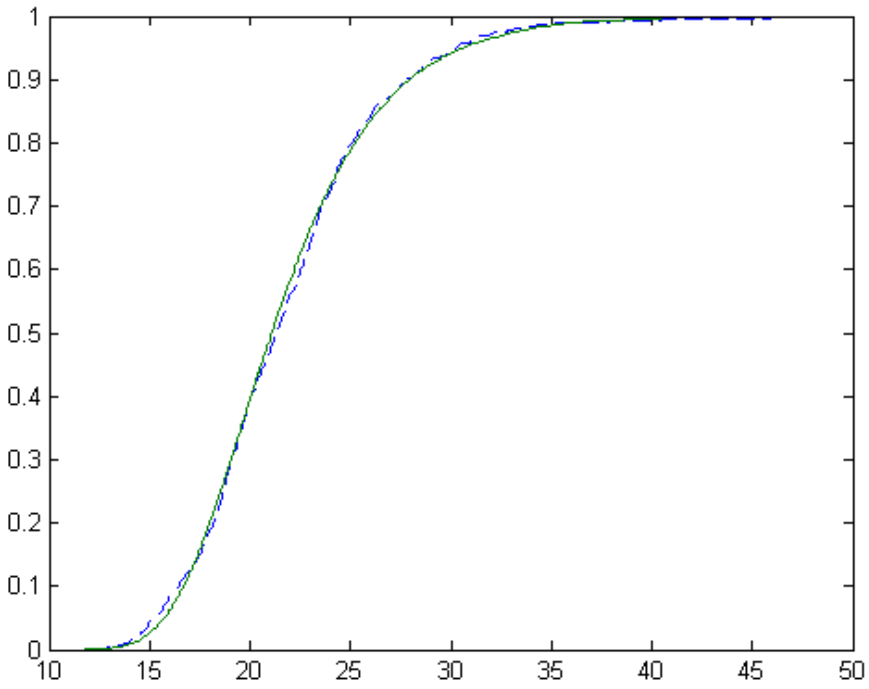


Figure 6: Gumbel and distribution of maximums

$$E = 140.4477$$

$$\Delta E = 12.8035$$

$$n_{eff} = 22.0000$$

$$n_{maxima} = 99.9460$$

$$\bar{x} = 15.3681$$

$$\sigma = 3.5654$$

$$a = 13.7635$$

$$b = 2.7799$$

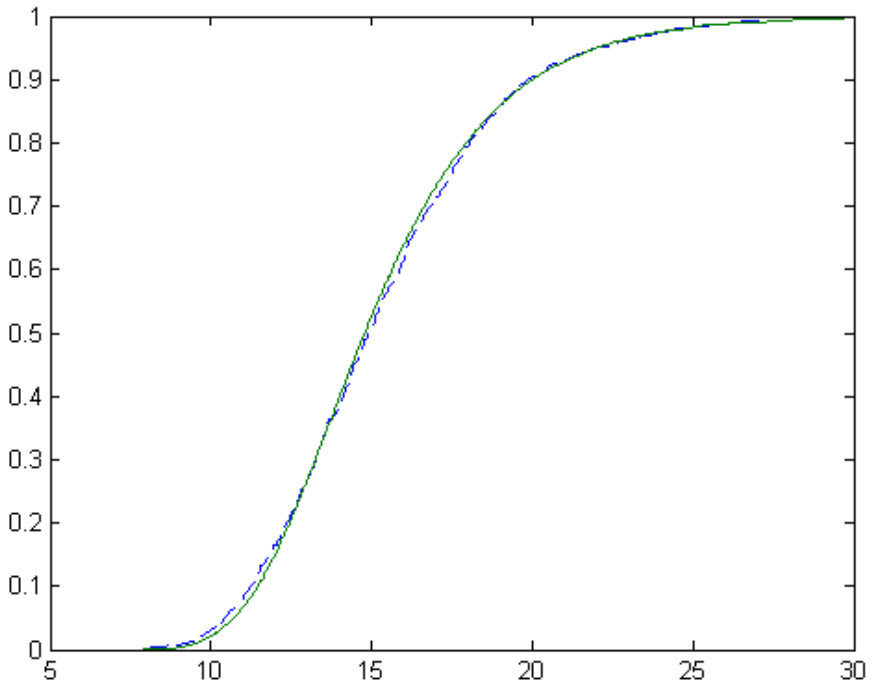


Figure 7: Gumbel and distribution of maximums

$$E = 73.8885$$

$$\Delta E = 9.2866$$

$$n_{eff} = 16.0000$$

$$n_{maxima} = 50.5520$$

$$\bar{x} = 11.7141$$

$$\sigma = 2.9421$$

$$a = 10.3900$$

$$b = 2.2940$$

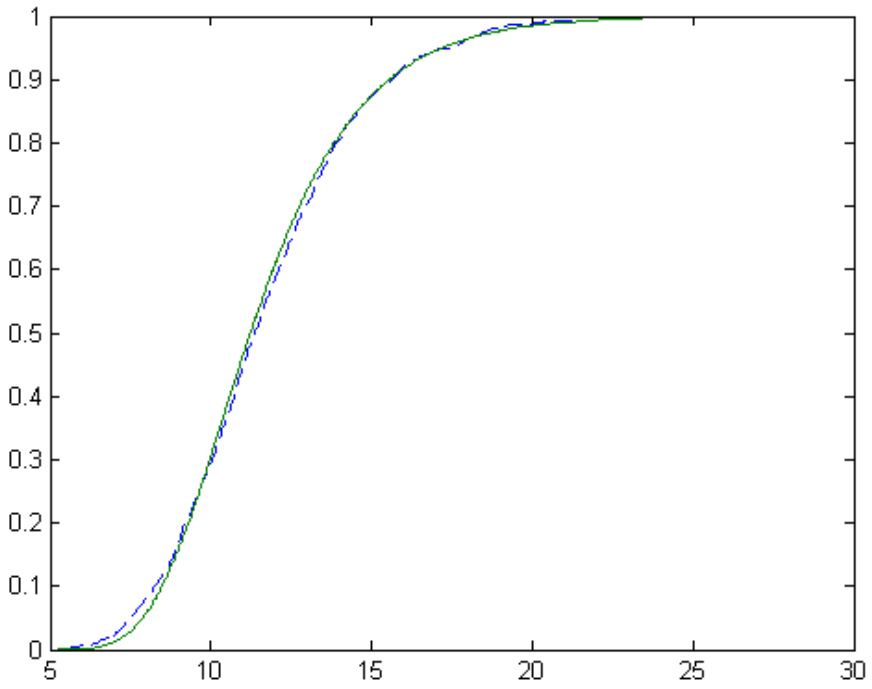


Figure 8: Gumbel and distribution of maximums

$$E = 38.4721$$

$$\Delta E = 6.7011$$

$$n_{eff} = 9.0000$$

$$n_{maxima} = 25.4700$$

$$\bar{x} = 7.5744$$

$$\sigma = 2.2453$$

$$a = 6.5639$$

$$b = 1.7506$$

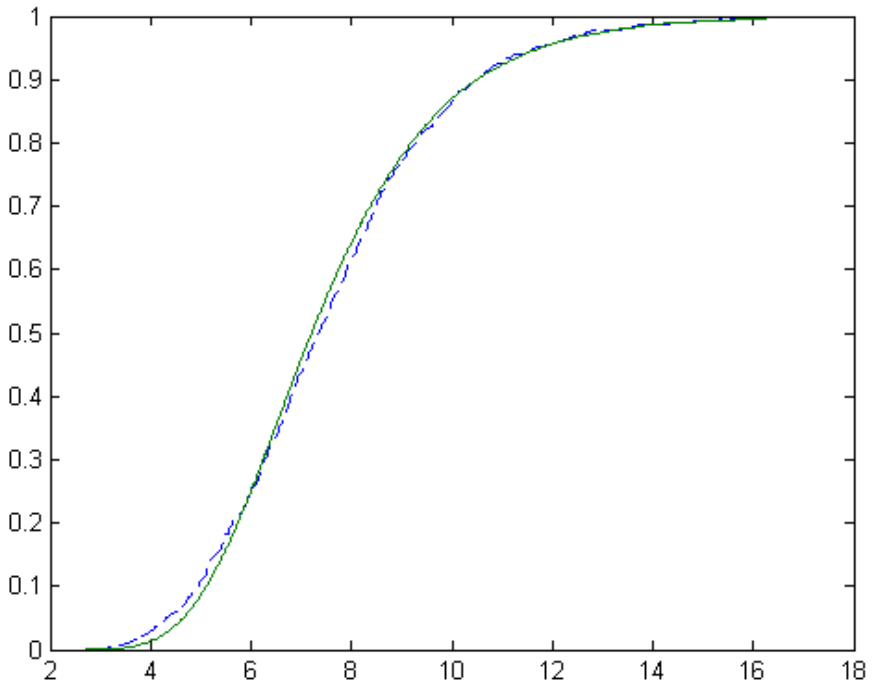


Figure 9: Gumbel and distribution of maximums