CS-E5740 Complex Networks, Answers to exercise set 1

Sara Cabodi, Student number: 784287

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Compile with `pdflatex ex_template.tex`

Problem 1

a) The adjacency matrix A is

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

b) The edge density of the graph is

$$\rho = \frac{2m}{N(N-1)} = \frac{2 * 9}{8(8-1)} = \frac{18}{56} = 0.32$$

where m is the number of edges and N is the number of nodes.

c) The degree k_i of a node i is is the number of edges it is incident to.

k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8
1	1	2	5	3	3	2	1

The degree distribution $P(k) = \frac{N_k}{N}$ are:

P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
0.125	0.125	0.25	0.625	0.375	0.375	0.25	0.125

d) The mean degree $\langle k \rangle$ of the graph is

$$\langle k \rangle = \sum_i \frac{k_i}{N} = \frac{2m}{N} = \frac{2 * 9}{8} = \frac{9}{4} = 2.25$$

e) The diameter d of the graph is the largest distance in the graph:

$$d = max_{i,j \in V} d_{i,j} = 4$$

The result is obtained calculating every distance between each couple of nodes and taking the maximum one.

f) The clustering coefficient defined for node i as the fraction of edges between its neighbours out of possible edges between its neighbours

$$c_i = \frac{2E_i}{k_i(k_i - 1)}$$

For the nodes with $k_i > 1$

- $c_3 = \frac{2*1}{2*1} = 1$
- $c_4 = \frac{2*2}{5*4} = \frac{1}{5} = 0.2$
- $c_5 = \frac{2*2}{3*2} = \frac{2}{3} = 0.67$
- $c_6 = \frac{2*1}{3*2} = \frac{1}{3} = 0.33$
- $c_7 = \frac{2*0}{2*1} = 0$

The average clustering coefficient is $C = \frac{0+0+1+0.2+0.67+0.33+0+0}{8} = 0.275$

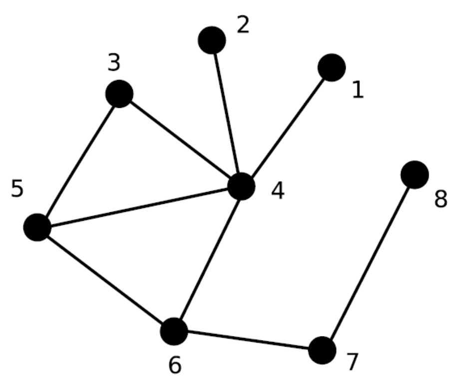

Figure 1: Graph for exercise 1.

Problem 2

a) The Karate club network is shown below

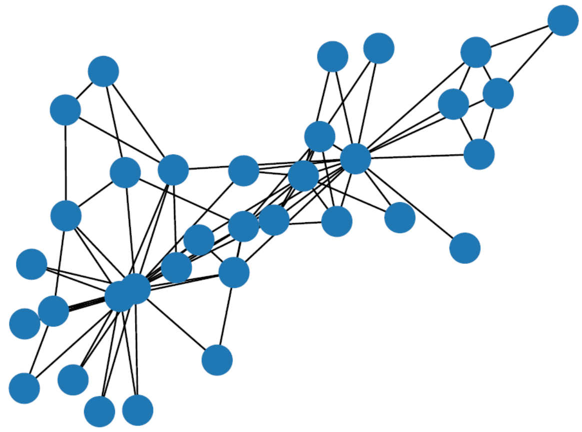

Figure 2: Karate club Network

b) The edge density of the network is $D = \frac{2*m}{N*(N-1)} = 0.13903743315508021$ and it's the same as the one computed with the NetworkX density function.

c) The average clustering coefficient computed with my own algorithm is $C = 0.5706384782076824$ and it's the same as the one obtained from the NetworkX function.

d) The two graphics are shown below

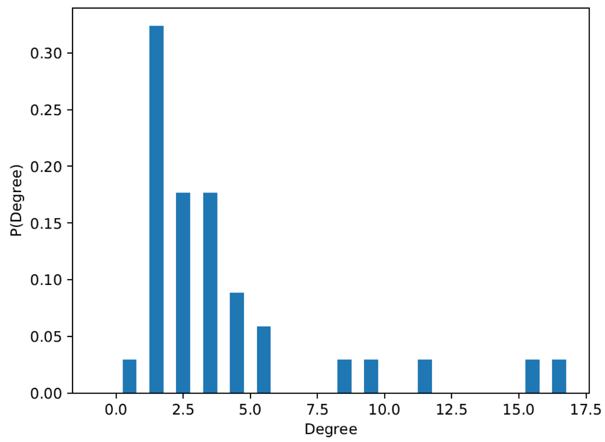

Figure 3: Degree distribution P(k)

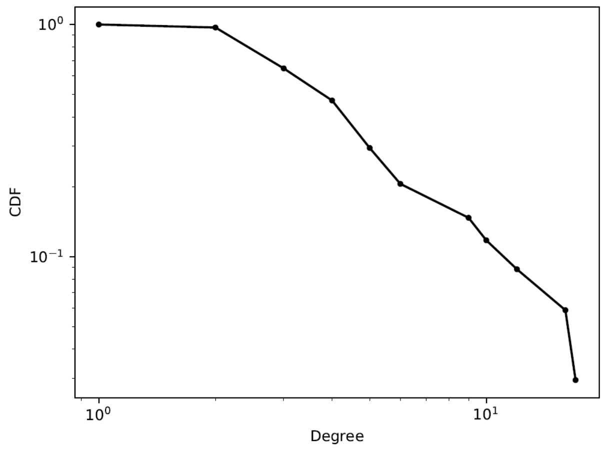

Figure 4: Complementary cumulative degree distribution 1-CDF

e) The average shortest path length $\langle l \rangle$ is 2.408199643493761

f) The scatter plot of C_i as a function of k_i is

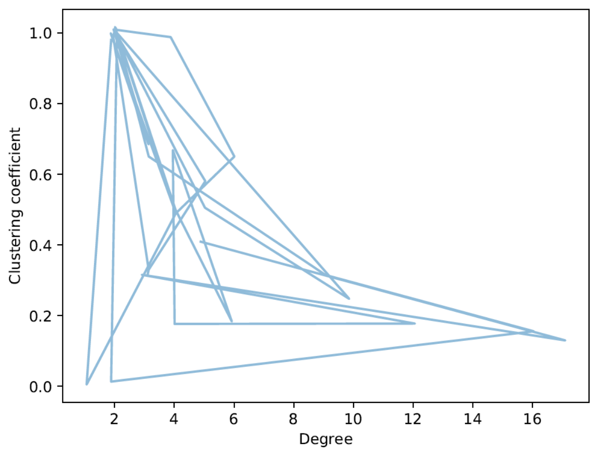

Figure 5: Scatter plot

Problem 3

a) The *induced subgraph* G* that is induced by vertices V* = {1,...,4} of network visualized in Figure 1.

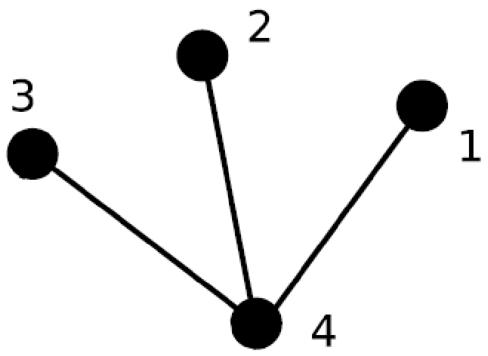

Figure 6: Induced subgraph G*.

The table below shows the number of walk of length two between all nodes pairs (i,j).

	1	**2**	**3**	**4**
1	1	1	1	0
2	1	1	1	0
3	1	1	1	0
4	0	0	0	3

The matrix A of the induced graph G* is

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

The matrix A^2 is

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

The two results, one from the first table and the other from matrix A^2, are equal.

b) There are 3 walks of length three from node 3 to node 4 (3-4-3-4, 3-4-2-4, 3-4-1-4). $(\mathrm{A}^3)_{3,4}$ is given by the dot product between the third row of A^2 and the fourth column of A^1.

$(\mathrm{A}^3)_{3,4} = a^2_{3,1} \cdot a^1_{1,4} + a^2_{3,2} \cdot a^1_{2,4} + a^2_{3,3} \cdot a^1_{3,4} + a^2_{3,4} \cdot a^1_{4,4} = 1 \cdot 1 + 1 \cdot 1 + 1 \cdot 1 + 0 \cdot 0 = 3$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

c) Given the matrix A^m and the adjacency matrix A^1, I show that the matrix $\mathrm{A}^{(m+1)}$ is composed by elements $a^{(m+1)}_{i,j}$ that are the scalar product of the i-th row of A^m and the j-th column of A^1.

A is a square matrix of size |V| x |V|, with |V| number of vertices.

$$a^{(m+1)}_{i,j} = \sum_{k=1}^{|V|} (a^m_{i,k} \cdot a^1_{k,j})$$

Where:

- $a^m_{i,k}$ is the number of walks of length m from vertex i to vertex k in matrix A^m
- $a^1_{k,j} \in \{0, 1\}$ depending if an edge exists between vertex k and vertex j
- $\sum_{k=1}^{|V|} (a^m_{i,k} \cdot a^1_{k,j})$ is the number of walks of length m+1 from vertex i to vertex j with last edge (k,j)

The inductive step is thus proved.