Scientific Diary

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Chapter 1

Probability

Theorem 1.0.1 (Kolmogorov). *Пусть* $\nu_{t_1 t_2 t_3 \dots t_k}$ *для* $\forall t_1, t_2, t_3 \dots t_k \in T, \quad k \in \mathrm{N}$ *являются вероятностными мерами на* R^{kn} *такими, что:*

$$\nu_{t_{\sigma(1)}, \cdots, t_{\sigma(k)}} (F_1 \times \cdots \times F_k) = \nu_{t_1, \cdots, t_k} \left(F_{\sigma^{-1}(1)} \times \cdots \times F_{\sigma^{-1}(k)}\right) \tag{1.1}$$

для всех перестановок $\sigma \in S_k$

$$\begin{aligned} &\nu_{t_1, \dots, t_k} (F_1 \times \cdots \times \cdots \times F_k) \\ &\quad = \nu_{t_1, \dots, t_k, t_{k+1}, \dots, t_{k+m}} (F_1 \times \cdots \times F_k \times \mathbf{R}^n \times \cdots \times \mathbf{R}^n) \end{aligned} \tag{1.2}$$

Тогда $\exists (\Omega, \mathcal{F}, \mathrm{P})$ *и случайный процесс* $\{X_t\}$ *на* Ω, $X_t : \Omega \to \mathrm{R}^n$

$$\nu_{t_1, \dots, t_k} (F_1 \times \cdots \times F_k) = P \left[X_{t_1} \in F_1, \cdots, X_{t_k} \in F_k\right] \tag{1.3}$$

Exercise 1.0.1 (Irwin-Hall distribution). *Try to derive Irwin-Hall distribution formula for pdf convolution formula.*

Chapter 2

Monte-Carlo Methods

Code Base

Heuristic 2.0.1. *There is one useful thing that i found while making Monte-Carlo simulations. Sometimes it is computatationaly costly to generate random variable from normal distribution and very good approximation for it appeared to be Irwin-Hall distribution.*

$$X_n = \sum_{i=0}^{n} U_k \quad \textit{where} \quad U_k \quad \textit{are independent random variables drawn from uniform distribution} \quad U(0,1) \tag{2.1}$$

The density function is given by:

$$f_X(x;n) = \frac{1}{2(n-1)!} \sum_{k=0}^{n} (-1)^k \left(\begin{array}{c} n \\ k \end{array} \right) (x-k)^{n-1} \operatorname{sgn}(x-k) \tag{2.2}$$

This pdf is basically piecewise polynomial function with $\mu = \frac{n}{2}$ *and* $\sigma = \frac{n}{12}$.

For $n = 12$ *it gives good approximation for normal distribution pdf.*

$$\phi(x) \approx \sqrt{\frac{12}{n}}(f_X(x;n) - \frac{n}{2}) - 6 \tag{2.3}$$

$$\phi(x) \approx f_X(x;n) - 6 = \sum_{i=0}^{12} U_k \tag{2.4}$$

2.1 Financial Instrument Pricing

2.1.1 Black-Scholes Model

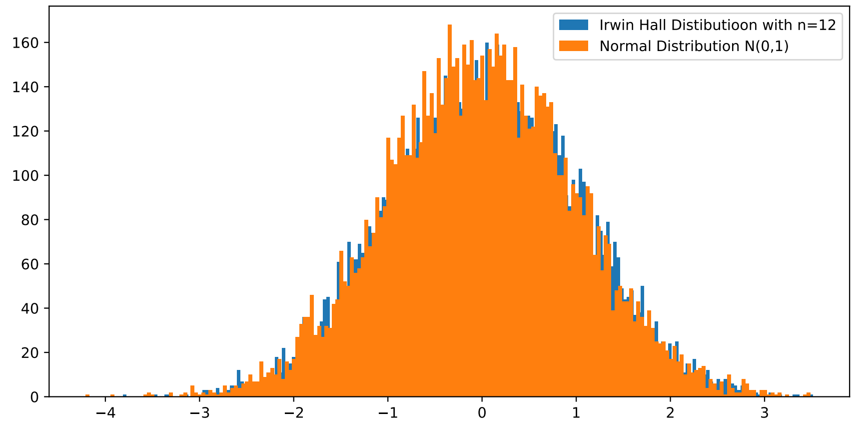

Figure 2.1:

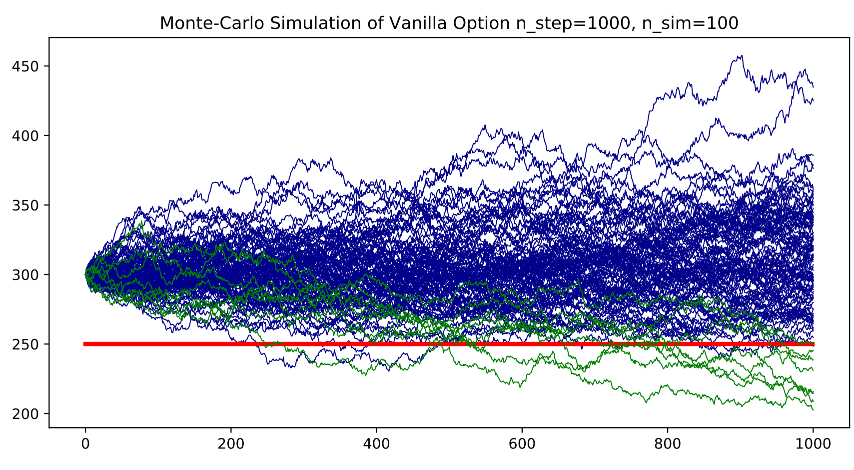

Figure 2.2: