Emergence of cooperation in structured networks

Eugenio Scafati 97959[1]
Ricardo Martins 67869[2]

Abstract
Cooperation has been crucial in the evolution of species. It's evident through history how species who learnt and managed the art of cooperation were able to evolve and be more prosperous than those who didn't. Here we introduce an agent-based simulation analysis on how the topology of the network plays a key role to determine whether a group of entities will cooperate between each other or not, and just seek to save their skin. We use the prisoner's dilemma model and test over several topologies and initial conditions how the fraction of cooperators changes while increasing the temptation to defect. Our work provides insights and shows how heterogeneous networks promote cooperation even under unfavorable conditions while more homogeneous ones tend to defect even when the temptation to defect is just above the reward of cooperating.

Keywords
network topologies — scale-free networks – evolution of cooperation — prisioners' dilemma

[1] *MEGI, Instituto Superior Técnico - Universidade de Lisboa, Portugal*
[2] *MECD, Instituto Superior Técnico - Universidade de Lisboa, Portugal*

1. Introduction

Cooperation, as working together for a common goal, is crucial for societies to evolve. Time has shown how societies and groups of entities who knew how to cooperate tend to evolve, and those who don't simply vanished. This is evident everywhere in nature, from ants working together as a group to carry resources and themselves to places that would be unreachable without the help of their peers, to humans cooperating to solve everyday tasks. Even now, in the middle of an epidemic, it's evident how countries with citizens that adopt a behavior more cooperative are showing lower number of infections than those who ignore it[1].

We find both crucial and stimulating to understand how cooperation works and persists in a world that is dominated by competition and with the famous Theory of Evolution presented by Charles Darwin [2], that stands on the premise of natural selection with the well-known phrase " survival of the fittest". That's why cheaters, those who benefit without making sacrifices, are likely to evolve because they will have an edge over individuals who spend energy on helping others, thus threatening the stability of any cooperative venturev[3]. So, why would we tend to cooperate?

Upon this inquiry we used Scale-free Network (SFN) [1] and more homogeneous networks [2] to model and see whether the structure of the network had any influence on the strengthening of cooperation and how cooperation evolves on time as the temptation to defect grows.

To represent human's cooperation first we have to understand how social networks are structured. This has been well studied in the past and we can be assertive when we express that they tend to imitate the behavior of a SFN. To sustain this, one can see Vilfredo Pareto's work, where he observed centuries ago that the wealth distributes asymmetrically in the world as the 80% of it its conserved by the 20% of the population[4]. Another example is shown by the father of scientometrics: Derek De Solla Price with his famous "Price Law", that states how value creation is not symmetric since 50% of the work is done by the square root of the total number of people who participated in the work[5]. Work is openly defined as anything you want, goals scored in a football match, subscriptions on YouTube channels, classical music reproductions on Spotify, and so on. The important thing here is to understand how social networks tend to structure themselves. In synthesis few individuals tend to have a larger number of connections (higher degree) and lots of individuals tend only few connections (lower degree) in Fig 1.

[1] SFN are networks with a small amount of highly connected nodes and lots of nodes with few connections

[2] In homogeneous networks the distribution of the connections follows a either regular where everyone has the same degree or follows a normal distribution, meaning that is poorly clustered

This study fits in the spectrum of Evolutionary Game Theory, where researchers tend to use Prisioners' Dilemma (PD) framework to model and test hypothesis [6]. PD is a game where two entities have to choose whether to cooperate or defect with another entity with whom they can't communicate. The best overall scenario is for both to cooperate but there's a big risk in taking this decision since if the other one defects, you won't get nothing out of it and the other party will get all of the reward.
In our study we are able to show, by running thousands of computer simulations how heterogeneous networks (SFN) tend to have higher fractions of cooperators even when unfavorable conditions are imposed, than more homogeneous and alike networks. As the Chinese philosopher Confucius said honest people coexist in a well-built society even if they are different [7]. Finally, we were able to perceive how cooperation tend to depend more in the structure of the ties between individuals than in the temptation to defect. This is concluded by observing how heterogeneous networks show robustness and persistence even in higher values of b (temptation to defect) while in more homogeneous networks, the cooperation vanishes quickly with low values of b. Regarding this behavior, the most assertive way to know if the cooperation in a society will persist is by knowing its structure and not the prize of defecting and just save your own skin.

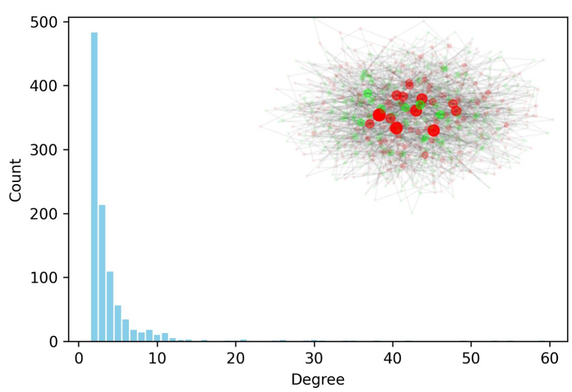

Figure 1. Degree distribution for the Barabási-Albert (BA) network and it's graphical representation with cooperators presented in green, defectors in red and the size of the nodes is proportional to its degree. The degree distribution follows a power law, having a low number of nodes with high degree and the majority with low degree. For this case the distribution of cooperators and defectors followed the trategy presented in 2.5.

2. Methods

2.1 Networks topologies

The networks we used to evaluate our hypothesis were: regular graph; random graph; and SFN. For consistency of results the size of the network was set to 10^4 and the average degree was fixed to 4 as sugested in [8]. The networks were created using the built-in function in NetworkX [9].

In order to create the regular graph we used the Watts-Strogatz model, considering the probability of rewiring each edge of the graph to 0 ($p_{sw} = 0$), so every node had exactly 4 connections. On the other hand, by setting $p_{sw} = 1$, we created a graph very similar to a random graph that is poorly clustered [10]. It is well known that the likelihood of cooperative strategies evolving out of an initial cooperative/non-cooperative mix also decreases with increasing p_{sw} [10].

To generate the SFN we used the BA formulation which considers growth and preferential attachment as mechanisms common to a number of complex systems, including business networks, social networks (describing individuals or organizations), transportation networks, and so on [11]. These large SFN share the common feature that the distribution of their local connectivity is free of scale, following a power law for large degree $P(k) \sim k^{-\gamma}$, with an exponent γ between 2.1 and 4 [11]. In order to have and average degree of 4 we set the number edges added in each iteration while the network is created as $m = 2$.

In all three topologies we fixed the seed so that the stochastic process of generating them is controlled and we created five different graphs for each of them to reduce the variability of the results since we encountered with significant evidence that this helped to have more accurate results.

2.2 Prisioners' Dilemma

As mentioned above, we used the well-known game theory metaphor PD. The implementation of the PD [6] followed a the simplified procedure proposed by Nowak and May [12], with a single parameter (b) represententing the temptation to defect. This free parameter is evaluated from 1 to 2 in steps of magnitude 0.1. The payoff matrix is shown in 1.

The strategy of the game implemented consists on a single play at each generation where the action of the nodes involved was pre-estabelished by either the random generation in the first iteration or the previous update of state in the evolution. The simple 2-state game was choosen, so the total number of games played per generation was two times the number of edges in the network.

Table 1. Prisioners' Dilemma payoff matrix with representation of the free parameter $b \in]1,2[$.

		Neighbour	
		Cooperate	Defect
Node	Cooperate	1	0
	Defect	b	0

Using the strategy of the game, at each generation the PD is evaluated for all pairs of nodes, and a cummulative payoff (fitness) is calculated for each node. This quantity allows to calculate the probability of the node of changing its strategy (C or D) for the following round.

2.3 Simulations

The study of the evolution of the state of each node was perfomed over 100 simulations and the results were averaged for consistency. For those simulations, 5 different network configurations were considered. Each simulation consisted of 10000 generations to overcome the transient stage and 1000 generations where we supervised the number of cooperators and defectors. All these parameters are summarized in 2.

Table 2. Parameters used in the simulations.

Number of nodes	10^3
Average degree	4
Number of simulations for transient stage	10^4
Number of supervised simulations	10^3
Number of simulations	100

Before starting the evolution process, the network is generated accordingly with the description on 2.1. A balanced and random distribution of cooperators and defectors were assigned to each node before starting the evolution. This phase was performed with two different strategies. First to evaluate the influence of the topology in the promotion of cooperation and later, as it is found that the SFN promotes cooperation, we tested those results placing defectors in the nodes with higher degree.

At the beggining of each iteration the PD game is played between all players. Then each node selects randomly a neighbour and in case the fitness of its opinion is lower than its opponent $(P_{node} - P_{neig})$, the node adopts the opponent's opinion with a probability that is a function based on the difference in the individual fitness of each player and the temptation to defect $(T - S)$ given by 1.

$$p = \frac{P_{node} - P_{neig}}{max\{k_{node}, k_{neig}\} \cdot (T - S)} \tag{1}$$

After all nodes play once with a random neighbour, a new generation starts. The evaluation of the fraction of cooperators is evaluated during the supervised generations after the trasient stage. For the purpose of this simulation we decided to follow the same parameters sugested by Santos e Pacheco [8].

2.4 Computing & Processing

The implementation of the procedure described was done in Python using the package NetworkX which has already implemented useful functions for the creation, manipulation, and study of complex networks [13].

Since this analysis is a stochastic process, to execute a fair number of simulations to have representative results and allow real time comuting, all the simulations were perfomed in parallel computing using the available online clusters in Google Colaboratory. These cloud computing technique proved to be at least 3 times faster than the local machines available.

For parallelization we splited the computations for different values of b in several files independent from each other. A deeper level of parallization was not developed as the maximum number of clusters available per user in the free version is 5, with the maximum sequentially runtime of 12 hours, which was in line with the performance of our code.

As the network was generated within the file, to ensure the that the same network was used for all the separate files we locked the seed number. The total computation time for generating these results was 500 hours. The output of each simulation was printed in a txt file that we then import it into another Python notebook for further analysis and visualization of the results.

2.5 Cooperation in Unfavorable Conditions

In order to simulate the cooperation in unfavorable conditions in a heterogeneous network, we did the same process as expressed before but, instead of distributing the states randomly at the start, we forced the top 1% of nodes with higher degrees to start as defectors. This constitutes an unfavorable scenario for cooperation since the hubs of the network start with a defector state (see 1). The rest of the nodes have their state assigned randomly. We only did this using the BA model since it's the only structured network where cooperations have emerged. This allows a direct comparison with the fraction of cooperators changes with an initial random distribution of states.

3. Results and Discussion

3.1 Topology influence on cooperation

The first relevant results obtained from the agent-based simulations we computed show strong evidence that heterogeneous networks such as the SFN of BA, which we used to model social networks, promote cooperation in a much more robust way than homogeneous networks as the temptation to defect gets higher.

As is shown in Figure 2 the fraction of cooperators for both Watts-Strogatz networks decreases once the benefit of defecting starts growing. In the case of the Regular graph (each node having exactly 4 connections) the fraction of cooperators hits nulls cooperation almost instantly, while in the case of random graphs (where connections of each node follow a normal distribution with mean value equal to 4) the fraction of cooperators show some resistance at the beginning but once the b=1.5 threshold is overcome, the temptation to defect plays gets more important than the structure of the network and the defectors win the majority in the pool.

This means that to generate a network that is robust and promotes a cooperative strategy, the way the connections are distributed plays a key role. At the same time, as the distribution of the connections get more and more heterogeneous, the fraction of cooperators after a transient stage tends to achieve higher values and resist the temptation to defect. This fact occurs even for high values of b, where b represents the reward of defecting.

On the other hand, having a heterogeneous network such as the BA model, we start having nodes few nodes with a lot of connections, that we refer as hub. One should not take it to the extreme because that would mean just one node connected to the rest and in that scenario, the fraction of cooperators outcome depends completely in the initial state of the hub.

This implication is relevant since one can partially understand why humans, living in a world based on competition that is naturally structured in hierarchies [14], the members of societies, such as humans, tend to cooperate with others and promote others work even when sometimes it doesn't seem to be the best individual option at the moment.

At the same time, it promotes the idea that everyone involved in the group benefits when cooperation plays a role in the network, and that the reward from cooperating and working as teams in the long term always beats defecting the society and trying to generating everything alone.

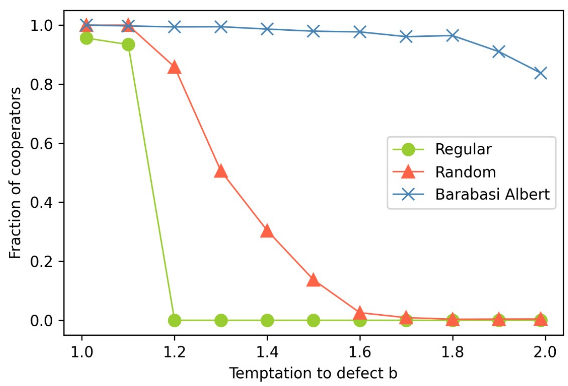

Figure 2. Average fraction of cooperators as a function of the temptation to defect (b), for 10^3 generations, after the trasient stage of 10^4 generations, for the networks created with Watts-Strogatz model setting $p_{ws} = 0$ for the regular graph (in green), $p_{ws} = 1$ for the random graph (in orange), and the Barabási-Albert model. All graphs having an average degree of 4. It is shown that the network topology influences the fraction of cooperators.

3.2 Unfavourable initial conditions

The second important result that we obtained from our analysis answers the statement proposed by the Chinese philosopher Confucius related to the honesty of humans even when diversity and unfavourable conditions are prevalent [7]. We put this into test by stating that the 1% of the higher connected nodes will start as defectors to prove that even with the generations starting in this unfair scenario, cooperation may emerge. The distribution of the connections is the same as the random case, but is clear how the higher connected nodes are classified as defectors at the beginning, as shown in Figure 3.

The evolution of the fraction of cooperators with the generations, shown for the two cases under analysis and considering b=1.8 in Figure 4, represents that having hubs classified as defectors almost vanishes cooperation in the first generations, until the structure of the network starts to play a role and cooperation rises again.

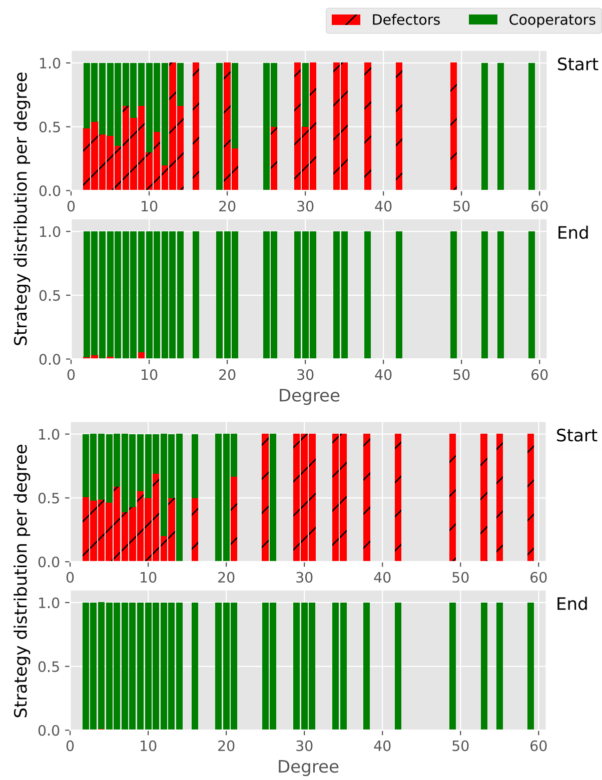

Figure 3. Distribution of cooperators and defectors per degree of the node for the BA graphs, considering with random distribution of states (top) and with defector occupying 1% of the nodes with higher degree (bottom). For each case its shown the initial distribution (before the strategies start to evolve) and at the end of the evolution (after $1.1 \cdot 10^4$ generations). The results considered are for one of the simulation and having $b = 1.8$. It is clear for both cases that for these networks the evolution promotes cooperation.

What we observe here, as shown in Figure 5, the network presents strong evidence that unfavourable initial conditions do not create a significant variation in the long term. Cooperation in this SFN is robust since the fraction of cooperation is similar for unfavourable and randomized distribution of defectors, independently of the value of b chosen.

We found this relevant when analysing the behaviour in societies where hubs are occupied by people with larger number of connections such as celebrities or politicians. When these hubs adopt strategies that are against a common good, the members of the society are often able to continue to cooperate between each other without getting much influence from the negative actions of the hubs. Not only that, but also influencing them to change their behaviour to a more cooperative-style as one can appreciate in Figure 3 where it is shown how the fraction of cooperators change at the start and at the end of the simulations for each value k of connections.

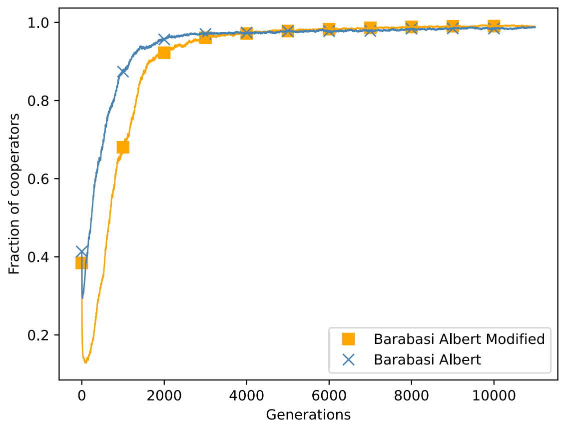

Figure 4. Evolution of opinion with time (generations) for the BA network considering both the random distribution of states (blue cross) and the defectors placed in nodes with higher degree (orange squares). The temptation to defect in this case is $b = 1.8$. We can observed that at the begining this temptation has an important role and cooperation reaches minimum values. Nevertheless the structure of the graph starts to play an important role promoting the evolution of cooperation. These results where obtained averaging over 10 simulations for each network.

This investigation was based in the paper ”A new route to the evolution of cooperation” [8]. We compared the results obtained in our work with those presented in [8] and they are consistent.

On the other hand these simulations contrast with the experiments found in ”Heterogeneous networks do not promote cooperation when humans play a Prisoner’s Dilemma” [15]. In this case, the authors instead of creating agent-based computer simulations, performed and experiment with 1229 humans, connected them in a lattice and a SFN, and make them play a spatial PD with parameters $T = 10 > R = 7 > S = P = 0$. Their works show that the network structure has little relevance as a cooperation promoter or inhibitor among humans. We found this interesting since it contradicts our theoretical work by using other methods to answer the same question.

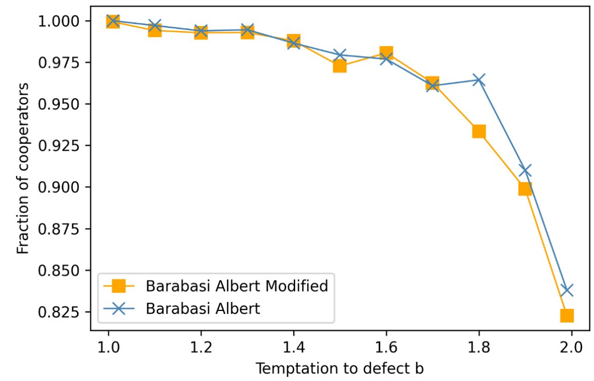

Figure 5. Average of the fraction of cooperators as a function of the temptation to defect (b) for 10^3 generations, after the trasient stage of 10^4 generations, for BA network considering both the random distribution of states (blue cross) and the defectors placed in nodes with higher degree (orange squares). Despite the distribution of defectors in the network a cooperative strategy is found in both cases for all values of b.

4. Conclusion

To sum up, our research shows how diversity in the structure of populations promote cooperation between the members, which in a practical sense means further evolution. We were able to contrast it and use as a benchmark the values of cooperation for less diverse topologies and see significance evidence to conclude that heterogeneous networks are much more robust against high values of the temptation to defect and even promote cooperation when the initial conditions are unfavourable.

We invite everyone to further analyse this stimulating field and we propose several paths for future investigations. We found interesting questions such as, what could be the possible threshold value of defectors for the initial configuration that makes nodes from a heterogeneous network not to cooperate with each other and behave as a homogeneous network? What other parameters influence the fraction of cooperators? Does the assortative or the clustering coefficient influences it? Finally, what happens if one fixes a certain number of nodes to always cooperate or defect no matter what their accumulated payoff is? We truly believe that all this questions and many others will add a lot of value to the understanding of evolution of biological species and will provide us with a further understanding of societies and why and how its members tend to cooperate even when it is not the best individual decision.

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