Problem Set 05 – Section 4.5 Applications of Congruences

All solutions must show all work and be written clearly and legibly. When applicable expand your answer to a short paragraph. Failure to not show work will result in no points awarded.

Due Date: October 2, 2017

Exercises – Section 4.5 Applications of Congruences

2. Which memory locations are assigned by the hashing function $h(k) = k$ **mod** 101 to the records of insurance company customers with these Social Security numbers?
a) 104578690 **b)** 432222187
c) 372201919 **d)** 501338753
3. A parking lot has 31 visitor spaces, numbered from 0 to 30.Visitors are assigned parking spaces using the hashing function $h(k) = k$ **mod** 31, where k is the number formed from the first three digits on a visitor's license plate.
a) Which spaces are assigned by the hashing function to cars that have these first three digits on their license plates: 317, 918, 007, 100, 111, 310?
b) Describe a procedure visitors should follow to find a free parking space, when the space they are assigned is occupied.
Another way to resolve collisions in hashing is to use *double hashing*. We use an initial hashing function $h(k) = k$ **mod** p where p is prime. We also use a second hashing function $g(k) = (k + 1)$ **mod** $(p - 2)$.When a collision occurs, we use a *probing sequence* $h(k, i) = (h(k) + i \cdot g(k))$ **mod** p.
4. Use the double hashing procedure we have described with $p = 4969$ to assign memory locations to files for employees with social security numbers $k1 = 132489971$,
$k2 = 509496993$, $k3 = 546332190$, $k4 = 034367980$,
$k5 = 047900151$, $k6 = 329938157$, $k7 = 212228844$,
$k8 = 325510778$, $k9 = 353354519$, $k10 = 053708912$.
6. What sequence of pseudorandom numbers is generated using the linear congruential generator $x_{n+1} = (4x_n + 1)$ **mod** 7 with seed $x_0 = 3$?
~~**16.** The ISBN-10 of the sixth edition of *Elementary Number Theory and Its Applications* is 0-321-500Q1-8, where Q is a digit. Find the value of Q.~~
The United States Postal Service (USPS) sells money orders identified by an 11-digit number $x_1x_2 \ldots x_{11}$. The first ten digits identify the money order; x_{11} is a check digit that satisfies $x_{11} = x_1 + x_2 + \cdots + x_{10}$ **mod** 9.
18. Find the check digit for the USPS money orders that have identification number that start with these ten digits.
a) 7555618873
b) 6966133421
~~**c)** 8018927435~~
~~**d)** 3289744134~~
20. One digit in each of these identification numbers of a postal money order is smudged. Can you recover the smudged digit, indicated by a Q, in each of these numbers?
a) Q1223139784
b) 6702120Q988
~~**c)** 27Q41007734~~
~~**d)** 213279032Q1~~

24. Determine the check digit for the UPCs that have these initial 11 digits.
a) 73232184434
b) 63623991346
~~**c)** 04587320720~~
~~**d)** 93764323341~~
Some airline tickets have a 15-digit identification number $a_1a_2 \ldots a_{15}$ where a_{15} is a check digit that equals $a_1a_2 \ldots a_{14}$ **mod** 7.
28. Find the check digit a_{15} that follows each of these initial 14 digits of an airline ticket identification number.
a) 10237424413392
b) 00032781811234
~~**c)** 00611232134231~~
~~**d)** 00193222543435~~
Periodicals are identified using an **International Standard Serial Number (ISSN)**. An ISSN consists of two blocks of four digits. The last digit in the second block is a check digit. This check digit is determined by the congruence $d_8 \equiv 3d_1 + 4d_2 + 5d_3 + 6d_4 + 7d_5 + 8d_6 + 9d_7 \pmod{11}$. When $d_8 \equiv 10 \pmod{11}$, we use the letter X to represent d_8 in the code.
32. For each of these initial seven digits of an ISSN, determine the check digit (which may be the letter X).
a) 1570-868
b) 1553-734
~~**c)** 1089-708~~
~~**d)** 1383-811~~