

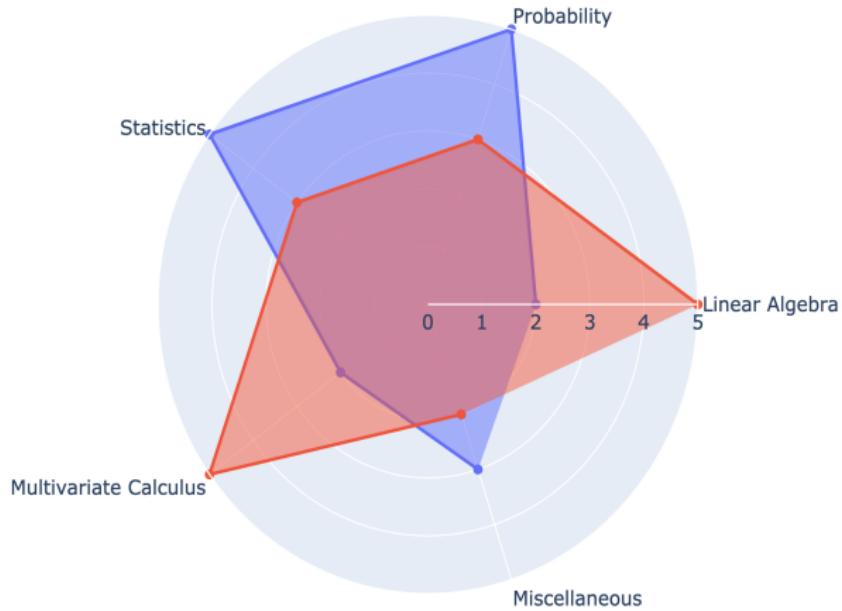
A GUIDED TOUR OF AI: FROM FOUNDATIONS TO LATEST APPLICATION

INTRODUCTION TO FUNCTIONS

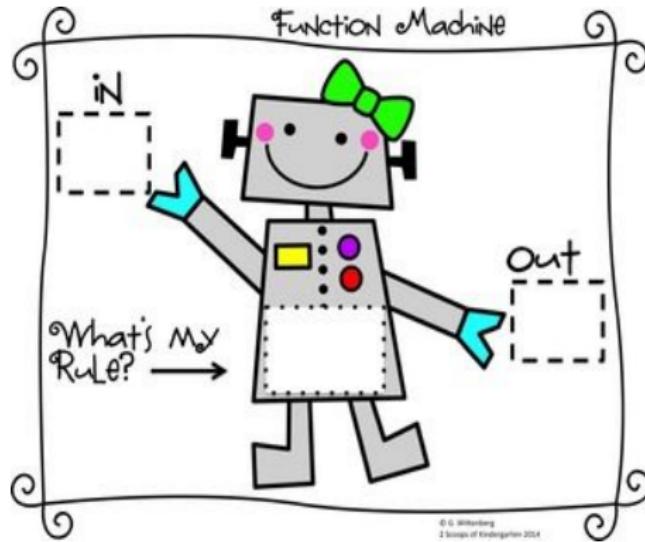
SCAI, Sorbonne University Abu Dhabi, Abu Dhabi, UAE



ROOTS OF MACHINE LEARNING

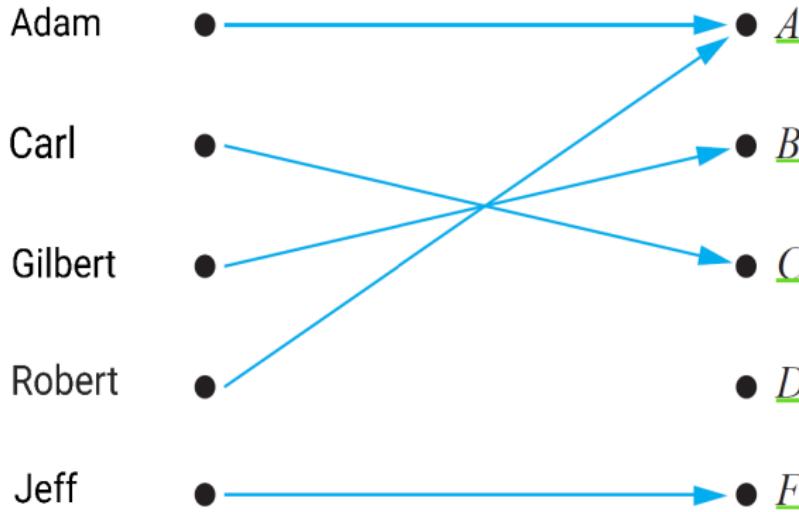


IMPORTANCE OF FUNCTIONS IN ML



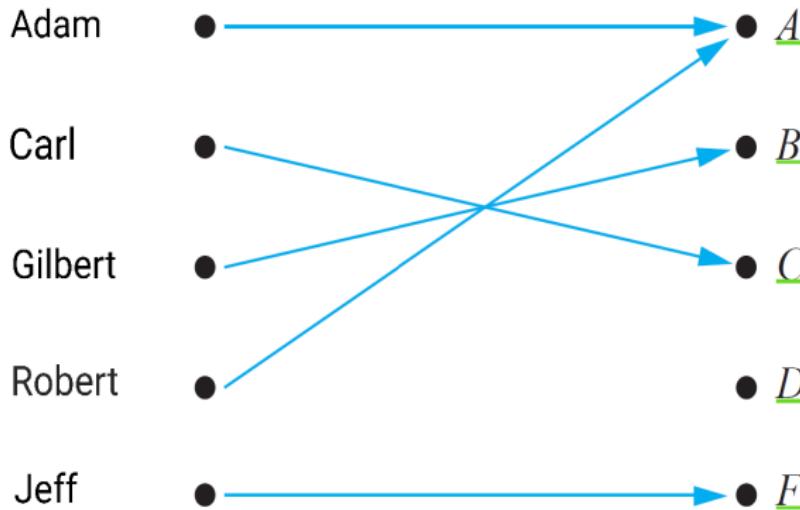
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Suppose that each student in a class is assigned a letter grade from the set $\{A, B, C, D, F\}$. And suppose that the grades are A for Adam, C for Carl, B for Gilbert, A for Robert, and F for Jeff.



FUNCTIONS

This assignment is an example of a function.



FUNCTIONS

- The concept of a function is extremely important in mathematics and computer science.
- For example, functions are used to represent how long it takes a computer to solve problems of a given size.
- Many computer programs and subroutines are designed to calculate values of functions.

DEFINITION

- Let A and B be collections of elements (that we call **sets**).
- A **function** f from A to B is an assignment of **exactly one element** of B to each element of A .
- We write $f(a) = b$ if b is the unique element of B assigned by the function f to the element a of A .
- If f is a function from A to B , we write

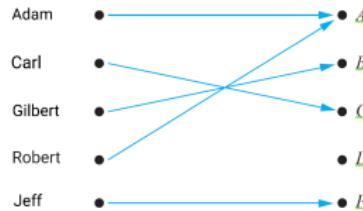
$$f : A \longrightarrow B$$

Remark: Functions are sometimes also called **mappings** or **transformations**.

FUNCTIONS

Functions are specified in many different ways.

- Sometimes we explicitly state the assignments, as in



- Often we give a formula, such as $f(x) = x + 1$, to define a function.

Some terminologies

- If f is a function from A to B , we say that A is the **domain** of f and B is the **codomain** of f .
- If $f(a) = b$, we say that b is the **image** of a and a is a **preimage** of b .
- The **range**, or **image**, of f is the collection of all images of elements of A .
- Also, if f is a function from A to B , we say that f **maps** A to B .

EXAMPLE

What are the **domain**, **codomain**, and **range** of the function that assigns grades to students described in the first slide?

EXAMPLE

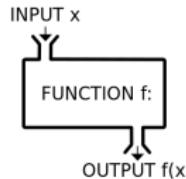
What are the **domain**, **codomain**, and **range** of the function that assigns grades to students described in the first slide?

Solution:

- Let G be the function that assigns a grade to a student in the class.
- Note that $G(\text{Adam}) = A$, for instance.
- The **domain** of G is the set Adam, Carl , Gilbert, Robert, Jeff.
- The **codomain** is the set A,B,C,D, F .
- The **range** of G is the set A,B,C, F , because each grade except D is assigned to some student.

FUNCTIONS

- To recap, a function is mainly a mathematical relationship from a set of inputs to a set of outputs.
 - The output value depends on (is a function of) the input value.
 - Each input produces exactly one output.



- Let's denote the input by the variable x . In this case, the output would be $f(x)$.
- From now on, we consider functions defined by their algebraic formula and depending on a variable x for example

$$f(x) = 3x^2 + 6$$

Function composition

- Given two functions $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ such that the domain of g is the codomain of f , their composition is the function $g \circ f: X \rightarrow Z$ defined by

$$(g \circ f)(x) = g(f(x)).$$

- That is, the value of $g \circ f$ is obtained by **first applying f** to x to obtain $y = f(x)$ and **then applying g** to the result y to obtain $g(y) = g(f(x))$.
- In the notation the function that is applied first is always written on the right.

FUNCTIONS

- Functions can be identified using **graphs**.
- Graphs display many input-output pairs in a small space. The visual information they provide often makes relationships easier to understand.
- We typically construct graphs with the input values along the horizontal axis and the output values along the vertical axis.

FUNCTIONS

- In general, the domain of a given function can be given by different sets (intervals) where for each set, a different rule is assigned.
- But in the sequel, we shall consider only the functions that are ruled by one algebraic formula all over the domain, that assigns to each input value one and only one output value.
- We shall also deal only with "nice enough" function, or mathematically, **continuous functions**, where we can draw the graph from start to finish without ever once picking up our pencil.

COMPUTATION OF VALUES

- For computing the image of a number (being in the domain of the given function), we replace x by the number in the algebraic form, then we simply compute.

EXAMPLE

- Given the function $f(x) = 3x^2 - 2$.
- What is the image of -2 by f ?

COMPUTATION OF VALUES

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EXAMPLE

- Given the function $f(x) = 3x^2 - 2$.
- what is the image of -2 by f ?

Solution:

- For computing the image of -2 by f , we replace x by -2 in the expression $3x^2 - 2$.
- In other words, we compute
$$f(-2) = 3(-2)^2 - 2 = 3 \cdot 4 - 2 = 12 - 2 = 10.$$
- Finally, the image of -2 by f is 10.

GRAPHIC REPRESENTATION OF A FUNCTION

In the coordinates plane, the **graphic representation** of a function consists of all the points of coordinates $(x, f(x))$, where x belongs to the domain of f .

EXAMPLE

Make a sketch of the graphic representation of the function
 $f(x) = 3x^2 - 2$.

EXAMPLE

Make a sketch of the graphic representation of the function $f(x) = 3x^2 - 2$.

Solution:

Since the **graphic representation** of a function consists of all the points of coordinates $(x, f(x))$, then

- We can start by computing the images of different points by f , and then gathering them eventually in a table of values.

x	-2	-1	-0.5	0	0.5	1	2
$f(x)$	10	1	-1.25	-2	-1.25	1	10

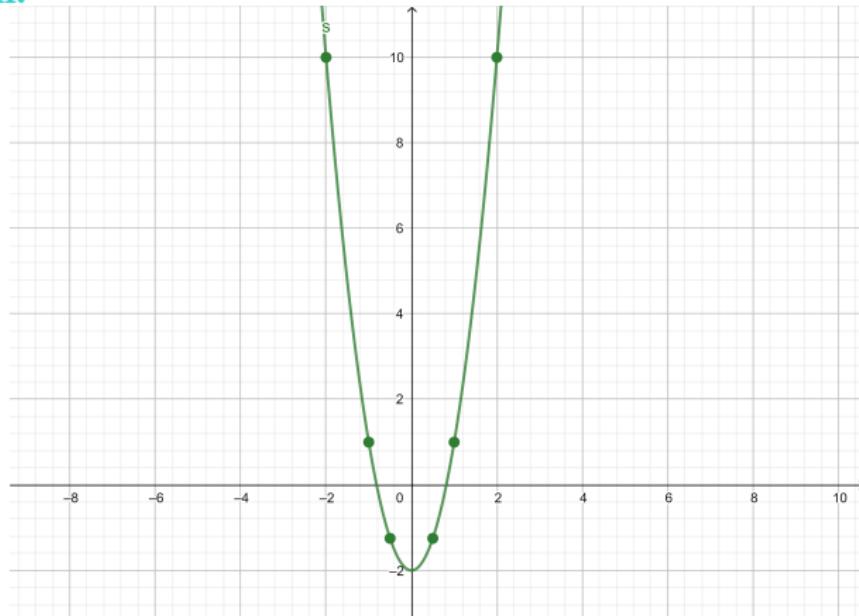
- Then, we can place the obtained points $(x, f(x))$ on the coordinate plane and connect them by considering that between two points, the evolution is progressive.

GRAPHIC REPRESENTATION OF A FUNCTION

EXAMPLE

Make a sketch of the graphic representation of the function $f(x) = 3x^2 - 2$.

Solution:



DIFFERENT TYPES OF FUNCTIONS

- In general, functions can have many different algebraic expressions ($\ln(x)$, $\exp(x)$, $\cos(x)$, $\sin(x)$...)
- The most **nice functions** are the polynomial ones. Their **domain** is the line of all the real values denoted \mathbb{R} .
- Generally speaking, a polynomial is a sum and subtraction of terms of the form kx^n where k is any number and n is a positive integer. In this case, the maximal occurring power is called **degree** of the polynomial. For example, $3x^2 + 2x - 5$ is a polynomial of degree 2.
- Polynomial functions of degree 1 are the affine function. Their graphs are straight lines.
- Polynomial functions of degree 2 are called quadratic functions. Their graphs are parabolas (U, \cap) .

EXAMPLE

Again, make a sketch of the graphic representation of the function

$$g(x) = 2x + 3$$

EXAMPLE

Again, make a sketch of the graphic representation of the function

$$g(x) = 2x + 3$$

Solution:

- Let's make a table of values.

x	-3	-2	-1	0	1	2	3
f(x)	-3	-1 (= -3 + 2)	1 (= -1 + 2)	3 (= 1 + 2)	5 (= 3 + 2)	7 (= 5 + 2)	9 (= 7 + 2)

- Then, we obtain the graph

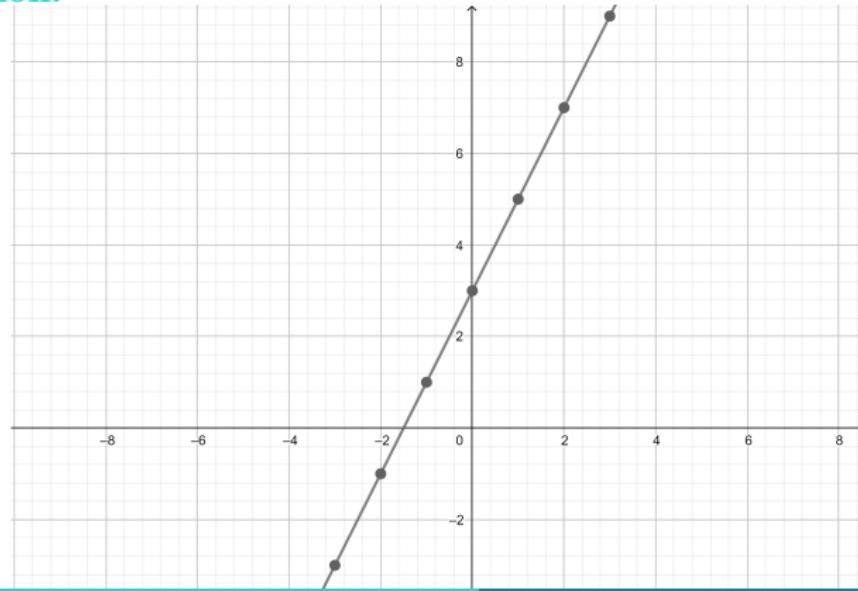
AFFINE FUNCTIONS

EXAMPLE

Again, make a sketch of the graphic representation of the function

$$g(x) = 2x + 3$$

Solution:



What can we notice?

- As we can clearly see, the graph of the function $f(x) = 2x + 3$ is a straight line.
- Moreover, from the table of values, we can notice that for $x = 0$, $f(0) = 3$, that is the **constant appearing in the function**.
- From the graph, we see that the line crosses the y -axis exactly at the point of coordinates $(0, f(0)) = (0, 3)$
- Also from the table of values, we can see that by increasing the value of x by 1, the value of $f(x)$ is increasing by **2**, that is the **the value attached to x** in the expression of $f(x)$.

In general

- The graph of any function of algebraic expression $f(x) = mx + b$ is a straight line.
- This algebraic expression can also be simply given by the linear equation $y = mx + b$ since any point on the graph (straight line) is of coordinates $(x, y) = (x, f(x))$.
- These functions are called **affine functions**.
- The number attached to x is called **the slope**.
- The number unattached to x is called **the y -intercept**.
- This specific form of an affine function $f(x) = mx + b$ is also called **slope-intercept** form.
- Cause simply by reading the expression, we can simply extract the slope and the y -intercept!

EXAMPLE

- Which of the following expressions is an affine function?

$$f(x) = 2x - \frac{1}{5}, \quad g(x) = 3x, \quad h(x) = \frac{1}{x} + 7, \quad s(x) = 4,$$

$$r(x) = 2(x - 2) - 5$$

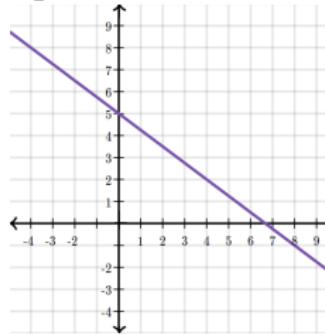
- When it applies, extract the slope and the y -intercept of each affine function.

Some properties about the slope

- Slope is a measure of the steepness of a line.
- It can be seen as

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x}$$

- Example, consider that we're given the graph of a line and asked to find its slope.

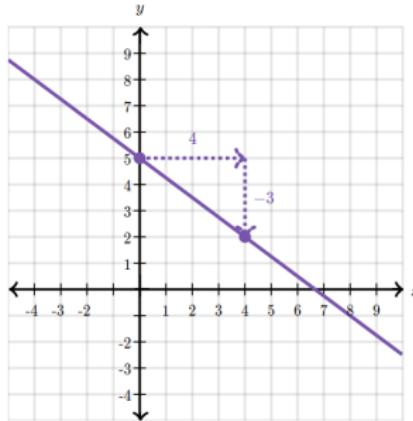


- The line appears to go through the points $(0, 5)$ and $(4, 2)$.

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{2 - 5}{4 - 0} = \frac{-3}{4}.$$

Some properties about the slope

- Slope is a measure of the steepness of a line.
- Example, consider that we're given the graph of a line and asked to find its slope.

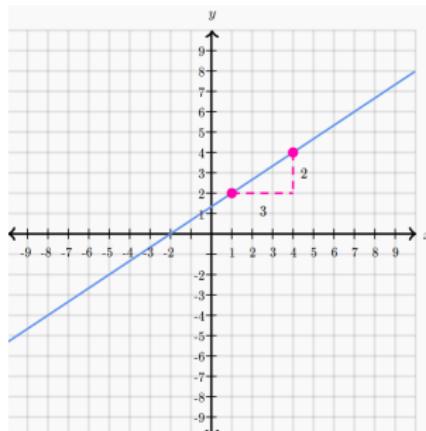


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FUNCTION VARIATION

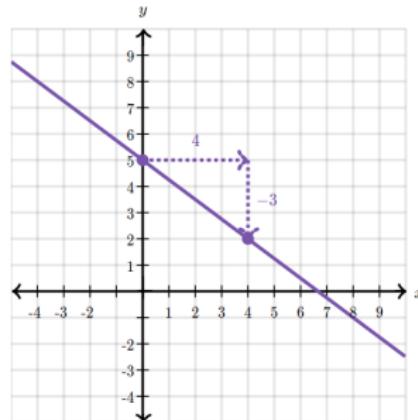
- We say that a function is **increasing** if "while running we are also rising!"
- In other words, when the slope is **positive**
- Take for example $f(x) = \frac{2}{3}x + \frac{4}{3}$.



- Here we see that running 3 steps along the x -axis yields to rising 2 steps along the y -axis.

FUNCTION VARIATION

- We say that a function is **decreasing** if "while running we are also dropping!"
- In other words, when the slope is **negative**
- Take for example $f(x) = \frac{-3}{4}x + 5$.



- Here we see that running 4 steps along the x -axis yields to dropping 3 steps along the y -axis.

FUNCTION VARIATION

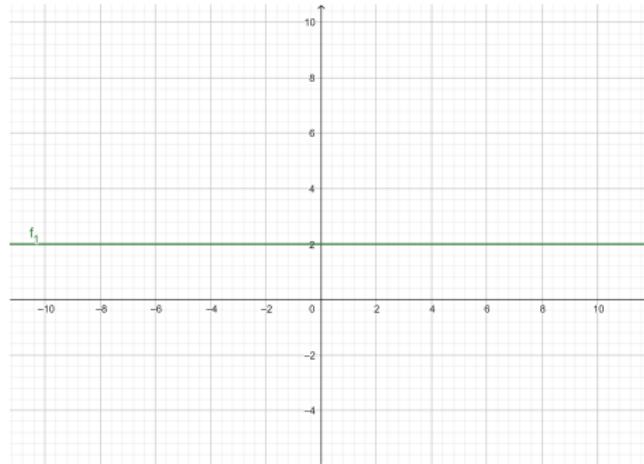
But what if the slope is 0?



FUNCTION VARIATION

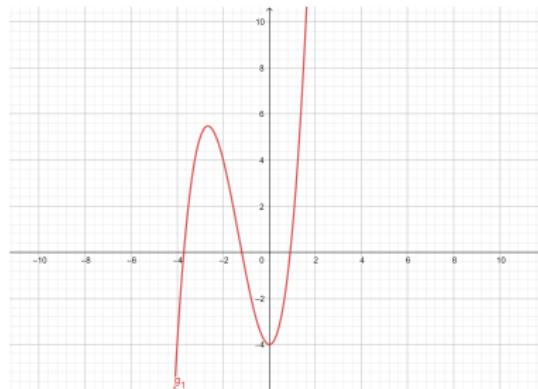
But what if the slope is 0?

- In this case, running along the x -axis will always keep us on the same level regarding the y axis.
- Our trajectory is thus a horizontal line.



Can you tell me in this case what is the expression of the corresponding function?

POLYNOMIAL FUNCTIONS (MAINLY QUADRATIC)



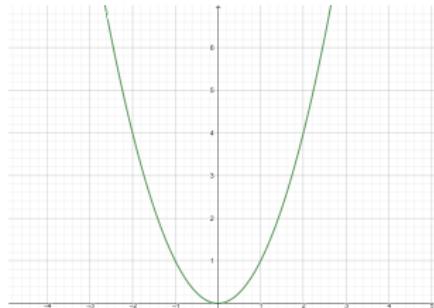
Now what if the function is not affine? Meaning when its graph is not a line.

What would be its variation?



POLYNOMIAL FUNCTIONS (MAINLY QUADRATIC)

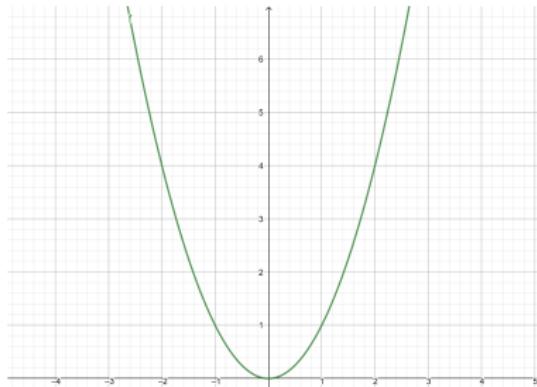
- Take for example $f(x) = x^2$. Its graph is given by



- As we can see, there is no constant variation all over the plane (*and when it's not constant, maybe it's a function of x .*) In fact
- when $x < 0$, i.e., on the left hand side of the plane, we "see" that the function is decreasing (while running we are dropping!),
- when $x > 0$, i.e., on the right hand side of the plane, we "see" that the function is increasing (while running we are rising!).

POLYNOMIAL FUNCTIONS (MAINLY QUADRATIC)

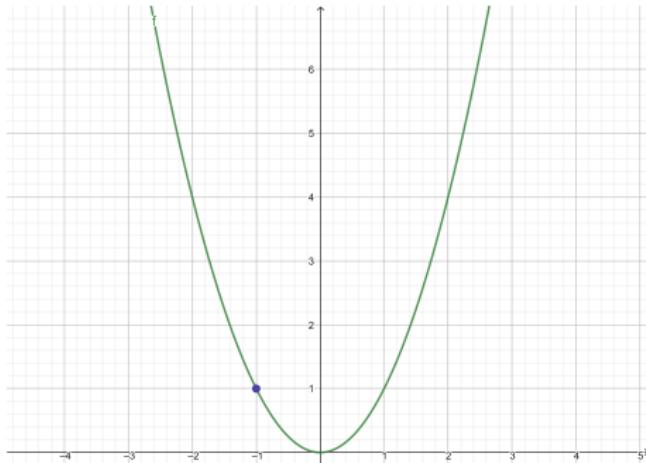
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- Thus, since there's no constant variation all over the plane, let's start by studying local variation.
- For this, let's choose a point on the graph (of coordinates $(x, f(x)) = (x, x^2)$), take for example the point $(-1, 1)$.

POLYNOMIAL FUNCTIONS (MAINLY QUADRATIC)

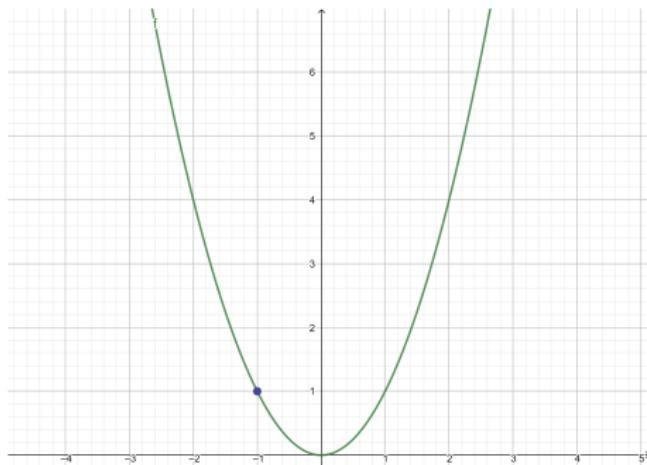
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- For this, let's chose a point on the graph (of coordinates $(x, f(x)) = (x, x^2)$), take for example the point $(-1, 1)$,
- and let's study the variation of the graph at the moment it passes by this point.

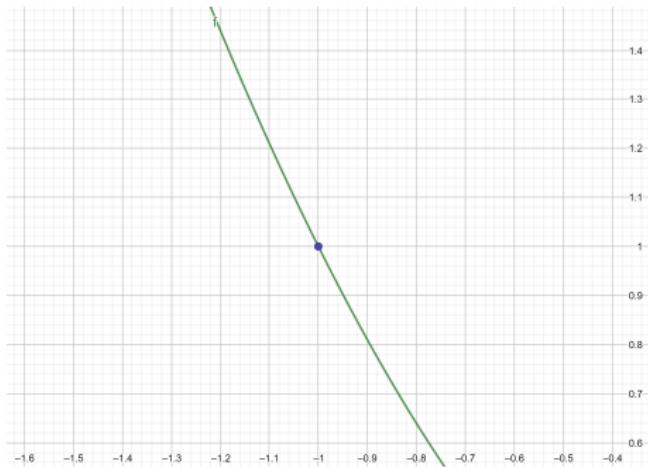
POLYNOMIAL FUNCTIONS (MAINLY QUADRATIC)

- Take for example $f(x) = x^2$. Its graph is given by



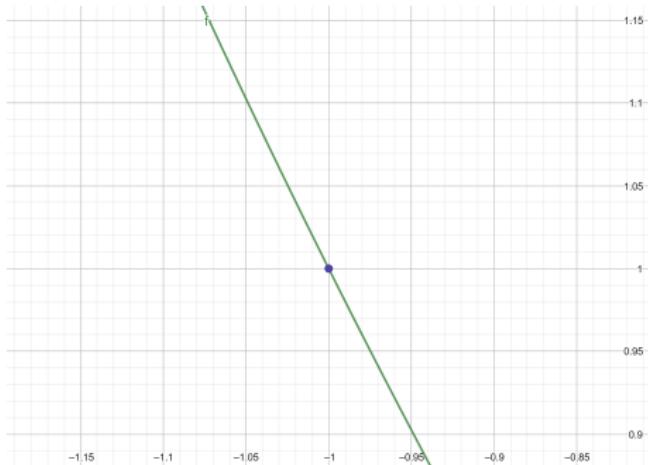
- Let's zoom in as much as we can to our point $(-1, 1)$

POLYNOMIAL FUNCTIONS (MAINLY QUADRATIC)



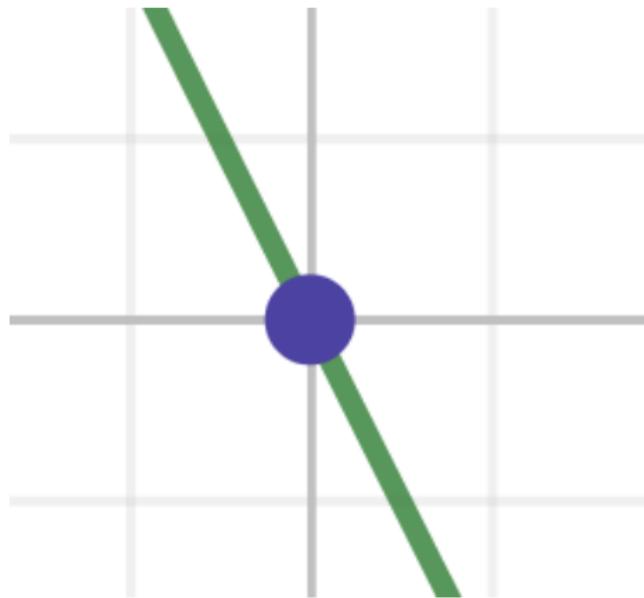
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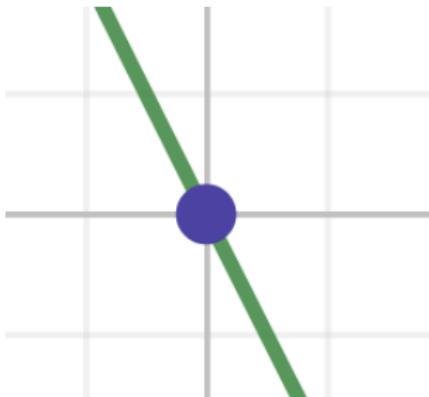
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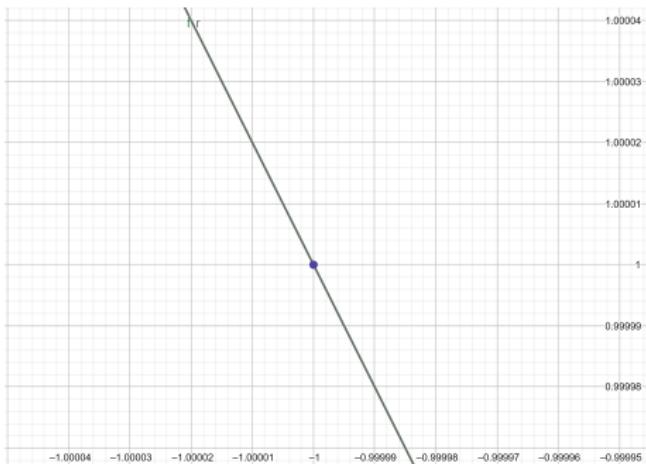
- Let's **zoom in** as much as we can to our point $(-1, 1)$

POLYNOMIAL FUNCTIONS (MAINLY QUADRATIC)



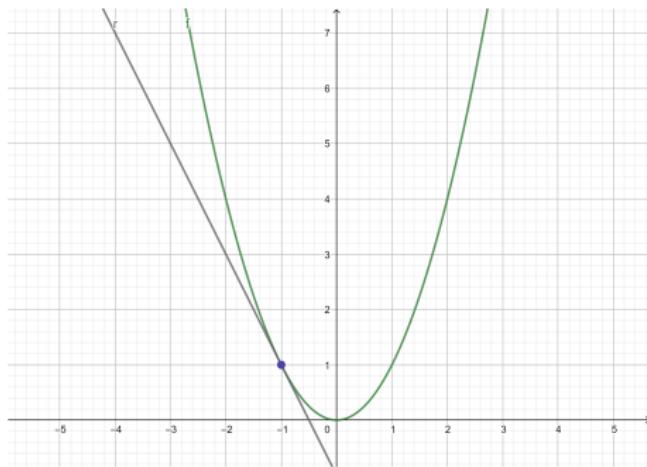
- As we can clearly see, around the point $(-1, 1)$, the graph resembles to a **straight line!** This is a good news since we already now how to find the slope of a line.
- We recall it however: given a line corresponding to a function $g(x)$ and two points on this line of coordinates $(a, g(a))$, $(b, g(b))$, then the slope is given by $\frac{g(b) - g(a)}{b - a}$.

POLYNOMIAL FUNCTIONS (MAINLY QUADRATIC)



- From this zoomed in place, let's draw the line we see, then zoom out.

POLYNOMIAL FUNCTIONS (MAINLY QUADRATIC)



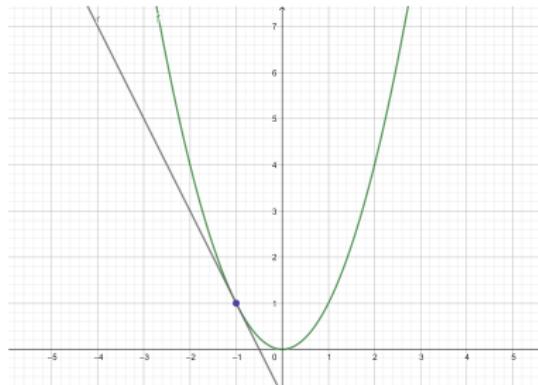
- From this zoomed in place, let's draw the line we see then zoom out.

POINT DERIVATIVE AND SLOPE OF A TANGENT

- The grey line that we've obtained is called the **tangent line** to the green curve at the point $(-1, 1)$.
- In general, the tangent line to a curve at a point is that straight line that best approximates (or “clings to”) the curve near that point.
- The term **slope** is defined for a straight line and the variation of the line is given by its slope.
- But for a curve, at a given point, its variation is given locally by the slope of the tangent at that point.
- Take the point $A = (a, f(a))$ on the graph of f . Then the slope of the tangent at the point A is said to be **the point derivative of f at a and is denoted by $f'(a)$** .
- For example for $f(x) = x^2$, when you are exactly at $x = -1$, i.e., on the point $(-1, 1)$, the slope of the tangent at that point is denoted by $f'(-1)$ and it is the point derivative of f at $x = -1$.

POINT DERIVATIVE AND SLOPE OF A TANGENT

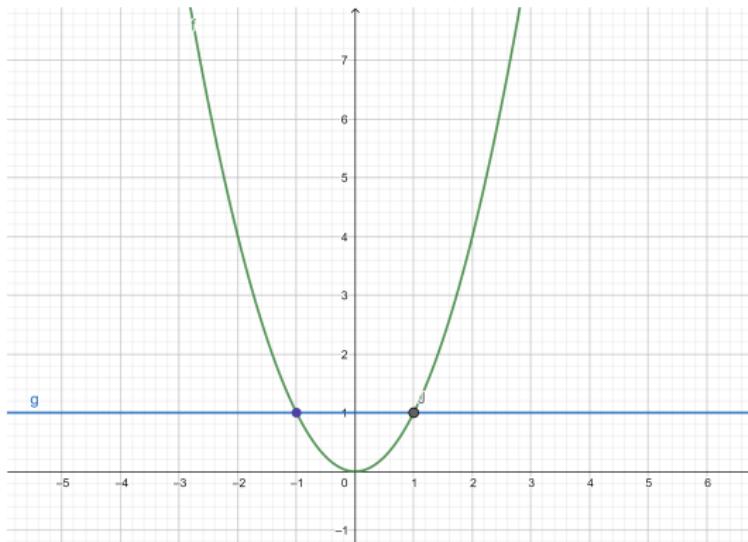
- Back to the example $f(x) = x^2$. Let's compute $f'(-1)$.



- Of course, we can compute the slope of the grey tangent at $(-1, 1)$ by reading the graph and considering two different points on the tangent. Can you tell me how and how much is it?
- But in general, We need the slope in order to be able to draw the tangent, that as we said, it best approximates the curve near that point.

POINT DERIVATIVE AND SLOPE OF A TANGENT

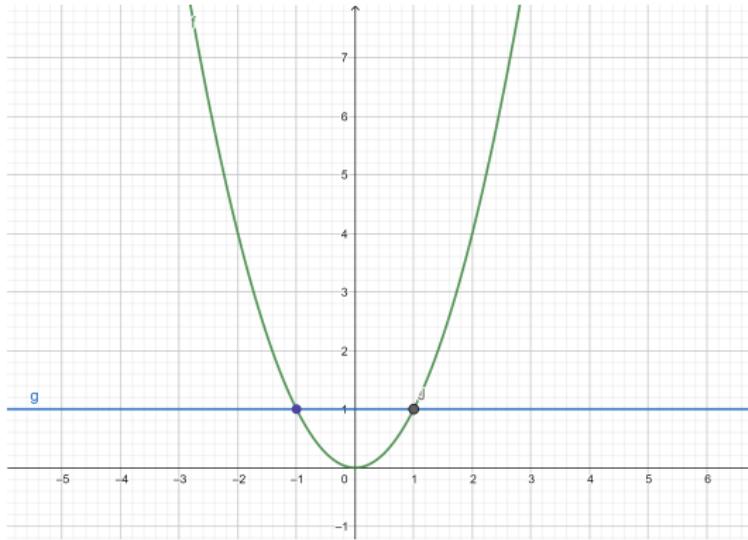
- Back to the example $f(x) = x^2$. Let's compute $f'(-1)$.



- Let's start by drawing a line from the point $(-1, 1)$ to any other point on the curve, say $(-1+2, f((-1+2))) = (1, 1)$.
- But, for simplifying, let's write h instead of 2 , i.e., let's write $(-1+h, f((-1+h)))$

POINT DERIVATIVE AND SLOPE OF A TANGENT

- Back to the example $f(x) = x^2$. Let's compute $f'(-1)$.

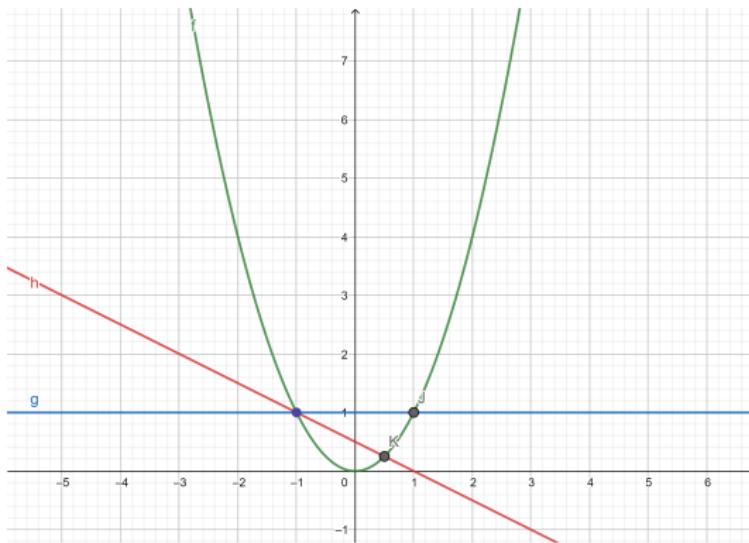


- The slope of the blue line passing by $(-1, 1)$ and $(-1+h, f(-1+h))$ is then

$$\frac{f(-1+h) - f(-1)}{(-1+h) - (-1)} = \boxed{\frac{f(-1+h) - f(-1)}{h}}$$

POINT DERIVATIVE AND SLOPE OF A TANGENT

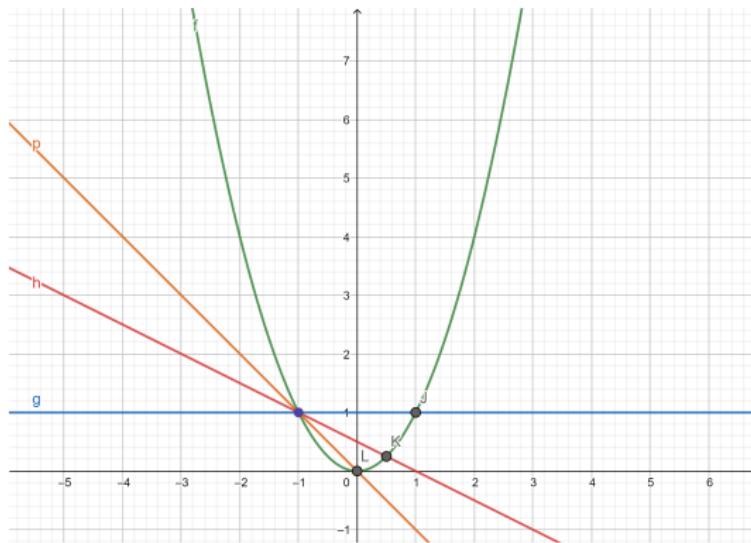
- Back to the example $f(x) = x^2$. Let's compute $f'(-1)$.



- Now, if we keep on decreasing the value of h , i.e., we take h from the value 2 to values closer and closer to zero, we get each time a line that is closer and closer to the tangent we aim at computing its slope.

POINT DERIVATIVE AND SLOPE OF A TANGENT

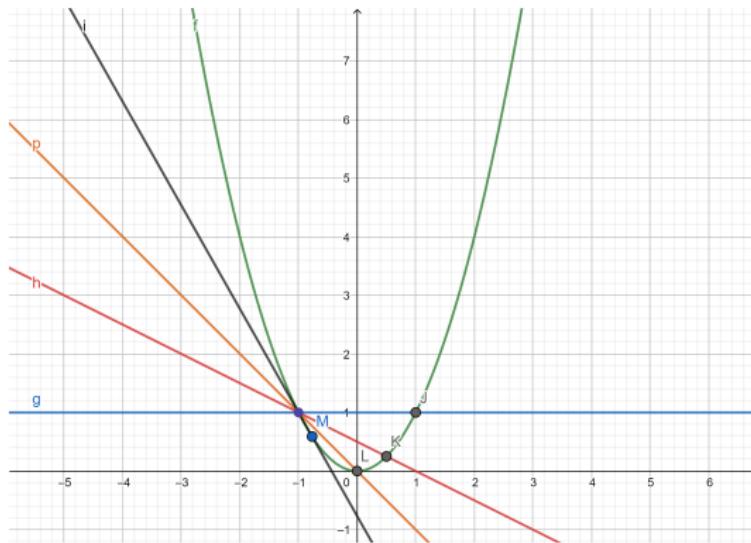
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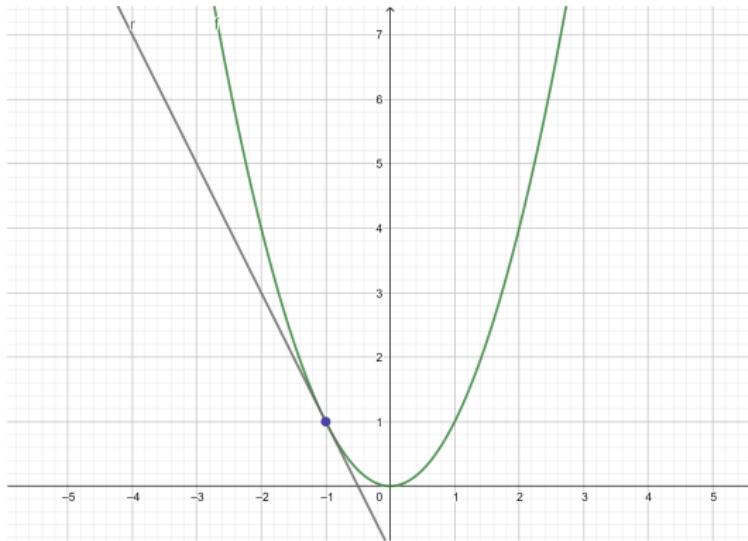
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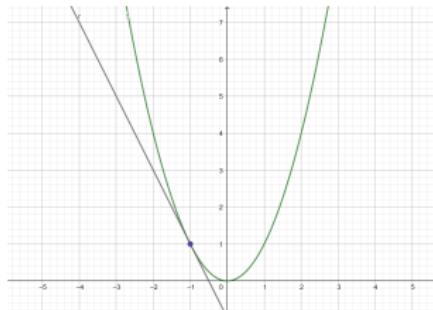
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POINT DERIVATIVE AND SLOPE OF A TANGENT

- Back to the example $f(x) = x^2$. Let's compute $f'(-1)$.



- That means that if we reduce h as much as possible to be extremely close to 0 (to the much we may think it's 0!) in the expression

$$\frac{f(-1+h) - f(-1)}{(-1+h) - (-1)} = \boxed{\frac{f(-1+h) - f(-1)}{h}}$$

then we get the slope of the tangent at $(-1, 1)$!

POINT DERIVATIVE AND SLOPE OF A TANGENT

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then we get the slope of the tangent at $(-1, 1)$!

- But for this value to be **defined** (we can't have a zero in the denominator), let's simplify the expression

$$\boxed{\frac{f(-1+h) - f(-1)}{h}}$$

then simply replace h by 0:

POINT DERIVATIVE AND SLOPE OF A TANGENT

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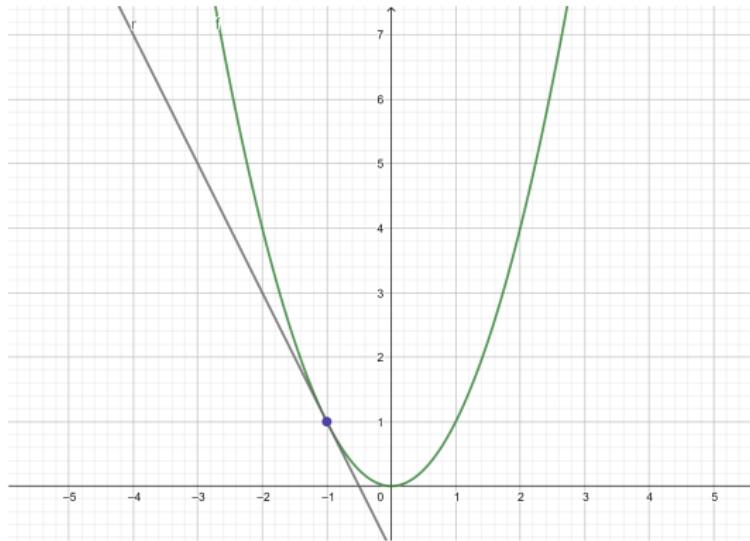
$$\frac{f(-1+h) - f(-1)}{h}$$

then simply replace h by 0:

$$\begin{aligned}\frac{f(-1+h) - f(-1)}{h} &= \frac{(-1+h)^2 - (-1)^2}{h} = \frac{(1-2h+h^2)-1}{h} \\ &= \frac{-2h+h^2}{h} \\ &= \frac{h(-2+h)}{h} \\ &= -2 + h \xrightarrow[h \rightarrow 0]{} \boxed{-2}\end{aligned}$$

POINT DERIVATIVE AND SLOPE OF A TANGENT

- $f(x) = x^2$:



$$f'(-1) = -2$$

POINT DERIVATIVE AND SLOPE OF A TANGENT

- For $f(x) = x^2$, $f'(-1) = -2$
- And here we go! We've found the slope of the tangent at $(-1, 1)$, that is $f'(-1)$ and we see it's negative, and the function f is decreasing when x is very close to the value -1 .

But what about all the other values on the x -axis?
Should we do this computation all over again each time we want to study the variation of a curve at a given point?

No!

We can simply define a new **function** and name it **the derivative function** denoted f' , that for any input value a returns $f'(a)$.

DERIVATIVE FUNCTION

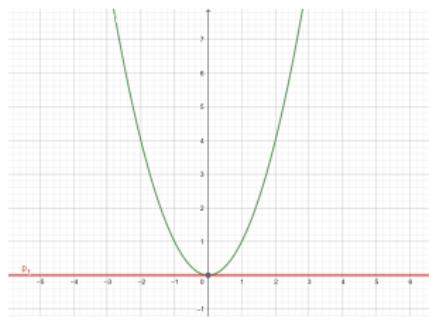
Function	Derivative
$f(x) = a$ (a constant value)	$f'(x) = 0$
$f(x) = x^n$	$f'(x) = nx^{n-1}$
$f(x) = \frac{1}{x}$	$f'(x) = -\frac{1}{x^2}$
$f(x) = a \cdot g(x)$	$f'(x) = a \cdot g'(x)$
$f(x) = g(x) + h(x)$	$f'(x) = g'(x) + h'(x)$
$f(x) = g(x) \cdot h(x)$	$f'(x) = g'(x)h(x) + h'(x)g(x)$
$f(x) = (g \circ h)(x)$ (chain rule)	$f'(x) = h'(x)(g'(h(x)))$
$f(x) = (g(x))^n$ (chain rule)	$f'(x) = ng'(x)(g(x))^{n-1}$

Example

Function	Derivative
$f(x) = 3$	$f'(x) = 0$
$f(x) = x^3$	$f'(x) = 3x^2$
$f(x) = 3 \cdot x^2$	$f'(x) = 3 \cdot (2x) = 6x$
$f(x) = 3x^2 + x^3$	$f'(x) = 6x + 3x^2$
$f(x) = (3x^2)(x^3)$	$f'(x) = 6x + 3x^2$
$f(x) = (x^2 + 2)^3$	$f'(x) = 3(2x)(x^2 + 2)^3$

VARIATION, MAXIMUMS, AND MINIMUM

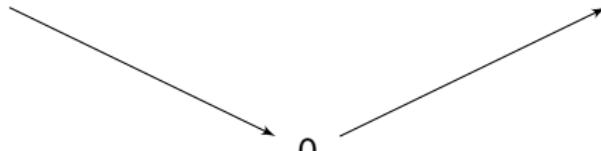
- Now back to our example $f(x) = x^2$.
- We have $f'(x) = 2x$.
- For example, $f'(-1) = -2$, $f'(2) = 2(2) = 4$, $f'(0) = 0$.
- In this case, we see that the slope of tangent at the point $(0,0)$ is equal to 0, i.e., the tangent at the point $(0,0)$ is a horizontal line!



- Moreover, for $x < 0$, $f'(x) = 2x < 0$, i.e., for $x < 0$, f is decreasing.
- For $x > 0$, $f'(x) = 2x > 0$, i.e., for $x > 0$, f is increasing.

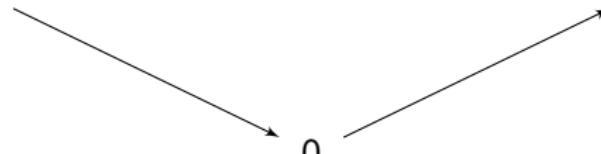
VARIATION, MAXIMUMS, AND MINIMUM

- Now back to our example $f(x) = x^2$, $f'(x) = 2x$.
- $f'(0) = 0$.
- Moreover, for $x < 0$, $f'(x) = 2x < 0$, i.e., for $x < 0$, f is decreasing.
- For $x > 0$, $f'(x) = 2x > 0$, i.e., for $x > 0$, f is increasing.
- Then we can draw a table that better illustrates the explained variation:

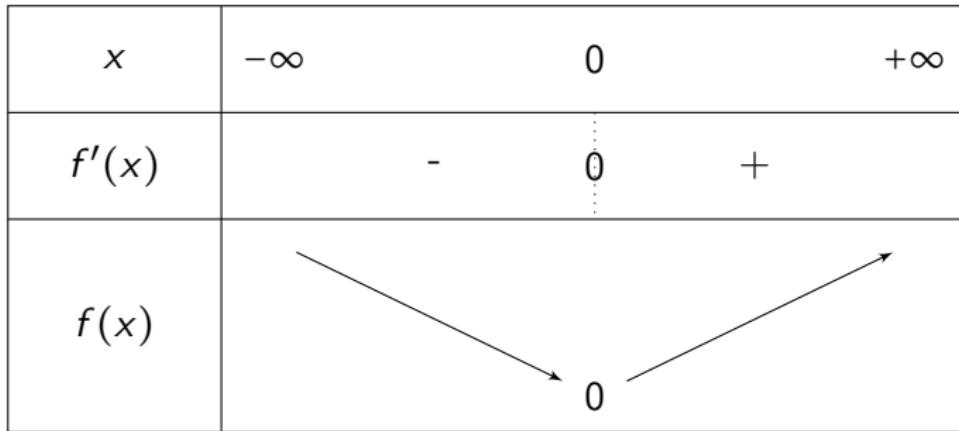
x	$-\infty$	0	$+\infty$
$f'(x)$	<i>negative</i>	0	<i>positive</i>
$f(x)$		0	

VARIATION, MAXIMUMS, AND MINIMUM

- Now back to our example $f(x) = x^2$, $f'(x) = 2x$.
- $f'(0) = 0$.
- Moreover, for $x < 0$, $f'(x) = 2x < 0$, i.e., for $x < 0$, f is decreasing.
- For $x > 0$, $f'(x) = 2x > 0$, i.e., for $x > 0$, f is increasing.
- Then we can draw a table that better illustrates the explained variation:

x	$-\infty$	0	$+\infty$
$f'(x)$	-	0	+
$f(x)$		0	

VARIATION, MAXIMUMS, AND MINIMUM

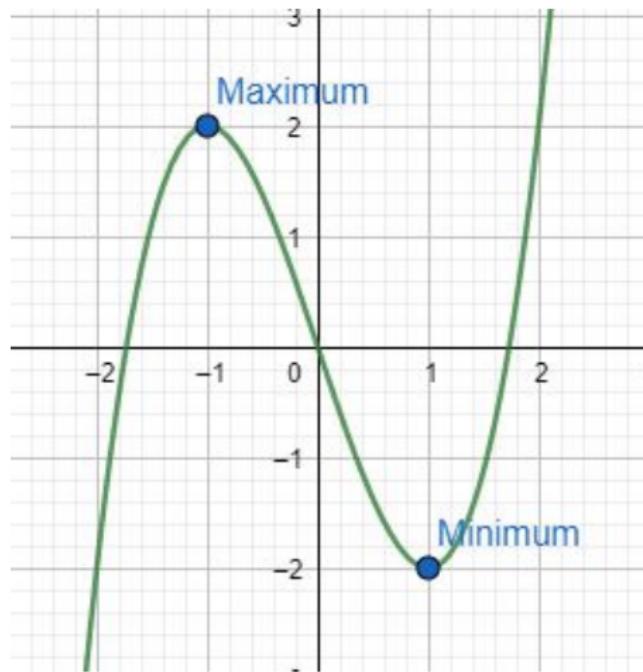


- The final point we see about the function $f(x) = x^2$ is that when $f'(x) = 0$, i.e., at $x = 0$, the function reaches its **minimal** y -value that is $f(0) = 0$.

VARIATION, MAXIMUMS, AND MINIMUM

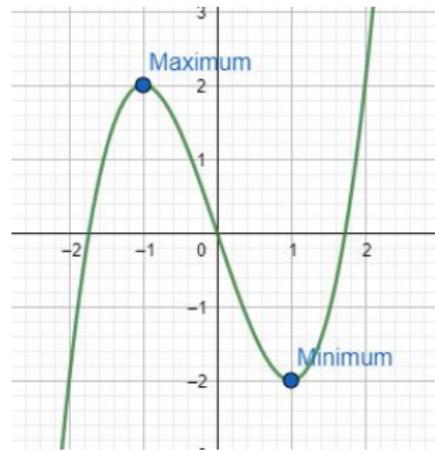
- In general the **local maximum value of a function** is the place where a function reaches its **highest point**, or **vertex**, on a graph, at a given interval. In other words, it is the maximal y -value that the function can reach, at a given interval.
- The **local minimum value of a function** is the place where the graph has a vertex at its lowest point, at a given interval. In other words, it is the minimal y -value that the function can reach.
- A **global maximum point** refers to the point with the largest value on the graph of a function, when a largest value exists. A **global minimum point** refers to the point with the smallest value. Together these two values are referred to as **global extrema**.

VARIATION, MAXIMUMS, AND MINIMUM



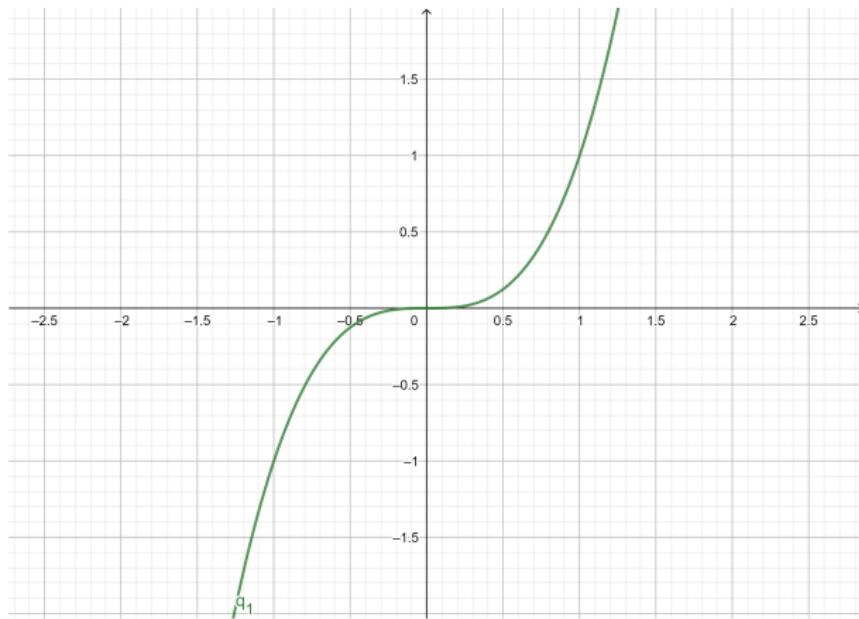
In this graph, are the maximum and the minimum local or global?

VARIATION, MAXIMUMS, AND MINIMUM



- In general, both the maximum and the minimum (either local or global) are reached at $x = a$ if $f'(a) = 0$ and the signs of f' (positive or negative) are different for $x < a$ and for $x > a$.
- In this case, the point $f(a)$ is either a maximum or a minimum at $x = a$.

VARIATION, MAXIMUMS, AND MINIMUM



In this graph (of the function x^3), do we have any maximum or minimum?

VARIATION, MAXIMUMS, AND MINIMUM QUADRATIC FUNCTIONS

- Take a quadratic function $f(x) = ax^2 + bx + c$ ($a \neq 0$).
- We recall that its graph is a parabola (\cup or \cap).
- Thus, it is clear that parabolas have either **global maximum** or **global minimum**.
- Let's draw its table of variation and find it's maximum or minimum.
- For this, let's compute $f'(x)$.
- We have $f'(x) = 2ax + b$
- We recall that the maximum or the minimum accrues at the value x where $f'(x) = 0$.

$$f'(x) = 2ax + b = 0 \iff 2ax = -b \iff x = -\frac{b}{2a}$$

- Thus, $f\left(-\frac{b}{2a}\right)$ is either the global minimum or the global maximum.

VARIATION, MAXIMUMS, AND MINIMUM QUADRATIC FUNCTIONS

Two cases arise:

- If $a > 0$ (positive), the the parabola is smiling, like 
- In this case, it's table of variation is as follows:

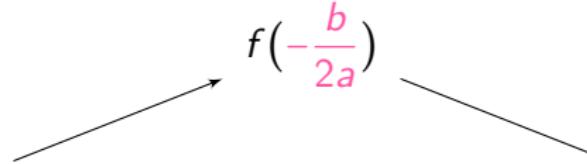
x	$-\infty$	$-\frac{b}{2a}$	$+\infty$
$f'(x)$	-	0	+
$f(x)$		$f\left(-\frac{b}{2a}\right)$	

VARIATION, MAXIMUMS, AND MINIMUM QUADRATIC FUNCTIONS

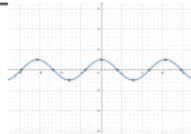
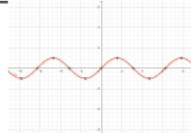
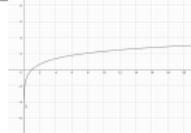
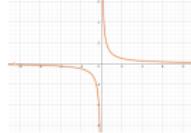
Two cases arise:

- If $a < 0$ (negative), the parabola is **frowning**, like 😞
- In this case, its table of variation is as follows:

x	$-\infty$	$-\frac{b}{2a}$	$+\infty$
$f'(x)$	+	0	-
$f(x)$		$f\left(-\frac{b}{2a}\right)$	



SOME INTERESTING FUNCTIONS

Function	Domain	Graph
cosine: $\cos(x)$	\mathbb{R}	
sin: $\sin(x)$	\mathbb{R}	
Exponential: $\exp(x)$	\mathbb{R}	
Logarithmic: $\ln(x)$	$]0, +\infty]$	
Hyperbolic $\frac{1}{x}$	$\mathbb{R} - \{0\}$	

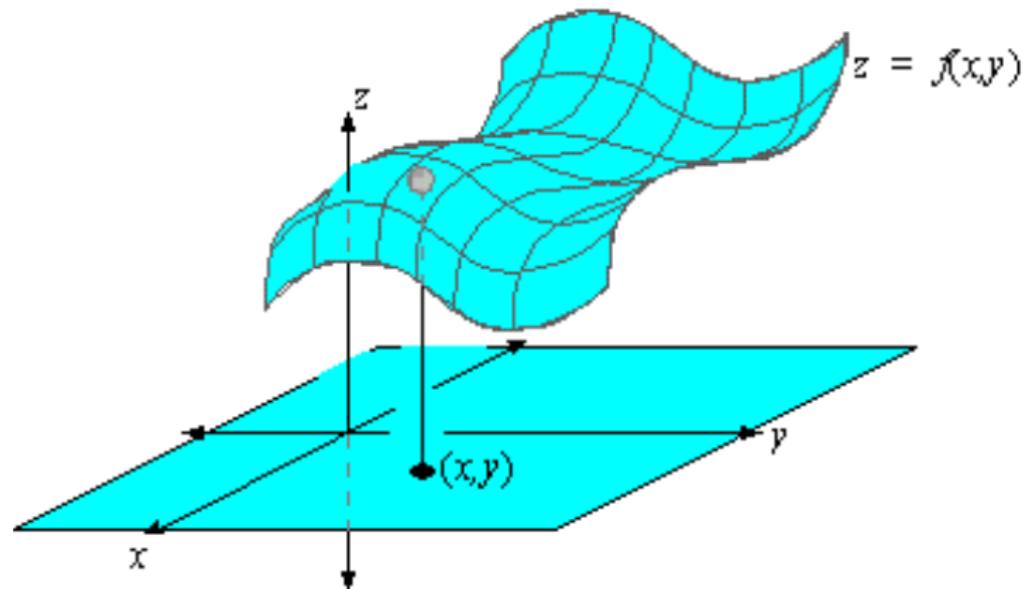
FUNCTION OF 2 VARIABLES

- A function of 2 variables is a function whose inputs are points (x, y) in the xy -plane and whose outputs are real numbers.
- We often denote functions of 2 variables by $f(x, y)$, which means "the output from an input of (x, y) ".
- We often define these functions in the form

$$f(x, y) = \text{"expression in } x \text{ and } y\text{"}$$

- Equivalently, we can consider $f(x, y)$ to be the assignment of a real number to a point (x, y) in the xy -plane.
- The graph of $f(x, y)$ is the set of points in \mathbb{R}^3 , i.e., point of the form (x, y, z) , that satisfy $z = f(x, y)$.
- The graph of $f(x, y)$ is the **surface** $z = f(x, y)$ and the output z is the **height of the surface** at the point (x, y) .

FUNCTION OF 2 VARIABLES



FUNCTION OF 2 VARIABLES

EXAMPLE

- Evaluate $f(1, 2)$ and $f(2, 5)$ if $f(x, y) = x^2 + 2xy$.

EXAMPLE

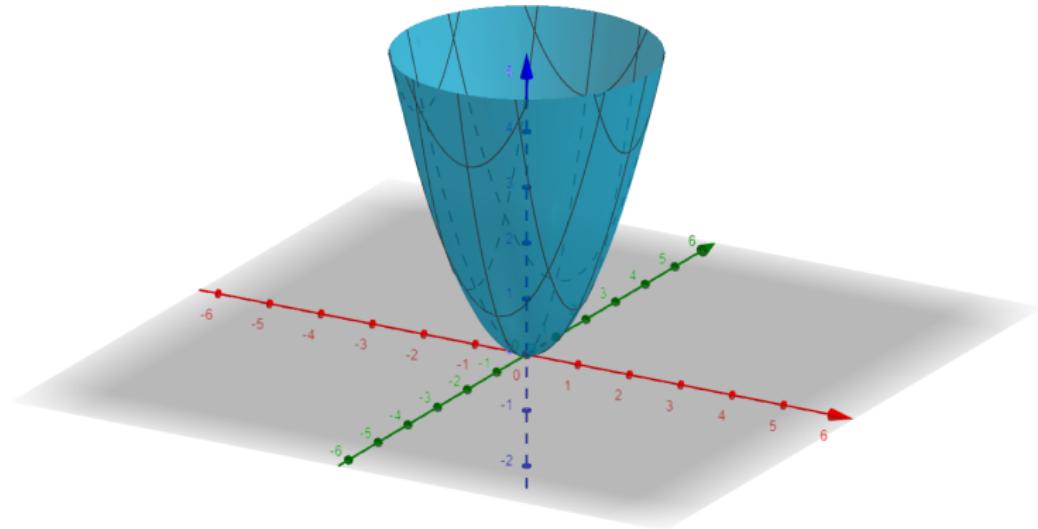
- Evaluate $f(1, 2)$ and $f(2, 5)$ if $f(x, y) = x^2 + 2xy + x$.

Solution:

- To begin with, $f(1, 2) = 1^2 + 2 \cdot 1 \cdot 2 + 1 = 6$, which is to say that $f(x, y) = x^2 + 2xy + x$ maps the point $(1, 2)$ to the number 6.
- Likewise, $f(2, 5) = 2^2 + 2 \cdot 2 \cdot 5 + 2 = 26$.

FUNCTION OF 2 VARIABLES

- A graphing calculator ([Geogebra](#)) or computer algebra system is often used to produce an approximation of the graph of a function.
- $f(x, y) = x^2 + y^2$:



THANK YOU!