Übungsblatt 7

Aufgabe 1

$$P = M_o \cdot M_p \quad \text{mit } M_o = S_{\frac{2}{r-l}, \frac{2}{t-b}, -\frac{2}{f-n}} \cdot T_{-\frac{1}{2}(r+l), -\frac{1}{2}(t+b), \frac{1}{2}(f+n)}$$

$$M_p = \begin{bmatrix} -n & 0 & 0 & 0 \\ 0 & -n & 0 & 0 \\ 0 & 0 & n+f & -nf \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

ges.: P^{-1}

$$P^{-1} = M_p^{-1} \cdot M_o^{-1}$$

$$\cdot \; M_o = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2}{t-b} & 0 & 0 \\ 0 & 0 & -\frac{2}{f-n} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{2}(r+l) \\ 0 & 1 & 0 & -\frac{1}{2}(t+b) \\ 0 & 0 & 1 & \frac{1}{2}(f+n) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_o^{-1} = T^{-1}_{(\dots)} \cdot S^{-1}_{(\dots)}$$

$$M_o^{-1} = \begin{bmatrix} 1 & 0 & 0 & \frac{1}{2}(r+l) \\ 0 & 1 & 0 & \frac{1}{2}(t+b) \\ 0 & 0 & 1 & -\frac{1}{2}(f+n) \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \frac{1}{\left(\frac{2}{r-l}\right)\cdot\left(\frac{2}{t-b}\right)\cdot\left(-\frac{2}{f-n}\right)} \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2}{t-b} & 0 & 0 \\ 0 & 0 & -\frac{2}{f-n} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_p^{-1} = \frac{1}{-nf} \begin{bmatrix} -n & 0 & 0 & 0 \\ 0 & -n & 0 & 0 \\ 0 & 0 & n+f & nf \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{f} & 0 & 0 & 0 \\ 0 & \frac{1}{f} & 0 & 0 \\ 0 & 0 & -\frac{n+f}{nf} & -1 \\ 0 & 0 & -\frac{1}{nf} & 0 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} \frac{1}{f} & 0 & 0 & 0 \\ 0 & \frac{1}{f} & 0 & 0 \\ 0 & 0 & -\frac{n+f}{nf} & -1 \\ 0 & 0 & -\frac{1}{nf} & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & \frac{1}{2}(r+l) \\ 0 & 1 & 0 & \frac{1}{2}(t+b) \\ 0 & 0 & 1 & -\frac{1}{2}(f+n) \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{4}(t-b)(f-n) & 0 & 0 & 0 \\ 0 & -\frac{1}{4}(r-l)\left(\frac{2}{f-n}\right) & 0 & 0 \\ 0 & 0 & \frac{1}{4}(r-l)(t-b) & 0 \\ 0 & 0 & 0 & -\frac{1}{8}(*) \end{bmatrix}$$

$* ((r-l)(t-b)(f-n))$

$$= \begin{bmatrix} \frac{1}{f} & 0 & 0 & \frac{1}{2f}(r+l) \\ 0 & \frac{1}{f} & 0 & \frac{1}{2f}(t+b) \\ 0 & 0 & -\frac{n+f}{nf} & \left(\frac{1}{2}\frac{(n+f)^2}{nf} - 1\right) \\ 0 & 0 & -\frac{1}{nf} & \frac{1}{2}\frac{(n+f)}{nf} \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{4}(t-b)(f-n) & 0 & 0 & 0 \\ 0 & -\frac{1}{4}(r-l)\left(\frac{2}{f-n}\right) & 0 & 0 \\ 0 & 0 & \frac{1}{4}(r-l)(t-b) & 0 \\ 0 & 0 & 0 & -\frac{1}{8}(r-l)(t-b)(f-n) \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} -\frac{1}{4f}(t-b)(f-n) & 0 & 0 & -\frac{1}{16f}(r+l)(r-l)(t-b)(f-n) \\ 0 & -\frac{1}{4f}(r-l)\left(\frac{2}{f-n}\right) & 0 & -\frac{1}{16f}(r-l)(t-b)(t+b)(f-n) \\ 0 & 0 & -\frac{1}{4nf}(r-l)(t-b)(n+f) & \left(\frac{1}{2}\frac{(n+f)^2}{nf} - 1\right)\cdot\left(-\frac{1}{8}(r-l)(t-b)(f-n)\right) \\ 0 & 0 & -\frac{1}{4nf}(r-l)(t-b) & -\frac{1}{16nf}(r-l)(t-b)(f-n)(f+n) \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} -\frac{(t-b)(f-n)}{4f} & 0 & 0 & -\frac{(r^2-l^2)(t-b)(f-n)}{16f} \\ 0 & -\frac{(r-l)}{2f(f-n)} & 0 & -\frac{(r-l)(t^2-b^2)(f-n)}{16f} \\ 0 & 0 & -\frac{(r-l)(t-b)(n+f)}{4nf} & -\frac{(n+f)^2(r-l)(t-b)(f-n)}{16nf} + \frac{(r-l)(t-b)(f-n)}{8} \\ 0 & 0 & -\frac{(r-l)(t-b)}{4nf} & -\frac{(r-l)(t-b)(f^2-n^2)}{16nf} \end{bmatrix}$$