

1. Many years ago, on a sultry July night in Omaha, it was raining heavily at midnight. Is it possible that 72 hours later the weather in Omaha was sunny?

2. A logician with some time to kill in a small town decided to get a haircut. The town had only two barbers, each with his own shop. The logician glanced into one shop and saw that it was extremely untidy. The barber needed a shave, his clothes were unkempt, his hair was badly cut. The other shop was extremely neat. The barber was freshly shaved and spotlessly dressed, his hair neatly trimmed. The logician returned to the first shop for his haircut. Why?

Proposition 1. *A proposition or statement is a sentence that is either true or false.*

Definition 2. *Let P and Q be propositions. The conjunction (AND, \wedge) of P and Q , the disjunction (OR, \vee) of P and Q , and the negation or denial (NOT, \neg , \sim ,) of P are defined by the truth tables.*

Example 3 (Implication). The following propositions all mean exactly the same thing:

- If you are born in Rome, then you are Italian.
- You are Italian if you are born in Rome.
- You are born in Rome only if you are Italian.
- Being born in Rome is sufficient to be Italian.
- Being Italian is necessary for being born in Rome.

3. “Feemster owns more than a thousand books,” said Albert.

“He does not,” said George. “He owns fewer than that.”

“Surely he owns at least one book,” said Henrietta.

If only one statement is true, how many books does Feemster own?

Definition 4 (The Converse and Contrapositive). *The converse of an implication $P \implies Q$ is the reversed implication $Q \implies P$. The contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.*

Example 5. Let P and Q be the following statements:

P . Claudia is holding a peach.

Q . Claudia is holding a piece of fruit.

The implication $P \implies Q$ is true, since all peaches are fruit. As a sentence, we have:

If Claudia is holding a peach, then Claudia is holding a piece of fruit.

The converse of $P \implies Q$ is the sentence:

If Claudia is holding a piece of fruit, then Claudia is holding a peach.

This is clearly false: Claudia could be holding an apple!

The contrapositive of $P \implies Q$ is the following sentence:

If Claudia is not holding any fruit, then she is not holding a peach.

This is clearly true.

4. A boy and a girl are sitting on the front steps of their commune.

“I’m a boy,” said the one with black hair.

“I’m a girl,” said the one with red hair.

If at least one of them is lying, who is which?

5. Make a statement about n that is true for, and only true for, all values of n less than 100.

6. A person either always answers truthfully, always answers falsely, or alternates true and false answers. How, in two questions, each answered by yes or no, can you determine whether this person is a truther, a liar, or an alternater?

7. Three men stand before you. One always answers questions truthfully, one always responds with lies, and one randomizes his answers, sometimes lying and sometimes not. You do not know which man does which, but the men themselves do. How can you identify all three men by asking three questions? Each question may be directed toward any man you choose, and each must be a question that is answered by yes or no.

8. Evaluate each of the 10 statements as to its truth or falsity:

1. Exactly one statement on this list is false.
2. Exactly two statements on this list are false.
3. Exactly three statements on this list are false.
4. Exactly four statements on this list are false.
5. Exactly five statements on this list are false.
6. Exactly six statements on this list are false.
7. Exactly seven statements on this list are false.
8. Exactly eight statements on this list are false.
9. Exactly nine statements on this list are false.
10. Exactly ten statements on this list are false.

9. There is an island upon which a tribe resides. The tribe consists of 1000 people, with various eye colours. Yet, their religion forbids them to know their own eye color, or even to discuss the topic; thus, each resident can (and does) see the eye colors of all other residents, but has no way of discovering his or her own (there are no reflective surfaces).

Every noon, a ferry stops at the island. If a tribesperson does discover his or her own eye color, then their religion compels them to leave the island at noon the following day in the village square for all to witness, and the rest stay. Every tribesperson can see everyone else at all times and keeps a count of the number of people they see with each eye color (excluding themselves). Every tribesperson on the island knows all the rules in these two paragraphs.

All the tribespeople are highly logical and devout, and they all know that each other is also highly logical and devout (and they all know that they all know that each other is highly logical and devout, and so forth).

(for the purposes of this logic puzzle, “highly logical” means that any conclusion that can logically deduced from the information and observations available to an islander, will automatically be known to that islander.)

Of the 1000 islanders, it turns out that 100 of them have blue eyes and 900 of them have brown eyes, although the islanders are not initially aware of these statistics (each of them can of course only see 999 of the 1000 tribespeople).

One day, a blue-eyed foreigner visits to the island and wins the complete trust of the tribe.

One evening, he addresses the entire tribe to thank them for their hospitality.

However, not knowing the customs, the foreigner makes the mistake of mentioning eye color in his address, remarking “how unusual it is to see another blue-eyed person like myself in this region of the world”.

What effect, if anything, does this *faux pas* have on the tribe?

