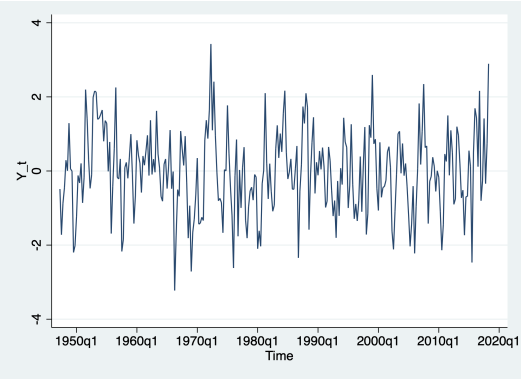
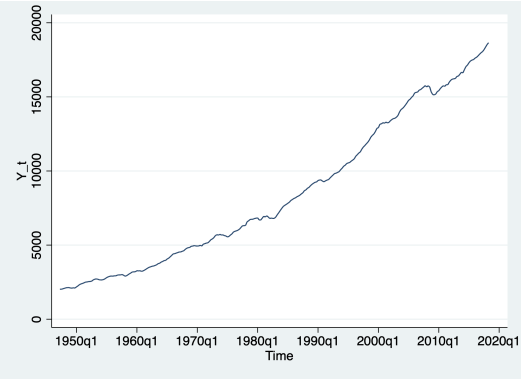
Dis 12: Time Series (Cont'd)

1 Stationarity vs. Nonstationarity

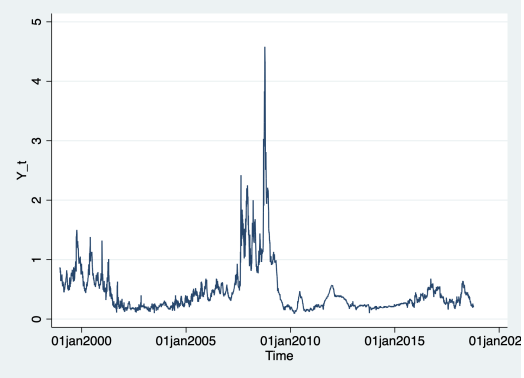
- Last week, we focused on forecasting variable Y_t that satisfies the stationarity assumption.
  - For stationary Y_t, two styles of models are proposed: $AR(p)$ and $ADL(p,q)$.
  - And we saw how to use AIC and BIC to determine how many lags to include in an $AR(p)$ or $ADL(p,q)$ model.
- But stationarity is not always a given. There are a couple cases where stationarity would not hold:
  1. **Trend**: Data exhibits a persistent long-term movement over time.
  2. **Breaks**: Parameters in a regression model would change during the span of the sample due to changes in the economy (policy enactment, innovation, specific event, or else).
  3. **Seasonality**: Data exhibits similar movement for the same month / same quarter / same day of the week / same time of the day / etc.
- How to detect nonstationarity?
  - A very informal method: **Plot the time series**



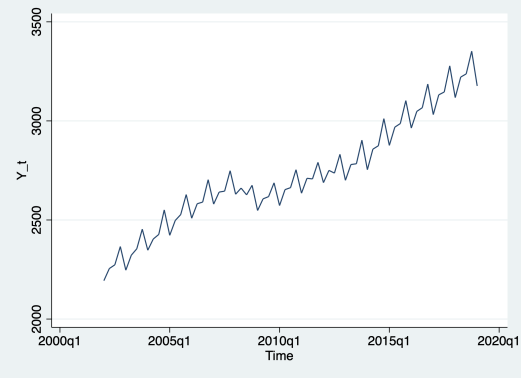
(a) Stationary series



(b) Nonstationarity: Trend



(c) Nonstationarity: Breaks



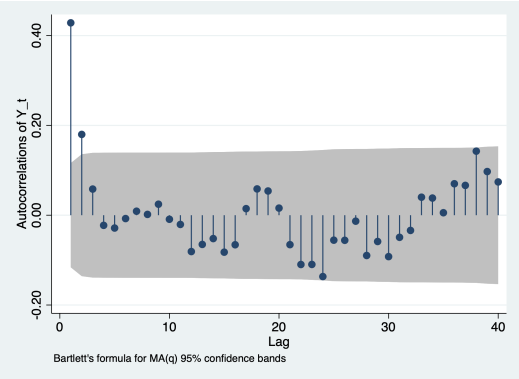
(d) Nonstationarity: Seasonality (and trend)

Stata command to plot time series (after `tsset` time variable in data):

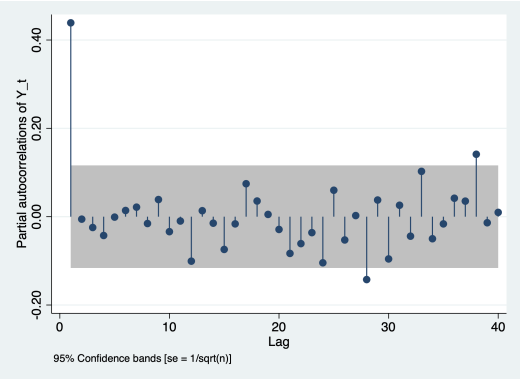
```
tsline VARNAME
```

– A slightly less informal method: **Plot the Autocorrelation Function (ACF) or Partial Autocorrelation Function (PACF)**

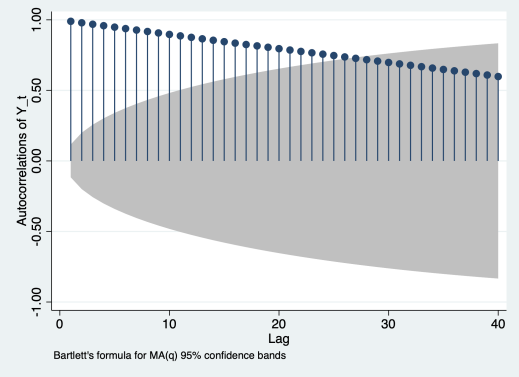
* Recall from last discussion:
  · j-th order autocorrelation $= Corr(Y_t, Y_{t-j}) = \frac{Cov(Y_t, Y_{t-j})}{\sqrt{Var(Y_t)Var(Y_{t-j})}}$
  · Replacement of OLS independence assumption for stationary time-series data: When time gap becomes large, Y_t should become independent with its lag value.
* These imply that, **if the data is stationary, then autocorrelation should quickly reduce to 0** (i.e. only after a few lags).
  If the data is nonstationarity, then autocorrelation will very slowly reduce to 0 (i.e. after many lags).
* What about partial autocorrelation?
  $\Rightarrow$ Similar to partial correlation: It controls for other X variables that could affect both the lag of Y and the current Y value.



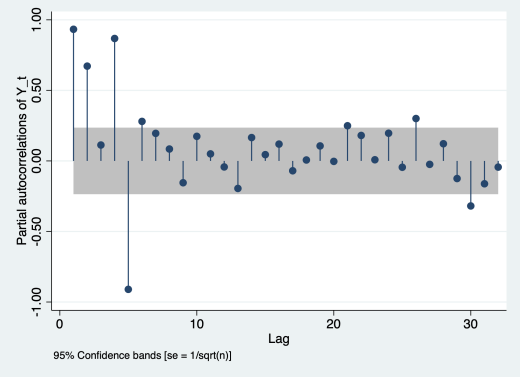
(a) Stationary series, ACF plot



(b) Stationary series, PACF plot



(c) Nonstationary series, ACF plot



(d) Nonstationary series, PACF plot

* How to create ACF and PACF plots in Stata (after `tsset` time variable in data)?

```
ac VARNAME
pac VARNAME
```

- A formal method to test stochastic trend: **Dickey-Fuller test**
  - Distinguish two types of trend:
    1. **Deterministic trend**: Trend is a nonrandom function of time.
    2. **Stochastic trend**: Trend is random and varies over time.
  - Although both types of trend imply nonstationarity ...
    - ... deterministic trend can simply be dealt with in the model by adding time term(s)
    - ... stochastic trend resembles a **random walk** process $\Rightarrow$ causes all sorts of trouble
  - Random walk:
    - $AR(1)$ random walk:

$$Y_t = Y_{t-1} + u_t$$

    - $AR(1)$ random walk with a drift:

$$Y_t = \beta_0 + Y_{t-1} + u_t$$

      where u_t is i.i.d. random error.
    - Both models imply that the best predictor of Y_t is its previous period value Y_{t-1} (with a constant β_0 term added or not), given that u_t is random.
    - Under random walk, coefficient on Y_{t-1} (call this coefficient β_1) has that $|\beta_1| = 1$, which is why random walk / stochastic trend is also called a **unit root** process.
      (*see Ch 15.7 for how the definition of unit root is adjusted for an $AR(p)$ model.)
  - Why is random walk / stochastic trend / unit root a bad thing?
    - **Makes forecasting useless**: The best forecastor of tomorrow's value is just today's value in an $AR(1)$ random walk process.
    - **Causes problems in estimation**: OLS creates downward bias on estimator*, and distribution of OLS estimator becomes nonnormal even in large sample time period.
    - **Leads to spurious regression**: Regress one random walk process onto another will imply a nonexisting relationship.
  - Dickey-Fuller test can test for unit root process for an $AR(p)$ model with and without deterministic trend component:
    - Without deterministic trend, for an $AR(1)$ model without drift:

$$\Delta Y_t = \delta Y_{t-1} + u_t$$

    - Without deterministic trend, for an $AR(1)$ model with drift:

$$\Delta Y_t = \beta_0 + \delta Y_{t-1} + u_t$$

    - With deterministic trend, for an $AR(1)$ model:

$$\Delta Y_t = \beta_0 + \alpha t + \delta Y_{t-1} + u_t$$

    - Hypothesis for all cases: $H_0 : \delta = 0$ (stochastic trend), $H_1 : \delta \neq 0$ (series is stationary).

*Explanation on why OLS estimator is biased downward (math warning)

- In Stata (after `tsset` time variable in data),

```
dfuller VARNAME // without deterministic trend, without drift
dfuller VARNAME, drift // without deterministic trend, with drift
dfuller VARNAME, trend // with deterministic trend

dfuller VARNAME, lags(4) // AR(4), without deterministic trend or drift
```

* What to do when Dickey-Fuller test concludes that the series is not stationary?
  - Easy solution: The difference of the series usually then is stationary.
    To see why this is the case, notice that a unit root process for an $AR(1)$ model with drift and without deterministic trend has

$$Y_t = \beta_0 + Y_{t-1} + u_t$$

Now, replace the predicted variable to be the first difference in Y is equivalent to moving Y_{t-1} to the left side of the equation:

$$Y_t - Y_{t-1} = \beta_0 + u_t$$
$$\Delta Y_t = \beta_0 + u_t$$

This model can now be consistently estimated by OLS.

```
. dfuller Y_t

Dickey-Fuller test for unit root                   Number of obs   =       284

                                 ———— Interpolated Dickey-Fuller ————
                 Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value            Value
——————————————————————————————————————————————————————————————————————————
 Z(t)             5.521            -3.457            -2.879           -2.570
——————————————————————————————————————————————————————————————————————————
MacKinnon approximate p-value for Z(t) = 1.0000

. gen Y_t_diff = Y_t - l.Y_t
(1 missing value generated)

. dfuller Y_t_diff

Dickey-Fuller test for unit root                   Number of obs   =       283

                                 ———— Interpolated Dickey-Fuller ————
                 Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value            Value
——————————————————————————————————————————————————————————————————————————
 Z(t)           -10.792            -3.457            -2.879           -2.570
——————————————————————————————————————————————————————————————————————————
MacKinnon approximate p-value for Z(t) = 0.0000
```

- A formal method to test breaks: **Chow test and QLR test**
  - If we **know** that data is breaking at certain date τ (usually due to policy enactment), consider the following model:

    $$Y_t = \beta_0 + \beta_1 Y_{t-1} + \gamma_0 D_t(\tau) + \gamma_1 [D_t(\tau) \times Y_{t-1}] + u_t$$

    where $D_t(\tau)$ is a binary variable that equals to 1 after the breakdate τ.
    In this case, perform a joint F-test on $\gamma_0 = \gamma_1 = 0$ tells us whether the forecast actually changes at breakdate τ. This test is called **Chow test**.
  - If we **don't know** that data is breaking at certain date, we can test whether between time τ_0 and τ_1, there exists a breakdate τ (so that $\tau_0 \leq \tau \leq \tau_1$).
    Idea: Search for all τs over τ_0 and τ_1, and find the largest F-statistic for a model similar to the one presented in Chow test. The date with the largest F-statistic can then be tested for statistical significance to determine whether it is a breakdate.
    This test is called a **Quandt likelihood ratio (QLR) test**.
  - How to deal with breaks?
    - If only some coefficients are breaking, include interaction terms between the predictors and the binary variable indicating breaking.
    - If all coefficients are breaking, run two separate regressions: one using the data before breakdate, one using the data after.

- Roadmap for dealing with time series data:
  1. Test for stationarity of series by any (or a combination) of the three following methods:
     (a) Plot time series
     (b) Plot ACF or PACF
     (c) Run Dickey-Fuller, Chow, and/or QLR test
  2. If the data is stationary, use $AR(p)$ or $ADL(p, q)$ model. Number of lags can be determined by qualitative reasons, or AIC or BIC.
  3. If the data is not stationary,
     - For stochastic trend, create a series of differences, and then regress on the differenced series.
     - For deterministic trend, include a time term t in your regression model.
     - For breaks, include interaction terms between the predictors and the binary variable indicating breaking, or estimating two separate regressions (using before and after breakdate data separately).

2 ARIMA Model

- An extension to $AR(p)$ model, where $ARIMA(p, d, q)$ stands for
  - $AR(p)$: Autoregression of order p
  - $I(d)$: Integration (referring to taking d differences to the dependent variable)
  - $MA(q)$: Moving average of order q (a way to model the error term in time-series)
- There's a Stata command, `arima` , that runs $ARIMA$ model. The main addition to our current way of modeling is the MA component, but it usually doesn't differ greatly from a model with just the ARI components.

3 Problems

1. This week's exercise is to help you come up with the appropriate model for a time-series data.

   (a) An initial hurdle when dealing with time-series data is to tell Stata which variable in my dataset records time. Let's practice using `tsset` on the following four sets of data:

   - Date information recorded in two columns: Cement data from Wooldridge
     `http://fmwww.bc.edu/ec-p/data/wooldridge/cement.dta`
   - Daily data: S&P 500 index
     `https://scaotravis.github.io/downloads/teaching/sp21-400/datasets/sp500.csv`
   - Monthly data: Personal savings rate
     `https://scaotravis.github.io/downloads/teaching/sp21-400/datasets/psr.csv`
   - Quarterly data: Private fixed investment
     `https://scaotravis.github.io/downloads/teaching/sp21-400/datasets/pfi.csv`

   A key step needed for each dataset is to transform the existing `date` variable to a variable of type Date.

   For the first dataset with date information recorded in two separate columns, Stata has function `ym(YEARVAR, MONTHVAR)` that gets the job done.

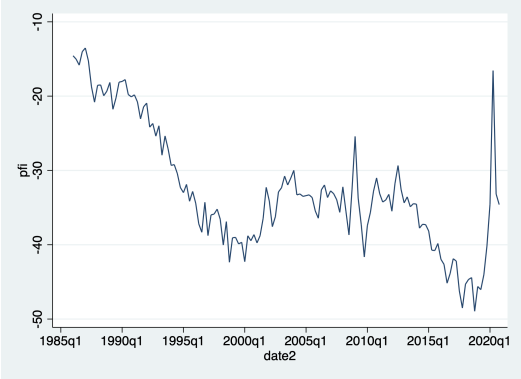
   For the rest three, if you look closely, the existing `date` variable in each of the three dataset is of type String. However, for Stata to recognize that this is a variable recording time period, it needs to be transformed into a Date object. Thankfully, we can directly convert String into Date object by using the `date(VARNAME, FORMAT)` function.

   After creating a variable of Date type, the remaining problem being that the new Date type variable looks like an integer, which is hard to interpret. Now, you can change this new Date type variable's format to make it more obvious that this is a Date variable, but depending on the frequency of your data, additional transformation is needed so that the change of format will make sense.

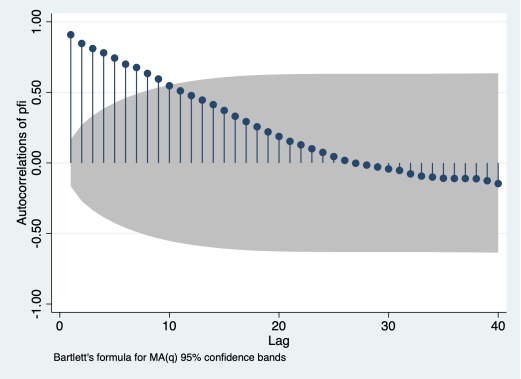
   See Do-file solution on how the transformation is done for each dataset.

   (b) Now that we have the quarterly data correctly `tsset` , let's figure out whether the `pfi` series is stationary before we propose any specific forecasting model. Plot the `pfi` series, its ACF and PACF. Can you guess whether this series is stationary or not?

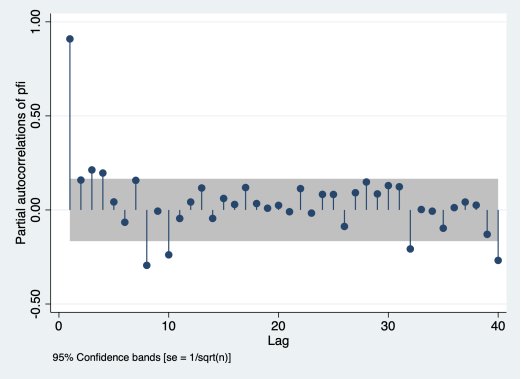
   The time series, ACF, and PACF plots are the following:



(a) Time series of `pfi`



(b) ACF plot of `pfi`



(c) PACF plot of `pfi`

Among the three, I'd say that the time series and the ACF plot are the most useful. The time series plot indicates the existence of some type of trend, and the autocorrelation plot suggests that autocorrelation reduces to 0 relatively slowly. Both plots point to the direction that `pfi` is nonstationary.

(c) Perform two Dickey-Fuller tests

- With drift
- With trend

Determine whether the series has stochastic trend component at 5% size for each test.

A Dickey-Fuller test with drift looks like the following

```
. dfuller pfi, drift

Dickey-Fuller test for unit root                    Number of obs   =       139

                                 ------------ Z(t) has t-distribution ------------
                       Test          1% Critical       5% Critical      10% Critical
                     Statistic          Value             Value            Value
------------------------------------------------------------------------------
 Z(t)                -2.844            -2.354            -1.656           -1.288
------------------------------------------------------------------------------
p-value for Z(t) = 0.0026
```

The p-value is less than 5%, meaning that we reject the null hypothesis and conclude that the

pfi series with drift is stationary.

What about the Dickey-Fuller test with trend?

```
. dfuller pfi, trend

Dickey-Fuller test for unit root                    Number of obs   =       139

                                 ---------- Interpolated Dickey-Fuller ----------
                    Test         1% Critical       5% Critical      10% Critical
                 Statistic           Value             Value            Value
------------------------------------------------------------------------------
 Z(t)              -3.414            -4.027            -3.445           -3.145
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0495
```

The p-value, albeit less than 5%, is very close to 5%. We will still reject the null hypothesis at 5% size, but we cannot confidently say that the series is stationary.

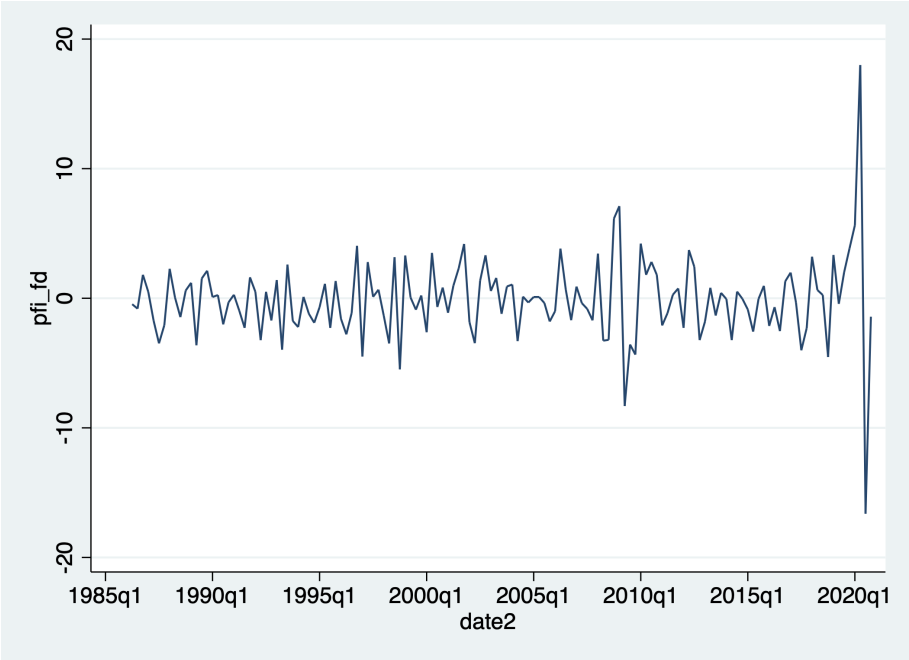
(d) Based on your conclusion in part (b) and part (c), can we directly predict the pfi series using an $AR(p)$ model? If not, what transformation on pfi needs to be done so that it is stationary?

The conclusion in (c) suggests that some nonstationarity might exist. While neither Dickey-Fuller test fully reject the nonstationary null hypothesis, the p-value being very close to 5% for the deterministic trend case suggests that some nonstationarity might still exist.

Additionally, with our conclusion from part (b), nonstationarity is certainly a concern.

So what do we do? Let's try taking the first difference of pfi, and run Dickey-Fuller test again and see what we get.

After obtaining the first difference of pfi, its time series plot looks like the following:



The data now looks a lot more stationary than the plain pfi graph.

Since the first difference of pfi doesn't seem to drift or have a time trend component, we only perform the Dickey-Fuller test without trend of drift. The result is the following:

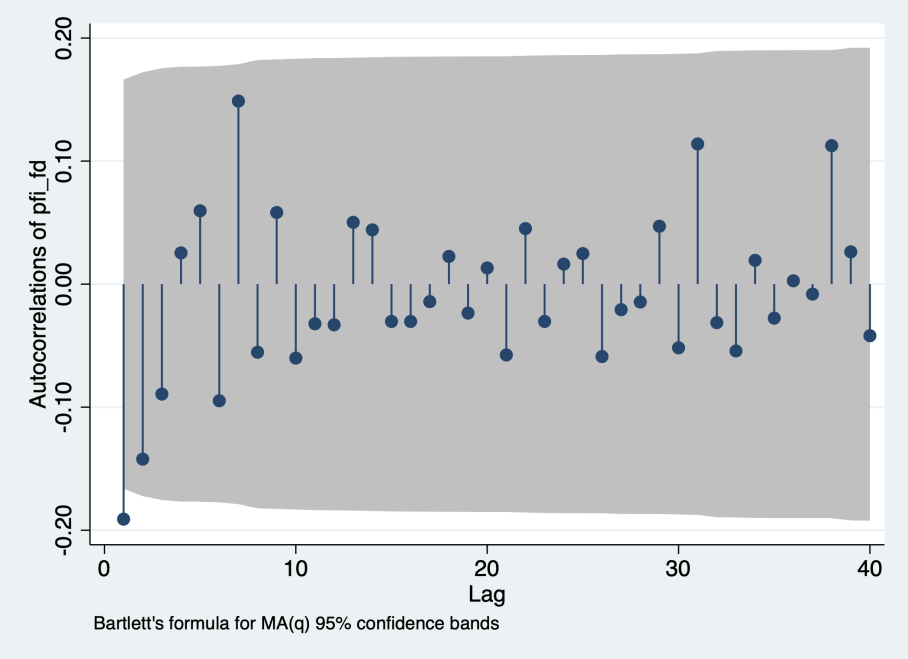
```
. dfuller pfi_fd

Dickey-Fuller test for unit root                    Number of obs   =       138

                                 ---------- Interpolated Dickey-Fuller ----------
                    Test         1% Critical       5% Critical      10% Critical
                 Statistic           Value             Value            Value
------------------------------------------------------------------------------
 Z(t)            -14.145            -3.497            -2.887           -2.577
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000
```

With the extremely small p-value, we can confidently conclude that the first difference of `pfi` is stationary.

(e) While we discussed last week that AIC and BIC can be used to determine number of lags to include in an $AR(p)$ model, an alternative way is to use the ACF plot to decide how many lags to include.

Plot the ACF for the variable that you decide to perform forecast on, and write down the corresponding model you're going to estimate.

The ACF plot of the first difference of `pfi` looks like the following:



Here, only the first lag is statistically significant (i.e. outside of the gray band), so let's use an $AR(1)$ model. Our proposed model is

$$\Delta \texttt{pfi}_t = \beta_0 + \beta_1 \Delta \texttt{pfi}_{t-1} + u_t$$

where $\Delta \texttt{pfi}_t = \texttt{pfi}_t - \texttt{pfi}_{t-1}$.

(f) Use the sample period before the first quarter of 2005 to estimate your proposed model, and use the rest of the data to construct pseudo out-of-sample estimate of MSFE (mean squared forecast error).

See Do-file for solution. Essentially, we estimate the model using data before the first quarter of 2005, and then obtain the residuals for period on and after the first quarter of 2005. The residuals here is the same as forecast error, since they are both

$$Y_t - \hat{Y}_t$$

where Y is the first difference of `pfi`.

Recall from last week that pseudo out-of-sample MSFE is estimated as

$$\widehat{MSFE}_{POOS} = \frac{1}{T-S} \sum_{j=S+1}^{T} e_j^2$$

where $S+1$ is the first pseudo out-of-sample observation (in this case, the observation corresponding to the first quarter of 2005), and $T-S$ is the number of pseudo out-of-sample observations (in this case, number of time periods between the first quarter of 2005 and the end date of the sample), and e_j is the forecast error / residual from regression.