Dis 14: TA Review Session

1 List of Concepts (Not Exhaustive)

- **Descriptive statistics**: set of methods used to summarize or present your data
  - How to calculate population parameters, given population data (mean, standard deviation, variance, covariance, correlation)
  - How to calculate the corresponding sample statistics, given sample data (keep in mind of potential degree of freedom adjustment)
  - How good are our descriptive statistics from the sample?
    - Evaluate whether a sample estimator is unbiased, relatively efficient, or consistent
- **Inferential statistics**: set of methods used to draw conclusion or make inference about the population using a sample data
  - Tools needed to make inference:
    - Probability (joint probability, conditional probability, Bayes law, etc.)
    - Random variable (assigns numbers to the outcomes of an experiment)
      - Discrete random variable, and the distributions they can follow (Binomial, Poisson)
      - Continuous random variable, and the distributions they can follow (Uniform, Normal, Exponential, Student-t, Chi-squared, F)
    - Sampling distribution: distribution of the sample statistic ($\bar{X}$, $\hat{p}$, $\bar{X}_1 - \bar{X}_2$)
  - How are inferences made:
    - Hypothesis testing – 3 different ways to testing whether the null hypothesis is rejected:
      1. Test statistic & rejection region (for one-tailed or two-tailed test): reject if test statistic is in the rejection region constructed using significance level α
      2. p-value (for one-tailed or two-tailed test): reject if p-value is less than α
      3. Confidence interval (for two-tailed test only): reject if the hypothesized value in null is outside of the confidence interval with $1 - \alpha$ confidence level
  - What population parameter(s) can we make inference on?
    - From one population:
      - μ_X, given that σ_X is known
      - μ_X, given that σ_X is unknown
      - σ_X^2
      - p (proportion of success from a Binomial experiment)
    - From two populations:
      - $\mu_1 - \mu_2$
      - $\frac{\sigma_1^2}{\sigma_2^2}$
      - $p_1 - p_2$
    - About relationships: simple linear regression
      - β_1: if β_1 is concluded to be non-zero, then the slope coefficient is statistically significant, meaning that the dependent and the independent variable share a nonzero relationship

2 Exercises

1. One of the food carts outside of Memorial Union sells spring rolls, and you want to study how many spring rolls this food cart sells in any given hour. Suppose that the amount of spring rolls sold in any hour is independent from other times, and you know that the true standard deviation of the number of spring rolls sold in an hour equals to 2.

   (a) Let X be a random variable denoting the number of spring rolls sold in an hour from this food cart. What distribution does X follow?

   Since X records the number of spring rolls sold in an hour at the said food cart, X can take on values such as $0, 1, 2, \ldots$. From this, X can be narrowed down to be a discrete random variable.

   Among the examples of discrete random variable from this class, a Poisson distribution seems to best describe X, since a Poisson distributed random variable records the number of success (i.e., selling spring rolls) within a fixed period of time (i.e., an hour).

   (b) What's the probability that the number of spring rolls sold in an hour would be less than 4?

   With $X \sim \text{Poisson}(\mu)$, the main obstacle now is to find out the parameter μ. Notice that the question stated that $\sigma_X = 2$. For a Poisson distributed random variable, we have that its variance equals to μ. Hence,

   $$V(X) = \sigma_X^2 = (\sigma_X)^2 = 4 = \mu$$

   So $X \sim \text{Poisson}(4)$.

   Now, the question asks about the probability that the number of spring rolls sold in an hour would be less than 4. Effectively, it is asking

   $$P(X < 4) = P(0 \le X \le 3)$$

   There are two ways that one can go about solving this question. Both methods yield the same answer.

   Method 1: Since X is discrete, we can write the probability as the following:

   $$\begin{aligned} P(X < 4) = P(0 \le X \le 3) &= P(X=0) + P(X=1) + P(X=2) + P(X=3) \\ &= \frac{e^{-4}4^0}{0!} + \frac{e^{-4}4^1}{1!} + \frac{e^{-4}4^2}{2!} + \frac{e^{-4}4^3}{3!} \\ &= e^{-4} \times \left(1 + 4 + 8 + \frac{32}{3}\right) \\ &= 0.4335 \end{aligned}$$

   where $P(X = x)$ is computed from the formula for a Poisson distributed X:

   $$P(X = x) = \frac{e^{-\mu}\mu^x}{x!}$$

   Method 2: Since X is Poisson distributed, and the probability that we want to solve $P(X < 4) = P(0 \le X \le 3) = P(X \le 3)$ is a common number, we can look up the probability value from the Poisson table:

TABLE 2 Poisson Probabilities

Tabulated values are $P(X \leq k) = \sum_{x=0}^{k} p(xi)$. (Values are rounded to four decimal places.)

	μ															
k	**0.10**	**0.20**	**0.30**	**0.40**	**0.50**	**1.0**	**1.5**	**2.0**	**2.5**	**3.0**	**3.5**	**4.0**	**4.5**	**5.0**	**5.5**	**6.0**
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.3679	0.2231	0.1353	0.0821	0.0498	0.0302	0.0183	0.0111	0.0067	0.0041	0.0025
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.7358	0.5578	0.4060	0.2873	0.1991	0.1359	0.0916	0.0611	0.0404	0.0266	0.0174
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9197	0.8088	0.6767	0.5438	0.4232	0.3208	0.2381	0.1736	0.1247	0.0884	0.0620
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9810	0.9344	0.8571	0.7576	0.6472	0.5366	0.4335	0.3423	0.2650	0.2017	0.1512
4		1.0000	1.0000	0.9999	0.9998	0.9963	0.9814	0.9473	0.8912	0.8153	0.7254	0.6288	0.5321	0.4405	0.3575	0.2851

This gives us $P(X \leq 3) = 0.4335$

Thus, regardless of the method, the probability that the number of spring rolls sold in an hour would be less than 4 is 0.4335.

2. App developers often perform A/B testing to see whether a change is well perceived. Suppose that 100 people are randomly selected to participate in an A/B testing. In this test, the developer randomly assigns all 100 users to one of the two equally-sized groups: the control group, and the experiment group. Users in the control group sees no change in app interface, while users in the experiment group is presented with a pop-up window that encourages them to change their current status.

   The app developer observes that, on average, the experiment group changes their status 5 more times compared with the control group. The estimated standard error of the mean is 9 for the control group, and 4 for the experiment group. The population standard deviations are unknown, and they are not assumed to be equal.

   (a) What's the sample variance of the amount of times users change their status for users in the experiment group?

   Denote the experiment group as the first group, and the control group as the second group.

   From the question, we learned the following:

$$n_1 = n_2 = \frac{100}{2} = 50, \quad \bar{X}_1 - \bar{X}_2 = 5, \quad \frac{s_1}{\sqrt{n_1}} = 4, \quad \frac{s_2}{\sqrt{n_2}} = 9$$

   Since $\frac{s_1}{\sqrt{n_1}} = 4$, and $n_1 = 50$, we have that

$$s_1 = 4 \times \sqrt{n_1} \quad \Rightarrow \quad s_1^2 = (4 \times \sqrt{n_1})^2 = 800$$

   Thus, the sample variance of the amount of times users change their status for users in the experiment group is 800.

   (b) Test whether the new pop-up window makes people change their status more on average, using a 5% significance level.

   i. Hypotheses:

   Since we are testing whether the new pop-up window makes people change their status **more on average**, our hypotheses are the following:

$$H_0 : \mu_1 - \mu_2 = 0$$
$$H_1 : \mu_1 - \mu_2 > 0$$

ii. Test statistic & its distribution:
The experiment and control group are not matched (people are randomly selected into each group), so we don't have a matched pair. Since the question stated that population standard deviations are unknown and are NOT assumed to be equal, we have

$$\text{test statistic} = \frac{(\bar{X}_1 - \bar{X}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(\bar{X}_1 - \bar{X}_2) - 0}{\sqrt{\left(\frac{s_1}{\sqrt{n_1}}\right)^2 + \left(\frac{s_2}{\sqrt{n_2}}\right)^2}} \overset{a}{\sim} t_{\text{Satterthwaite}}$$

where

$$\text{Satterthwaite} = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1-1} + \frac{(s_2^2/n_2)^2}{n_2-1}} = 68 \text{ for this sample}$$

And Satterthwaite is the degree of freedom for the t-distribution.

iii. Rejection rule:
Looking up the t-table for the critical value with a right-tail area of 5%, we find that the critical value 1.667. (Note: Though the t-table doesn't have a row where degree of freedom equals to 68 exactly, you can find the nearest degree of freedom where it exists in the table. On the exam, we will allow margin of error of this kind.)
Hence, the rejection region is

$$\text{test statistic} > 1.667$$

Calculating the value of test statistic with this sample yields $\text{test statistic} = 0.507$. Since $\text{test statistic} > 1.667$ is NOT satisfied, we fail to reject the null hypothesis at 5% significance level, and we fail to conclude that the new pop-up window makes people change their status more on average.

3. You're interested in learning the average amount of daily coffee consumption among Econ students taking stats and econometrics classes. There are in total 1400 students enrolled in these classes, and you collected a random sample of size 100.

Suppose that the true average amount of coffee consumed by such Econ students is 11 oz per day, and the population variance of such Econ student's coffee consumption is known, and it is 4 oz per day.

(a) Let X be the amount of coffee consumed daily by Econ students enrolled in stats and econometrics classes. What distribution does the sample average $\bar{X}$ follow?
Since $n = 100 \geq 30$, by Central Limit Theorem, $\bar{X} \overset{a}{\sim} N(\mu_{\bar{X}}, \sigma^2_{\bar{X}})$.
Now, from what we learned about sampling distribution, $\mu_{\bar{X}} = \mu_X = 11$.
However, in terms of $\sigma^2_{\bar{X}}$, since $N = 1400$, and $n = 100$, we do NOT have $N \geq 20n$ satisfied, meaning that we need finite population correction factor (which equals to $\sqrt{\frac{N-n}{N-1}}$) to adjust for $\sigma_{\bar{X}}$ (i.e., the standard error at the mean).

This gives us the following:

$$\sigma_{\bar{X}}^2 = \left(\frac{\sigma_X}{\sqrt{n}} \times \sqrt{\frac{N-n}{N-1}} \right)^2 = \left(\frac{\sqrt{4}}{\sqrt{100}} \times \sqrt{\frac{1400-100}{1400-1}} \right)^2 = (0.1928)^2$$

Thus, $\bar{X} \overset{a}{\sim} N(11, (0.1928)^2)$.

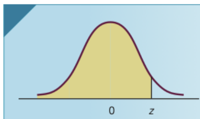
(b) What's the probability that the average found in your sample ends up being greater than 10.7 oz per day?

This question asks for

$$\begin{aligned} P(\bar{X} > 10.7) &= P\left(\frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} > \frac{10.7 - 11}{0.1928} \right) \\ &= P(Z > -1.56) = P(Z < 1.56) \qquad \text{(based on distribution's symmetry around 0)} \\ &= 0.9406 \end{aligned}$$

The last value is found by looking up the standard normal z-table:

TABLE **3** ***(Continued)***



$P(-\infty < Z < z)$

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	$\boxed{0.9406}$	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545

Thus, the probability that the average found in your sample ends up being greater than 10.7 oz per day is 0.9406.

Probability table for a Poisson distribution

TABLE 2 Poisson Probabilities

Tabulated values are $P(X \leq k) = \sum_{x=0}^{k} p(xi)$. (Values are rounded to four decimal places.)

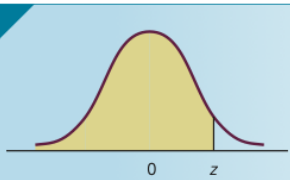
	μ															
k	**0.10**	**0.20**	**0.30**	**0.40**	**0.50**	**1.0**	**1.5**	**2.0**	**2.5**	**3.0**	**3.5**	**4.0**	**4.5**	**5.0**	**5.5**	**6.0**
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.3679	0.2231	0.1353	0.0821	0.0498	0.0302	0.0183	0.0111	0.0067	0.0041	0.0025
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.7358	0.5578	0.4060	0.2873	0.1991	0.1359	0.0916	0.0611	0.0404	0.0266	0.0174
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9197	0.8088	0.6767	0.5438	0.4232	0.3208	0.2381	0.1736	0.1247	0.0884	0.0620
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9810	0.9344	0.8571	0.7576	0.6472	0.5366	0.4335	0.3423	0.2650	0.2017	0.1512
4		1.0000	1.0000	0.9999	0.9998	0.9963	0.9814	0.9473	0.8912	0.8153	0.7254	0.6288	0.5321	0.4405	0.3575	0.2851
5				1.0000	1.0000	0.9994	0.9955	0.9834	0.9580	0.9161	0.8576	0.7851	0.7029	0.6160	0.5289	0.4457
6						0.9999	0.9991	0.9955	0.9858	0.9665	0.9347	0.8893	0.8311	0.7622	0.6860	0.6063
7						1.0000	0.9998	0.9989	0.9958	0.9881	0.9733	0.9489	0.9134	0.8666	0.8095	0.7440
8							1.0000	0.9998	0.9989	0.9962	0.9901	0.9786	0.9597	0.9319	0.8944	0.8472
9								1.0000	0.9997	0.9989	0.9967	0.9919	0.9829	0.9682	0.9462	0.9161
10									0.9999	0.9997	0.9990	0.9972	0.9933	0.9863	0.9747	0.9574
11									1.0000	0.9999	0.9997	0.9991	0.9976	0.9945	0.9890	0.9799
12										1.0000	0.9999	0.9997	0.9992	0.9980	0.9955	0.9912
13											1.0000	0.9999	0.9997	0.9993	0.9983	0.9964
14												1.0000	0.9999	0.9998	0.9994	0.9986
15													1.0000	0.9999	0.9998	0.9995
16														1.0000	0.9999	0.9998
17															1.0000	0.9999
18																1.0000
19																
20																

Probability table for a Poisson distribution (continued)

	μ												
k	**6.50**	**7.00**	**7.50**	**8.00**	**8.50**	**9.00**	**9.50**	**10**	**11**	**12**	**13**	**14**	**15**
0	0.0015	0.0009	0.0006	0.0003	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0113	0.0073	0.0047	0.0030	0.0019	0.0012	0.0008	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000
2	0.0430	0.0296	0.0203	0.0138	0.0093	0.0062	0.0042	0.0028	0.0012	0.0005	0.0002	0.0001	0.0000
3	0.1118	0.0818	0.0591	0.0424	0.0301	0.0212	0.0149	0.0103	0.0049	0.0023	0.0011	0.0005	0.0002
4	0.2237	0.1730	0.1321	0.0996	0.0744	0.0550	0.0403	0.0293	0.0151	0.0076	0.0037	0.0018	0.0009
5	0.3690	0.3007	0.2414	0.1912	0.1496	0.1157	0.0885	0.0671	0.0375	0.0203	0.0107	0.0055	0.0028
6	0.5265	0.4497	0.3782	0.3134	0.2562	0.2068	0.1649	0.1301	0.0786	0.0458	0.0259	0.0142	0.0076
7	0.6728	0.5987	0.5246	0.4530	0.3856	0.3239	0.2687	0.2202	0.1432	0.0895	0.0540	0.0316	0.0180
8	0.7916	0.7291	0.6620	0.5925	0.5231	0.4557	0.3918	0.3328	0.2320	0.1550	0.0998	0.0621	0.0374
9	0.8774	0.8305	0.7764	0.7166	0.6530	0.5874	0.5218	0.4579	0.3405	0.2424	0.1658	0.1094	0.0699
10	0.9332	0.9015	0.8622	0.8159	0.7634	0.7060	0.6453	0.5830	0.4599	0.3472	0.2517	0.1757	0.1185
11	0.9661	0.9467	0.9208	0.8881	0.8487	0.8030	0.7520	0.6968	0.5793	0.4616	0.3532	0.2600	0.1848
12	0.9840	0.9730	0.9573	0.9362	0.9091	0.8758	0.8364	0.7916	0.6887	0.5760	0.4631	0.3585	0.2676
13	0.9929	0.9872	0.9784	0.9658	0.9486	0.9261	0.8981	0.8645	0.7813	0.6815	0.5730	0.4644	0.3632
14	0.9970	0.9943	0.9897	0.9827	0.9726	0.9585	0.9400	0.9165	0.8540	0.7720	0.6751	0.5704	0.4657
15	0.9988	0.9976	0.9954	0.9918	0.9862	0.9780	0.9665	0.9513	0.9074	0.8444	0.7636	0.6694	0.5681
16	0.9996	0.9990	0.9980	0.9963	0.9934	0.9889	0.9823	0.9730	0.9441	0.8987	0.8355	0.7559	0.6641
17	0.9998	0.9996	0.9992	0.9984	0.9970	0.9947	0.9911	0.9857	0.9678	0.9370	0.8905	0.8272	0.7489
18	0.9999	0.9999	0.9997	0.9993	0.9987	0.9976	0.9957	0.9928	0.9823	0.9626	0.9302	0.8826	0.8195
19	1.0000	1.0000	0.9999	0.9997	0.9995	0.9989	0.9980	0.9965	0.9907	0.9787	0.9573	0.9235	0.8752
20			1.0000	0.9999	0.9998	0.9996	0.9991	0.9984	0.9953	0.9884	0.9750	0.9521	0.9170
21				1.0000	0.9999	0.9998	0.9996	0.9993	0.9977	0.9939	0.9859	0.9712	0.9469
22					1.0000	0.9999	0.9999	0.9997	0.9990	0.9970	0.9924	0.9833	0.9673
23						1.0000	0.9999	0.9999	0.9995	0.9985	0.9960	0.9907	0.9805
24							1.0000	1.0000	0.9998	0.9993	0.9980	0.9950	0.9888
25									0.9999	0.9997	0.9990	0.9974	0.9938
26									1.0000	0.9999	0.9995	0.9987	0.9967
27										0.9999	0.9998	0.9994	0.9983
28										1.0000	0.9999	0.9997	0.9991
29											1.0000	0.9999	0.9996
30												0.9999	0.9998
31												1.0000	0.9999
32													1.0000

Probability table for a standard normal distribution ($z \geq 0$)

TABLE **3** ***(Continued)***

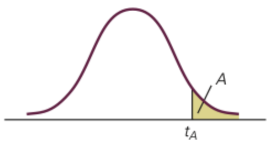


$P(-\infty < Z < z)$

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Probability table for a t-distribution

TABLE **4**
Critical Values of the Student *t* Distribution



Degrees of Freedom	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
35	1.306	1.690	2.030	2.438	2.724
40	1.303	1.684	2.021	2.423	2.704
45	1.301	1.679	2.014	2.412	2.690
50	1.299	1.676	2.009	2.403	2.678
55	1.297	1.673	2.004	2.396	2.668
60	1.296	1.671	2.000	2.390	2.660
65	1.295	1.669	1.997	2.385	2.654
70	1.294	1.667	1.994	2.381	2.648
75	1.293	1.665	1.992	2.377	2.643
80	1.292	1.664	1.990	2.374	2.639
85	1.292	1.663	1.988	2.371	2.635
90	1.291	1.662	1.987	2.368	2.632
95	1.291	1.661	1.985	2.366	2.629
100	1.290	1.660	1.984	2.364	2.626
110	1.289	1.659	1.982	2.361	2.621
120	1.289	1.658	1.980	2.358	2.617
130	1.288	1.657	1.978	2.355	2.614
140	1.288	1.656	1.977	2.353	2.611
150	1.287	1.655	1.976	2.351	2.609
160	1.287	1.654	1.975	2.350	2.607
170	1.287	1.654	1.974	2.348	2.605
180	1.286	1.653	1.973	2.347	2.603
190	1.286	1.653	1.973	2.346	2.602
200	1.286	1.653	1.972	2.345	2.601
∞	1.282	1.645	1.960	2.326	2.576