Dis 2: Coase Theorem; Bargaining †

1 Review: Coase Theorem

Theorem 1 (Coase Theorem)**.** If property rights are well-defined and tradable, then in the absence of transaction costs, voluntary negotiations will lead to efficiency.

- Notice the conditions for achieving efficiency:

  Property rights must be **well-defined**
  & Property rights must be **tradable** $\longrightarrow$ Efficiency achieved by voluntary negotiation
  & Transaction costs must be **absent**

- The three conditions stated above are **sufficient conditions**, but not **necessary***:
  - If voluntary negotiations do not lead to efficient outcomes, then at least one of these three conditions should fail.
  - If one or more of the conditions do not obtain, then voluntary negotiations may not lead to efficient outcomes (but this **does not imply that voluntary negotiations never** lead to efficient outcomes).
- What does Coase theorem predict?
  - **In terms of the final allocation:** Coase theorem predicts that, regardless of how property right is assigned in the first place, as long as the right is well-defined + transferable + transaction costs are low, then the final allocation is always efficient.
  - **In terms of actual payoff to parties involved:** Coase theorem doesn't tell us anything about how each involved person's payoffs are under different property right assignment.

    (Recall: Example of farmer and rancher from lecture 4 and 5)

† Adapted from Jonathan Becker's Fall 2018 handout

* In mathematical logic, if $A \to B$, but not $B \to A$, then A is the sufficient but not necessary conditions for B. Additionally, $A \to B$ is logically equivalent to its contrapositive ($\neg B \to \neg A$). Notice that $A \to B$ does not imply $B \to A$.

2 Review: Bargaining

2.1 Terminology

Coase theorem gives us conditions allowing for efficiency to be achieved through voluntary bargaining. When bargaining takes place, here are some important terminologies to help us figure out the outcome of bargaining:

- **Threat point** (also known as **outside option** / **reservation utility**): the payoff each player can guarantee for themselves if a deal cannot be reached
- **Gains from cooperation**: the amount increased in the combined payoffs if a deal is reached

2.2 One Obstacle to Bargaining: Transaction Costs

Definition 1 (Transaction Costs)**.** Any cost that makes it difficult or expensive for two or more parties to achieve a mutually beneficial trade

- Examples of transaction costs:
  - **Search costs**: difficulty or cost of finding a trading partner
  - **Enforcement costs**: difficulty or expense of enforcing an agreement after the fact
  - **Bargaining costs**: difficulty in reaching an agreement with the trading partner
    - Asymmetric information / Adverse selection (you know something that I don't)
    - Private information (either person not knowing the other's threat point)
    - Uncertainty about property rights or threat points
    - Large number of buyers / sellers (causing holdout or freeriding)
    - Hostility
- How to deal with transaction costs? Two approaches:
  - **Normative Coase Approach**: Design the law to minimize the transaction costs
    - Goal is to "lubricate" private bargaining
      (ex. Bargaining with multiple people is too costly, so the law can direct people to appoint a representative, and let the representative engage in bargaining)
    - When optimal? $\rightarrow$ transaction costs are low, and information costs are high
  - **Normative Hobbes Approach**: Design the law to minimize the need for bargaining
    - Goal is to allocate the rights efficiently to begin with, or to allow for ways to reallocate rights without bargaining
      (ex. City planning)
      Notice: this requires us to research and figure out what is the efficient outcome, so it incurs information costs
    - When optimal? $\rightarrow$ transaction costs are high, and information costs are low

3 Problems

1. Adam is a heavy smoker. He obtains utility as a function of the number of cigarettes (X) he smokes and the amount of money (m_A) he has:

$$U_A = 36X - 2X^2 + m_A$$

The costs of smoking X cigarettes are $C(X) = X^2$.

Bob, who is Adam's roommate, detests smoking. His utility is a decreasing function in the number of cigarettes Adam smokes:

$$U_B = 128 - X^2 + m_B$$

where m_B represents the amount of money Bob has.

Assume that Adam and Bob each starts with a sufficiently large amount of money M, such that their budget constraints never bind.

(a) How many cigarettes will Adam choose to smoke, if he lives alone and makes rational decisions?

Since the budget constraints never bind, the amount of money Adam has $m_A = M - C(X) = M - X^2$.

So, by himself, Adam faces the following maximization problem:

$$\max_X U_A \quad \Leftrightarrow \quad \max_X (36X - 2X^2 + M - X^2)$$

Since U_A is strictly concave, the FOC yields $36 - 4X - 2X = 0$, so $X = 6$.

The resulting utility is $36 * 6 - 2 * (6^2) + M - (6^2) = M + 108$.

(b) What is the efficient number of cigarettes when they live together?

The efficient X should maximizes the total utility of Adam and Bob. Here, notice that $m_B = M$. So the total utility between the two $U_A + U_B = 36X - 2X^2 + M - X^2 + 128 - X^2 + M$.

Efficiency thus solves

$$\max_X (U_A + U_B) \quad \Leftrightarrow \quad \max_X (36X - 2X^2 + M - X^2 + 128 - X^2 + M)$$

Since $U_A + U_B$ is strictly concave, the FOC yields $36X - 4X - 2X - 2X = 0$. We thus have $X = 4.5$ as the efficient number of cigarettes.

Suppose that Adam owns the apartment, and Bob has to bribe Adam to stop him from smoking.

(c) What is the threat point for Adam? For Bob?

Adam has the exclusive property rights to the apartment. If Bob does not bribe him, Adam will choose to smoke 6 cigarettes, gaining a utility of $M + 108$. Therefore, Adam will not accept any outcome that leaves him with utility less than $M + 108$. In other words, $(M + 108)$ is Adam's threat point.

Without bargaining, Bob has to accept the Adam's decision of smoking 6 cigarettes, resulting in his utility of $128 - 6^2 + M = M + 92$. Any bargaining outcome must leave Bob a utility equal to or higher than $M + 92$, otherwise he will not participate in bargaining. Hence $(M + 92)$ is Bob's threat point.

(d) How many cigarettes does Coase theorem predict that Adam will smoke?

By Coase Theorem, under the assumption that the right to smoke by Adam is transferable, and transaction costs are absent, bargaining will reach the efficient outcome, i.e. Adam smoking 4.5 cigarettes.

(e) How much is the gains from cooperation?

Let S be the amount of bribe that Bob pays Adam.

When a deal is reached,

- Adam's utility will be

$$\begin{aligned}
&U_A(X = 4.5, m_A = M - C(4.5) + S) \\
&= 36 \times 4.5 - 2 \times 4.5^2 + M - 4.5^2 + S \\
&= 101.25 + M + S
\end{aligned}$$

- Bob's utility will be

$$\begin{aligned}
&U_B(X = 4.5, m_B = M - S) \\
&= 128 - 4.5^2 + M - S \\
&= 107.75 + M - S
\end{aligned}$$

Gains from cooperation is the increase in payoffs (utility) from bargaining:

$$\begin{aligned}
&[U_A(X = 4.5, m_A = M - C(4.5) + S) + U_B(X = 4.5, m_B = M - S)] \\
&\quad - [U_A(X = 6, m_A = M - C(6)) + U_B(X = 6, m_B = M)] \\
&= [101.25 + M + S + 107.75 + M - S] - [M + 108 + M + 92] \\
&= 9
\end{aligned}$$

Thus, gains from cooperation is 9.

(f) How much money will Bob be willing to pay Adam to make him smoke the efficient number of cigarettes?

To make sure that bargaining can take place, we need both of the following conditions to hold:

$$U_A(X = 4.5, m_A = M - C(4.5) + S) \geq \text{A's threat point: } U_A(X = 6, m_A = M - C(6)) \quad (1)$$
$$U_B(X = 4.5, m_B = M - S) \geq \text{B's threat point: } U_B(X = 6, m_B = M) \quad (2)$$

Solving for (1):

$$101.25 + M + S \geq M + 108 \quad \Leftrightarrow \quad S \geq 6.75$$

Solving for (2):

$$107.75 + M - S \geq M + 92 \quad \Leftrightarrow \quad S \leq 15.75$$

Thus, S could be any value between 6.75 and 15.75. The exact amount depends on the bargaining power that Adam and Bob each has.

(g) If the gains from cooperation are split evenly between Adam and Bob, how much then should

Bob pay Adam to make him smoke the efficient number of cigarettes?

Gains of cooperation split evenly is $9/2 = 4.5$ (i.e. Each person gets 4.5 higher utility compared with their threat points).

Payoff for Adam after bargaining is thus

$$\begin{aligned} & U_A(X = 6, m_A = M - C(6)) + 4.5 \\ &= M + 108 + 4.5 = M + 112.5 \end{aligned}$$

which should exactly equal to the payoff for Adam after a deal being made when he receives payment S:

$$\begin{aligned} U_A(X = 4.5, m_A = M - C(4.5) + S) &= U_A(X = 6, m_A = M - C(6)) + 4.5 \\ 101.25 + M + S &= M + 112.5 \\ S &= 11.25 \end{aligned}$$

Note: You can also do this from Bob's perspective. Bob's payoff after bargaining is exactly the same as his payoff after a payment S is made to Adam:

$$\begin{aligned} U_B(X = 4.5, m_B = M - S) &= U_B(X = 6, m_B = M) + 4.5 \\ 107.75 + M - S &= M + 92 + 4.5 \\ S &= 11.25 \end{aligned}$$

We arrive at the same answer that Bob needs to pay Adam 11.25 dollars.

(h) Suppose that Adam owns the apartment, and Bob can only bargain with the help of a lawyer. What is the maximum amount that the lawyer is able to charge, if Bob pays the lawyer fees? What if Adam pays the fees instead?

Suppose that the lawyer charges L.

If Bob pays the fee: If the bargaining by lawyer works, then each person's utility after bargaining should exceed his threat point:

$$\begin{cases} U_A(X = 4.5, m_A = M - C(4.5) + S) \geq \text{A's threat point: } U_A(X = 6, m_A = M - C(6)) \\ U_B(X = 4.5, m_B = M - S - L) \geq \text{B's threat point: } U_B(X = 6, m_B = M) \end{cases}$$

$$\Rightarrow \begin{cases} 101.25 + M + S \geq 108 + M \\ 107.75 + M - S - L \geq M + 92 \end{cases}$$

$$\Rightarrow \begin{cases} S \geq 6.75 \\ S \leq 15.75 - L \end{cases}$$

Here, for $6.75 \leq S \leq 15.75 - L$ to hold, we need $15.75 - L \geq 6.75$, which yields that $L \leq 9$.

Thus, at most, the lawyer can charge a fee of 9 dollars.

If Adam pays the fee:

Again, each person's utility after bargaining should exceed his threat point. We now have

$$\begin{cases} U_A(X = 4.5, m_A = M - C(4.5) + S - L) \geq \text{A's threat point: } U_A(X = 6, m_A = M - C(6)) \\ U_B(X = 4.5, m_B = M - S) \geq \text{B's threat point: } U_B(X = 6, m_B = M) \end{cases}$$

$$\Rightarrow \begin{cases} 101.25 + M + S - L \geq 108 + M \\ 107.75 + M - S \geq M + 92 \end{cases}$$

$$\Rightarrow \begin{cases} S \geq 6.75 + L \\ S \leq 15.75 \end{cases}$$

Here, for $6.75 + L \leq S \leq 15.75$ to hold, we need $15.75 \geq 6.75 + L$, which again yields that $L \leq 9$. Notice that 9 is also the gains from cooperation. Thus, the lawyer can at most extract the gains from cooperation (which is the increase in total surplus from successful bargain).