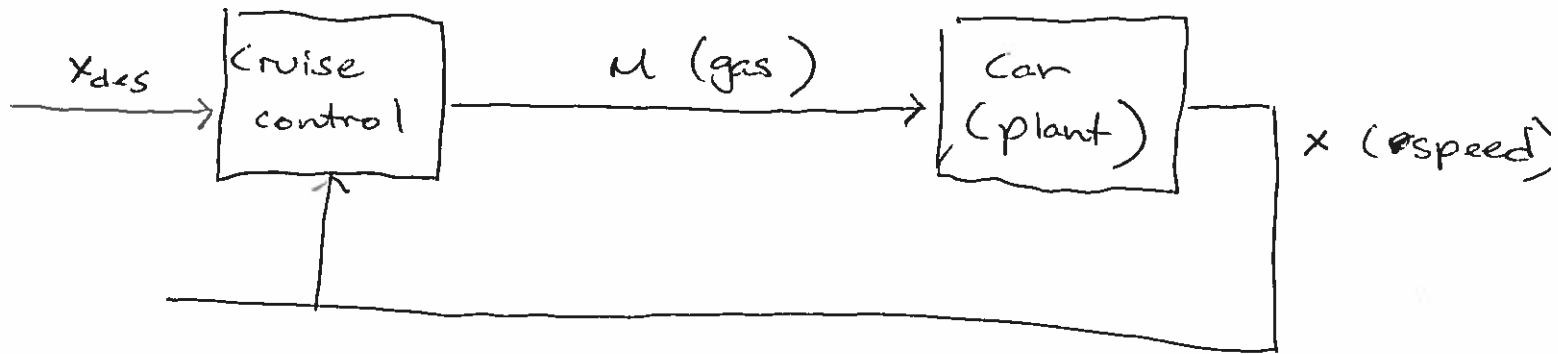


# Feedback control



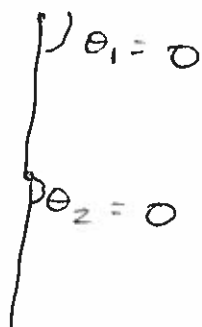
- $x$  is state
- $u$  is control

This is a proportional controller:  
amount of gas proportional to error

$$u = K \cdot (x_{des} - x)$$

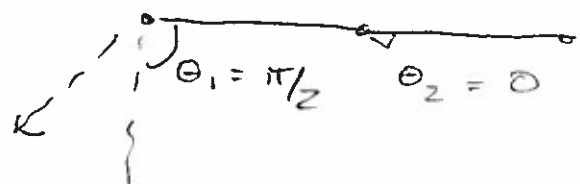
Problem with P-control:

- Rise time too long or too much overshoot (depending on size of  $K$  term)



object of study:  
double pendulum

Desired set point:



State for pendulum:  $\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2$

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Notation:  $\dot{\theta}_i = d\theta/dt$  ( $\theta$  is a function of time)

Control for pendulum:  $\tau_1, \tau_2$

• We'll ignore actual dynamics of pendulum for now except to say that:

$$\{\ddot{\theta}_1, \ddot{\theta}_2\} = f(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2, \tau_1, \tau_2)$$

Still complicated, but given large enough positive  $\tau_1, \tau_2$  then  $\ddot{\theta}_1, \ddot{\theta}_2$  will be positive

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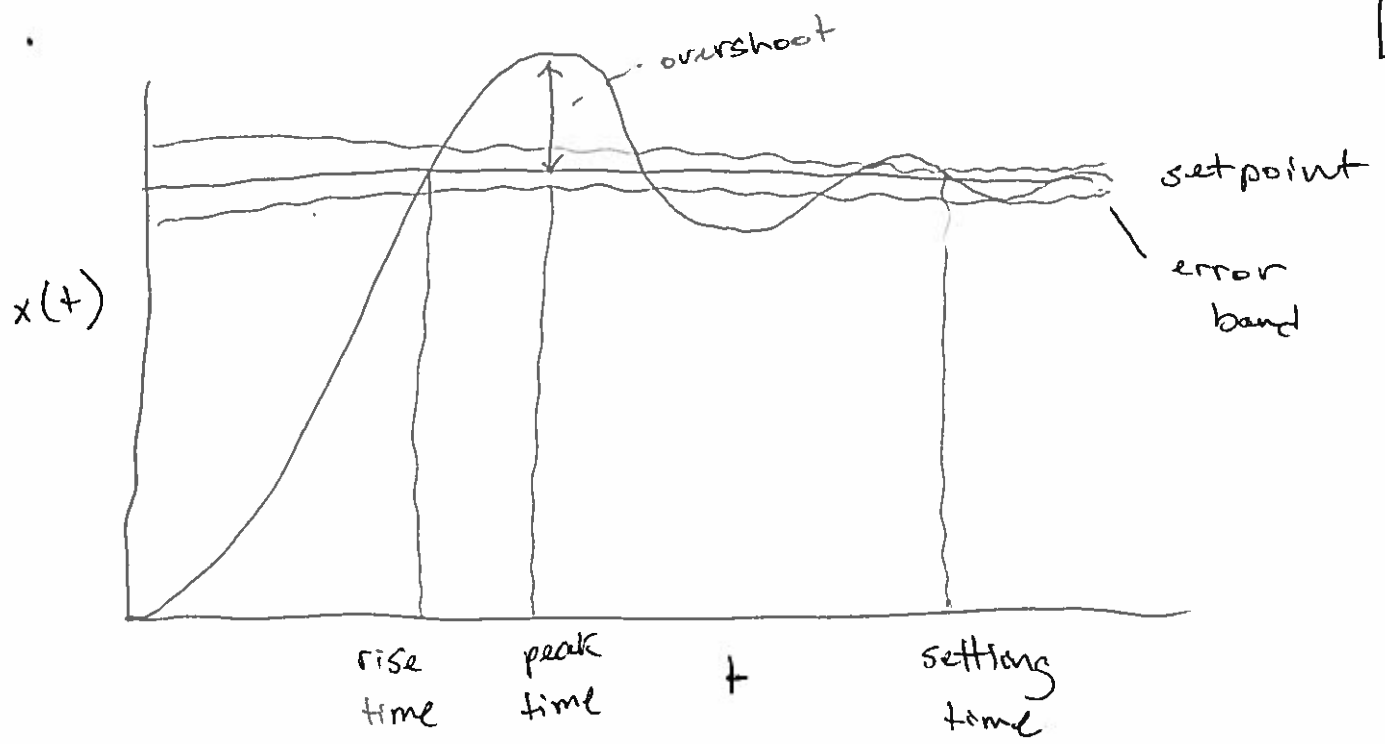
Proportional-derivative controller:

$$\tau_1 = K_p^1 \cdot (\theta_1^{des} - \theta_1) + K_d^1 \cdot (\dot{\theta}_1^{des} - \dot{\theta}_1)$$

$$\tau_2 = K_p^2 \cdot (\theta_2^{des} - \theta_2) + K_d^2 \cdot (\dot{\theta}_2^{des} - \dot{\theta}_2)$$

• How to pick  $K_p, K_d$ ?

- Rule of thumb  $K_d$  order of mag smaller than  $K_p$
- Ziegler-Nichols
- Optimization

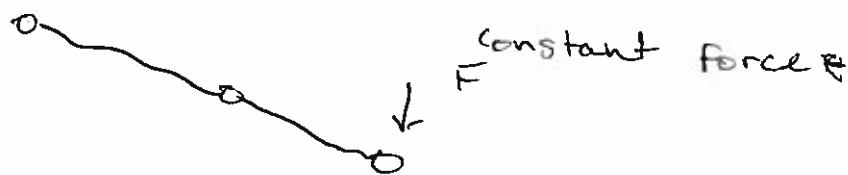


rise time: time to reach the set point from start

peak time: time at maximum overshoot

settling time: time at which system enters and remains within error band

Integral control:



Steady state error will make it difficult for robot to get to the set point

Add integral term:  $K_i \int_0^t (\theta^{des}(t) - \theta(t)) dt$

Can also have  $PIDD^2$  controllers, integrals on velocity error, etc.

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Exercise : write a <sup>PD</sup> controller (in class) function using this template:

```
double PD-controller (double theta-des,  
double dtheta-des, double theta,  
double dtheta)  
{  
  
}
```