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Dissipation and θ_{13} in neutrino oscillations

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Abstract We obtain a complete survival and transition probability involving three neutrino flavors when dissipation effects in vacuum are taken into consideration. In an approach that presents decoherence and relaxation effects, we study the behavior of the probabilities obtained from complete positivity constraints. Making the von Neumann entropy increase in time, many cases can be obtained and studied with the Lindblad master equation with addition of only one or two parameters related to dissipation. New possibilities are obtained when we take into account two decoherence parameters with different magnitudes which are given by reactor and accelerator neutrino oscillation experiments. We also present a model with only one parameter that has an important symmetry property, which can be used when the effective matter potential is important. Furthermore, the dissipation effects can contribute to the appearance of neutrinos that can hide or imitate the θ_{13} effects and we study these possibilities showing that dissipative effects have an important role in three-neutrino oscillations.

1 Introduction

The hypothesis of considering neutrinos evolving as an open quantum system has been evoked in several analyses resulting in nonstandard neutrino oscillations which proved to be successful to describe experimental data in different circumstances [1–5]. In fact, physical systems are not completely isolated. Consequently, it is important to employ a formalism which incorporates a description of such an open quantum system in a reliable way. In general, the Lindblad approach to these systems is very useful and its master equation appears in several articles including phenomenological parameters which describe decoherence or other dissipation effects [6–10].[1] In the present analysis we will use the Lindblad formalism imposing complete positivity which ensures that the time evolution of the open system can always be physically interpreted in a reasonable way. We will discuss how decoherence and relaxation effects change the behavior of the standard survival and transition probability of quantum oscillations in a system involving three neutrino families.

Some articles exist in the literature which use the Lindblad master equation to describe the decoherence effects between two neutrino families [13]. There are papers that present experimental data analysis for oscillations between electron and muon neutrinos and also for oscillations between muon and tau neutrinos [4, 5]. From these studies it is clear that decoherence can present different intensity for oscillations between different neutrino families. Experimental data analyses were made with open quantum system approach in three-neutrino oscillations in the articles [1, 2] where some limits to the decoherence effects were found.

Our interest along the present article is to investigate the dissipation effects in a three neutrino subsystem that is allowed to interact with the environment. A three neutrino subsystem implies many more new parameters in the Lindblad master equation. These parameters may incorporate several new phenomena like loss of coherence, relaxation, entropy increasing, irreversibility, and statistical mixing between the quantum states involved in the time evolution of the system.

In order to make our analysis more realistic and simpler, we will impose energy conservation and add relaxation effects in the neutrino subsystem. These two constraints eliminate a great number of the new parameters. We will find five different resulting models which will present complete positivity and present only one or two new parameters.

In summary, we will analyze a general model where decoherence is evoked as coherence suppressor and relaxation

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[1] There are other similar approaches to treat open systems, like the one developed in [11]. A discussion of the differences and problems in these approaches can be found in [12].

effects lead the system to the maximal mixing states. Thus, following Lindblad theory, with the constraints discussed above, we obtain a finite number of models which do not violate the complete positivity. A general survival and transition three neutrino oscillation probability will be presented.

Furthermore, we will study the appearance of neutrinos due to θ_{13} and dissipation effects. Daya Bay and Double Chooz experiments confirm that $\sin^2(2\theta_{13}) \sim 0.1$ [14, 15]. Such a value implies that dissipative effects can be relevant together or not with θ_{13} to the appearance neutrinos. This could be useful to new data analysis and to determine new constraints on decoherence and relaxation parameters as well as on θ_{13} itself.

2 Formalism

The dissipation effects in neutrino oscillation can be studied by Lindblad master equation. We treat the neutrinos as a subsystem and together with the environment compose a global system. To apply Lindblad theory in a correct form, we must suppose that initially the subsystems of interest and the environment are not correlated. This ensures that only the subsystem of interest is unambiguously evolved. The product tensor of the states that form the global system can be written as $\rho = \rho_\nu \otimes \rho_e$, where ρ, ρ_ν and ρ_e are the density matrices describing the global system, the neutrino subsystem and the environment, respectively [9, 16].

In the Lindblad master equation approach, we do not need further knowledge about environment, once the trace is taken over the environment degrees of freedom, and the dynamic equation for the interest subsystem is written as

$$\frac{d\rho_\nu(t)}{dt} = -i\big[H_m, \rho_\nu(t)\big] + D\big[\rho_\nu(t)\big], \tag{1}$$

where H_m is the interest subsystem Hamiltonian and $D[\rho_\nu(t)]$ is given by

$$D[\rho_\nu] = \frac{1}{2}\sum_{k=1}^{N^2-1}\big(\big[V_k, \rho_\nu V_k^\dagger\big] + \big[V_k\rho_\nu, V_k^\dagger\big]\big), \tag{2}$$

where N is the dimension of Hilbert space of the interest subsystem, the operators V_k, with $k = 1, 2, \dots, N^2 - 1$, preserve the trace of ρ_ν only if and $\sum_k V_k^\dagger V_k = 1$. The V operator acts only on the interest subsystem states and it arises from the interaction with the environment. Equation (1) is obtained when the weak coupling limit approach is applied. This condition is necessary to avoid possible inconsistencies like the appearance of negative probabilities.[2] The evolution from Eq. (1) leads from an initial density matrix to a new density matrix and the time evolution is complete positive, transforming pure states into mixed states due to dissipation effects [9, 17, 18]. We can impose that the von Neumann entropy, $S = -\mathrm{Tr}[\rho_\nu \ln \rho_\nu]$ increases in time for the density matrix of interest subsystem doing $V_k^\dagger = V_k$ [19]. The entropy must be increasing in time because the environment is understood as a reservoir which remains unchanged in time.

In current circumstances, we want to study three neutrino oscillations. Therefore, we can expand the dynamical Eq. (1) in the basis of the $SU(3)$ matrices defined as

$$F_0 = \frac{1}{\sqrt{6}} I_3, \qquad F_j = \frac{1}{2}\lambda_j \quad (j = 1, 2, \dots, 8) \tag{3}$$

where λ_i are the Gell-Mann matrices. This basis satisfies

$$[F_j, F_k] = i\sum_l f_{jkl} F_l, \tag{4}$$

such that f_{jkl} are the structure constants of SU(3). So, Eq. (1) is written as

$$\dot\rho_\vartheta\lambda_\vartheta = f_{ijk} H_i \rho_j \lambda_\vartheta \delta_{\vartheta k} + \rho_\nu D_{\vartheta\nu}\lambda_\vartheta, \tag{5}$$

where ρ_ϑ and H_i are the vector coefficients of each term of the expansion of $\rho(t)$ and H_m in the basis of the $SU(3)$ matrices. The term $D_{\mu\nu}$ is derived from the following expansion: $V_k = a_\eta^k \lambda_\eta$ by solving the commutators in (2) with $V_k = V_k^\dagger$ [19]. In this way, we obtain a real, symmetric, and positive matrix. This term must be carefully treated to keep the consistency of time evolution, where one has to transform a positive density operator into positive density operator [8, 9]. The dissipation matrix $D_{\mu\nu}$ is 8×8 in the present case, with 36 parameters bounded among each other by inequalities which guarantee positivity of the evolution, but the complete relations among these elements will not be reproduced here. One of these relations is given by

$$D_{ii} \le \sum_{i\ne j=1}^{8} D_{jj}, \tag{6}$$

where diagonal parameters are positive definite, $D_{kk} \ge 0$. This is the main relation because off diagonal parameters must be smaller than the diagonal parameter to keep $D_{\mu\nu}$ positive definite. Furthermore, it has an important role in models that we will introduce in the next section.

On the other hand, we impose two constraints: first that the average energy of subsystem, $\mathrm{Tr}[H\rho]$, is conserved and second, relaxation effects will be included to break down this previous symmetry because the global system must obey the energy conservation. So, the relations among the dissipative parameters are easily obtained this form.

In other words, these constraints imply that the quantum dissipator $D_{\mu\nu}$ will have a diagonal form. A discussion about complete positivity to the general case of a diagonal quantum dissipator is shown in Appendix A. To impose

[2] A more rigorous study about this can be found in [8, 9, 17].

these constraints we start with $[V_k, H] = 0$ which implies a diagonal matrix such as

$$D_{\mu\nu} = -\operatorname{diag}\left\{0, 2(\mathbf{a}_3)^2, 2(\mathbf{a}_3)^2, 0, \frac{1}{2}(\mathbf{a}_3+\mathbf{a}_8)^2, \frac{1}{2}(\mathbf{a}_3+\mathbf{a}_8)^2, \frac{1}{2}(\mathbf{a}_3-\mathbf{a}_8)^2, \frac{1}{2}(\mathbf{a}_3-\mathbf{a}_8)^2, 0\right\}, \tag{7}$$

or, equivalently

$$\begin{aligned} D_{11} &= D_{22} = 2a_3^2, \\ D_{44} &= D_{55} = \frac{1}{2}\left(a_3^2+a_8^2\right) + a_3 a_8 \cos\phi, \\ D_{66} &= D_{77} = \frac{1}{2}\left(a_3^2+a_8^2\right) - a_3 a_8 \cos\phi, \end{aligned} \tag{8}$$

where ϕ is the angle between the vectors $\mathbf{a}_3$ and $\mathbf{a}_8$. Due to the equations in (8) it is possible to show that there exist 11 different models that include a decoherence effect in the oscillation neutrino model and keep the complete positivity of the time evolution. These models only depend on the parameterizations of a_3, a_8, and $\cos\phi$ to keep the complete positivity.

The relaxation effects are obtained by imposing nonvanishing D_{33} and D_{88} which are renamed as $D_{33} = \Gamma_{33}$ and $D_{88} = \Gamma_{88}$. Also, all entries are changed in (8) to a new form as is shown in Appendix A. This increases considerably the number of allowed models and we will study some of them to impose experimental constraints. In fact, some of these analysis have already been performed [4, 5].

We rewrite the new quantum dissipator with the renamed parameters as

$$D_{\mu\nu} = -\operatorname{diag}\{0, \Gamma_{12}, \Gamma_{12}, \Gamma_{33}, \Gamma_{13}, \Gamma_{13}, \Gamma_{23}, \Gamma_{23}, \Gamma_{88}\}. \tag{9}$$

The Hamiltonian describing the neutrino propagation in vacuum is diagonal when written in the mass basis, $H_m = \operatorname{diag}[E_1, E_2, E_3]$, with $E_i^2 = m_i^2 + p_i^2$ and the relation between flavor and mass basis is the usual one, $\nu_\alpha = U_{\alpha i}\nu_i$, where α is the flavor neutrino, e, μ, τ and i is the index for each mass state, $1, 2, 3$. Furthermore, the mixing matrix U is written as

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}d^* \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}d & c_{12}c_{23} - s_{12}s_{23}s_{13}d & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}d & -c_{12}s_{23} - s_{12}c_{23}s_{13}d & c_{23}c_{13} \end{pmatrix}, \tag{10}$$

where $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$, and $d = \operatorname{Exp}[i\delta]$, where δ is the Dirac CP phase.

Solving Eq. (5), we can write the density matrix after time evolution summing $\rho(t) = \rho_\vartheta \lambda_\vartheta$ to arrive at the following state:

$$\rho(t) = \begin{pmatrix} \tilde{\rho}_{11}(t) & \rho_{12}(0)e^{-\widetilde{\Delta}_{12}^* t} & \rho_{13}(0)e^{-\widetilde{\Delta}_{13}^* t} \\ \rho_{21}(0)e^{-\widetilde{\Delta}_{12} t} & \tilde{\rho}_{22}(t) & \rho_{23}(0)e^{-\widetilde{\Delta}_{23} t} \\ \rho_{31}(0)e^{-\widetilde{\Delta}_{13} t} & \rho_{32}(0)e^{-\widetilde{\Delta}_{23}^* t} & \rho_{33}(0)e^{-(\Gamma_{88})t} \end{pmatrix}, \tag{11}$$

where the functions $\tilde{\rho}_{11}(t)$ and $\tilde{\rho}_{22}(t)$ present damping term and depend on Γ_{33} and Γ_{88}. The $\rho_{ij}(0)$ are elements of the initial state, $\widetilde{\Delta}_{12} = \Gamma_{12} + i\Delta_{12}$, $\widetilde{\Delta}_{13} = \Gamma_{13} + i\Delta_{13}$, $\widetilde{\Delta}_{23} = \Gamma_{23} + i\Delta_{23}$ and $\Delta_{ij} = E_j^2 - E_i^2 \sim \frac{\Delta m_{ij}^2}{2E}$ for relativistic neutrinos.

The survival probability of a neutrino created in flavor state α is given by

$$P_{\nu_\alpha\to\nu_\alpha} = \langle\nu_\alpha|\rho(t)|\nu_\alpha\rangle = \sum_{ij} U_{\alpha i}U_{\alpha j}^*\rho_{ij}^\alpha(t), \tag{12}$$

where $\rho_{ij}^\alpha(t)$ is given by (11) with $\rho_{ij}^\alpha(0) = U_{\alpha i}U_{\alpha j}^*$. The transition to the flavor β is obtained replacing ν_α for ν_β in (12), i.e.,

$$P_{\nu_\alpha\to\nu_\beta} = U_{\beta i}^* U_{\beta j}\rho_{ij}^\alpha(t). \tag{13}$$

Note that we first impose the energy conservation in neutrino subsystems and after we include the relaxation effects. As discussed earlier, many models can be obtained imposing the first constraint, but physically this constraint is hard to justify, because the interaction between the interest subsystem and the environment occurs during the whole evolution without our knowledge by hypothesis. So, the relaxation entries make the models more realistic and treat the neutrinos subsystem like a true quantum open system.

In this context, we arrived at one general quantum dissipator given by Eq. (9) that is diagonal. This characteristic implies that it is the most relevant quantum dissipator because the off diagonal elements should be smaller than the diagonal elements to keep the complete positivity satisfied. So, other dissipative effects can occur, but their intensities must always be smaller than the effects generated by quantum dissipator in Eq. (9).

In the next section, in order to describe decoherence and relaxation effects, we will include two parameters in survival and appearance probabilities considering a muon neutrino source in our examples.

3 The complete survival and transition probabilities

In three neutrino oscillations the Lindblad master equation is very difficult to use due to the large number of parameters that the dissipator can have. However, if only decoherence and relaxation effects are taken into account in the neutrino subsystem, this large number of parameters can be reduced and the quantum dissipator is diagonal presenting five new phenomenological parameters. Here, we present a very simple form of the general survival and appearance probabilities considering a ν_μ source. The survival probability is written as

$$\begin{aligned}P_{\nu_\mu\to\nu_\mu} = {} & \frac{1}{3} + e^{-\Gamma_{33}x} f_{33} + e^{-\Gamma_{88}x} f_{88} \\ & + e^{-\Gamma_{12}x} f_{12} \cos\left[\frac{\Delta m_{21}^2}{2E}x\right] \\ & + e^{-\Gamma_{13}x} f_{13} \cos\left[\frac{\Delta m_{31}^2}{2E}x\right] \\ & + e^{-\Gamma_{23}x} f_{23} \cos\left[\frac{\Delta m_{32}^2}{2E}x\right],\end{aligned} \tag{14}$$

such that Γ_{ii} are responsible for relaxation effects and Γ_{ij} are responsible for decoherence effects. The coefficients f_{ii} and f_{ij} are defined in Appendix B and they depend only on the mixing angles θ_{12}, θ_{13}, θ_{23}, and Dirac CP phase, δ. The general appearance probability is given by

$$\begin{aligned}P_{\nu_\mu\to\nu_e} = {} & \frac{1}{3} + e^{-\Gamma_{33}x} A_{11} - e^{-\Gamma_{88}x} A_{33} + e^{-\Gamma_{12}x} \\ & \times \left(A_{12} \sin\left[\frac{\Delta m_{21}^2}{2E}x\right] + B_{12} \cos\left[\frac{\Delta m_{21}^2}{2E}x\right]\right) \\ & + e^{-\Gamma_{13}x}\left(A_{13} \cos\left[\delta + \frac{\Delta m_{31}^2}{2E}x\right] + B_{13}\right. \\ & \left.\times \cos\left[\frac{\Delta m_{31}^2}{2E}x\right]\right) + e^{-\Gamma_{23}x}\left(A_{23}\right. \\ & \left.\times \cos\left[\delta + \frac{\Delta m_{32}^2}{2E}x\right] + B_{23} \cos\left[\frac{\Delta m_{32}^2}{2E}x\right]\right),\end{aligned} \tag{15}$$

where the coefficients A_{ii}, A_{ij} and B_{ij}, also depend only on the mixing angles θ_{12}, θ_{13}, θ_{23}, and the Dirac CP phase, δ. These coefficients are defined in Appendix B.

Again, terms containing Γ_{ii} can be understood as relaxation effects while terms containing Γ_{ij} $(i \neq j)$ can be thought as decoherence phenomenon. The relaxation phenomenon is very intriguing in neutrino oscillations, because it is a flavor changing mechanism even when the mixing angles are null. This can be seen below in the probabilities

$$P_{\nu_\mu\to\nu_\mu} = \frac{1}{3} + \frac{1}{2}e^{-\Gamma_{33}x} + \frac{1}{6}e^{-\Gamma_{88}x}, \tag{16}$$

$$P_{\nu_\mu\to\nu_e} = \frac{1}{3} - \frac{1}{2}e^{-\Gamma_{33}x} + \frac{1}{6}e^{-\Gamma_{88}x} \tag{17}$$

and

$$P_{\nu_\mu\to\nu_\tau} = \frac{1}{3} - \frac{1}{3}e^{-\Gamma_{88}x}. \tag{18}$$

Note that there is always a flavor change in the probabilities above, in other words, even when we make the approximation in two neutrino families, we still have the appearance of the family decoupled due only to dissipative effects. This is the main difference: that relaxation effect holds for these probabilities, when compared with the usual oscillation probability. Furthermore, in the asymptotic limit these probabilities do not depend on the mixing angle and its value is $1/3$. This shows that after a long propagation distance the neutrino state becomes a maximal statistic mixing.

Another important point about relaxation effect can be seen through Eqs. (16–18). This model presents two relaxation parameters and Eqs. (16–18) show that in case of $\Gamma_{33} = \Gamma_{88}$, the relaxation effect acts exactly in the same way in the conversions between $\nu_\mu \to \nu_e$ and $\nu_\mu \to \nu_\tau$, but if $\Gamma_{33} \neq \Gamma_{88}$, then the conversions are different. The transition probability $P_{\nu_\mu\to\nu_\tau}$ depends only on Γ_{88}, and, for example, considering $\Gamma_{88} > \Gamma_{33}$, the tau neutrinos achieve firstly the equilibrium state rather than the other flavors.

The energy conservation constraint makes the neutrino subsystem to be subject to the decoherence effect. When this is done to two neutrino families only one model is granted by complete positivity [10, 13], but to three neutrino families there are other possible models. Adding the relaxation effects in Eq. (9) further increases the amount of possible models.

Once the energetic dependence of the dissipative parameters is not known, because we are not assuming a microscopic dissipative model, we can parameterize it using a power-law: $\Gamma = \Gamma_0(E_\nu/E_0)^n$, in the same way as it was done in references [4, 20]. Therefore, the decoherence effect appears in two intensity scales, being lower in electron to muon neutrino oscillations than in muon to tau neutrino oscillations [4, 5, 20]. In the first case the decoherence effect is $\sim 10^{-25}$ GeV [4] while in the second in the case $\sim 10^{-23}$ GeV [20] both at 95 % confidence level (or 2σ), with $n = 0$. In the present paper, we maintain this value for n.

In two neutrino oscillations the complete positivity constraint compels the relaxation and decoherence parameters to have at most the same order of magnitude, because in this case Eq. (6) reduces to

$$D_{ii} \leq \sum_{i\neq j=1}^{3} D_{jj}. \tag{19}$$

We will only keep the solution to solar and reactor neutrinos valid if we use the limit to Γ_{12} found in [4] and imposing that Γ_{33} is very close to its limit. We can take for simplicity $\Gamma_{33} = \Gamma_{12}$ that satisfies Eq. (19). We can think the same way of the relation between Γ_{88} and Γ_{23}, once the solution to accelerator and atmospheric neutrinos is not affected if we use the limit to Γ_{23} found in [20]. On the other hand, as $\Gamma_{23} > \Gamma_{12}$ by two orders of magnitude, then Γ_{88} can be directly defined by the constraint given in Eq. (6). Thus, we can see that Γ_{88} will be at most of the same order of magnitude as Γ_{23}, but we assume $\Gamma_{88} \leq \Gamma_{23}$ because this restriction together with complete positivity entails all interesting physical situations.

Thus, while one relaxation parameter is fixed the other relaxation parameter can vary, but within a limit. With these constraints we can obtain only five models having only two or less dissipative parameters that satisfy complete positivity. In this situation, the interaction between neutrino and environment is easily understood. We will present these quantum dissipators below as well as the survival and appearance probabilities obtained when the usual approximation is made in two families, i.e., we will consider $\theta_{13} = 0$ and $\Delta m^2_{12} \sim 0$ to decouple the ν_e, but due to the relaxation effect an appearance probability is expected. But, as discussed before, if there is always an appearance probability, a two family approximation does not exist in common sense when we take into account dissipative effects. This approximation here is important to probe the appearance probability due to only dissipative effects.

3.1 Model 1: normal case

As the first model we will consider in our analysis we take one obtained in references [4, 5], where ν_e oscillating to ν_μ and ν_μ oscillating to ν_τ present different magnitudes to decoherence effect. Nevertheless, the early results do not take into account the relaxation effects, which we will include through the quantum dissipator written as follows:

$$D_{\mu\nu} = -\,\mathrm{diag}\{0, \Gamma_{12}, \Gamma_{12}, \Gamma_{12}, \Gamma_{23}, \Gamma_{23}, \Gamma_{23}, \Gamma_{23}, \Gamma_{23}\}. \tag{20}$$

Note that we assume $\Gamma_{88} \to \Gamma_{23}$ and $\Gamma_{33} \to \Gamma_{12}$, so these new definitions change the survival probability in (14) and the appearance probability in (15). Furthermore, in agreement with the experimental analysis, the intensity of decoherence effects is given by $\Gamma_{13} = \Gamma_{23} > \Gamma_{12}$ [4, 5].

The survival and appearance probability in the two neutrinos approximation can be rewritten as

$$\begin{aligned} P_{\nu_\mu \to \nu_\mu} &= \frac{1}{3} + \frac{1}{2} e^{-\Gamma_{12} x} \cos[\theta_{23}]^4 \\ &\quad + \frac{1}{24} e^{-\Gamma_{23} x} \biggl(\bigl(1 - 3\cos[2\theta_{23}]\bigr)^2 \\ &\quad + 12 \cos\left[\frac{\Delta m^2_{23} x}{2E}\right] \sin[2\theta_{23}]^2 \biggr) \end{aligned} \tag{21}$$

and

$$\begin{aligned} P_{\nu_\mu \to \nu_e} &= \frac{1}{3} - \frac{1}{24} e^{-\Gamma_{23} x} \bigl(2 - 6\cos[2\theta_{23}]\bigr) \\ &\quad - \frac{1}{2} e^{-\Gamma_{12} x} \cos[\theta_{23}]^2. \end{aligned} \tag{22}$$

Note that the appearance probability is nonvanishing due to dissipative effects. Here, we can see an interesting peculiarity. When we take into account the relaxation effects and make the approximation of three into two families neutrino, although ν_e family is not decoupled, the survival probability does not depend on the θ_{12} mixing angle. So, the survival probability oscillates with the same number of parameters known from oscillation theory plus the parameters related with the dissipative effects. If relaxation effects are not present on the quantum dissipator, but decoherence effects are taken into account, the survival and appearance probabilities depend on θ_{12} mixing angle and, therefore, this model need of one parameter additional to describe the neutrino evolution with less effects than the our model.

In the approach that we are presenting the ν_e family is correctly decoupled only when dissipative parameters are null. In the approach with decoherence it is possible to decouple the ν_e family and arrive at the survival probability found in [10, 13, 20] by setting $\Gamma_{12} \to 0$. Thus, this shows that the dissipative effects in neutrino oscillation between two families must be treated carefully, since the decoupling of three into two families does not happens trivially.

3.2 Model 2: when $\Gamma_{13} \to \Gamma_{12}$

This model is also inspired by the previous model, but here Γ_{13} can be different from Γ_{23}. Model 1 assumes that decoherence effect due to Γ_{13} suppresses the quantum superposition, which depends on θ_{13} [4, 5], but this does not need to be always this way. The complete positivity is kept when we consider $\Gamma_{13} \to \Gamma_{12}$, which brings about a new model with quantum dissipator written as

$$D_{\mu\nu} = -\,\mathrm{diag}\{0, \Gamma_{12}, \Gamma_{12}, \Gamma_{12}, \Gamma_{12}, \Gamma_{12}, \Gamma_{23}, \Gamma_{23}, \Gamma_{23}\}, \tag{23}$$

where decoherence occurs with Γ_{12} intensity for ν_e oscillating to ν_μ and ν_e oscillating to ν_τ.

In the two family approximation, the survival probability is written as

$$\begin{aligned} P_{\nu_\mu \to \nu_\mu}(x, E) &= \frac{1}{3} + \frac{1}{2} e^{-\Gamma_{12} x} \cos[\theta_{23}]^2 \biggl(\cos[\theta_{23}]^2 \\ &\quad + 4 \cos\left[\frac{\Delta m^2_{32} x}{2E}\right] \sin[\theta_{12}]^2 \sin[\theta_{23}]^2 \biggr) \end{aligned}$$

$$+\frac{1}{24}e^{-\Gamma_{23}x}\bigg(\big(1-3\cos[2\theta_{23}]\big)^2 + 12\cos\bigg[\frac{\Delta m^2_{32}x}{2E}\bigg]\cos[\theta_{12}]^2\sin[2\theta_{23}]^2\bigg), \tag{24}$$

which depends on θ_{12} angle.

The behavior of the probability above implies that the neutrinos are going to oscillate for longer distances than is the case for the probability in (21), because when the relaxation and decoherence term due to Γ_{23} tends to zero, there will still be oscillation due to the second term on the right side of this probability. The appearance probability to this model is identical to the previous model.

3.3 Model 3: universal relaxation effect $\Gamma_{88} \to \Gamma_{33} = \Gamma_{12}$

The complete positivity in three neutrino oscillations allows that two parameters describe relaxation effect. We discuss now when the parameter Γ_{88} is not equal to Γ_{23}. We assume $\Gamma_{88} \to \Gamma_{33} = \Gamma_{12}$. Using this assumption, the quantum dissipator can be written as

$$D_{\mu\nu} = -\operatorname{diag}\{0, \Gamma_{12}, \Gamma_{12}, \Gamma_{12}, \Gamma_{23}, \Gamma_{23}, \Gamma_{23}, \Gamma_{23}, \Gamma_{12}\}, \tag{25}$$

where decoherence must occur with Γ_{12} intensity for ν_e oscillating to ν_μ and with Γ_{23} in other oscillation modes.

Using the quantum dissipator written in Eq. (25), the two family approximation has the following survival probability:

$$P_{\nu_\mu\to\nu_\mu} = \frac{1}{3} + \frac{1}{12}e^{-\Gamma_{12}x}\big(5+3\cos[4\theta_{23}]\big) + \frac{1}{2}e^{-\Gamma_{23}x}\cos\bigg[\frac{\Delta m^2_{32}x}{2E}\bigg]\sin[2\theta_{23}]^2 \tag{26}$$

and the appearance probability is given by

$$P_{\nu_\mu\to\nu_e} = \frac{1}{3} - \frac{1}{3}e^{-\Gamma_{12}x}, \tag{27}$$

Note that this equation coincides with Eq. (22) when $\Gamma_{23} = \Gamma_{12}$. This occurs because this model has only one parameter that describes the relaxation effect. Furthermore, if we have the universal relaxation effect it is easy to prove that Eqs. (17) and (18) are identical, as we have indicated before.

This model is very motivating because the appearance probability depends only on Γ_{12}. Thus, it could be used to constrain the scale of relaxation effects in the electron neutrino appearance, once this probability does not depend on masses and mixing angles. Indeed, if we use the neutrino approximation as here was done, the relaxation parameter must also determine the quantum decoherence scale in $\nu_\mu \to \nu_e$ oscillations due to simplification done between Γ_{33} and Γ_{12}.

3.4 Model 4: no relaxation effect in Γ_{88}

In this model $\Gamma_{88} = 0$. This is allowed by the complete positivity constraint in case of oscillation in three neutrinos. This is the most peculiar situation because only a part of the oscillation system has the relaxation effect. The quantum dissipator is written as follows:

$$D_{\mu\nu} = -\operatorname{diag}\{0, \Gamma_{12}, \Gamma_{12}, \Gamma_{12}, \Gamma_{23}, \Gamma_{23}, \Gamma_{23}, \Gamma_{23}, 0\}, \tag{28}$$

where Γ_{23} parameter corresponds to decoherence effects only. In this model, the survival and appearance probabilities in two family approximation are written, respectively, as

$$P_{\nu_\mu\to\nu_\mu} = \frac{1}{3} + \frac{1}{96}\big(-2+6\cos[2\theta_{23}]\big)^2 + \frac{1}{2}e^{-\Gamma_{12}x}\cos[\theta_{23}]^4 + \frac{1}{2}e^{-\Gamma_{23}x}\cos\bigg[\frac{\Delta m^2_{32}x}{2E}\bigg]\sin[2\theta_{23}]^2 \tag{29}$$

and

$$P_{\nu_\mu\to\nu_e} = \frac{1}{2}\big(1-e^{-\Gamma_{12}x}\big)\cos[\theta_{23}]^2. \tag{30}$$

Note that, in this case, the asymptotic values of these probabilities are θ_{23} mixing angle dependent. This happens since the relaxation effect given by Γ_{88} is not present on the probabilities and the oscillation between the second and third neutrino generation is subjected to only decoherence effect. Furthermore, the appearance probability implies that if $\theta_{23} = \pi/4$, the asymptotic value of the appearance probability is 25 % and the rate among the families in the asymptotic case is not $1/3 : 1/3 : 1/3$ but $2/8 : 3/8 : 3/8$.

This can constitute a new interesting scenario for astrophysical and cosmic neutrinos [23]. It is known that the decoherence effect due to dynamic or dissipation mechanism can alter the ratios of the flavors during the neutrino propagation and irrespective of the source the ratio value is changed to $1/3 : 1/3 : 1/3$. This model allows a different ratio and strong constraints could be placed on the dissipative effects. Interesting enough, the first neutrino generation, even in the vacuum, interacts most effectively with the environment compared with the other generations and thus, in the present case, the value of the ratio of astrophysical and cosmic neutrinos is given by $2/8 : 3/8 : 3/8$.

3.5 Model 5: universal relaxation and decoherence effects

This model has an important symmetry property. The universal dissipative effect, here, means that all parameters have the same intensity, so the quantum dissipator has the form

$$D_{\mu\nu} = -\operatorname{diag}\{0, \Gamma, \Gamma, \Gamma, \Gamma, \Gamma, \Gamma, \Gamma, \Gamma\}, \tag{31}$$

note that this dissipator is invariant under change of representation once that the dissipator is proportional to unity. Thus, we can verify that if ρ^m is the mass representation and ρ^i is the other representation, the dissipation term in the evolution Eq. (1) under any unitary transformation, such that $\tilde{U}$ leads from ρ^m into ρ^i, changes as follows:

$$\tilde{U} D^m\big[\rho^m(x)\big]\tilde{U}^\dagger = D^m\big[\rho^i(x)\big]. \tag{32}$$

This model is very useful when there are other effects for taking into account in neutrino oscillation, for instance, matter effects. In this case, states or probabilities can be obtained easily without any approximation method.

In this model, the survival and appearance probabilities in two family approximation are given by

$$P_{\nu_\mu\to\nu_\mu} = \frac{1}{3} + \frac{1}{12}e^{-\Gamma x}\big(5 + 3\cos[4\theta_{23}]\big) + \frac{1}{2}e^{-\Gamma x}\sin[2\theta_{23}]^2\cos\left[\frac{\Delta m_{32}^2 x}{2E}\right] \tag{33}$$

and

$$P_{\nu_\mu\to\nu_e} = \frac{1}{3} - \frac{1}{3}e^{-\Gamma x}. \tag{34}$$

It is important to recall that the decoherence effect has a smaller magnitude in reactor and solar neutrinos than atmospheric and accelerator neutrinos. Thus, in this model, dissipative effects must be subtle and be more relevant in solar and reactor sectors than in atmospheric and accelerator sectors.

4 Summarizing the models and behaviors

In order to clearly see the differences among the models previously studied, we discuss some of the behavior in two different situations. First, we show the survival probability behavior depending on the propagation distance and second, we call attention to the appearance probability behavior varying the energy of the neutrino beam with the propagation distance fixed in 735 km, thus as in the neutrino long base line experiments, like MINOS experiment [24]. In the plots we have set the following values to the oscillation parameters: $\Delta m_{12}^2 = 7.58 \times 10^{-5}$ eV^2, $\Delta m_{23}^2 = 2.32 \times 10^{-3}$ eV^2, $\Delta m_{13}^2 = \Delta m_{23}^2 - \Delta m_{12}^2$, $\theta_{12} = 0.6$ and $\theta_{23} = \pi/4$, which can be obtained from [21, 22], furthermore, the dissipative parameters are energy independent.

In general, as $\Gamma_{23} > \Gamma_{12}$, the behavior of each model has two different moments. First, when the damping term due to Γ_{23} is more effective than Γ_{12} eliminating more rapidly the coherence between ν_μ and ν_τ and when the damping term due to Γ_{23} vanishes there no longer exists coherence between the ν_μ and ν_τ families. Second, when damping term due to Γ_{23} vanishes, the damping term due to Γ_{12} is not null yet and thus, it suppresses the coherence between the other families for most long distances.

As the behavior general of the models discussed above are similar. In order to know this behavior we take as example model 1 and we made a plot that can be seen on the left in Fig. 1, where the survival probability behavior for this appears with three different intensities to Γ_{12} and Γ_{23}. As shown by the thin solid line, with the intensities taken from references [4, 5], there exists strong suppression until 10^5 km due to Γ_{23} and after only Γ_{12} is responsible for eliminating the oscillation.

Model 4 has asymptotic behavior different from other models. So, in order to verify this difference we made a plot that one can see on the right side of Fig. 1, where we introduce the survival probability behavior for model 4, but we have done the approximation of three into two families. One

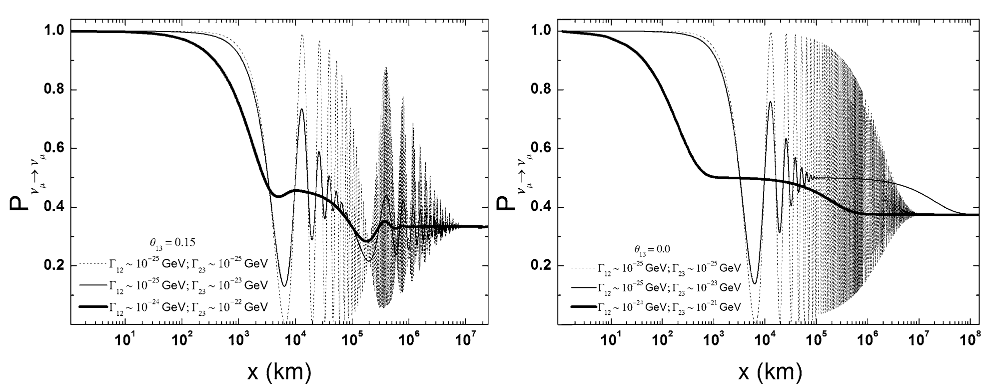

Fig. 1 The figures on the *left* and on the *right* show the survival probability of model 1 and 4 with $\theta_{13} = 0.15$ obtained from [14, 15] and $\theta_{13} = 0$, respectively. To a beam with 3, the behavior with the dissipative effects are indicated as follows: the *dot line* represents the universal dissipation with $\Gamma_{12} = \Gamma_{23}$, the *thin* and *thick solid lines* represent the behavior when $\Gamma_{12} < \Gamma_{23}$ with energy independent Γ_{ij} [4, 5]. To both figures the values used for the oscillation parameters are: $\Delta m_{12}^2 = 7.58 \times 10^{-5}$ eV^2, $\Delta m_{23}^2 = 2.32 \times 10^{-3}$ eV^2, $\Delta m_{13}^2 = \Delta m_{23}^2 - \Delta m_{12}^2$, $\theta_{12} = 0.6$ and $\theta_{23} = \pi/4$, which can be obtained from [21, 22]

can see clearly by thin solid line, when the damping term due to Γ_{23} becomes null, coherence no longer exists and in this case the damping term due to Γ_{12} is responsible for the decline behavior of this probability.

The most significant differences among these models can be seen in the following way. The survival probabilities of models 1 and 3 have the same behavior, where first Γ_{23} eliminates the oscillation between the families that depends on this damping term and then Γ_{12} becomes important suppressing $\nu_e \to \nu_\mu$ oscillation. The relaxation effect to these cases mixes the states until to achieve the flavor ratio $(\nu_e : \nu_\mu : \nu_\tau =)$ $1/3 : 1/3 : 1/3$. Model 4 is analog to models 1 and 3, but the relaxation effect leads the states until the flavor ratio $2/8 : 3/8 : 3/8$.

The survival probability of model 2 has a little different behavior from the cases previously presented. First Γ_{23} only eliminates $\nu_\mu \to \nu_\tau$ oscillation and after Γ_{12} becomes important suppressing the remaining oscillation between the families. The relaxation effect in this case mixes the states until achieving the flavor ratio $1/3 : 1/3 : 1/3$. Moreover, model 2 in two family approximation is an exception and depends on θ_{12} as we can see in Eq. (25).

The survival probability of model 5 has decoherence and relaxation effects given by the same universal parameter Γ. So, as this parameter must be obtained in experimental data analysis of the solar or reactor neutrinos, it causes small dissipative effects and the system can oscillate for long distances. The relaxation effect to this case also mixes the states until to achieve the flavor ratio $1/3 : 1/3 : 1/3$.

Last, all these models, even when we make the two family approximation and put $\Gamma_{12} \to 0$, do not recover the survival probabilities of models 1 and 2 found in [13]. There, the asymptotic values of this probabilities are $1/2$ and here, the models present different value. It must be so, because here, even when we make the two family approximation, there is appearance of the first family due to dissipative effects.

The dissipative effects induce the appearance of the first family. This can see in Fig. 2, where we test some values of Γ_{12} and Γ_{23} in the appearance probability of model 1, where we suppose a propagation distance of neutrino beam like MINOS experiment, i.e., $x = 735$ km.

On the left side of Fig. 2, the mixing angle is chosen as $\theta_{13} = 0.15$ in the appearance probability. We can verify by the dotted line that, when dissipative parameter is null, there is not appearance of the first family when the neutrino energy is greater than $\sim$2 GeV, but when quantum dissipation is not null, we can see that the neutrino appears for any energy. On the right side of Fig. 2, the mixing angle $\theta_{13} = 0$ in the appearance probability and the neutrino appearance is due only to dissipative effects.

As the dissipative effects due to Γ_{23} have greater intensity than Γ_{12}, so, the dissipative process due to Γ_{23} occurs faster than the process due to Γ_{12}. This is an important consequence, once the last results for reactor neutrino indicate neutrino flavor change in short propagation distances. Indeed, this was decisive for determining the value of θ_{13} [14], but the dissipative effects have potential to explain the flavor conversion in short distances with the models presented in this work and thus, an investigation of the problem using the open quantum system approach could be interesting.

At the moment there is not a complete experimental data analysis to three neutrino oscillation plus dissipation effects. In another work, we will make this including a study about the limits on the Γ parameters.

5 Comments and conclusion

The open quantum systems formalism was used in this work to describe the neutrino oscillation phenomenon in

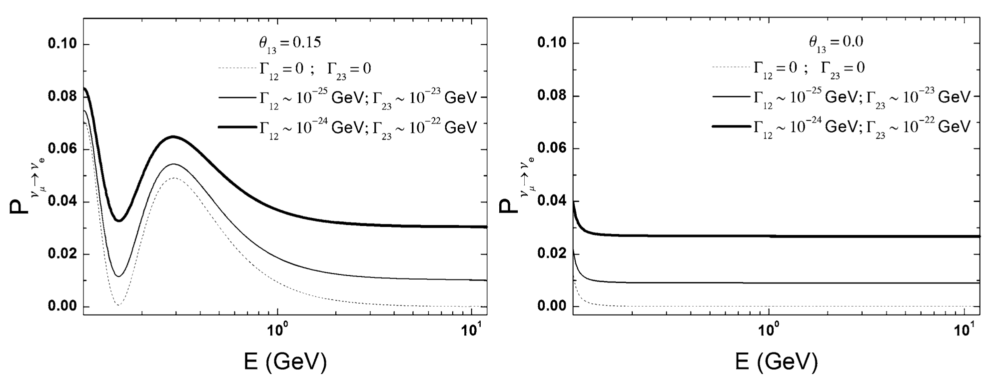

Fig. 2 The figure on the *left* shows the appearance probability of model 1 with $\theta_{13} = 0.15$ obtained from [14, 15] while the figure on the *right* shows the appearance probability of model 1 with $\theta_{13} = 0$. In both, to *dot line* there is not dissipative effects and to *thin* and *thick solid line* the dissipation effects have $\Gamma_{12} < \Gamma_{23}$ with energy independent Γ_{ij}. To both figures the values used to the oscillation parameters are: $\Delta m^2_{12} = 7.58 \times 10^{-5}$ eV^2, $\Delta m^2_{23} = 2.32 \times 10^{-3}$ eV^2, $\Delta m^2_{13} = \Delta m^2_{23} - \Delta m^2_{12}$, $\theta_{12} = 0.6$ and $\theta_{23} = \pi/4$, which can be obtained from [21, 22]

three families. The neutrino propagation was done with the Lindblad master equation, in which dissipative effects are taken into account. These dissipative effects were described by means of quantum dissipators which respect the complete positivity constraint. In the form of five different models described for one or two phenomenological parameters, we studied the main dissipative effects, decoherence and relaxation, which can occur in neutrino oscillation.

The general survival and appearance probabilities were derived for a source of muon neutrinos and the five different quantum dissipators were analyzed in order to appreciate the physical consequences of the dissipative effects in neutrino oscillation. We point out the characteristics and behavior of each model.

Subsequently, we showed the two family approximation of each model and thus, it was possible to observe the characteristics of the open quantum system approach in neutrino oscillation when we initially consider three or two neutrino families [13].

When we coupled the dissipation effects in neutrino oscillations for the three generation case, the two family approximation did not occur, because, the excluded family continues appearing due to dissipative effect. Interest enough, this scheme of the approximation reduced the survival probabilities expression and, with exception of model 2, they did not depend on θ_{12}, like in the case without dissipative effects. In the case of coupling of the dissipative effects in two neutrino oscillation the previous phenomena could not be seen. In other words, the treatment in three neutrinos is not reducible to a two survival probability in the sense often used in neutrino oscillations.

We conclude this work presenting an important relation between θ_{13} mixing angle and dissipative effects. We discussed how the dissipative effects can be important together or not with the mixing angle θ_{13} to the neutrino appearance.[3]

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Appendix A: Complete positivity

The general quantum dissipator, $D_{\mu\nu}$, is taken diagonal and present its elements as

$$
\begin{aligned}
D_{11} &= 2a_2^2 + 2a_3^2 + \frac{1}{2}a_4^2 + \frac{1}{2}a_5^2 + \frac{1}{2}a_6^2 + \frac{1}{2}a_7^2 \geq 0;\\
D_{22} &= 2a_1^2 + 2a_3^2 + \frac{1}{2}a_4^2 + \frac{1}{2}a_5^2 + \frac{1}{2}a_6^2 + \frac{1}{2}a_7^2 \geq 0;\\
D_{33} &= 2a_1^2 + 2a_2^2 + \frac{1}{2}a_4^2 + \frac{1}{2}a_5^2 + \frac{1}{2}a_6^2 + \frac{1}{2}a_7^2 \geq 0;\\
D_{44} &= \frac{1}{2}a_1^2 + \frac{1}{2}a_2^2 + \frac{1}{2}a_3^2 + 2a_5^2 + \frac{1}{2}a_6^2 + \frac{1}{2}a_7^2\\
&\quad + a[3]a[8] + \frac{1}{2}a_8^2 \geq 0;\\
D_{55} &= \frac{1}{2}a_1^2 + \frac{1}{2}a_2^2 + \frac{1}{2}a_3^2 + 2a_4^2 + \frac{1}{2}a_6^2 + \frac{1}{2}a_7^2\\
&\quad + a[3]a[8] + \frac{1}{2}a_8^2 \geq 0;\\
D_{66} &= \frac{1}{2}a_1^2 + \frac{1}{2}a_2^2 + \frac{1}{2}a_3^2 + \frac{1}{2}a_4^2 + \frac{1}{2}a_5^2 + 2a_7^2\\
&\quad - a[3]a[8] + \frac{1}{2}a_8^2 \geq 0;\\
D_{77} &= \frac{1}{2}a_1^2 + \frac{1}{2}a_2^2 + \frac{1}{2}a_3^2 + \frac{1}{2}a_4^2 + \frac{1}{2}a_5^2 + 2a_6^2\\
&\quad - a[3]a[8] + \frac{1}{2}a_8^2 \geq 0;\\
D_{88} &= \frac{1}{2}3a_4^2 + \frac{1}{2}3a_5^2 + \frac{1}{2}3a_6^2 + \frac{1}{2}3a_7^2 \geq 0.
\end{aligned}
\tag{A.1}
$$

The main configuration of the quantum dissipator happens when it can be expressed in the diagonal form, given the fact that off diagonal elements should be smaller than the diagonal elements to keep the complete positivity satisfied.

Under the hypothesis $\Gamma_{12} = D_{11} = D_{22}$, $\Gamma_{13} = D_{44} = D_{55}$, $\Gamma_{23} = D_{66} = D_{77}$, which results from the $[V_k, H_{\mathrm{osc}}] = 0$ restriction, and adding the relaxation parameters, one can easily show that each diagonal element of the quantum dissipator is written as

$$
\begin{aligned}
\Gamma_{12} &= 2a_1^2 + 2a_3^2 + a_5^2 + a_7^2 \geq 0;\\
\Gamma_{12} &= 2a_1^2 + 2a_3^2 + a_5^2 + a_7^2 \geq 0;\\
\Gamma_{33} &= 4a_1^2 + a_5^2 + a_7^2 \geq 0;\\
\Gamma_{13} &= a_1^2 + \frac{1}{2}\big(2\big(2a_5^2 + a_7^2\big) + \big(a[3] + a[8]\big)^2\big) \geq 0;\\
\Gamma_{13} &= a_1^2 + \frac{1}{2}\big(2\big(2a_5^2 + a_7^2\big) + \big(a[3] + a[8]\big)^2\big) \geq 0;\\
\Gamma_{23} &= \frac{1}{2}\big(2a_1^2 + 2\big(a_5^2 + 2a_7^2\big) + \big(a[3] - a[8]\big)^2\big) \geq 0;\\
\Gamma_{23} &= \frac{1}{2}\big(2a_1^2 + 2\big(a_5^2 + 2a_7^2\big) + \big(a[3] - a[8]\big)^2\big) \geq 0;\\
\Gamma_{88} &= 3\big(a_5^2 + a_7^2\big) \geq 0.
\end{aligned}
\tag{A.2}
$$

[3] Elsewhere, we will present a data analysis on reactor antineutrinos taking into account the dissipative effects and exploiting the possibilities discussed in this work.

We assume $\Gamma_{33} = D_{33}$ and $\Gamma_{88} = D_{88}$. These are the relaxation parameters.

Appendix B: Probability coefficients

The survival probability coefficients from Eq. (14) are given by

$$f_{33} = \frac{1}{32}\big(\cos[2\theta_{12}]\big(2\cos^2[\theta_{13}] - (-3+\cos[2\theta_{13}]) \times \cos[2\theta_{23}]\big) - 4\cos[\delta]\sin[2\theta_{12}]\sin[\theta_{13}] \times \sin[2\theta_{23}]\big)^2, \tag{B.1}$$

$$f_{88} = \frac{1}{96}\big(1 - 3\cos[2\theta_{13}] + 6\cos^2[\theta_{13}]\cos[2\theta_{23}]\big)^2, \tag{B.2}$$

$$f_{12} = \frac{1}{32}\big(\big(1+3\cos[2\theta_{23}] + 2\cos[2\theta_{13}]\sin^2[\theta_{23}]\big)^2 \times \sin^2[2\theta_{12}] + 8\sin[\theta_{13}]\sin[2\theta_{23}]\big(\cos[\delta]\cos[2\theta_{12}] \times \big(1+3\cos[2\theta_{23}] + 2\cos[2\theta_{13}]\sin^2[\theta_{23}]\big) \times \sin[2\theta_{12}] + 2\big(\cos^2[\delta]\cos^2[2\theta_{12}] + \sin^2[\delta]\big) \times \sin[\theta_{13}]\sin[2\theta_{23}]\big)\big), \tag{B.3}$$

$$f_{13} = \cos^2[\theta_{13}]\sin^2[\theta_{23}]\big(2\cos^2[\theta_{23}]\sin^2[\theta_{12}] + \sin[\theta_{13}] \times \big(2\cos^2[\theta_{12}]\sin[\theta_{13}]\sin^2[\theta_{23}] + \cos[\delta]\sin[2\theta_{12}] \times \sin[2\theta_{23}]\big)\big), \tag{B.4}$$

$$f_{23} = \cos^2[\theta_{13}]\sin^2[\theta_{23}]\big(2\cos^2[\theta_{12}]\cos^2[\theta_{23}] + \sin[\theta_{13}] \times \big(2\sin^2[\theta_{12}]\sin[\theta_{13}]\sin^2[\theta_{23}] - \cos[\delta]\sin[2\theta_{12}] \times \sin[2\theta_{23}]\big)\big). \tag{B.5}$$

Appearance probability coefficients from Eq. (15) are given by

$$A_{11} = \frac{1}{8}\cos^2[\theta_{13}]\big(\cos^2[2\theta_{12}]\big((-3+\cos[2\theta_{13}]) \times \cos[2\theta_{23}] - 2\cos^2[\theta_{13}]\big) + 2\cos[\delta]\sin[4\theta_{12}] \times \sin[\theta_{13}]\sin[2\theta_{23}]\big), \tag{B.6}$$

$$A_{33} = \frac{1}{48}\big(-1+3\cos[2\theta_{13}]\big)\big(-1+3\cos[2\theta_{13}] - 6\cos^2[\theta_{13}]\cos[2\theta_{23}]\big), \tag{B.7}$$

$$A_{12} = -\frac{1}{2}\cos^2[\theta_{13}]\sin[\delta]\sin[2\theta_{12}]\sin[\theta_{13}]\sin[2\theta_{23}], \tag{B.8}$$

$$B_{12} = -\frac{1}{4}\cos[\theta_{12}]\cos^2[\theta_{13}]\sin[\theta_{12}]\big(\big(1+3\cos[2\theta_{23}] + 2\cos[2\theta_{13}]\sin^2[\theta_{23}]\big)\sin[2\theta_{12}] + 4\cos[\delta] \times \cos[2\theta_{12}]\sin[\theta_{13}]\sin[2\theta_{23}]\big), \tag{B.9}$$

$$A_{13} = -A_{23} = -\frac{1}{2}\cos^2[\theta_{13}]\sin[2\theta_{12}]\sin[\theta_{13}]\sin[2\theta_{23}], \tag{B.10}$$

$$B_{13} = -2\cos^2[\theta_{12}]\cos^2[\theta_{13}]\sin^2[\theta_{13}]\sin^2[\theta_{23}], \tag{B.11}$$

$$B_{23} = -\frac{1}{2}\sin^2[\theta_{12}]\sin^2[2\theta_{13}]\sin^2[\theta_{23}]. \tag{B.12}$$

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