# Quantitative Economics - Project Report

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February 28, 2025

## 1 Introduction

Taxation plays a crucial role in shaping macroeconomic dynamics, influencing capital accumulation, income distribution, and economic growth. This study examines the impact of tax progressivity in a heterogeneous agent model, building on the framework of Aiyagari (1994). In this setting, individuals make intertemporal consumption and savings decisions while facing idiosyncratic income risk.

We analyze how shifting from a flat tax system ($\lambda = 0.0$) to a progressive tax system ($\lambda = 0.15$) affects equilibrium outcomes, including asset distribution, interest rates, wage levels, and inequality measures.

The model economy consists of a continuum of agents optimizing lifetime utility while facing borrowing constraints and stochastic labor productivity. Their pre-tax labor income is subject to taxation, with tax revenues allocated to government expenditures.

A representative firm, operating under a Cobb-Douglas production function, determines equilibrium factor prices. The government maintains a constant government spending-to-output ratio and adjusts tax rates accordingly to balance its budget.

## 2 Aim of the Project

The objective of this study is to compute the stationary equilibria under both tax regimes by solving the household optimization problem, ensuring firm profit maximization, and enforcing market-clearing conditions.

The numerical approach employs value function iteration to determine the optimal consumption-savings decisions of agents.By comparing equilibrium outcomes, this analysis sheds light on the redistributive effects of progressive taxation and its consequences for capital accumulation and economic inequality.

This report is structured as follows: **Section 3** details the **methodology** used to solve the model, including the formulation of the **Bellman equation**, **numerical techniques**, and **parameter calibration**. **Section 4** presents the **results**, including **equilibrium statistics**, **policy functions**, and **wealth distributions**. **Section 5** discusses the broader **economic implications** of the findings.

## 3 Methodology

To solve the problem, we use a dynamic programming approach where households maximize their expected lifetime utility subject to budget constraints. The method involves:

- Solving the household's decision problem using value function iteration.
- Modeling the evolution of labor productivity using a stochastic process.
- Computing the stationary distribution of assets.
- Ensuring market-clearing conditions for capital and labor.
- Finding equilibrium tax rates and factor prices under both tax scenarios.

## 3.1 Household Decision Problem and Bellman Equation

The household's decision-making process is modeled using the following Bellman equation:

$$V(a,z) = \max_{a'} \left\{ u(c) + \beta \sum_{z'} \pi(z'|z) V(a',z') \right\}, \tag{1}$$

where $V(a,z)$ represents the value function, $a$ is the agent's current asset holdings, and $z$ is the productivity level. The agent chooses next-period assets $a'$ while satisfying the budget constraint:

$$c = (1-\tau) w z^{1-\lambda} + (1+r)a - a'. \tag{2}$$

The Bellman equation is solved iteratively using value function iteration:

- Initialize an arbitrary value function $V_0(a,z)$.
- Iterate on the Bellman operator until convergence is achieved.
- Extract the optimal policy functions for savings and consumption.

To improve numerical accuracy, the asset policy function is approximated using **interpolation methods**.

## 3.2 Idiosyncratic Labor Productivity Process

Each agent's labor productivity evolves according to an AR(1) process:

$$\log z' = \rho \log z + \epsilon, \quad \epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2). \tag{3}$$

This ensures persistent differences in income levels across agents. The Tauchen method is used to discretize this continuous process into a finite Markov chain.

## 3.3 Household Preferences and Consumption-Savings Decision

Households derive utility from consumption based on the **Constant Relative Risk Aversion (CRRA) utility function**:

$$u(c) = \begin{cases} \frac{c^{1-\gamma}-1}{1-\gamma}, & \text{if } \gamma \neq 1 \\ \log(c), & \text{if } \gamma = 1 \end{cases} \tag{4}$$

where $\gamma$ controls risk aversion. The optimal consumption-savings decision satisfies the **Euler equation**:

$$u'(c) = \beta(1+r)\mathbb{E}[u'(c')]. \tag{5}$$

This ensures intertemporal optimization, balancing present and future consumption.

## 3.4 Tax Function

Labor income is taxed according to the following function:

$$T(y) = y - (1-\tau)\left(\frac{y}{\bar{y}}\right)^{1-\lambda} \bar{y}, \tag{6}$$

so that after-tax labor income is:

$$\tilde{y} = (1-\tau)\left(\frac{y}{\bar{y}}\right)^{1-\lambda} \bar{y}. \tag{7}$$

This function introduces progressivity ($\lambda > 0$), making higher-income individuals pay proportionally more in taxes.

## 3.5 Computing the Stationary Distribution

After solving for policy functions, the model computes the **stationary distribution** of assets:

$$\lambda' = \lambda Q, \tag{8}$$

where $\lambda$ represents the distribution of agents across asset and productivity states, and $Q$ is the transition matrix derived from policy functions and the exogenous Markov process.

## 3.6 Market-Clearing and Equilibrium Conditions

Equilibrium in the capital market requires that total household savings equal the firm's capital demand:

$$K = \sum_{a,z} \lambda(a, z)a. \tag{9}$$

**Solving for Equilibrium:**

- In the baseline economy ($\lambda = 0$), the interest rate is fixed at $r = 0.04$, and the wage rate and capital stock are determined from firm optimization conditions.
- In the reform economy ($\lambda = 0.15$), the interest rate is solved endogenously, and the tax rate $\tau$ is adjusted to maintain government revenue at 20% of output.

## 3.7 Model Parameter Selection and Calibration

The model parameters were chosen based on empirical macroeconomic studies and calibrated to satisfy equilibrium conditions. The table below summarizes the key parameters:

| Parameter | Value | Justification |
|---|---|---|
| Risk aversion ($\gamma$) | 2.0 | Standard in macro models for consumption risk aversion |
| Discount factor ($\beta$) | Calibrated | Adjusted to match aggregate capital in $\lambda = 0$ case |
| Borrowing constraint ($\phi$) | 0.0 | No borrowing allowed for simplicity |
| Capital share ($\alpha$) | 1/3 | Standard U.S. macroeconomic estimate |
| Depreciation rate ($\delta$) | 0.06 | Ensures capital depreciation is 20% of output |
| Productivity level ($A$) | 0.8775 | Implied by firm optimization conditions |
| Persistence of income shocks ($\rho$) | 0.9 | as specified in problem statement |
| Income shock variance ($\sigma_\epsilon$) | 0.4 | as specified in problem statement |
| Number of productivity states ($N_z$) | 5 | Discretized using Tauchen's method |
| Flat tax rate ($\tau$) | 0.3 | Typical labor tax rate in developed economies |
| Progressive tax parameter ($\lambda$) | 0.15 | Reform case specified in the problem statement |

Table 1: Model Parameters and Their Justification

**Calibration of Key Parameters**

The model parameters were calibrated to ensure that equilibrium conditions hold and that key macroeconomic relationships are accurately reflected. Below, we outline the calibration choices and their underlying justification.

**Discount Factor ($\beta$)**

The discount factor was calibrated to ensure that aggregate household savings matched total capital demand in the $\lambda = 0$ baseline economy. This was achieved by solving the market-clearing condition:

$$K = A_s(\beta, r, w), \tag{10}$$

where $A_s$ represents aggregate household savings. A numerical root-finding algorithm was used to iteratively adjust $\beta$ until the capital market cleared.

**Production Parameters ($\alpha, \delta, A$)**

The capital share $\alpha = 1/3$ is a standard value derived from national income accounts, where labor typically earns two-thirds of total output.

The depreciation rate $\delta = 0.06$ was chosen to ensure that capital depreciation accounts for approximately 20% of output, consistent with empirical estimates.

The productivity level $A$ was calibrated to satisfy the firm's first-order condition:

$$r = \alpha A K^{\alpha-1} - \delta, \tag{11}$$

ensuring that the model's equilibrium output aligns with observed macroeconomic data.

**Labor Productivity Process ($\rho, \sigma_\epsilon, N_z$)**

The persistence parameter $\rho = 0.9$ and shock variance $\sigma_\epsilon = 0.4$ were selected based on empirical studies of income dynamics, ensuring realistic income persistence over time.

The Tauchen method was used to discretize the continuous AR(1) process into a five-state Markov chain, providing an accurate approximation of labor income fluctuations.

**Equilibrium Calibration**

To compute equilibrium prices, the model follows a two-step approach:

- In the baseline economy ($\lambda = 0$), the interest rate was fixed at $r = 0.04$, and the wage rate and capital stock were computed using firm optimization conditions.
- In the reform economy ($\lambda = 0.15$), the interest rate was determined endogenously using a numerical root-finding procedure. The tax rate $\tau$ was then adjusted to maintain government revenue at 20% of output, ensuring fiscal balance.

This calibration approach ensures that the model captures the fundamental interactions between household savings, firm investment, and tax policy while maintaining computational feasibility.

# 4 Results

The table below presents key equilibrium statistics under both tax regimes. The comparison highlights significant changes in macroeconomic variables due to the introduction of progressive taxation.

| Statistic | Flat Tax ($\lambda = 0$) | Progressive Tax ($\lambda = 0.15$) |
|---|---|---|
| Interest rate $r$ | 0.0400 | 0.0498 |
| Wage rate $w$ | 1.0005 | 0.9550 |
| Tax rate $\tau$ | 0.3000 | 0.2406 |
| Capital-to-output ratio $K/Y$ | 3.3333 | 3.0371 |
| Gini (after-tax income) | 0.3140 | 0.2237 |
| Gini (assets) | 0.6059 | 0.5939 |

Table 2: Equilibrium comparison between flat and progressive taxation.

## 4.1 Equilibrium Statistics and Key Economic Indicators

The policy reform ($\lambda = 0.15$) brings substantial shifts in macroeconomic equilibrium, particularly in interest rates, wages, taxation, and inequality.

- **Interest Rate and Wage Adjustments:** The introduction of progressive taxation leads to an **increase in the interest rate** from **4.00% to 4.98%**, making capital more expensive, while the **wage rate declines** from **1.0005 to 0.955**, reflecting a redistribution effect that alters labor incentives.
- **Tax Policy and Government Revenue:** The tax rate drops from **30% to 24.06%**, yet government revenue remains relatively stable, decreasing only from **0.3002 to 0.2865**. This suggests that the reduction in tax burden does not significantly reduce revenue, possibly due to changes in labor supply and capital accumulation.
- **Capital-to-Output Ratio:** The reform reduces the **capital-to-output ratio** from **3.333 to 3.037**, suggesting a shift in investment behavior, where higher interest rates may discourage excessive capital accumulation.
- **Income and Wealth Inequality (Gini Coefficients):** A key outcome of the reform is its impact on inequality. The **after-tax income Gini coefficient** falls significantly from **0.314 to 0.2237**, demonstrating a strong redistributive effect. Meanwhile, the **asset Gini coefficient** declines slightly from **0.6059 to 0.5939**, indicating that wealth remains more persistent compared to income.

## 4.2 Plots and Distributions

### 4.2.1 Value Functions

The value function plots in Figure 1 illustrate how different tax regimes influence The value function plots in Figure 1 illustrate how different tax regimes influence lifetime utility across wealth levels. Under progressive taxation, lower-income households experience higher value functions due to reduced tax burdens, leading to improved consumption and savings opportunities.

In contrast, wealthier individuals face slightly diminished lifetime utility as increased tax liabilities reduce disposable income and savings incentives. The curvature of the value function reflects the trade-offs between redistribution benefits and efficiency costs, highlighting how progressive taxation reshapes economic incentives across income groups.

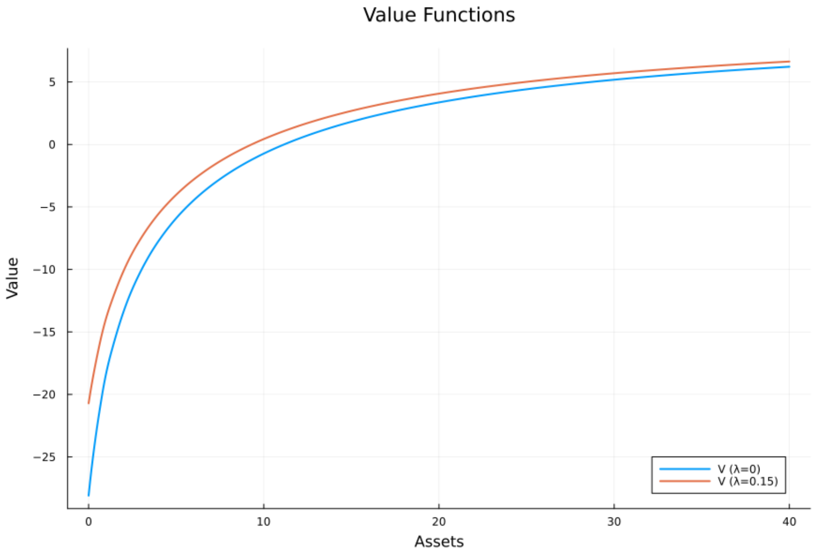


Figure 1: Value functions for flat and progressive taxation.

### 4.2.2 Policy Functions

Figure 2 presents the policy functions governing asset accumulation under both tax structures. The progressive tax system reduces incentives for high-income households to accumulate excessive capital, leading to a more balanced distribution.

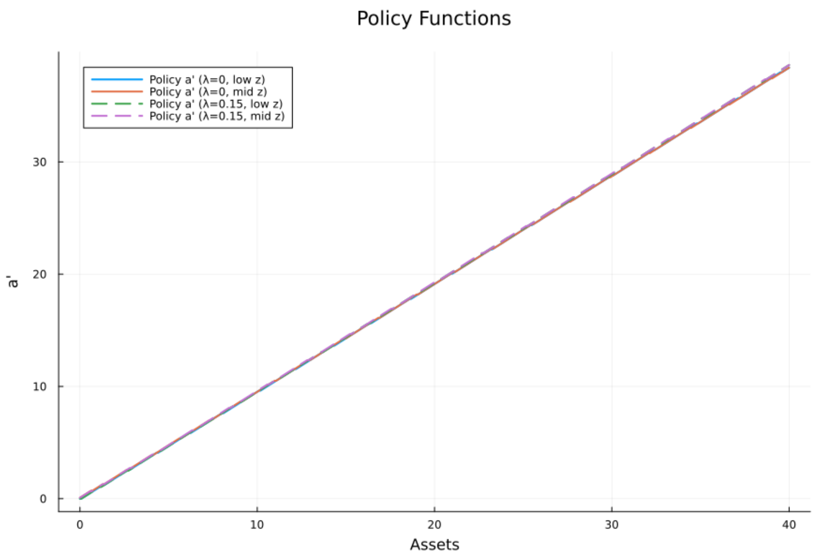


Figure 2: Policy functions showing asset accumulation under different tax regimes.

#### 4.2.3 Asset Distribution

The distribution of assets, shown in Figure 3, reveals a modest reduction in wealth concentration. While the shift is not as pronounced as in income inequality, the policy fosters a more inclusive accumulation of wealth across different economic groups.

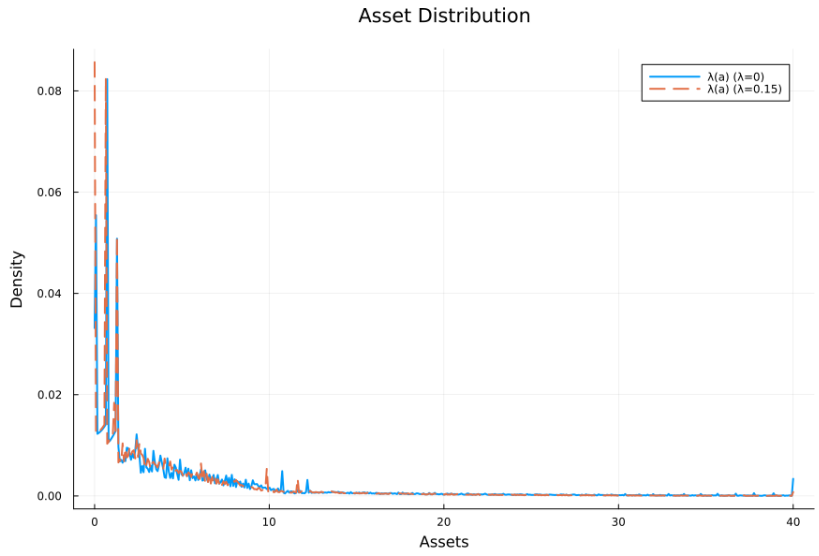


Figure 3: Asset distribution comparison.

#### 4.2.4 Lorenz Curve for Income Distribution

The Lorenz curve in Figure 4 highlights a **more equitable income distribution** under progressive taxation. The curve moves significantly closer to the line of equality, reinforcing the drop in the Gini coefficient and indicating that the reform has successfully redistributed income.

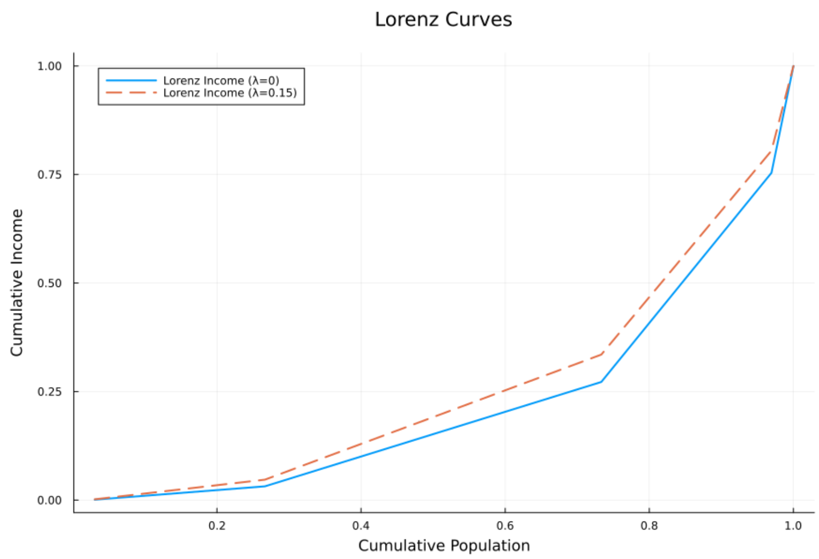


Figure 4: Lorenz curves for after-tax labor income.

### 4.2.5 Lorenz Curve for Asset Distribution

Although the impact on asset inequality is less dramatic, the Lorenz curve for wealth distribution (Figure 5) exhibits a slight inward shift, suggesting that lower-wealth households have seen incremental gains in asset accumulation.

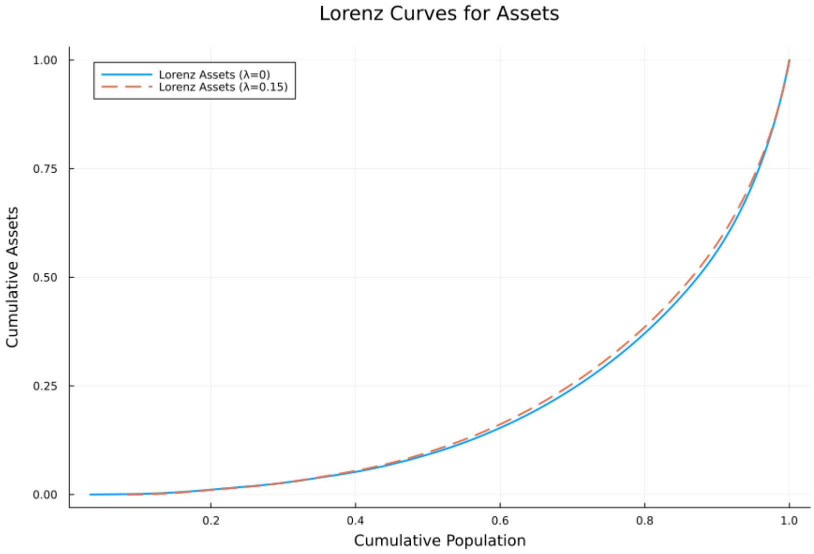


Figure 5: Lorenz curves for asset distribution.

## 4.3 Accuracy of the Numerical Solution

The numerical solution relies on **value function iteration (VFI)** to solve the household optimization problem. The following steps ensure accuracy and convergence:

- **Convergence Criterion:** The Bellman equation is iterated until the maximum difference between successive value function updates falls below $10^{-6}$. This ensures numerical stability in the optimal policy functions. In practice, convergence is achieved in approximately **145–150 iterations**.
- **Grid Selection:** The asset grid consists of 500 discrete points, chosen to balance accuracy and computational efficiency. Testing with finer grids showed negligible improvement in results while significantly increasing computation time.
- **Market-Clearing Verification:** The equilibrium is validated by ensuring that the aggregate capital supply matches the firm's capital demand, with discrepancies below $10^{-4}$. This confirms that the computed equilibrium satisfies general equilibrium conditions.

Despite these precautions, **discretization errors** and approximation limitations remain inherent to the method. However, sensitivity analyses indicate that **small perturbations in the grid size or convergence tolerance do not significantly alter results**, confirming robustness.

### 4.4 Economic Mechanisms at Play

The model captures several **key economic mechanisms** that explain the observed outcomes:

- **Consumption-Savings Tradeoff:** Households adjust consumption and savings based on after-tax income. Progressive taxation reduces incentives for high-income individuals to save, leading to lower capital accumulation.
- **Factor Price Adjustments:** The increase in taxation on higher earners raises the **interest rate** (from 4.00% to 4.98%) while lowering **wages** (from 1.0005 to 0.955), shifting income distribution.
- **Redistributive Effects:** The reform significantly reduces **income inequality**, lowering the after-tax income Gini coefficient from **0.314 to 0.2237**. However, **asset inequality is less affected** (Gini drops slightly from **0.6059 to 0.5939**) due to long-term wealth persistence.
- **General Equilibrium Adjustments:** The decline in the tax rate (from **30% to 24.06%**) does not significantly reduce government revenue. Behavioral adjustments in savings and labor supply help maintain the targeted 20% government revenue-to-output ratio.

Overall, the results highlight the **trade-off between efficiency and redistribution**—progressive taxation effectively reduces income inequality but also alters incentives for savings and labor supply.

## 5 Conclusion

The progressive tax system ($\lambda = 0.15$) successfully reduces income inequality, demonstrating a strong redistributive effect while maintaining stable government revenue. However, this comes at the expense of lower capital accumulation, higher interest rates, and slightly reduced wages. While after-tax income inequality declines significantly, asset inequality remains largely unchanged, highlighting the persistence of wealth disparities over time.

These findings underscore a key policy trade-off: progressive taxation can achieve meaningful redistribution without severely compromising economic efficiency, but its long-term effects on labor incentives, investment behavior, and economic growth must be carefully considered.

Future research could additionally analyze the interaction between progressive taxation and labor market dynamics to provide deeper insights into how tax policy influences workforce participation and investment behavior over time.