```matlab
% Modelo        Parámetros                    Nombre
% OE            na, nb, nk                    Output Error
% ARX           na, nb, nk                    Auto-regresivo con variable exógena (v.e.)
% ARMAX         na, nb, nc, nk                Auto-regresivo, media móvil con v.e.
% BJ            nb, nc, nd, na, nk            Box-Jenkins
%
% Nota: orden de los parámetros de acuerdo al formato de clase

modelo = 'OE';
na = 1;
nb = 2;
nc = 0;
nd = 0;
nk = 1;

syms a [1, na]
syms b [1, nb]
syms c [1, nc]
syms d [1, nd]
syms q y(t) u(t) e v N

modelos = {'OE', 'ARX', 'ARMAX', 'BJ'};

sympref('PolynomialDisplayStyle','descend');

A = q^(-na)*poly2sym([1, a], q);
B = q^(-nb)*poly2sym(b, q)*q^(-nk);
C = q^(-nc)*poly2sym([1, c], q);
D = q^(-nd)*poly2sym([1, d], q);


if strcmp(modelo, modelos{1}) % Modelo OE
    A, B
    modelo_OE = y == (B/A)*u + e
    e = solve(modelo_OE, e)
    v = (1/N)*symsum(e^2, t, 1, N)
    gradiente_OE = gradient(v, [a b])
    hessiano_OE = hessian(v, [a b])

elseif strcmp(modelo, modelos{2}) % Modelo ARX
    A, B
    modelo_ARX = y == (B/A)*u + (1/A)*e
    e = solve(modelo_ARX, e)
    v = (1/N)*symsum(e^2, t, 1, N)
    gradiente_ARX = gradient(v, [a b])
    hessiano_ARX = hessian(v, [a b])

elseif strcmp(modelo, modelos{3}) % Modelo ARMAX
    A, B, C
    modelo_ARMAX = y == (B/A)*u + (C/A)*e
    e = solve(modelo_ARMAX, e)
    v = (1/N)*symsum(e^2, t, 1, N)
    gradiente_ARMAX = gradient(v, [a b c])
```

```
    hessiano_ARMAX = hessian(v, [a b c])

else % Modelo BJ
    A, B, C, D
    modelo_BJ = y == (B/A)*u + (C/(A*D))*e
    e = solve(modelo_BJ, e)
    v = (1/N)*symsum(e^2, t, 1, N)
    gradiente_BJ = gradient(v, [a b c d])
    hessiano_BJ = hessian(v, [a b c d])

end
```

```
A =
```

$$\frac{a_1 + q}{q}$$

```
B =
```

$$\frac{b_1\, q + b_2}{q^3}$$

```
modelo_OE(t) =
```

$$y(t) = e + \frac{u(t)\ (b_1\, q + b_2)}{q^2\ (a_1 + q)}$$

```
e =
```

$$y(t) - \frac{u(t)\ (b_1\, q + b_2)}{q^2\ (a_1 + q)}$$

```
v =
```

$$\frac{\sum_{t=1}^{N} \left(y(t) - \frac{u(t)\ (b_1\, q + b_2)}{q^2\ (a_1 + q)}\right)^2}{N}$$

```
gradiente_OE =
```

$$\begin{pmatrix} \frac{2\,\sigma_1\ (b_1\, q + b_2)}{N\, q^2\ (a_1 + q)^2} \\ -\frac{2\,\sigma_1}{N\, q\ (a_1 + q)} \\ -\frac{2\,\sigma_1}{N\, q^2\ (a_1 + q)} \end{pmatrix}$$

where

$$\sigma_1 = \sum_{t=1}^{N} u(t)\ \left(y(t) - \frac{u(t)\ (b_1\, q + b_2)}{q^2\ (a_1 + q)}\right)$$

```
hessiano_OE =
```

$$\begin{pmatrix} \frac{2\,\sigma_6\,(b_1\,q+b_2)^2}{N\,q^4\,(a_1+q)^4}-\frac{4\,\sigma_5\,(b_1\,q+b_2)}{N\,q^2\,(a_1+q)^3} & \sigma_2 & \sigma_1 \\ \sigma_2 & \frac{2\,\sigma_6}{\sigma_4} & \sigma_3 \\ \sigma_1 & \sigma_3 & \frac{2\,\sigma_6}{N\,q^4\,(a_1+q)^2} \end{pmatrix}$$

where

$$\sigma_1 = \frac{2\,\sigma_5}{\sigma_4} - \frac{2\,\sigma_6\,(b_1\,q+b_2)}{N\,q^4\,(a_1+q)^3}$$

$$\sigma_2 = \frac{2\,\sigma_5}{N\,q\,(a_1+q)^2} - \frac{2\,\sigma_6\,(b_1\,q+b_2)}{N\,q^3\,(a_1+q)^3}$$

$$\sigma_3 = \frac{2\,\sigma_6}{N\,q^3\,(a_1+q)^2}$$

$$\sigma_4 = N\,q^2\,(a_1+q)^2$$

$$\sigma_5 = \sum_{t=1}^{N} u(t)\,\left(y(t) - \frac{u(t)\,(b_1\,q+b_2)}{q^2\,(a_1+q)}\right)$$

$$\sigma_6 = \sum_{t=1}^{N} u(t)^2$$

La estimación del vector de parámetros está dada por:

$\theta:$ vector de parámetros del modelo (OE, ARX, ARMAX, Box-Jenkins).

$$\hat{\theta}\,(k+1) = \hat{\theta}\,(k) - \alpha_k * \nabla v$$

$$\nabla v = \frac{\partial v}{\partial\,\theta}$$