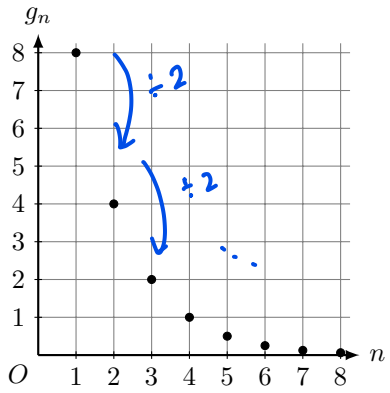
Practice

1. The values of a sequence g_n are given in the graph below.



$a_1 = 8$

$r = 1/2$

Answer each of the following.

(a) Find a formula for the nth term of the sequence.

$a_n = 8\left(\frac{1}{2}\right)^{n-1}$ or $a_n = 16\left(\frac{1}{2}\right)^{n}$ or $a_n = \left(\frac{1}{2}\right)^{4-n}$

(b) Does the sum $\sum_{n=1}^{\infty} g_n$ converge? If so, what is the sum?

yes, $S = \frac{8}{1-1/2} = 16$

2. The terms of an increasing arithmetic sequence a_n are positive. The terms of the increasing geometric sequence b_n are positive. The values of the first term of both sequences are the same, and the values of the fourth terms are the same. Which of the following statements describes the values of the second terms of the sequences?

   A. The second term of the arithmetic sequence must be less than the second term of the geometric sequence.

   B. The second term of the arithmetic sequence must be greater than the second term of the geometric sequence.

   C. The second term of the arithmetic sequence must be equal to the second term of the geometric sequence.

   D. The relationship between the values of the second terms cannot be determined from the given information.

3. Consecutive terms of a sequence have the values $6, 2, -2, -6, \ldots$. What is the sum of the first 100 terms of the sequence?

-4 -4 -4

$S_{100} = 100\left(\frac{2(6) + 99(-4)}{2}\right) = -19200$

$a_n = 6 - 4(n-1)$

$a_1 + a_n = 6 + 6 - 4(n-1)$

$2(6) + 99(-4)$

4. Determine if the following sequences are arithmetic, geometric, or neither. For all, find a formula (explicit or recursive) for the sequence.

(a) 1, 3, 5, 7, 9, ...

arith

$a_n = 1 + 2(n-1)$

(b) 2, 4, 8, 16, 32, ...

geo

$a_n = 2 \cdot 2^{n-1}$

or

$= 2^n$

(c) $1^2, 2^2, 3^2, \ldots$

neither

$a_n = n^2$

(d) $2, \frac{4}{\sqrt{3}}, \frac{8}{3}, \frac{16}{3\sqrt{3}}, \ldots$

geo

$a_n = 2 \cdot \left(\frac{2}{\sqrt{3}}\right)^{n-1}$

or

$= \frac{2^n}{3^{\frac{n-1}{2}}}$

(e) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

neither

$a_n = \frac{1}{n+1}$

(f) $\frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \ldots$

$\frac{6}{4}$ $\frac{8}{4}$

arith

$a_n = \frac{5}{4} + \frac{1}{4}(n-1)$

or

$a_n = \frac{5+(n-1)}{4}$

(g) $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \ldots$

neither

$a_n = \frac{1+2(n-1)}{2+2(n-1)}$

or

$a_n = \frac{2n-1}{2n}$

(h) 1, 0.9, 0.99, 0.999, ...

neither

$a_n = 1 - (0.1)^{n-1}$

(i) Write a seqence with positve and negative terms that has a sum less than zero.

EX

$r = -1/2$

$a = -50$

$\sum_{n=1}^{\infty} -50\left(\frac{-1}{2}\right)^{n-1} = \frac{-50}{1+1/2}$

$= -\frac{100}{3}$