Name: ______________________________ Grade: ______/56

Answer the questions in the spaces provided on the following pages. If you run out of room for an answer, continue on the back of the page. Show **all** your work to be able to receive full credit on any question. **YOU ARE A MISSILE**

Part I - Formulas

For each of the following questions, complete the specified trigonometric identity by filling in the blank(s).

1. The Pythagorean Identity is [1]

$$\sin^2 x + \underline{\hspace{3cm}} = \underline{\hspace{3cm}}.$$

2. The Power Reducing Identity for $\sin^2 x$ is [1]

$$\sin^2 x = \underline{\hspace{6cm}}.$$

3. The Double Angle Identity for $\sin 2x$ is [1]

$$\sin 2x = \underline{\hspace{6cm}}.$$

4. The Angle Difference Identity for $\cos(x - y)$ is [1]

$$\cos(x - y) = \underline{\hspace{6cm}}.$$

5. The Angle Sum Identity for $\sin(x + y)$ is [1]

$$\sin(x + y) = \underline{\hspace{6cm}}.$$

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Part II - No Calculator

Recall: *Fundamental Trigonometric Identities*

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$
$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

Sum and Difference Identities

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

Even/Odd Identities

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$
$$\tan(-\theta) = -\tan\theta$$

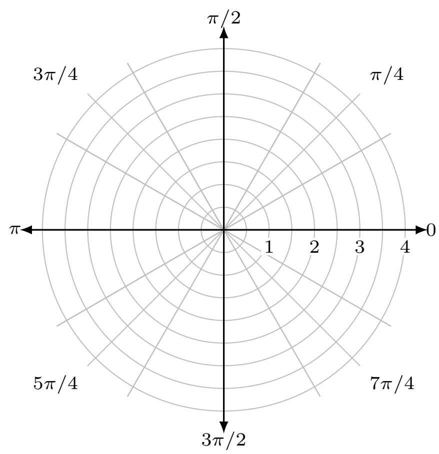
Double Angle Identities

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

Power Reducing Identities

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$
$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$
$$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

1. Plot the coordinate $\left(2, \frac{2\pi}{3}\right)$ and find two additional polar representations of this point with $-2\pi \le \theta \le 2\pi$. [3]



(-2,-pi/3) or (2,-4pi/3)

2. Convert the rectangular coordiant $(4,4)$ to polar form. [2]

(4sqrt2,pi/4)

3. Convert $\left(-6, -\frac{7\pi}{6}\right)$ to rectangular form. [2]

x=-6cos(-7pi/6)=-6*(-sqrt3/2)=3sqrt3

y=-6sin(-7pi/6)=-6*(1/2)=-3

(3sqrt3,-3)

4. Convert the following rectangular equations to polar form. Leave your answer in the form $r = f(\theta)$.

(a) $y^3 = x^2$ [3]

(rsintheta)^3=(rcostheta)^2

r=cos^2theta/sin^3theta

(b) $2xy = 5$ [3]

2(rcostheta)(rsintheta)=5

r^2=5/(2costhetasintheta)

5. Convert the following polar equations to rectangular form. Leave your answer in the form $y = f(x)$ or the general form for the equation of a circle.

   (a) $r = 9\cos\theta$ [3]

   r^2=9rcostheta
   x^2+y^2=9x
   (x-3)^2+y^2=9

   (b) $\theta = \dfrac{11\pi}{6}$ [3]

   tan(11pi/6)=y/x

   y=-(sqrt3/3)x

6. Given that $\tan u = \frac{3}{4}$ and $\sec v = -\frac{13}{5}$ with $0 < u < \frac{\pi}{2}$ and $\sin v < 0$, find the exact value of $\cos(u+v)$. [3]

7. Verify the following trigonometric identities.

   (a) $2\sin\theta\cos\theta\sec 2\theta = \tan 2\theta$ [3]

   sin(2theta)(1/cos(2theta))=
   tan(2theta) = tan(2theta)

   (b) $\sin x\left(1 - 2\cos^2 x + \cos^4 x\right) = \sin^5 x$ [3]

   sinx(1-cos^2 x)^2=
   sinx(sin^2 x)^2=
   sinx*sin^4 x=
   sin^5 x= sin^5 x

8. Solve the following trigonometric equations for $0 \leq x < 2\pi$.

   (a) $\cos 4x + \sin 2x = 0$ [3]

   cos^2(2x)-sin^2(2x) + sin(2x)=0
   1-2sin^2(2x)+sin(2x)=0
   2sin^2(2x)-sin(2x)-1=0
   (2sin(2x)+1)(sin(2x)-1)=0

   2sin(2x)+1=0 sin(2x)-1=0
   sin(2x)=-1/2 sin(2x)=-1
   2x=7pi/6+2pi n 2x=3pi/2+2pi n
   =11pi/6 + 2pi n

   x=7pi/12 + pi n x = 3pi/4 + pi n
   = 11pi/12 + pi n

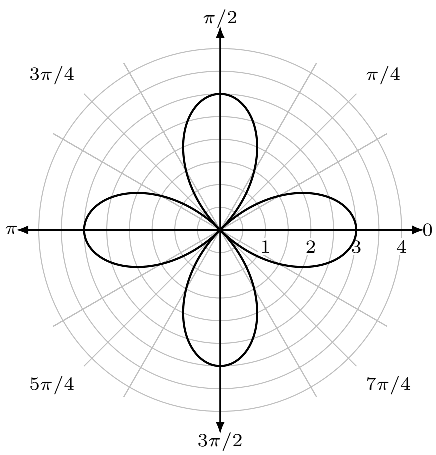
   x={7pi/12,3pi/4,11pi/12,19pi/12,7pi/4,23pi/12}

   (b) $1 + 2\cos 3x = 0$ [3]

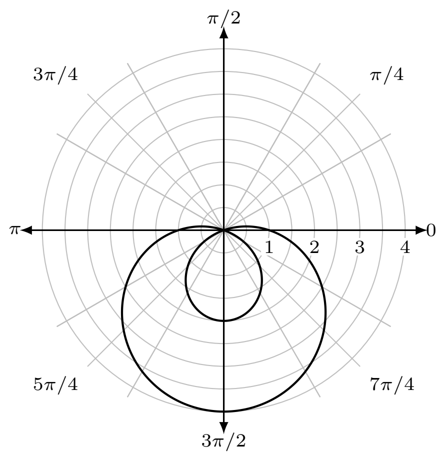
   omit

9. Given the graph of a polar function, write the equation. [4]

   (a)

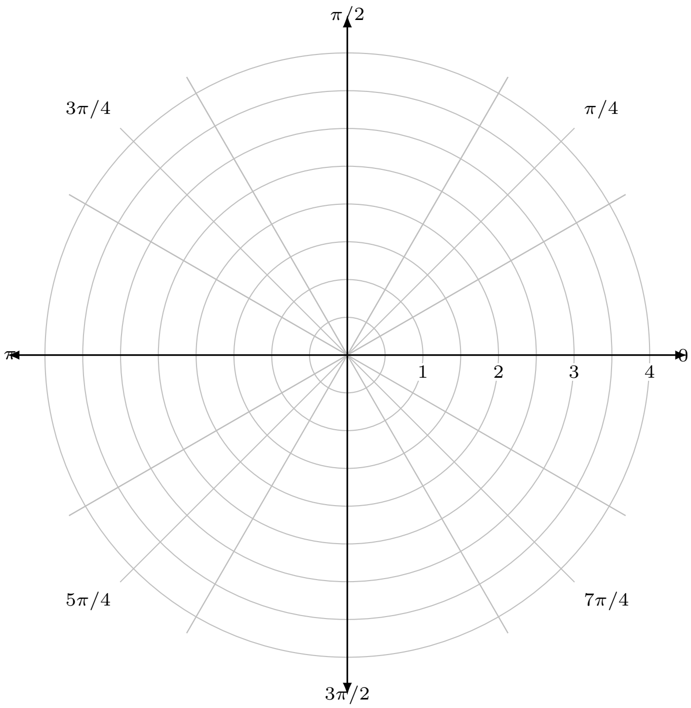


   (b)



   3cos2theta

10. On the polar axes below, graph the function $r = -2\sin 3\theta$ for $0 \leq \theta \leq 2\pi$. Then, specify what lines of symmetry the curve has. [5]



Part III - Calculator Allowed

1. Consider the graph of $f(\theta) = 1 + 2\sin\theta$ for $0 \le \theta \le 2\pi$. Which of the following statements is true about the distance between $f(\theta)$ and the origin? [2]

   A. The distance is increasing on $0 \le \theta \le \frac{\pi}{2}$, becuase $f(\theta)$ is positive and increasing on the interval.

   B. The distance is increasing on $\frac{3\pi}{2} \le \theta \le \frac{11\pi}{6}$, becuase $f(\theta)$ is negative and increasing on the interval.

   C. The distance is decreasing on $0 \le \theta \le \frac{\pi}{2}$, becuase $f(\theta)$ is positive and decreasing on the interval.

   D. The distance is decreasing on $\frac{3\pi}{2} \le \theta \le \frac{11\pi}{6}$, becuase $f(\theta)$ is negative and decreasing on the interval.

2. What is the average rate of change of the polar curve $r = 2 + 4\cos\theta$ on the interval $\left[0, \dfrac{\pi}{2}\right]$? [2]

3. Consider the function $f(\theta) = -2 + 4\sin\theta$ for $0 \le \theta \le 2\pi$.

   (a) For what interval(s) of θ is $f(\theta)$ increasing? [2]

   (b) On the interval $\left[\frac{\pi}{2}, \frac{2\pi}{3}\right]$, is the distance between $f(\theta)$ and the origin increasing or decreasing? Justify your answer. [2]