Rates of Change of Polar Functions

Due to the nature of changing variables, talking about rates of change requires slightly more focus. This is becuase our normal understanding of the rate of change can be interpreted as...

$$\text{Rate of Change} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}.$$

However, when it comes to polar coordinates, it could be different.

Defintion: *Rates of Change in Polar Coordinates*

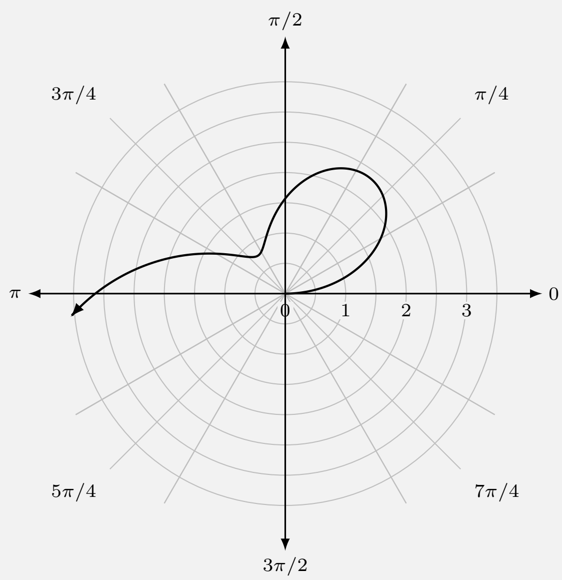
Let $r = f(\theta)$ be a polar function. Then, the **rate of change of** r can be interpreted as

$$\frac{\Delta r}{\Delta \theta}$$

This is the rate of change of r with respect to θ. We determine this by considering "as θ increases, does r increase or decrease?"

$$\frac{\Delta y}{\Delta x}$$

This is the rate of change of y with respect to x. As θ and r change, is the point going up or down in relation to the x-axis?



Consider the polar function $r(\theta) = 3\cos 3\theta$. Graph this function on your calculator and locate the points at the specified values of θ. Is the value of r increasing or decreasing over this interval?

(a) $\theta = \dfrac{\pi}{6}$,

(b) $\theta = \dfrac{\pi}{4}$,

(c) $\theta = \dfrac{\pi}{3}$.

Defintion: *Distance from the Origin*

Let $r = f(\theta)$ be a polar function. Then the distance from the origin is **increasing** when either

- r is positive and increasing
- r is negative and decreasing

Similarly, the distance from the origin is **decreasing** when either

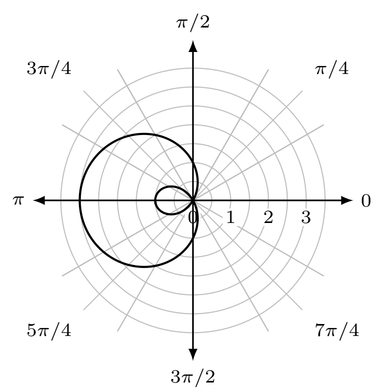
- r is positive and decreasing
- r is negative and increasing

Examples

1. Consider the curve gievn by $r = 3$ on the interval $0 \leq \theta \leq 2\pi$.
   (a) Sketch a graph of the curve r.
   (b) What is the rate of change of r with respect to θ? How do you know?

   (c) For what interval(s) of θ is y increasing?

2. The graph of $r(\theta) = 1 - 2\cos\theta$ is shown alongside. Determine if the polar curve is getting closer to the origin, futher from the origin, or neither when $\theta = \pi/6$.



For the example above, is there another way we can visualize the rate of change of r with respect to θ?

Defintion: *Average Rate of Change of a Polar Function*
Let $r = f(\theta)$ be a polar function. Then the average rate of change of r on the interval $[\theta_1, \theta_2]$ is given by $$\Delta r = \frac{f(\theta_2) - f(\theta_1)}{\theta_2 - \theta_1}.$$

3. Consider the curve given by $r = -3 + 5\sin\theta$ on the interval $0 \le \theta \le 2\pi$.
   (a) Is the distance between $f(\theta)$ and the origin increasing or decreasing on the interval $\pi/2 \le \theta \le 3\pi/4$?

   (b) Find the average rate of change of $f(\theta)$ between $\theta = \pi/4$ and $\theta = \pi/2$.

The table alongside gives select values of a polar curve r.

(a) Is r increasing or decreasing on the interval $\dfrac{\pi}{4} \le \theta \le \dfrac{\pi}{2}$?

(b) Is the distance betwen $f(\theta)$ and the pole increasing or decreasing on the interval $0 \le \theta \le \pi/4$?

(c) Is the rate of change of r faster on the interval $\left[0, \frac{\pi}{8}\right]$ or $\left[\frac{\pi}{8}, \frac{\pi}{4}\right]$? Justify your answer.

θ	r
0	0
$\frac{\pi}{8}$	-1.41
$\frac{\pi}{4}$	-2
$\frac{3\pi}{8}$	-1.41
$\frac{\pi}{2}$	0