Name: ______ Grade: ____/200

Answer the questions in the spaces provided on the following pages. If you run out of room for an answer, continue on the back of the page. Show all your work to be able to receive full credit on any question. YOU ARE A MISSILE

For each statement below, circle whether they are true or false. As a reminder, a statement that is not always true is considered false. No work is needed for the following problems.

1. True (False) If
$$\lim_{n\to\infty} a_n = 0$$
, then $\sum a_n$ is convergent. [3]

2. True False If
$$\sum a_n$$
 is divergent, then $\sum |a_n|$ is divergent. [3]

3. True False If
$$\sum a_n$$
 is divergent, then $\sum a_n$ is divergent. [3]

4. True False The ratio test can be used to determine whether
$$\sum 1/n!$$
 converges. [3]

5. True False The ratio test can be used to determine whether
$$\sum 1/n^3$$
 converges. [3]

6. True False If
$$0 \le a_n \le b_n$$
 and $\sum b_n$ converges, then $\sum a_n$ converges. [3]

7. True False If
$$-1 < \alpha < 1$$
, then $\sum \alpha^n$ is convergent. [3]

8. True False If
$$a_n > 0$$
 and $\lim_{n \to \infty} (a_{n+1}/a_n) = 1$, then $\sum a_n$ converges. [3]

9. True False
$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \frac{1}{e}$$
 [3]

13. True False If
$$a_n > 0$$
 and $b_n > 0$, $\sum b_n$ converges and $\lim_{n \to \infty} \frac{a_n}{b_n} = 0$, then $\sum a_n$ is convergent. [3]

14. True False
$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$
 [3]

16. Determine whether the following series are convergent or divergent. Show all work to recieve full credit. An answer of convergent or divergent alone will yield no credit.

(a)
$$\sum_{n=2}^{\infty} \frac{\ln n}{n^3} \qquad b_n = \frac{n}{\sqrt{3}} \geqslant \frac{\ln n}{\sqrt{3}}$$
 [10]

(b)
$$\sum_{i=1}^{\infty} \left(\sqrt[i]{i} - 1 \right)^{i} = \lim_{i \to \infty} \left(\sqrt[i]{i} - 1 \right)^{i}$$

(c)
$$\sum_{n=0}^{\infty} (-1)^{n+1} e^{\left(\frac{2}{n+1}\right)}$$

$$\lim_{N \to \omega} e^{\left(\frac{2}{n+1}\right)} = e^{c} = 1 \implies D = D = D$$
[10]

$$\begin{array}{ccc}
\text{(d)} & \sum_{k=1}^{\infty} \frac{\arctan(k)}{k^{1.5}} & & & & \\
& & \downarrow_{n} & = \frac{\pi \sqrt{2}}{\sqrt{1.5}} & \geqslant \alpha \, n
\end{array} \tag{10}$$

[10]

[5]

[10]

17. Find the sum of the following convergent series.

Find the sum of the following converge
(a)
$$\sum_{n=0}^{\infty} \frac{3^{n+1}}{(-4)^n} = \frac{3}{3} \left(-\frac{3}{4}\right)^n$$

$$= \frac{3}{(+\frac{3}{4})^4}$$

$$= \frac{12}{7}$$

(b) $\sum_{i=0}^{\infty} \left[\frac{1}{i^2} - \frac{1}{(i+2)^2} \right]$

$$= \left(\frac{1}{2^{2}} - \frac{1}{4^{2}}\right) + \left(\frac{1}{3^{2}} - \frac{1}{3^{2}}\right) + \left(\frac{1}{4^{2}} - \frac{1}{3^{2}}\right) + -\frac{1}{4^{2}}$$

$$\frac{1}{4} + \frac{1}{9}$$

- 18. Find the raidus of convergence for the given power series.

$$\sum_{n=0}^{\infty} \frac{2^n}{n!} (x-1)^n$$

$$\sum_{n=0}^{\infty} \frac{2^n}{n!} (x-1)^n \qquad \lim_{n \to \infty} \left| \frac{2^{n+1}}{(n-1)!} (x-1)^n - \frac{n!}{2^n (x-1)^n} \right| = \lim_{n \to \infty} \frac{2^n}{n!} (x-1)^n$$

19. Find the interval of convergence for the given power series.

$$\sum_{n=2}^{\infty} \frac{n}{n+1} \left(\frac{x}{4}\right)^{n-1}$$

$$\lim_{N \to \infty} \left| \frac{nH}{NT2} \left(\frac{x}{4} \right)^{N} \cdot \left(\frac{x}{x} \right)^{N-1} \cdot \frac{n}{N+1} \right| = \left| \frac{x}{4} \right| \leq 1 \quad \Rightarrow \quad -4 \leq x \leq 4$$

20. Consider the function given by

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x+2)^{n+1}}{n+1}.$$

Answer the following.

(a) Find a series for f'(x). Then, find its interval of convergence.

(b) Write the series $\int_{-2}^{-1.3} f(x)dx$. You do not need to find the interval of convergence. $\int_{-2}^{-1.3} f(x)dx = \frac{(-1)(x+2)^{n+2}}{(n+1)(n+2)} \Big|_{-2}^{-1.3}$

$$\int_{-2}^{-1.3} f(x) dx = \int_{-2}^{-2} \frac{(-1)^2 (x+2)^{n+2}}{(n+1)(n+2)} \Big|_{-2}^{-1.3}$$

$$\leq \frac{\left(-1\right)^{n}\left(.7\right)^{n+2}}{\left(n+1\right)\left(n+2\right)}$$

(c) Show that the maximum error associated with the approximation of S_7 for the series obtained in part (b) is less than $\frac{29}{500}$.

$$\frac{1}{3125}$$
 ASRT: $|E| \leq \alpha_8 = \frac{.7^{10}}{9.10} = 0.000313...$

21. Consider the following functions. Using any means, construct a power series for the given function at the specified center. Leave your answer in summation notation. Simplify completely.

(a)
$$h(x) = 3xe^{-\frac{x^2}{3}}, c = 0$$

$$=3\chi \sum_{n!} \frac{(-x^2/3)^n}{n!}$$

$$= \sum_{n=0}^{\infty} \frac{(n)^n \times^{2n+1}}{3^{n-1} \cdot n!}$$

(b)
$$r(x) = \frac{1}{1+x}, c = 3$$

$$= + (x-1+3)$$

$$=\frac{1}{4} \left[\left(-\left(\frac{\mathsf{X}-3}{4} \right) \right)^{\mathsf{N}} \right]$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n (x-3)^n}{4^{n+1}}$$

(c)
$$t(x) = \sin 3x \cos 3x, c = 0$$

$$= \frac{1}{2} Sin(6x)$$

$$= \frac{\sqrt{-10^{\circ} (2nt)} \times 2nt}{2 \cdot (2nt)!}$$

[10]

[15]

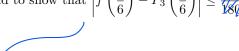
- 22. Let f be the function given by $f(x) = \frac{1}{1-2x}$ and let $P_3(x)$ be the third degree MacLaurin Polynomial for f.
 - (a) Find $P_3(x)$.

$$f = 2^{7}(2x)^{n}$$

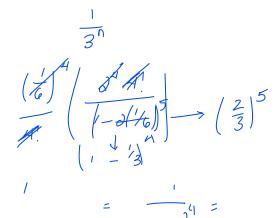
$$= 1 + 2x + 4x^{2} + 8x^{3} \left(+ 16x^{4} \right)$$

$$f^{(4)} = \frac{2^4 4!}{(1-2x)^5}$$

(b) Use the Lagrange Error Bound to show that $\left| f\left(\frac{1}{6}\right) - P_3\left(\frac{1}{6}\right) \right| \le \frac{1}{1800}$.



$$\frac{\left(\frac{1}{6}-0\right)^{4}}{4!}$$
 max $\left|\frac{1}{4}^{(4)}(2)\right|$



23. The function ϕ has a Taylor series about x=2 that converges for all x in it's interval of convergence. The nth derivative of ϕ at x=2 is given by

$$\phi^{(n)}(2) = \frac{(n+1)!}{3^n}$$

for $n \ge 1$.

of $\phi(3)$ using the *n*th [10]

(a) Find an expression for the Lagrange Error bound associated with the approximation of $\phi(3)$ using the *n*th [10] degree Taylor polynomial centered about x = 2.

$$| \frac{4}{3} | - \frac{1}{n} \frac{3}{3} | \leq \frac{3 - 2}{(n + 1)!} \max | \frac{4}{9} \frac{(n + 1)!}{(n + 1)!}$$

$$\leq \frac{1}{(n + 1)!} \frac{(n + 1)!}{3^{n + 1}}$$

$$\leq \frac{1}{2^{n + 1}}$$

(b) How many terms are needed for the error to be less than $\frac{1}{1200}$? [10]

$$\frac{1+2}{3^{n+1}} \stackrel{?}{\sim} \frac{1}{1200}$$

Extra Credit (10pts): Consider the following series.

$$\sum_{n=0}^{\infty} \frac{n! \cdot (n+1)! \cdot 3^n}{\left(1 \cdot 3 \cdot 6 \cdot \dots \cdot (3n)\right)^2}.$$

Determine the convergence or divergence of the series. Show all work.

$$\begin{array}{c|c} L_{1M} & \frac{(n+1)!(n+2)!(3^{N+1})}{(1.3-(3n)!(3n+2))^2} & \frac{(13-(3n)!)^2}{N!(n+1)!(3^{N+1})} \end{array}$$

= Lim
$$\frac{(n+1)(n+2)(3)}{(3n+3)(3n+3)} = \frac{3}{9} = \frac{1}{3} \angle 1$$