Name: ______________________________ Grade: ______/55

Answer the questions in the spaces provided on the following pages. If you run out of room for an answer, continue on the back of the page. Show **all** your work to be able to receive full credit on any question. **YOU ARE A MISSILE**

Part I - Formulas

For each of the following questions, complete the specified trigonometric identity by filling in the blank(s).

1. The Pythagorean Identity is [1]

$$\sin^2 x + \underline{\hspace{3cm}} = \underline{\hspace{3cm}}.$$

2. The Power Reducing Identity for $\sin^2 x$ is [1]

$$\sin^2 x = \underline{\hspace{6cm}}.$$

3. The Double Angle Identity for $\sin 2x$ is [1]

$$\sin 2x = \underline{\hspace{6cm}}.$$

4. The Angle Difference Identity for $\cos(x - y)$ is [1]

$$\cos(x - y) = \underline{\hspace{6cm}}.$$

5. The Angle Sum Identity for $\sin(x + y)$ is [1]

$$\sin(x + y) = \underline{\hspace{6cm}}.$$

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Part II - No Calculator

Recall: *Fundamental Trigonometric Identities*

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$
$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

Sum and Difference Identities

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

Even/Odd Identities

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$
$$\tan(-\theta) = -\tan\theta$$

Double Angle Identities

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

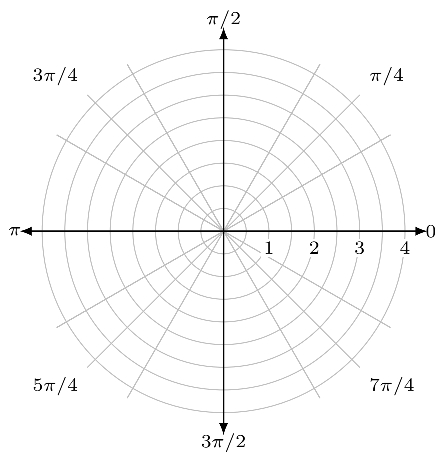
$$\begin{aligned}\cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Power Reducing Identities

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$
$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$
$$\tan^2\theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

1. Plot the coordinate $\left(2, \frac{2\pi}{3}\right)$ and find two additional polar representations of this point with $-2\pi \le \theta \le 2\pi$. [3]



2. Convert the rectangular coordiant $(4, 4)$ to polar form. [2]

3. Convert $\left(-6, -\dfrac{7\pi}{6}\right)$ to rectangular form. [2]

4. Convert the following rectangular equations to polar form. Leave your answer in the form $r = f(\theta)$.

   (a) $y^3 = x^2$ [3]

   (b) $2xy = 5$ [3]

5. Convert the following polar equations to rectangular form. Leave your answer in the form $y = f(x)$ or the general form for the equation of a circle.

   (a) $r = 9\cos\theta$ [3]

   (b) $\theta = \dfrac{11\pi}{6}$ [3]

6. Given that $\tan u = \frac{3}{4}$ and $\sec v = -\frac{13}{5}$ with $0 < u < \frac{\pi}{2}$ and $\sin v < 0$, find the exact value of $\cos(u + v)$. [3]

7. Verify the following trigonometric identities.
   (a) $2 \sin\theta \cos\theta \sec 2\theta = \tan 2\theta$ [3]

   (b) $\sin x \left(1 - 2\cos^2 x + \cos^4 x\right) = \sin^5 x$ [3]

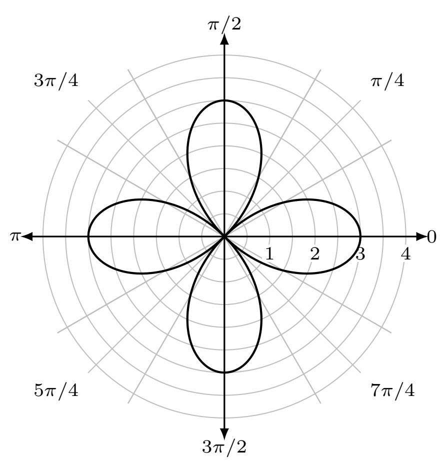
8. Solve the following trigonometric equation [5]

$$\cos 4x + \sin 2x = 0.$$

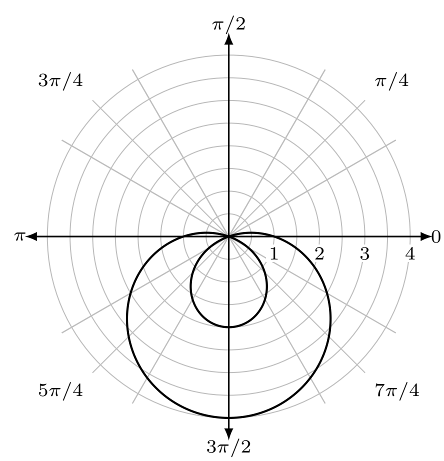
Find all solutions on the interval $0 \leq x < 2\pi$.

9. Given the graph of a polar function, write the equation. [4]

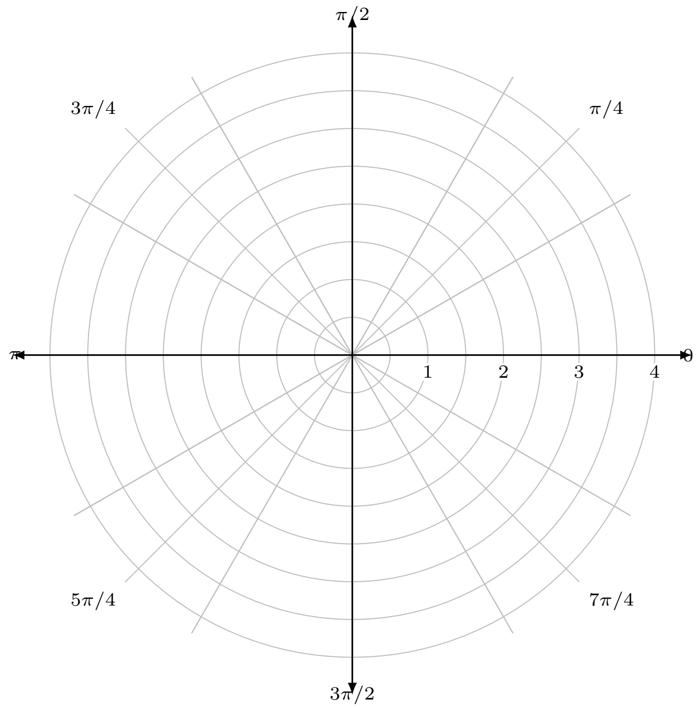
(a)



(b)



10. On the polar axes below, graph the function $r = -2\sin 3\theta$ for $0 \leq \theta \leq 2\pi$. Then, specify what lines of symmetry the curve has. [5]



Part III - Calculator Allowed

1. Consider the graph of $f(\theta) = 1 + 2\sin\theta$ for $0 \leq \theta \leq 2\pi$. Which of the following statements is true about the distance between $f(\theta)$ and the origin? [2]

   A. The distance is increasing on $0 \leq \theta \leq \frac{\pi}{2}$, becuase $f(\theta)$ is positive and increasing on the interval.

   B. The distance is increasing on $\frac{3\pi}{2} \leq \theta \leq \frac{11\pi}{6}$, becuase $f(\theta)$ is negative and increasing on the interval.

   C. The distance is decreasing on $0 \leq \theta \leq \frac{\pi}{2}$, becuase $f(\theta)$ is positive and decreasing on the interval.

   D. The distance is decreasing on $\frac{3\pi}{2} \leq \theta \leq \frac{11\pi}{6}$, becuase $f(\theta)$ is negative and decreasing on the interval.

2. What is the average rate of change of the polar curve $r = 2 + 4\cos\theta$ on the interval $\left[0, \dfrac{\pi}{2}\right]$? [2]

3. Consider the function $f(\theta) = -2 + 1\sin\theta$ for $0 \leq \theta \leq 2\pi$.
   (a) On the inteval when $\theta = 0$ to $\theta = \frac{\pi}{6}$, is $r = f(\theta)$ increasing, decreasing, or neither? How do you know? [2]

   (b) On the interval $\left[\frac{\pi}{2}, \frac{2\pi}{3}\right]$, is the distance between $f(\theta)$ and the origin increasing or decreasing? Justify your answer. [2]