Other Common Sequences and Series

Factorials

While this particular class does not focus on them much, they become important in calculus and probability.

Definition: *Factorial*

For a nonnegative integer n, the **factorial** of n is defined as

$$n! = n(n-1)(n-2)\cdots(3)(2)(1)$$

with the special case that $0! = 1$.

The factorial of n is the product of all positive integers less than or equal to n, which is a recursive sequence: $n! = n(n-1)!$.

Simplify the following factorials.

1. $\dfrac{8!}{2! \cdot 6!}$
2. $\dfrac{n!}{(n-1)!}$
3. $\dfrac{4!(n+2)!}{6!n!}$

Binomial Coefficient

Definition: *Combinations*

For a nonnegative integers n and r, with $0 \leq r \leq n$, then

$${}_nC_r = \frac{n!}{(n-r)!r!}.$$

The symbol $\dbinom{n}{r}$ is often used of the C notation when referring to the binomial coefficient.

Evaluate each of the following expressions.

1. ${}_8C_2$
2. $\dbinom{10}{3}$
3. ${}_7C_4$
4. ${}_7C_3$

The Binomial Theorem

Introductory Example:

By repeated foiling, expand the expression $(x+y)^3$. Then, compute the values of $\binom{3}{0}$, $\binom{3}{1}$, $\binom{3}{2}$, and $\binom{3}{3}$. What do you notice?

Definition: *The Binomial Theorem*

For all $n \in \mathbb{Z}^+$, the expansion of $(x+y)^n$ is

$$(x+y)^n = x^n + nx^{n-1}y + \cdots + \binom{n}{r}x^{n-r}y^r + \cdots + nxy^{n-1} + y^n.$$

The Binomial Theorem allows us to expand, or partially expand, binomials quickly or to higher degrees without foiling repeatedly.

Practice: .

1. Expand each expression
   (a) $(2x+1)^4$

   (b) $(3x-2y)^5$

   (c) $\left(x^2 + \frac{1}{x}\right)^6$

2. Find the sixth term of the expansion of $(a + 2b)^8$.

3. Find the coefficient of the a^6b^5 term in the expansion of $(2a - 5b)^1 1$.

4. Find the coefficient of the constant term in the expansion of $\left(3x - \frac{2}{x^2}\right)^{10}$.