3.E - Trigonometric Identities and Equations

Fundamental Identities

Recall: *Fundamental Trigonometric Identities*

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$
$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

Sum and Difference Identities

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

Even/Odd Identities

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$
$$\tan(-\theta) = -\tan\theta$$

Double Angle Identities

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Power Reducing Identities

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$
$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$
$$\tan^2\theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

Proving Identities

Recall, to prove an identity, we start with one side of the equation and manipulate it until it looks like the other side.

1. $\dfrac{\sin\theta}{1 + \cos\theta} = \dfrac{1 - \cos\theta}{\sin\theta}$

2. $\dfrac{\sec^2\theta - 1}{\sec^2\theta} = \sin^2\theta$

3. $\dfrac{1}{1-\sin x}+\dfrac{1}{1+\sin x}=2\sec^2 x$

4. $\left(\tan^2\theta+1\right)\left(\cos^2\theta-1\right)=-\tan^2\theta$

5. $\tan^3 x+1=(\tan x+1)(\sec^2 x-\tan x)$

6. $\sin^3 x\cos^4 x=\left(\cos^4 x-\cos^6 x\right)\sin x$

7. $\cos^4\theta-\sin^4\theta=\cos 2\theta$

8. $\csc(2x)=\dfrac{\sec x}{2\sin x}$

Solving Equations

For each of the following, solve the equation and list all solutions on the interval $[0, 2\pi)$.

1. $2\sin 3\theta = -\sqrt{3}$

2. $2\cos^2\theta + \cos\theta - 1 = 0$

3. $\sin\theta = \sqrt{3}\cos\theta$

4. $\dfrac{\sin 2x}{\cos x} - 1 = 0$

5. $\sec x + \tan x = 1$

6. $\cos 2\theta - \sin^2\theta = \cos^2\theta + 3\cos\theta$

Other Problems

Using identities, and without the use of a calculator, solve the following problems. Your answers should be exact values.

1. $\sin(75°)$

2. $\cos\left(\frac{5\pi}{12}\right)$

3. Given $\sin\theta = \frac{5}{14}$ and θ is in Quadrant II, find $\cos 2\theta$.

4. Given that $\theta = -105°$, find the exact values of sine, cosine, and tangent of θ.

3.F - Polar Coordinates and Equations

Converting Between Rectangular and Polar Coordinates

Recall: *Converting Between Rectangular and Polar Coordinates*

Rectangular to Polar	**Polar to Rectangular**
$r = \sqrt{x^2 + y^2}$	$x = r\cos\theta$
$\theta = \arctan\left(\frac{y}{x}\right)$	$y = r\sin\theta$

Convert each of the following points to rectangular coordinates.

1. $(3, \pi/6)$
2. $(4, 7\pi/4)$
3. $(2, 5\pi/3)$
4. $(-3, 5\pi/6)$
5. $(-4, -7\pi/6)$
6. $(0, 1\pi/3)$

Convert each of the following points to polar coordinates.

1. $(3, 4)$
2. $(-3, 4)$
3. $(-3, -4)$
4. $(2\sqrt{3}, -2)$
5. $(-1, \sqrt{3})$
6. $(8, 0)$

Converting Equations

Convert each of the following equations from rectangular to polar form.

1. $y^2 = -x^2 + 16$
2. $xy = -(x-2)^2$

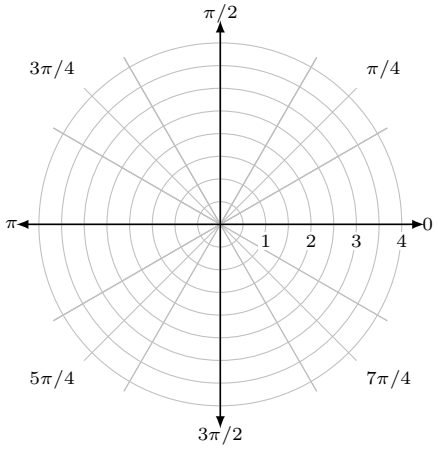
Convert each of the following equations from polar to rectangular form.

1. $r = 3$
2. $r = 2\sec\theta$
3. $\theta = -\frac{11\pi}{6}$
4. $r = -2\sin\theta$
5. $r = 5\sin\theta\cos\theta$

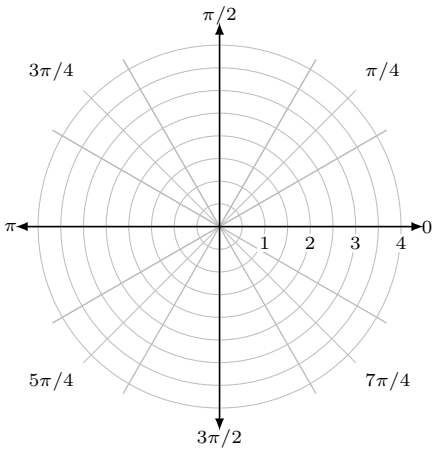
Graphing Polar Equations

Sketch a graph of the given polar functions. Confirm your answer with a calculator or Desmos.com.

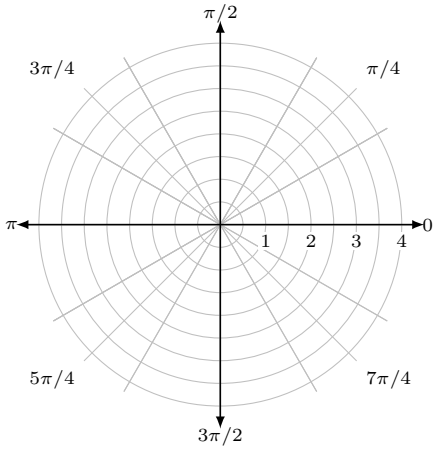
1. $r = 3\cos\theta$



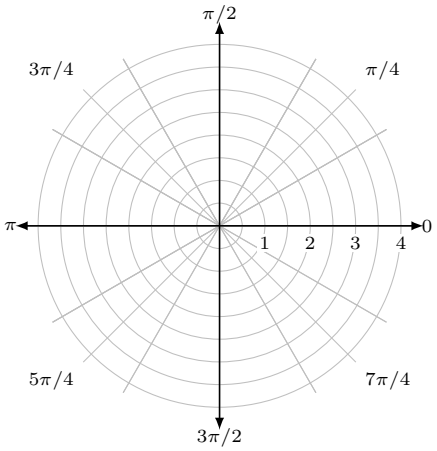
2. $r = -2\sin\theta$



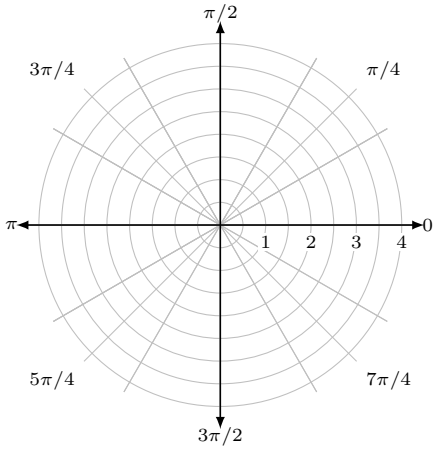
3. $r = 4\sin 2\theta$



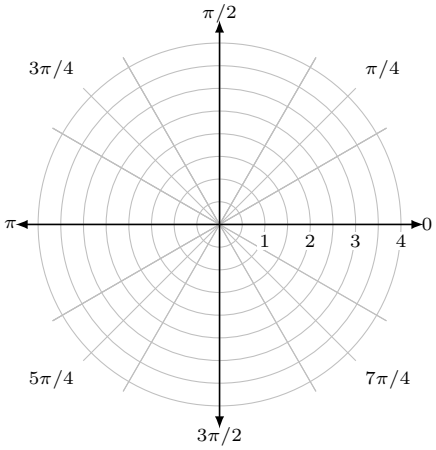
4. $r = 1 + 3\cos\theta$



5. $r = -3 - 3\sin\theta$

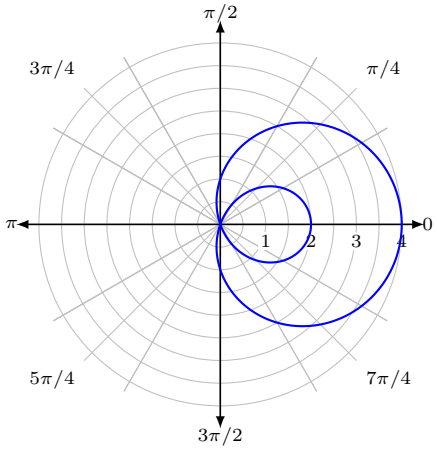


6. $r = -3\cos(3\theta)$

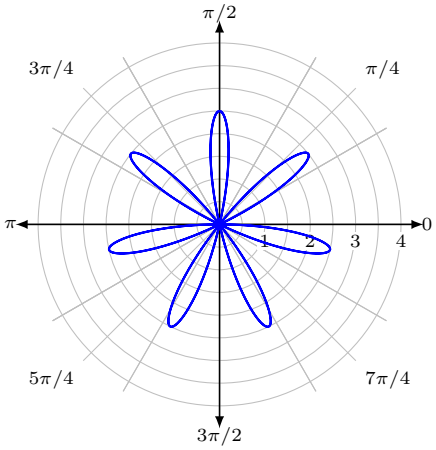


Given the graph of a polar curve, write the equation $r = f(\theta)$ that describes it. There may be multiple answers.

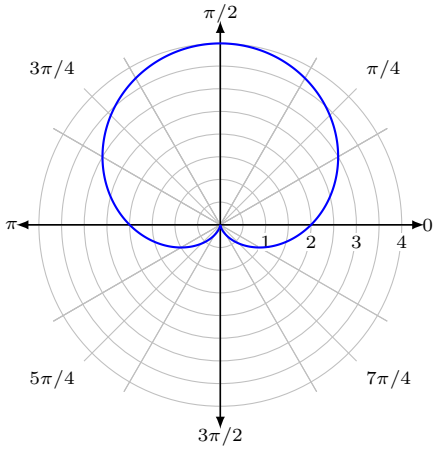
1.



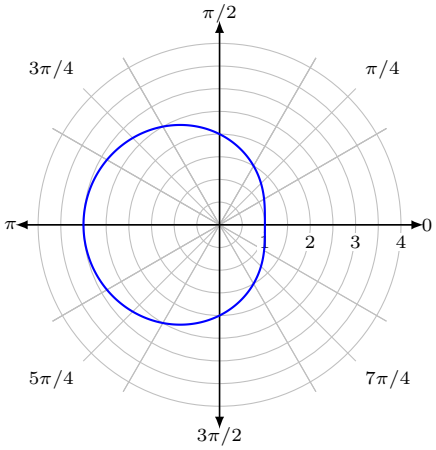
2.



3.



4.



Symmetry

Find all axes or lines of symmetry for the provided polar functions.

1. $r = 3\sin\theta$

2. $r = 4\cos 2\theta$

Rates of Change

Find the average rate of change of the given polar functions over the given interval $[a, b]$. Round to 3 decimal places.

1. $r = 2\sin(3\pi\theta)$ over $\left[1, \frac{7}{6}\right]$

2. $r = 2\cos\left(\frac{5}{3}\theta\right) + 3$ over $\left[-\frac{12\pi}{5}, -\frac{9\pi}{4}\right]$

For each of the following polar functions, determine **(1)** if r and **(2)** the distance from the origin are increasing, decreasing, or neither at the given value of θ.

1. $r = 2\sin\theta$ at $\theta = \pi/4$

2. $r = 3\cos\theta$ at $\theta = 3\pi/2$

3. $r = 4\sin 2\theta$ at $\theta = \pi/6$

4. $r = 1 + 3\cos\theta$ at $\theta = 0$