

Name: _____

Grade: ____/200

Answer the questions in the spaces provided on the following pages. If you run out of room for an answer, continue on the back of the page. Show **all** your work to be able to receive full credit on any question. **YOU ARE A MISSILE**

For each statement below, circle whether they are true or false. As a reminder, a statement that is not always true is considered false. No work is needed for the following problems.

1. **TRUE** False If $\lim_{n \rightarrow \infty} a_n = 0$, then $\sum a_n$ is convergent. [3]
2. **TRUE** False If $\sum a_n$ is divergent, then $\sum |a_n|$ is divergent. [3]
3. **TRUE** False If $\sum a_n$ is divergent, then $\sum |a_n|$ is divergent. [3]
4. **TRUE** False The ratio test can be used to determine whether $\sum 1/n!$ converges. [3]
5. **TRUE** False The ratio test can be used to determine whether $\sum 1/n^3$ converges. [3]
6. **TRUE** False If $0 \leq a_n \leq b_n$ and $\sum b_n$ converges, then $\sum a_n$ converges. [3]
7. **TRUE** False If $-1 < \alpha < 1$, then $\sum \alpha^n$ is convergent. [3]
8. **TRUE** False If $a_n > 0$ and $\lim_{n \rightarrow \infty} (a_{n+1}/a_n) = 1$, then $\sum a_n$ converges. [3]
9. **TRUE** False $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \frac{1}{e}$ [3]
10. **TRUE** False If a series converges, then the sequence of its terms converges. [3]
11. **TRUE** False If a series converges, then the sequence of its partial sums converge. [3]
12. **TRUE** False A series can be convergent without its sequence being convergent. [3]
13. **TRUE** False If $a_n > 0$ and $b_n > 0$, $\sum b_n$ converges and $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 0$, then $\sum a_n$ is convergent. [3]
14. **TRUE** False $\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ [3]
15. **TRUE** False The alternating harmonic series is conditionally convergent. [3]

16. Determine whether the following series are convergent or divergent. Show all work to receive full credit. An answer of convergent or divergent alone will yield no credit.

(a) $\sum_{n=2}^{\infty} \frac{\ln n}{n^3}$ [10]

Solution:

$$\frac{\ln n}{n^3} < \frac{n}{n^3} \Rightarrow \sum_{n=2}^{\infty} \frac{\ln n}{n^3} \text{ converges by DCT since } \sum_{n=2}^{\infty} \frac{n}{n^3} \text{ converges by } p\text{-series}$$

(b) $\sum_{i=1}^{\infty} \left(\sqrt[i]{i} - 1\right)^i$ [10]

Solution:

$$L = \lim_{i \rightarrow \infty} \sqrt[i]{\left|\left(\sqrt[i]{i} - 1\right)^i\right|} = \lim_{i \rightarrow \infty} \left|\sqrt[i]{i} - 1\right| = 0$$

$\therefore L < 1$ implies that the series is convergent by the root test

(c) $\sum_{n=0}^{\infty} (-1)^{n+1} e^{\left(\frac{2}{n+1}\right)}$ [10]

Solution:

$$\lim_{n \rightarrow \infty} e^{\left(\frac{2}{n+1}\right)} = 1 \neq 0$$

\therefore since the limit of the non alternating term is not, 0, the series diverges by the AST

(d) $\sum_{k=1}^{\infty} \frac{\arctan(k)}{k^{1.5}}$ [10]

Solution: Let $b_n = \frac{1}{k^{1.5}}$

$$\lim_{k \rightarrow \infty} \frac{a_k}{b_k} = \lim_{k \rightarrow \infty} \frac{\arctan(k)}{k^{1.5}} = \frac{k^{1.5}}{k^{1.5}} \frac{\pi}{2} = \frac{\pi}{2}$$

\therefore the series converges by LCT

17. Find the sum of the following convergent series.

[10]

(a) $\sum_{n=0}^{\infty} \frac{3^{n+1}}{(-4)^n}$

(b) $\sum_{i=2}^{\infty} \left[\frac{1}{i^2} - \frac{1}{(i+2)^2} \right]$

18. Find the radius of convergence for the given power series.

[5]

$$\sum_{n=0}^{\infty} \frac{2^n}{n!} (x-1)^n$$

19. Find the interval of convergence for the given power series.

[10]

$$\sum_{n=2}^{\infty} \frac{n}{n+1} \left(\frac{x}{4} \right)^{n-1}$$

20. Consider the function given by

[15]

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x+2)^{n+1}}{n+1}.$$

Answer the following.

(a) Find a series for $f'(x)$. Then, find its interval of convergence.

(b) Write the series $\int_{-2}^{-1.3} f(x)dx$. You do not need to find the interval of convergence.

(c) Show that the maximum error associated with the approximation of S_7 for the series obtained in part (b) is less than $\frac{29}{500}$.

21. Consider the following functions. Using any means, construct a power series for the given function at the specified center. Leave your answer in summation notation. Simplify completely. [30]

(a) $h(x) = 3xe^{-\frac{x^2}{3}}, c = 0$

(b) $r(x) = \frac{1}{1+x}, c = 3$

(c) $t(x) = \sin 3x \cos 3x, c = 0$

22. Let f be the function given by $f(x) = \frac{1}{1-2x}$ and let $P_3(x)$ be the third degree MacLaurin Polynomial for f .

(a) Find $P_3(x)$.

[10]

(b) Use the Lagrange Error Bound to show that $\left| f\left(\frac{1}{6}\right) - P_3\left(\frac{1}{6}\right) \right| \leq \frac{1}{1800}$.

[15]

23. The function ϕ has a Taylor series about $x = 2$ that converges for all x in its interval of convergence. The n th derivative of ϕ at $x = 2$ is given by

$$\phi^{(n)}(2) = \frac{(n+1)!}{3^n}$$

for $n \geq 1$.

- (a) Find an expression for the Lagrange Error bound associated with the approximation of $\phi(3)$ using the n th degree Taylor polynomial centered about $x = 2$. [10]

- (b) How many terms are needed for the error to be less than $\frac{1}{1200}$? [10]

Extra Credit (10pts): Consider the following series.

$$\sum_{n=0}^{\infty} \frac{n! \cdot (n+1)! \cdot 3^n}{(1 \cdot 3 \cdot 6 \cdot \dots \cdot (3n))^2}.$$

Determine the convergence or divergence of the series. Show all work.