Volatility Arbitrage

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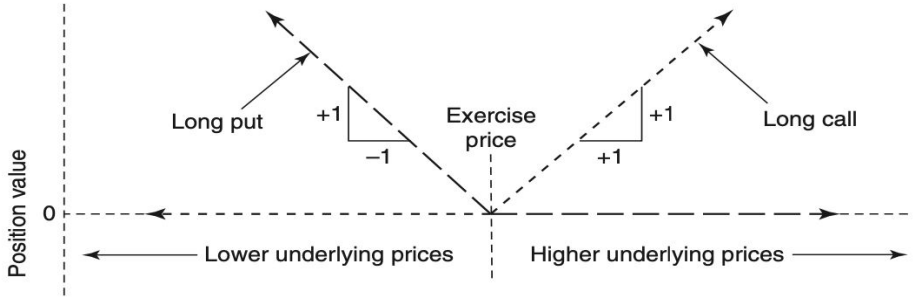
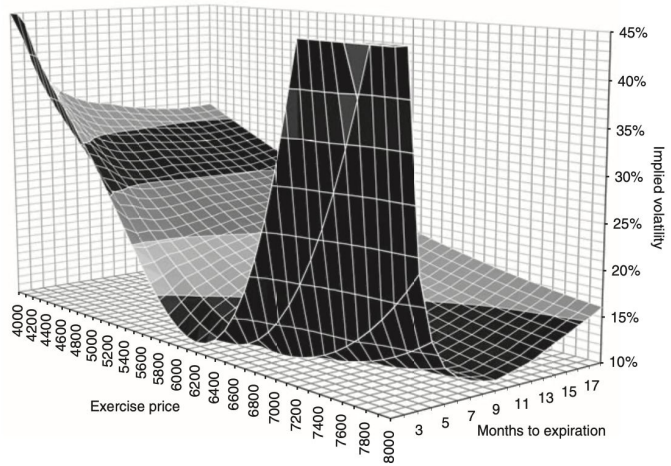
Intro to Volatility Arbitrage

- Volatility arbitrage seeks to exploit mispriced volatility in options markets
- Option prices imply the expected level of volatility between now and expiration
- An option's replicating portfolio is given by the option's delta
- If volatility is mispriced, there is an arbitrage opportunity between the replicating portfolio (dynamic delta hedge) and the option
    - Implied volatility is rich: Sell the option, delta hedge (buy the RP)
    - Implied volatility is cheap: Buy the option, delta hedge (sell the RP)
- We will buy or sell a delta-hedged at-the-money straddle if we see a discrepancy between our predicted realized and the market-implied volatility in SPX options
    - ATM straddles best isolate gamma and vega risk, which are what we intend to trade
    - We will delta hedge with the SPY ETF because it tracks this index

Goals

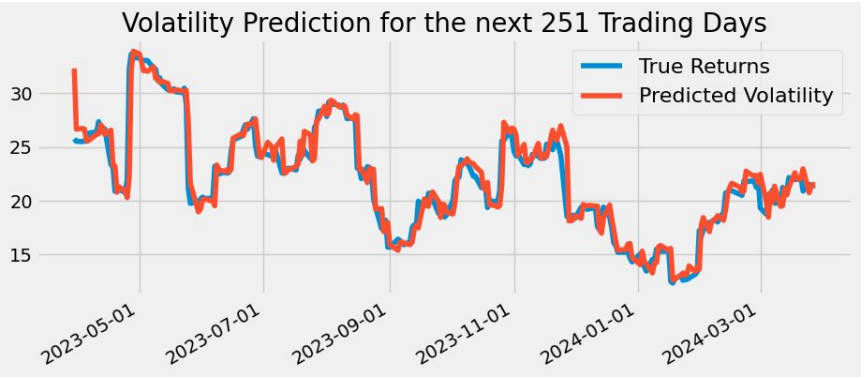
- Learn about option and volatility theory
- Learn about volatility forecasting techniques
- Gain hands-on experience with wrangling options data
- Gain experience with the process of backtesting

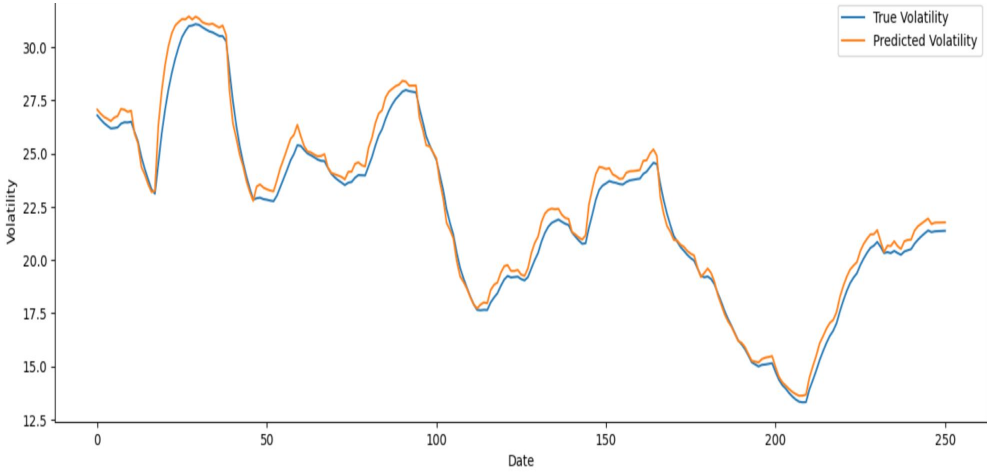




Model Testing

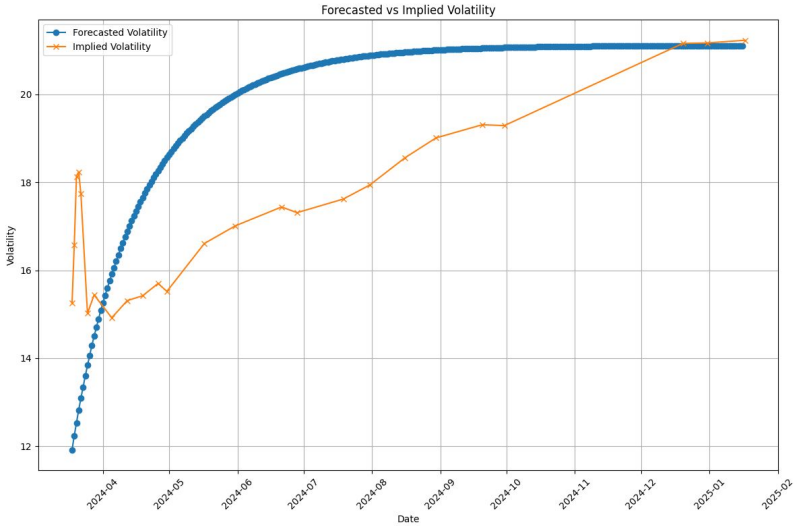
- LSTM (Singular and Bivariate)
- XGBoost
- ARCH
- GARCH
- Volatility cones



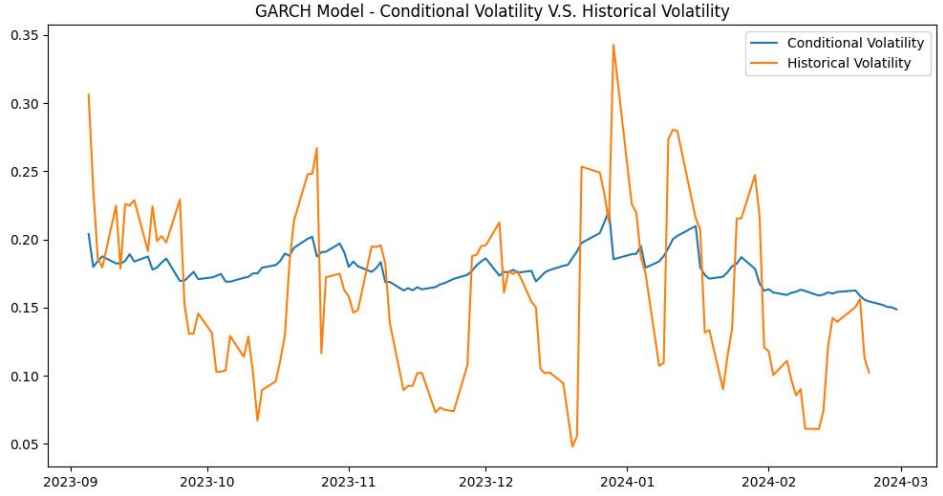


ARCH/GARCH

- Capture time-varying volatility in financial markets
- Better at capturing volatility clustering and mean reversion than other models
- Gives a rough approximation of the implied volatility term structure

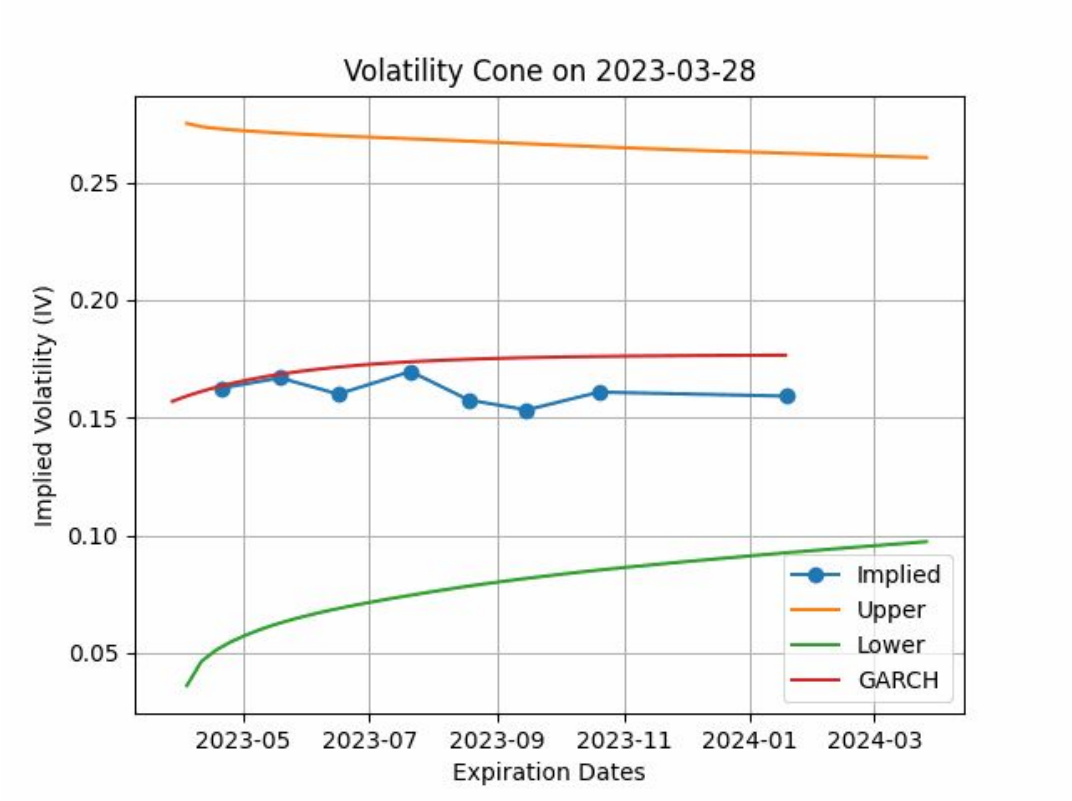


Forecasted volatility term structure



Rolling volatility forecast

Volatility Cones



Backtest Data Preprocessing

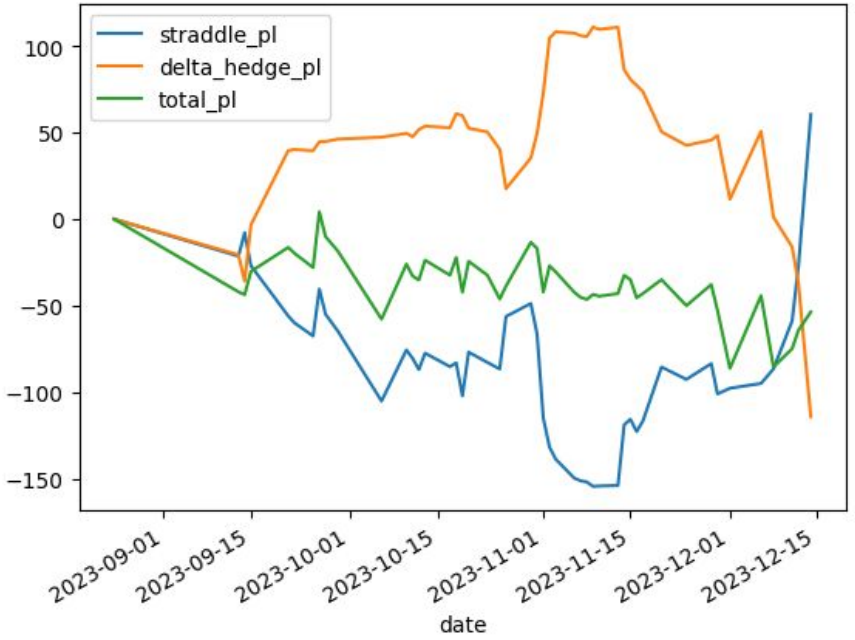
- Downloaded a year's worth of SPX options price and other historical data
  - SPX index, SPY price, SOFR, S&P 500 dividend yield
- Parsed data and formatted the dataframe
- Calculated straddle price at each strike and expiration
- Calculated Black-Scholes inputs and Greeks
  - Time to expiration, implied volatility, and straddle delta

date	exp	strike	call	put	spot	SPY	tte	straddle	r	q	iv	delta
2023-03-28	2023-04-21	3500	486.80	3.32	3971.270020	395.600006	0.095238	490.12	0.0484	0.0171	0.227145	0.938315
2023-03-28	2023-04-21	3650	343.30	8.00	3971.270020	395.600006	0.095238	351.30	0.0484	0.0171	0.198201	0.853903
2023-03-28	2023-04-21	3655	334.62	10.02	3971.270020	395.600006	0.095238	344.64	0.0484	0.0171	0.207053	0.830254
2023-03-28	2023-04-21	3675	319.27	10.95	3971.270020	395.600006	0.095238	330.22	0.0484	0.0171	0.201752	0.813264
2023-03-28	2023-04-21	3720	268.27	12.70	3971.270020	395.600006	0.095238	280.97	0.0484	0.0171	0.186539	0.774742
...	...	...	...	...	...	...	...	...	...	...	...	...
2024-04-05	2025-12-19	4100	1430.23	110.68	5204.339844	518.429993	2.472222	1540.91	0.0532	0.0136	0.209225	0.739520
2024-04-05	2025-12-19	6000	233.25	651.20	5204.339844	518.429993	2.472222	884.45	0.0532	0.0136	0.103935	-0.145539
2024-04-05	2028-12-15	5100	1278.56	458.82	5204.339844	518.429993	6.805556	1737.38	0.0532	0.0136	0.221667	0.520149
2024-04-05	2028-12-15	5200	1227.39	483.92	5204.339844	518.429993	6.805556	1711.31	0.0532	0.0136	0.220499	0.502559
2024-04-05	2028-12-15	5400	1117.81	542.44	5204.339844	518.429993	6.805556	1660.25	0.0532	0.0136	0.107143	0.609770

Backtest Profit and Loss Calculation

- Initial cash flows: pay/receive straddle premium, open delta hedge position
- Intermediate cash flows: delta hedge adjustments
- Calculate cumulative sum of cash flows
- Calculate daily mark-to-market PL

straddle_cf	delta_change	delta_hedge_cf	cumulative_straddle_cf	cumulative_delta_cf	straddle_pl	delta_hedge_pl	total_pl
-300.92	0.000000	935.751375	-300.92	935.751375	0.00	0.000000	0.000000
0.00	0.181193	809.044721	-300.92	1744.796096	-21.40	-20.604553	-42.004553
0.00	0.069605	313.473060	-300.92	2058.269156	-7.83	-35.826493	-43.656493
0.00	-0.106872	-473.840141	-300.92	1584.429015	-26.90	-3.324254	-30.224254
0.00	-0.292787	-1263.053315	-300.92	321.375700	-55.92	39.577279	-16.342721
0.00	-0.028673	-123.412339	-300.92	197.963361	-59.92	40.210916	-19.709084
0.00	0.044674	193.094877	-300.92	391.058238	-67.52	39.547537	-27.972463
0.00	-0.166599	-709.512687	-300.92	-318.454449	-40.42	44.711675	4.291675
0.00	-0.010802	-46.023577	-300.92	-364.478026	-54.94	44.856626	-10.083374
0.00	0.042554	181.909571	-300.92	-182.568455	-64.80	46.230545	-18.569455
0.00	0.008967	38.516073	-300.92	-144.052382	-105.12	47.333112	-57.786888



Results

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Short implied volatility 0.17798015748732676
Realized volatility: 0.11670193729620755
Profit: 264.529483322454
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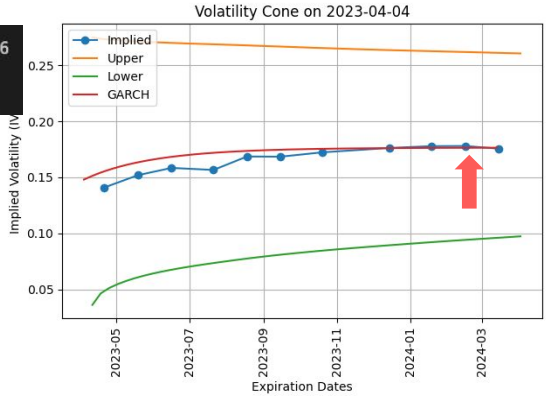

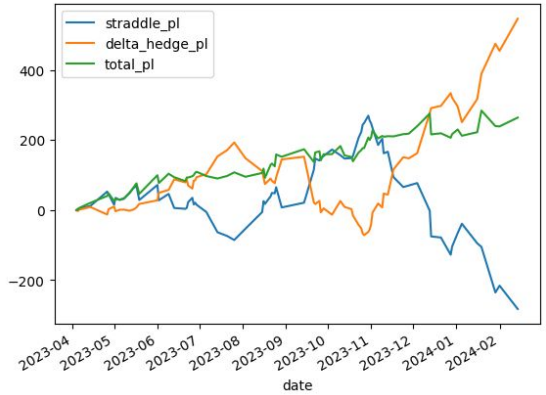


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Short implied volatility 0.1714819199129184
Realized volatility: 0.1154090008208446
Profit: 202.3596977606161
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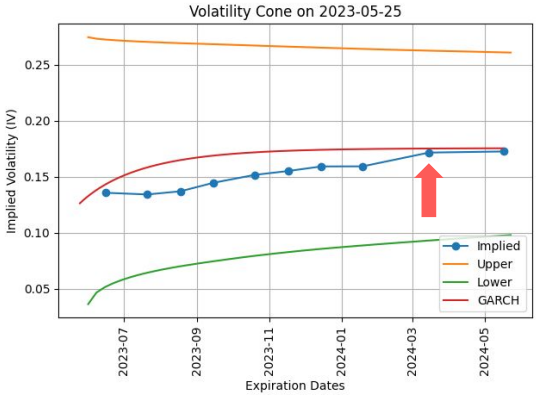

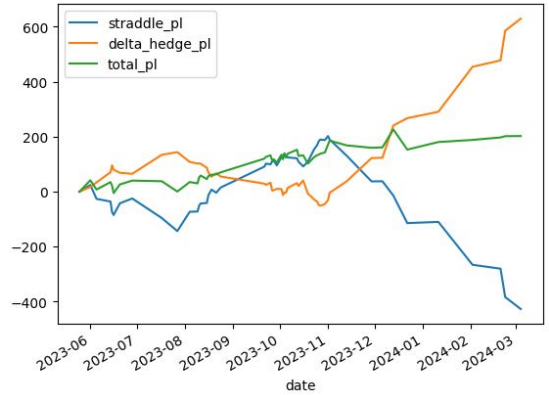

Short straddles performed well while long straddles performed poorly; likely due to delta hedging intervals not capturing enough volatility

Reflections

What we learned

- Learned about option theory and volatility trading in a short amount of time
- Learned about backtesting an option trading strategy and PL calculation

Challenges

- We struggled with building a systematic, quantitative model to predict volatility
  - Volatility is a measure of uncertainty, which is often idiosyncratic and fundamentally driven
  - Options markets are generally efficiently priced; edge is found in niche areas of expertise
  - Probably need to incorporate more features beyond relying on the time series' memory
- Acquiring accurate pricing data was difficult; we could only use approximations for implied volatility and delta