Assignment 2 - A Little Slice of π Writeup

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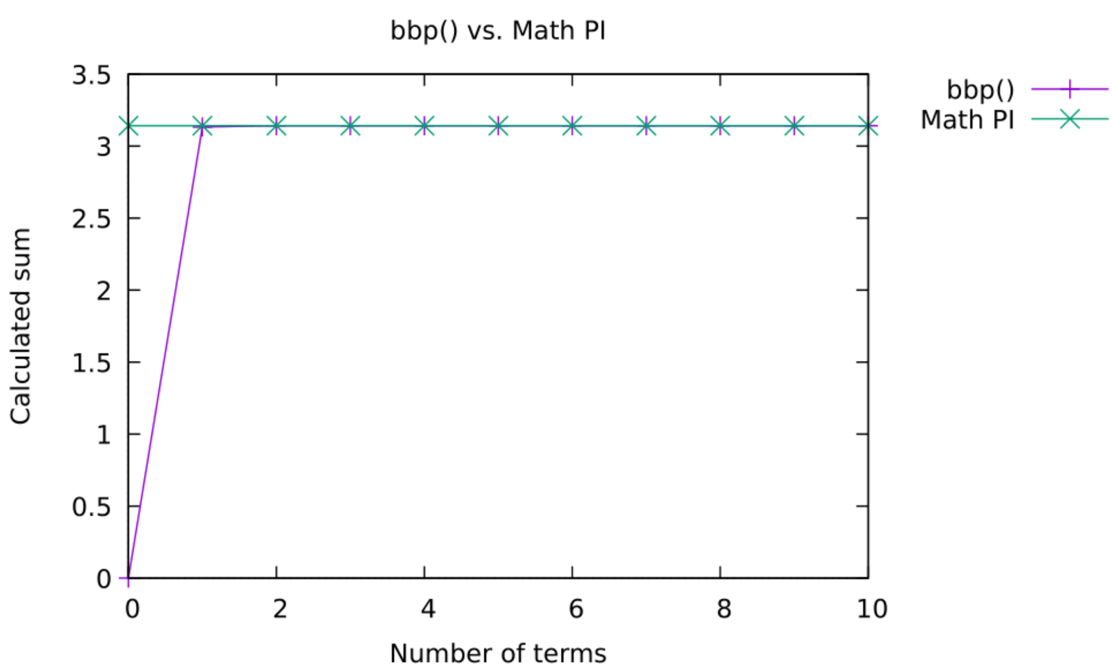
1 Introduction

This is my Writeup for the A Little Slice of π assignment. It includes 6 graphs of the 6 different functions I wrote. It compares them to the math library version of each function.

2 bbp

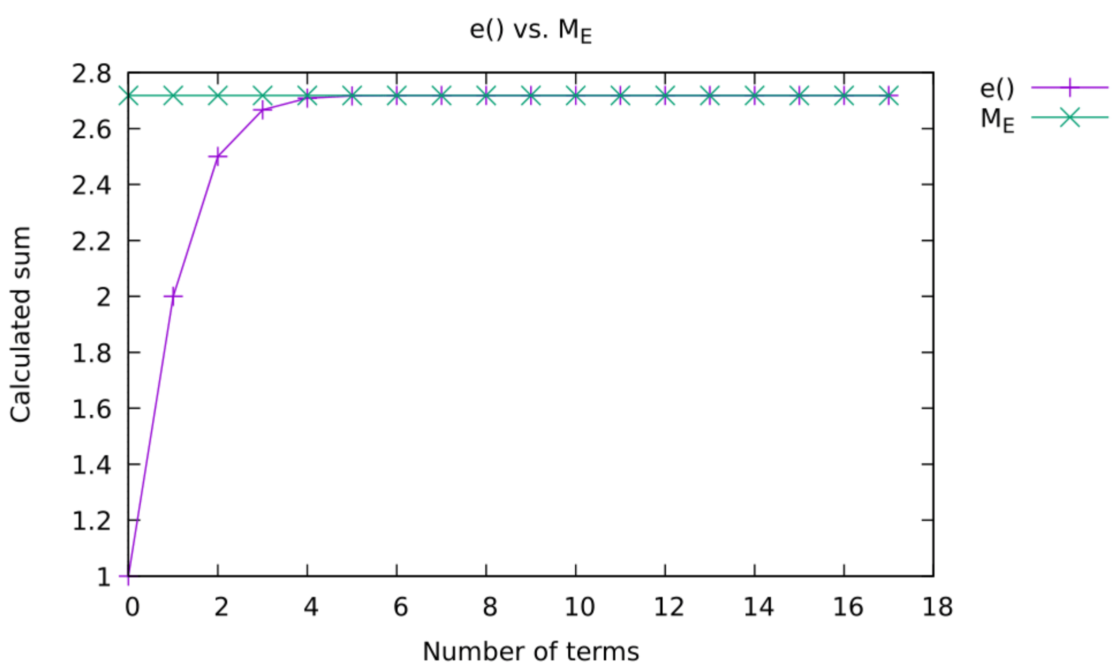


bbp uses the Baily-Borwein-Plouffe Formula to find π. This formula is:

$\sum_{k=0}^{n} 16^{-k}(\frac{(k(120k+151)+47)}{k(k(k(512k+1024)+712)+194)+15}) = \pi$.

bbp calculates pi through summation where the last term of the sum is smaller than epsilon (in this case epsilon is 0.00000000000001). The summation approximates π in only 10 terms. The graph above looks like it reaches π in one term, but in reality it just gets very close to π in one term. Then for the next 9 terms it keeps getting closer in very small increments that are unable to be seen by the graph. Finally at the 10th term of bbp is less than epsilon, so bbp has approximated π.

3 e

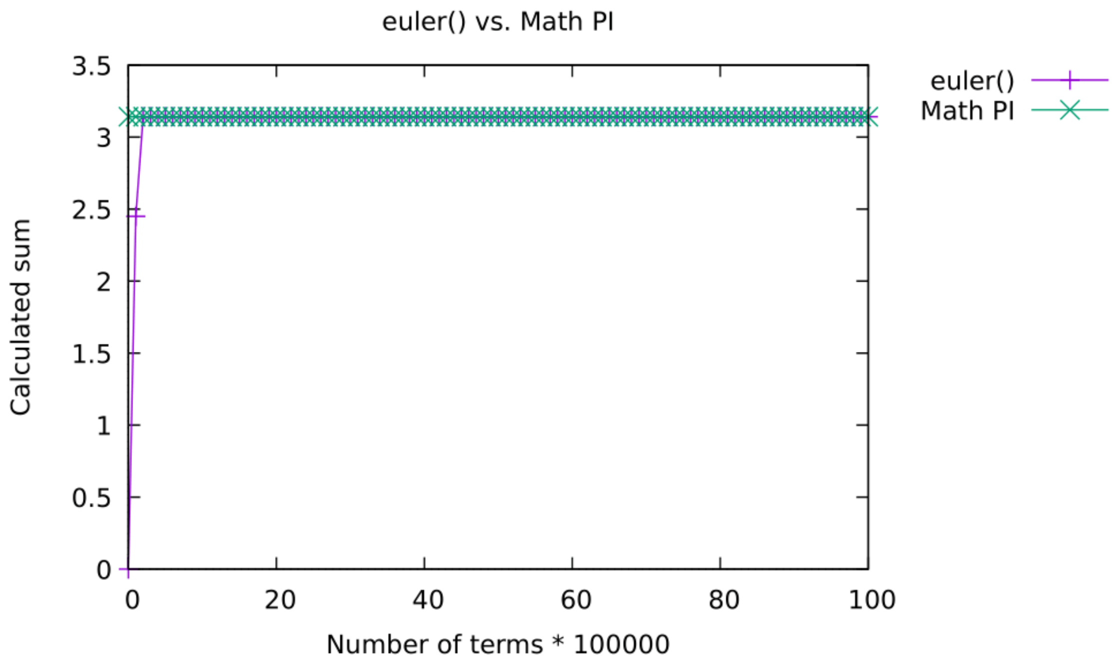


e uses the formula:

$\sum_{k=0}^{\infty}(\frac{1}{k!}) = e$.

the graph shows that e slowly gets closer and closer to e over 17 terms. Since 1/k! gets smaller and smaller every term at around 5 terms each term gets so small that you cannot see it on the graph. So adding each term from then on looks likes nothing on the graph. The last term is not less than epsilon however until 17 terms. So, I have approximated e at 17 terms.

4 euler

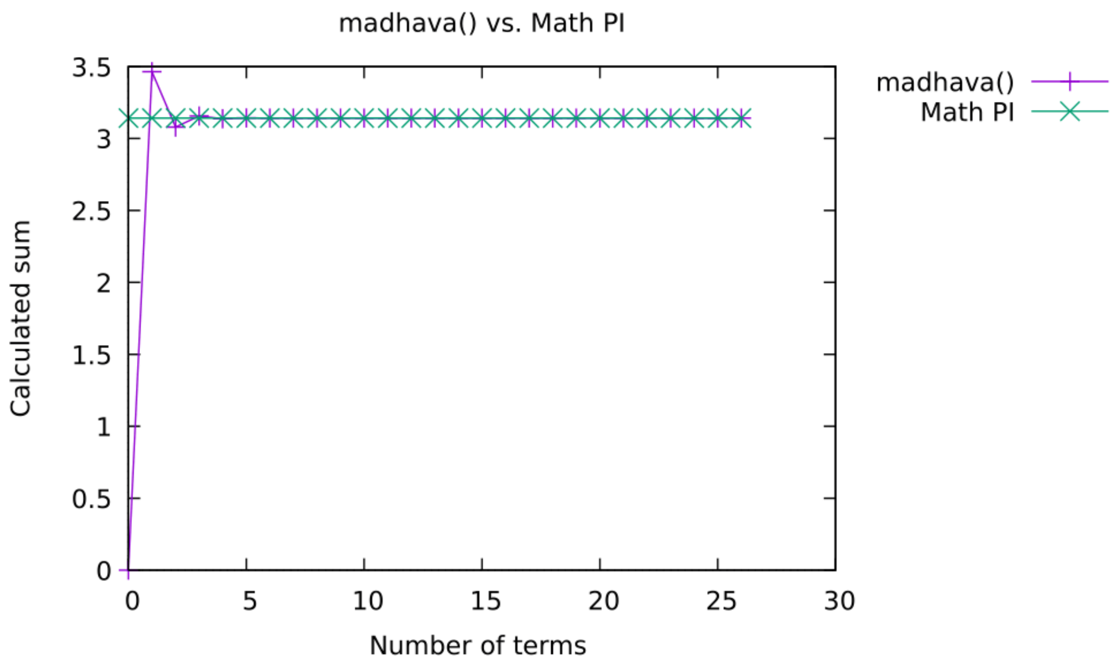


The euler function uses the formula:

$\sqrt{6\sum_{k=0}^{n}(\frac{1}{k!})} = \pi$.

euler actually takes around 10,000,000 terms to approximate π. I only graphed every 100,000 terms because graphing 10,000,000 was too much to compute for my computer. Still, euler reaches around π relatively quickly around 400,000 terms. At this point we are unable to see how much closer euler gets every term but we know it is slowly approaching π as each term is added. At 10,000,000 terms euler's terms become less than epsilon and euler has approximated π.

5 madhava

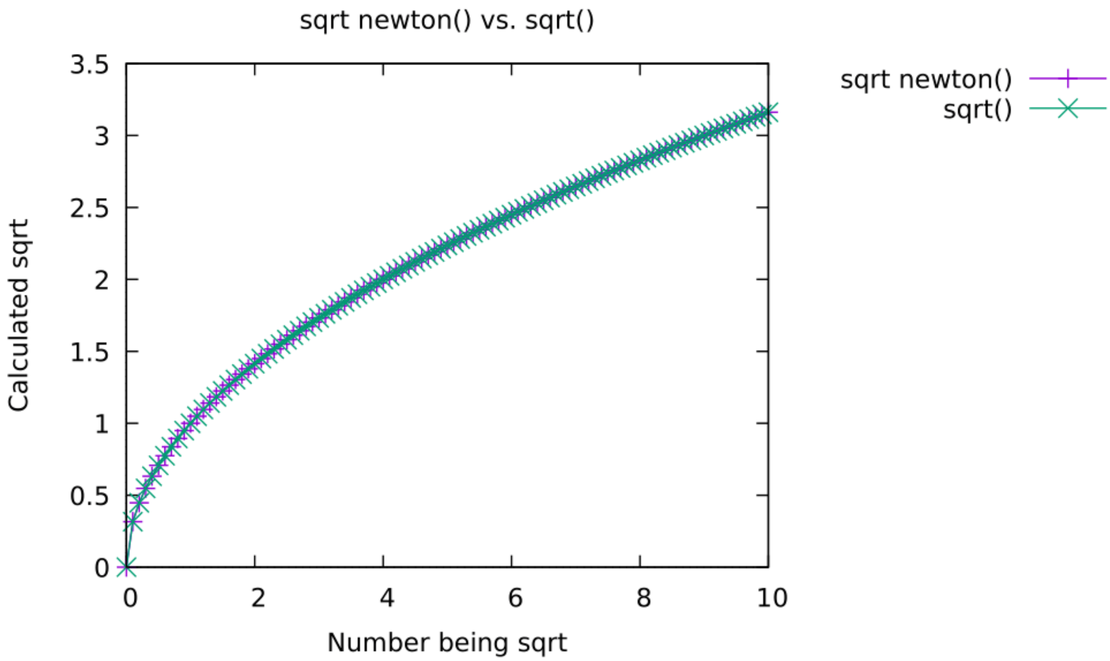


madhava uses the formula:

$\sqrt{12}\sum_{k=0}^{n}(\frac{(-3)^{-k}}{2k+1}) = \pi$.

madhava is interesting because some of the terms make the summation go above π. It turns out that some of the terms are negative numbers which allows the summation to go from above π to below π. Eventually the summation averages out at π in 26 terms. At around 4 terms the terms become so small that they are almost invisible to the graph. Yet, it still takes 26 terms for madhava approximate π.

6 newton

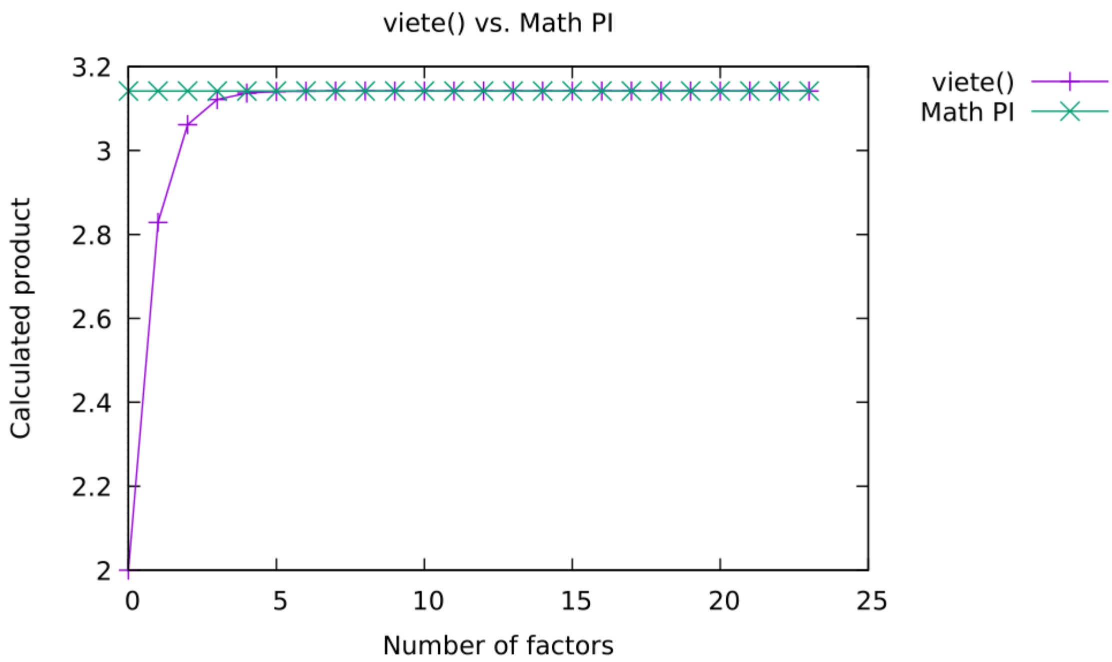


newton approximates our own square root function using the formula:

$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$

This equation keep iterating over itself until the estimated value on the left of the equation is within an epsilon of the last value on the right of the equation. With this we will have found a good approximation of the square root of the given number. As we can see our sqrt newton values are almost exactly the same as the math.h sqrt values. This means that our newton function accurately determines the sqrt values from 0 to 10 in increments of 0.1.

7 viete



viete uses this equation:

$\prod_{k=1}^{\infty}(\frac{a_i}{2}) = \frac{2}{\pi}$ where $a_1 = \sqrt{2}$ and $a_k = \sqrt{2 + a_{k-1}}$ for all $k > 1$

This graph shows $\frac{2}{product}$. This is because the original equation finds $\frac{2}{\pi}$ and we are looking to approximate π. The graph shows that this function takes 23 factors to approximate π. At around 5 factors the multiplied factors become so close to the product that it barely changes the number. If we look at the original equation the numerator approaches the number 2. Once the numerator is 2 then the product is always multiplied by a factor of 1 and does not change. So, viete approaches until the numerator is within an epsilon of 2. At that point, we have approximated $\frac{2}{\pi}$.