Problem 1

Q1. a) given :- $S_0 = \$100$

$K = \$105$

$T = 1$ yr

$r = 5\%$ per annum

$\sigma = 20\%$ per annum

Call price $= S_0 N(d_1) - Ke^{-rT} N(d_2)$

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = 0.106$$

$$d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} = -0.094$$

$N(d_1) = 0.5422$, $N(d_2) = 0.4626$

so $C = 100 \times 0.5422 - 105 \times e^{-0.05 \times 1} (0.4626)$

$= 54.22 - 4620$

$= \$8.02$ - Price of European call option

b) Put price $= Ke^{-rT} N(-d_2) - S_0 N(-d_1)$

$N(-d_1) = 0.45779$, $N(-d_2) = 0.53745$

So, $P = 105 \times e^{-0.05} \times 0.53745 - 100 \times 0.45779$

$= 53.680 - 45.779$

$= \$7.901$ - Price of European put option

c) $C - P = S(0) - Ke^{-rT}$

or $C + Ke^{-rT} = P + S_0$

LHS $= C + Ke^{-rT} = 8.02 + 105 \cdot e^{-0.05} = 8.02 + 99.88 = \$107.90$

RHS $= P + S_0 = \$7.90 + \$100 = \$107.90$

LHS = RHS - Hence put call parity relationship verified d)

Call option Delta $(\Delta c) = N(d_1) = \underline{0.5422}$

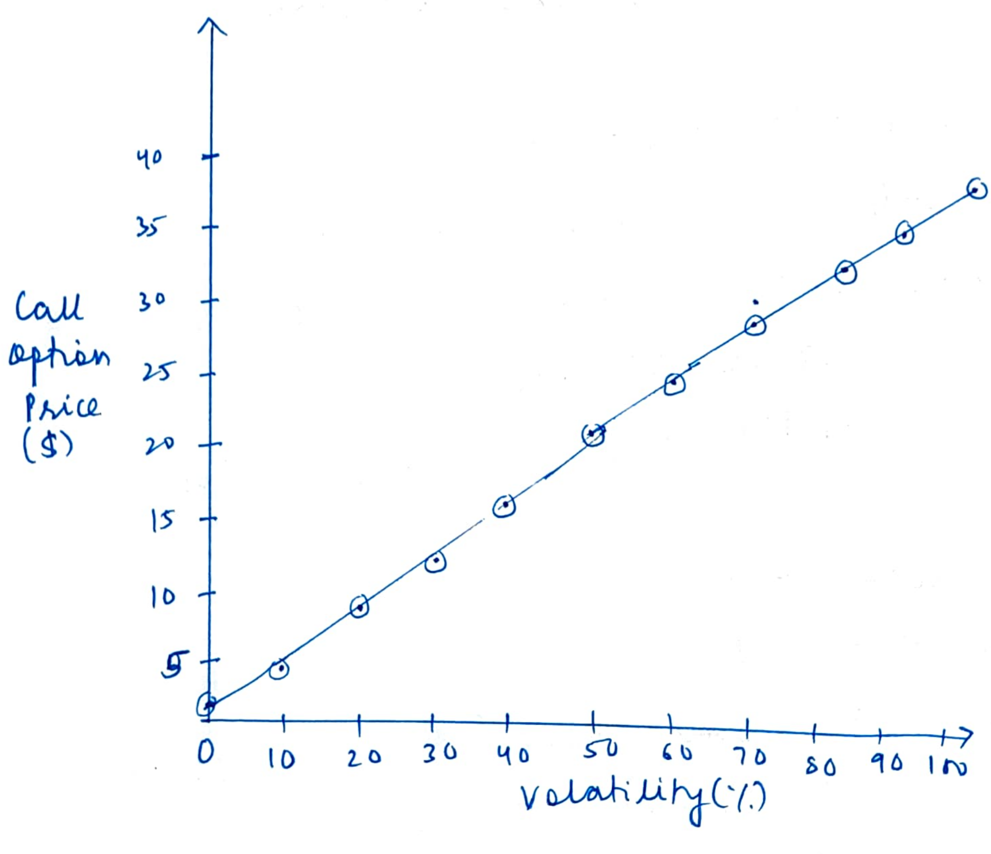
Put option Delta $(\Delta p) = -N(-d_1) = N(d_1) - 1 = \underline{-0.4578}$

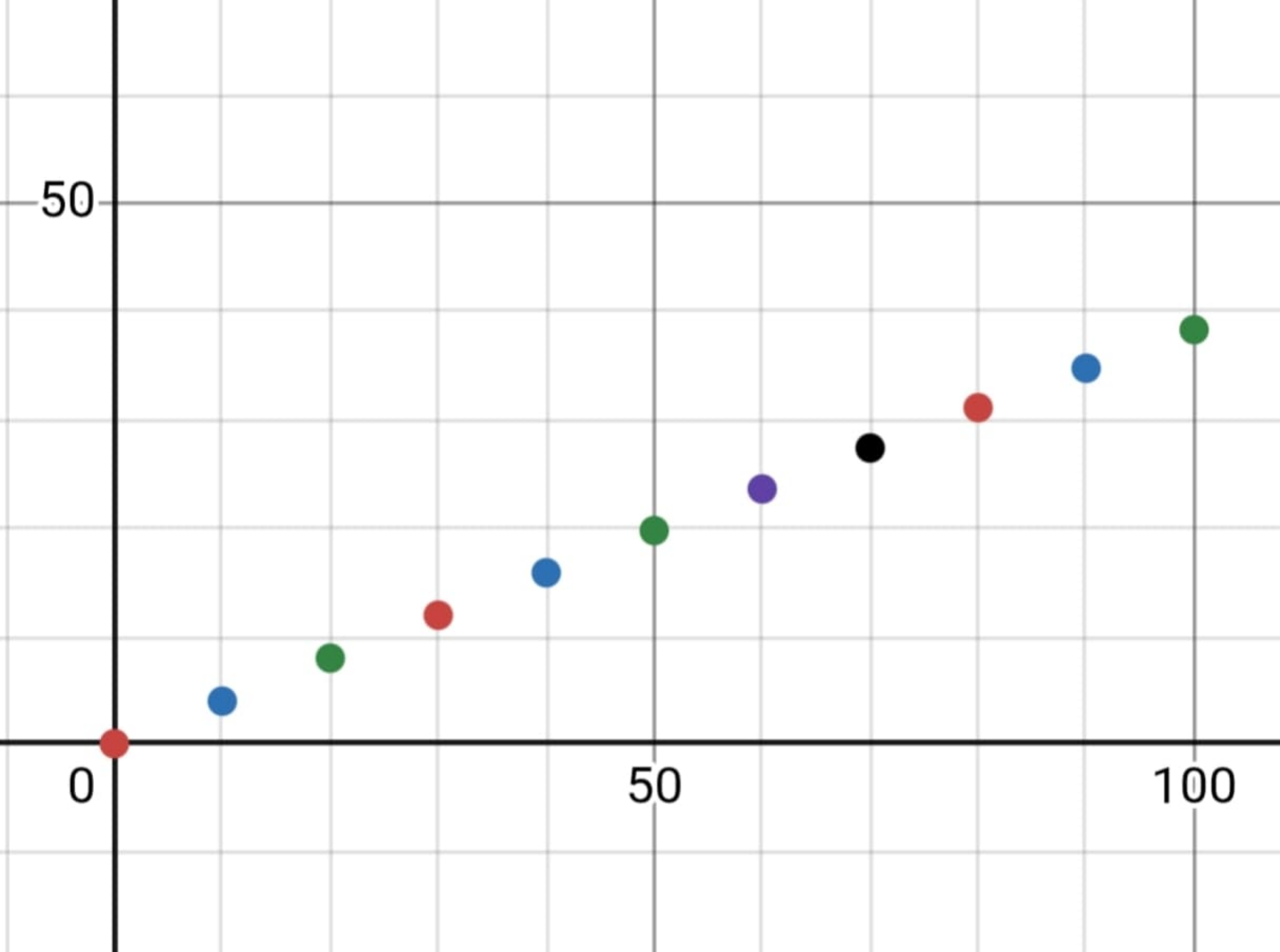
Relation b/w the two $\rightarrow \Delta c - \Delta p = 1$

or $\boxed{\Delta p = \Delta c - 1}$

e)

Volatility (%)	Call Price ($)
0%	0.12
10%	4.05
20%	8.02
30%	11.98
40%	15.90
50%	19.79
60%	23.63
70%	27.41
80%	31.13
90%	34.77
100%	38.33




50
0
50
100

- The plot shows a clear, nearly linear increase in call option price as volatility rises from 0% to 100%.
- Higher volatility increases the option price as there is a greater chance the option will finish in the money.
- The relationship is monotonic and nearly linear..

f)

$$C_E(S) = S_0 N(d_1) - K e^{-rT} N(d_2)$$

$$P_E(S) = K e^{-rT}(N(-d_2)) - S_0 (N(-d_1))$$

S_0 is current stock price

Delta $= \frac{d}{dS} C_E(S) = N(d_1)$ and $\frac{d}{dS} P_E(S) = -N(-d_1)$

gamma $= \frac{\partial^2 V}{\partial S^2}$, Theta $= \frac{\partial V}{\partial t}$, Vega $= \frac{\partial V}{\partial \sigma}$, Rho $= \frac{\partial V}{\partial r}$

gamma For call $= \frac{\partial^2 C}{\partial S^2} = \frac{\partial}{\partial S}(N(d_2)) = N'(d_1)\frac{\partial}{\partial S}(d_1)$

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \Rightarrow \frac{\partial}{\partial S}(d_1) = \frac{1}{S_0 \sigma \sqrt{T}}$$

where $N'(d_1) = \frac{1}{\sqrt{2\pi}} e^{-d_1^2/2}$

So. $$\frac{\partial^2 C}{\partial S^2} = \frac{1}{S_0 \sigma \sqrt{2\pi T}} e^{-\frac{d_1^2}{2}}$$

gamma For put $\rightarrow \frac{\partial^2 P}{\partial S^2} = \frac{\partial}{\partial S}(-N(-d_1)) = -N'(-d_1)\frac{\partial}{\partial S}(-d_1)$

$$= \frac{1}{S_0 \sigma \sqrt{T}} N'(-d_1)$$

where $N'(-d_1) = \frac{1}{\sqrt{2\pi}} e^{-(-d_1)^2/2} = N'(d_1)$

So $$\frac{\partial^2 P}{\partial S^2} = \frac{1}{S_0 \sigma \sqrt{2\pi T}} e^{-\frac{d_1^2}{2}}$$

Theta For call option $= -\dfrac{\partial C}{\partial t} = -\dfrac{\partial}{\partial t}\left(S_0 N(d_1) - Ke^{-rt}N(d_2)\right)$

$$= -S_0 N'(d_1)\frac{\partial d_1}{\partial t} + Ke^{-rt}\frac{\partial}{\partial t}N(d_2) + K(-r)e^{-rt}N(d_2)$$

$$= -S_0 \cdot N'(d_1) \cdot \frac{\partial d_1}{\partial t} + K(-r)e^{-rt}N(d_2) + Ke^{-rt}\left(N'(d_2) \cdot \frac{\partial}{\partial t}(d_1 - \sigma\sqrt{t})\right]$$

$$= \left[-S_0 \cdot N'(d_1) + Ke^{-rt}N'(d_2)\right]\frac{\partial}{\partial t}(d_1) + K(-r)e^{-rt}N(d_2) + Ke^{-rt} \cdot \left(\frac{-\sigma}{2\sqrt{t}}\right)N'(d_2)$$

Relation b/w $N'(d_1)$ and $N'(d_2) \Rightarrow \left[S_0(N'(d_1)) = Ke^{-rt}N'(d_2)\right]$

So Theta $(\theta) = -Kre^{-rt}N(d_2) + S_0(N'(d_1)) \cdot \left(\dfrac{-\sigma}{2\sqrt{t}}\right)$

$$= -Kre^{-rt}N(d_2) + S_0\left(\frac{-\sigma}{2\sqrt{t}}\right) \times \frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}$$

$$= -Kre^{-rt}N(d_2) - \frac{S_0\sigma}{2\sqrt{2\pi t}}e^{-d_1^2/2}$$

For Put option $= -\dfrac{\partial}{\partial t}\left(Ke^{-rt}N(-d_2) - S_0 N(-d_1)\right)$

$$= S_0 N'(-d_1)\left(-\frac{\partial d_1}{\partial t}\right) - Ke^{-rt}N'(-d_2)\left(-\frac{\partial d_2}{\partial t}\right) - Ke^{-rt}(-r)N(-d_2)$$

$$= \underbrace{-S_0 N'(-d_1)\left(\frac{\partial d_1}{\partial t}\right) + Ke^{-rt}N'(-d_2)\frac{\partial}{\partial t}d_1} - Ke^{-rt}N'(-d_2) \cdot \frac{\sigma}{2\sqrt{t}} + Ke^{-rt} \cdot r \cdot N(-d_2)$$

Relation b/w $N'(d_1)$ and $N'(d_2) \Rightarrow \left[S_0(N'(-d_1)) = Ke^{-rt}N'(-d_2)\right]$

So $\theta = Kre^{-rt} \cdot N(-d_2) - S_0(N'(-d_1)) \cdot \dfrac{\sigma}{2\sqrt{t}}$

$$= Kre^{-rt}N(d_2) - \frac{S_0\sigma}{2\sqrt{2\pi t}}e^{-d_1^2/2}$$

Vega (For call option) $\Rightarrow \dfrac{\partial C}{\partial \sigma} = \dfrac{\partial [S_0 N(d_1) - K e^{-rt} N(d_2)]}{\partial \sigma}$

$$= S_0 N'(d_1) \cdot \frac{\partial d_1}{\partial \sigma} - K e^{-rt} N'(d_2) \frac{\partial d_2}{\partial \sigma}$$

$$= S_0 N'(d_1) \frac{\partial d_1}{\partial \sigma} - K e^{-rt} N'(d_2) \frac{\partial (d_1 - \sigma\sqrt{t})}{\partial \sigma}$$

$$= S_0 N'(d_1) \frac{\partial d_1}{\partial \sigma} - K e^{-rt} \cdot N'(d_2) \frac{\partial d_1}{\partial \sigma} + K e^{-rt} \cdot N'(d_2) \cdot \sqrt{t}$$

$$= \Big(\underbrace{S_0 (N'(d_1)) - K e^{-rt} \cdot N'(d_2)}_{0}\Big) \frac{\partial d_1}{\partial \sigma} + K e^{-rt} \cdot N'(d_2) \cdot \sqrt{t}$$

$$= K e^{-rt} \cdot N'(d_2) \sqrt{t} = \underline{\underline{S_0 N'(d_1) \sqrt{t}}} = \underline{\underline{\frac{S_0 \cdot e^{-d_1^2/2} \sqrt{t}}{\sqrt{2\pi}}}}$$

For put option $\rightarrow \dfrac{\partial P}{\partial \sigma} = \dfrac{\partial [K e^{-rt} N(-d_2) - S_0 N(-d_1)]}{\partial \sigma}$

$$= K e^{-rt} N'(-d_2) \cdot \left(- \frac{\partial d_2}{\partial \sigma}\right) - S_0 N'(-d_1) \cdot \left(- \frac{\partial d_1}{\partial \sigma}\right)$$

$$= \underbrace{S_0 N'(-d_1) \left(\frac{\partial d_1}{\partial \sigma}\right) - K e^{-rt} N'(-d_2) \frac{\partial d_1}{\partial \sigma}}_{0} + \sqrt{t} \cdot K e^{-rt} N'(-d_2)$$

$$= K e^{-rt} N'(-d_2) \sqrt{t}$$

$$= \underline{\underline{S_0 N'(d_1) \sqrt{t}}} = \underline{\underline{\frac{S_0 e^{-d_1^2/2} \sqrt{t}}{\sqrt{2\pi}}}}$$

Rho

For call option $\rightarrow \dfrac{\partial C}{\partial r} = \dfrac{\partial [S_0(N(d_1)) - Xe^{-rt}N(d_2)]}{\partial r}$

$$= S_0 N'(d_1)\frac{\partial d_1}{\partial r} - Xe^{-rt}N'(d_2).\frac{\partial d_2}{\partial r} - Xe^{-rt}.(-t)N(d_2)$$

$$= \left(S_0 N'(d_1) - Xe^{-rt}N'(d_2)\right)\frac{\partial d_1}{\partial r} + Xe^{-rt}.t.N(d_2)$$

$$= \underline{\underline{Xe^{-rt}.t.N(d_2)}}$$

For put option $\rightarrow \dfrac{\partial P}{\partial r} = \dfrac{\partial [Xe^{-rt}N(-d_2) - S_0 N(-d_1)]}{\partial r}$

$$= Xe^{-rt}N'(-d_2).\left(-\frac{\partial d_2}{\partial r}\right) + Xe^{-rt}(-t)N(d_2)$$

$$- S_0 N'(-d_1)\left(-\frac{\partial d_1}{\partial r}\right)$$

$$= -Xe^{-rt}N'(-d_2)\left(\frac{\partial d_1}{\partial r}\right) + S_0 N'(-d_1)\left(\frac{\partial d_1}{\partial r}\right) + Xe^{-rt}(-t).N(d_2)$$

$$= \Big(\underbrace{-Xe^{-rt}N'(-d_2) + S_0 N'(-d_1)}_{0}\Big)\frac{\partial d_1}{\partial r} + Xe^{-rt}(-t).N(d_2)$$

$$= \underline{\underline{-Xe^{-rt}.t.N(d_2)}}$$

Problem 2

- Initial margin per contract = $5000
- Maintenance margin = $4000
- Current futures prize: $70 per barrel
- Contract size: 1000 barrels/contract
- Number of contracts: 10

1. Total initial margin = $5000 × 10 = $50000
2. Daily margin balance :- Tracked in Table below.
3. Margin call determination
   - Day 2: $20000 required to restore margin
   - Day 3: $20000 required to restore margin (Details in table)
4. Final margin balance = $80000 (Details in Table)
5. Total loss = $10000 (Details in Table)

Note

- After each margin call, the balance is restored to the initial margin requirement of $50000, from which the next day's gain/loss is calculated.
- Margin calls are made when balance falls below the maintenance margin.

Day	Futures Price	Day gain/Loss	Total gain/Loss	Margin Balance	Margin call	Explanation
0	$70	$ —	—	$50000	~~$50000~~ —	Initial Position with 10 contracts at $5000 each.
1	$72	$20000	$20000	$70000	No	Price rose due to market optimism (eg-demand spike). Profit increased so margin balance increase naturally. No call needed.
2	$68	-$40000	- $20000	$30000	Yes $20000	Price dropped due to oversupply worries. Loss brought margin below required $40000. you had to top up $20,000
3	$66	- $20000	- $40000	$ 30000	Yes $20000	Further price drop due to weak demand. Again, balance below $40000- another margin call.
4	$69	$30000	- $10000	$80000	No	Price bounced back. Margin way above req. level now, so no call.