Rates QRMP Assignment

1. *Suppose that zero interest rates with continuous compounding are as follows:*

Maturity (months)	*Rate (% per annum)*
3	3.0
6	3.2
9	3.4
12	3.5
15	3.6
18	3.7

Calculate forward interest rates for the second, third, fourth, fifth, and sixth quarters.

2. *Companies and B have been offered the following rates per annum on a $20 million five-year loan:*

	Fixed Rate	*Floating Rate*
Company A	5.0%	LIBOR+0.1%
Company B	6.4%	LIBOR+0.6%

Company A requires a floating-rate loan; company B requires a fixed-rate loan. Design a swap that will net a bank, acting as intermediary, 0.1% per annum and that will appear equally attractive to both companies.

3. *Companies X and Y have been offered the following rates per annum on a $5 million 10-year investment:*

	Fixed Rate	*Floating Rate*
Company X	*8.0%*	*LIBOR*
Company Y	*8.8%*	*LIBOR*

Company X requires a fixed-rate investment; company Y requires a floating-rate investment. Design a swap that will net a bank, acting as intermediary, 0.2% per annum and will appear equally attractive to X and Y.

4. Simple Curve Building Exercise

Our aim here is to construct a Curve which we will construct using a set of swap rate (Fixed vs Float) quotes of different maturities (signifying the term structure of Interest rates). Please feel free to assume the coupon payment frequency as you desire (could assume it to be annual or quarterly for example). *Please assume that the Floating Rate Index is also read off this curve for swaps in consideration. This means that discounting and floating rate are based off a single curve which we aim to construct. The utility of such a curve is that it can be used to discount any expected cash flows to compute its PV (present value) and also for floating rate projection.*

We know that if we know Discount factor (DF) from today (t) to say a future time (T), we know the spot rate (R) applicable from t to T and vice versa. We could also speak in terms of forward rates from (T1,T2) at time t (Today) i.e. F(t,T1,T2) as well – which we can get from either the DFs DF(t,T1) and DF(t,T2). Forward rates are very useful when we need to estimate the floating rates in the future. So, if we know the function which specifying T (time in future) gives us discount factor, we have the complete definition of the curve. Let's say the function CurveDiscountFactor(t, T) -> Discount Factor from t to T. Our aim is to find such a function which can return us the Discount factors or Forward Rates – which could be used to price instruments like swaps.

The table shows swap rate quotes for 4 dates (3 this year and 1 from 2022). Once we have the setup to generate curves, we can generate the curve for any date and visualize it. We could use curve bootstrapping technique where we start by adding lower tenor segments 1y and then keep on adding segments such that all swaps till that segment are priced.

a. To start with, we can assume rates to be constant in a segment – these would be forward rates. Visualize the 4 curves.
b. Repeat the exercise assuming some interpolation scheme.
c. Assume a parametric form like Nelson Siegel or any polynomial and estimate the best fit parameters for each of the curves.

The success criterion for our curve fitting is how well we are able to reprice the swap benchmarks which we used to construct the IR Curve. It would be interesting look at how the shape of the curve has evolved over time when we look at the 4 curves.

	Current	07 Apr 2023	10 Mar 2023	11 Apr 2022
1 Year	4.646%	4.624%	5.043%	1.842%
2 Year	3.979%	3.953%	4.500%	2.441%
3 Year	3.602%	3.566%	4.090%	2.561%
5 Year	3.264%	3.209%	3.651%	2.538%
7 Year	3.143%	3.093%	3.460%	2.524%
10 Year	3.093%	3.049%	3.335%	2.530%
15 Year	3.107%	3.073%	3.264%	2.530%
30 Year	2.895%	2.872%	2.923%	2.338%

ANSWERS

Q1.

If:

- $R1$ = zero rate for time $T1$
- $R2$ = zero rate for time $T2$

Then the forward rate between T1and T2 is:

$f=(R2*T2-R1*T1)/(T2-T1)$

Where:

- All rates are annual (continuous compounding),
- T1,T2 are in years.

Forward Rate Calculations-

1.**Second Quarter** (from 3 to 6 months)-

$f= ((3.2)(0.5)-(3.0)(0.25))/0.25 = (1.6-0.75)/0.25= 3.40\%$

2. **3rd Quarter** (from 6 to 9 months)-

$f= ((3.4)(0.75)-(3.2)(0.5))/ 0.25 = (2.55-1.6)/0.25= 3.80\%$

3. **Fourth quarter** (from 9 to 12 months):

$f=(3.5)(1.0)-(3.4)(0.75)/ 0.25 = (3.5-2.55)/0.25 =3.80\%$

4. **Fifth quarter** (from 12 to 15 months):

$f= (3.6)(1.25)-(3.5)(1.0)/0.25 = (4.5-3.5)/0.25 = 4\%$

5.**Sixth quarter** (from 15 to 18 months):

$f= ((3.7)(1.5)-(3.6)(1.25))/0.25 = (5.55-4.5)/0.25 = 4.2\%$

Final Answers:

Quarter	Forward Rate (%)
Q2	3.40%
Q3	3.80%
Q4	3.80%
Q5	4.00%
Q6	4.20%

Q2.

Designing the interest rate swap-
Company A needs floating. Company B needs fixed.

- **Company A** has **comparative advantage in fixed rate** (has more advantage: 1.4% in fixed vs 0.5% in floating).
- **Company B** has **comparative advantage in floating rate** (relatively smaller disadvantage: 0.5% worse vs 1.4% worse in fixed)
- Fixed-rate difference: **1.4%**
  Floating-rate difference: **0.5%**
  Total potential gain from swap: **1.4% - 0.5% = 0.9%**

So, the maximum net gain from the swap is 0.9%, out of which **0.1%** goes to the bank, leaving 0.8% to be shared equally between companies -> each gets **0.4%** benefit.

Swap Structure-
Company A:

1. Borrows fixed at 5.0% directly.
2. Enters a swap to:
    - **Pay LIBOR − 0.3% to the bank.**
    - **Receive 5.0% from the bank.**

3. Company A saves: (LIBOR+0.1%)−(LIBOR−0.3%)=0.4%

Company B:

1. Borrows floating at LIBOR + 0.6% directly.
2. Enters a swap to:
   - **Pay 6.0% to the bank.**
   - **Receive LIBOR + 0.6% from the bank**.
3. Company B saves: 6.4%−6.0%=0.4%

Bank:

1. Pays 5.0% to A and receives LIBOR − 0.3%.
2. Pays LIBOR + 0.6% to B and receives 6.0%.
3. **Net profit**: (6.0%−(LIBOR+0.6%))+((LIBOR−0.3%)−5.0%)=0.1%

Q3

- Company Y wishes to invest in floating rate but will invest the same amount at fixed rate due to the comparative advantage, Company X wishes to invest at fixed rate but might get a comparative advantage in investing in floating rate if entered into a swap, so Company X invests the same amount at floating rate (LIBOR).
- This is different from the previous case as here a higher rate would be more advantageous.
- X vs Y fixed-rate spread: 8.8 – 8.0 = **0.8%**,
- Floating spread =0% (as both LIBOR),total potential gain from a swap can be = 0.8% – 0% = **0.8%** per annum**.**
- Since we as an intermediary bank need to receive 0.2 % gain per annum**,** Net gain available to split between companies X and Y = 0.8 - 0.2 % = 0.6%, 0.3% each company gain.
- Company X and Y would get equally appealing deals , that is , each would get a gain of 0.3% from entering the swap,Company X aims for 8%+0.3% fixed investment rate,and Company Y aims for LIBOR +0.3%
- **Company X:**
  - invests $5 M at floating LIBOR externally → receives LIBOR
  - Enters into a swap,through the swap,Pays bank a floating LIBOR
  - Receives 8.3%(net gain = 0.3% as without swap it would get 8%)
- **Company Y:**
  - invests $5 M at fixed 8.8% externally → receives 8.8%
  - Enters into a swap,through the swap,Pays bank a fixed <u>S</u>**%**

- Receives LIBOR(net gain = 8.8% - S% = 0.3%)
- S =8.5%

- **Bank:**
  - For the bank to gain 0.2%:
    - Total gain of bank = money received from X and Y - money given to X and Y = LIBOR+S -(LIBOR +8.3) = S%-8.3% = 0.2% (S = 8.5%)

  Therefore, the swap offers X and Y equally attractive offers and a gain of 0.2% per annum for intermediary bank , if its conditions are :
  - **Company X pays LIBOR to bank and receives 8.3% from bank**
  - **Company Y pays 8.5% to the bank and the bank pays LIBOR to company Y.**

Q4. Notebook Link- Rates_Q4.ipynb

(a)

- Objective:
  Building a discount curve using fixed-vs-floating swap quotes for different maturities and extracting the forward rate curve from it.

- Method:
  Used bootstrapping to compute:
  - Discount Factors for each maturity.
  - Forward Rates between maturities (assuming constant forward rate within each interval).

  Performed this for 4 different dates: Current, 07 Apr 2023, 10 Mar 2023, 11 Apr 2022.

Approach:

- Assumed swap payments are made annually.
- Started from the 1-year point using the formula:

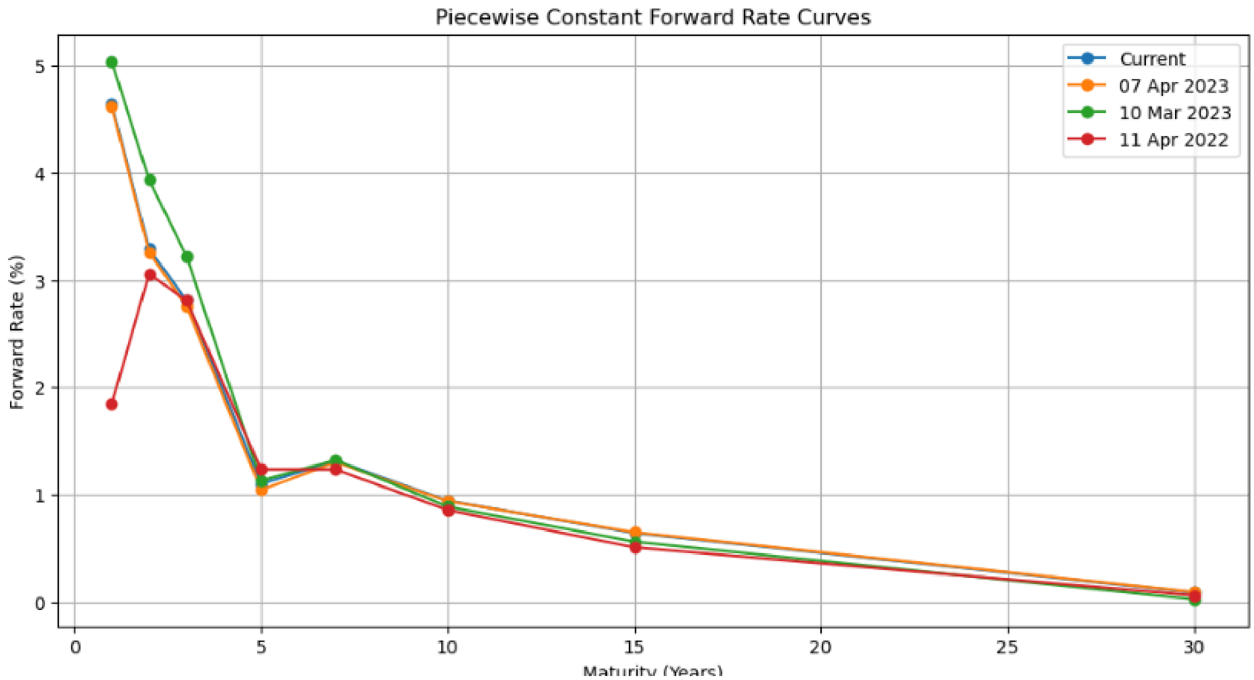
$$DF(1) = \frac{1}{1 + S_1}$$

- Then, iteratively used the formula:

$$DF(T_i) = \frac{1 - S_i \cdot \sum_{j=1}^{i-1} DF(T_j)}{1 + S_i}$$

- Forward rate between Ti-1 and Ti:

$$f_i = \frac{DF(T_{i-1})}{DF(T_i)} - 1$$



Current Forward Rates:

Maturity	Swap Rate	Forward Rate	Discount Factor
1	4.646	4.646000	0.955603
2	3.979	3.290055	0.925164
3	3.602	2.814030	0.899842
5	3.264	1.098301	0.880501
7	3.143	1.313334	0.857965
10	3.093	0.940736	0.834416
15	3.107	0.639939	0.808545
30	2.895	0.083932	0.798493

07 Apr 2023 Forward Rates:

Maturity	Swap Rate	Forward Rate	Discount Factor
1	4.624	4.624000	0.955804
2	3.953	3.260125	0.925627
3	3.566	2.757691	0.900786
5	3.209	1.041689	0.882402
7	3.093	1.299366	0.860052
10	3.049	0.937004	0.836537
15	3.073	0.646356	0.810348
30	2.872	0.088065	0.799783

10 Mar 2023 Forward Rates:

Maturity	Swap Rate	Forward Rate	Discount Factor
1	5.043	5.043000	0.951991
2	4.500	3.935630	0.915943
3	4.090	3.226883	0.887310
5	3.651	1.128533	0.867725
7	3.460	1.320731	0.845395
10	3.335	0.885585	0.823516
15	3.264	0.559001	0.801124
30	2.923	0.021411	0.798560

11 Apr 2022 Forward Rates:

Maturity	Swap Rate	Forward Rate	Discount Factor
1	1.842	1.842000	0.981913
2	2.441	3.058319	0.952774
3	2.561	2.811521	0.926719
5	2.538	1.232616	0.904423
7	2.524	1.232135	0.882672
10	2.530	0.854136	0.860619
15	2.530	0.506000	0.839383
30	2.338	0.058211	0.832117

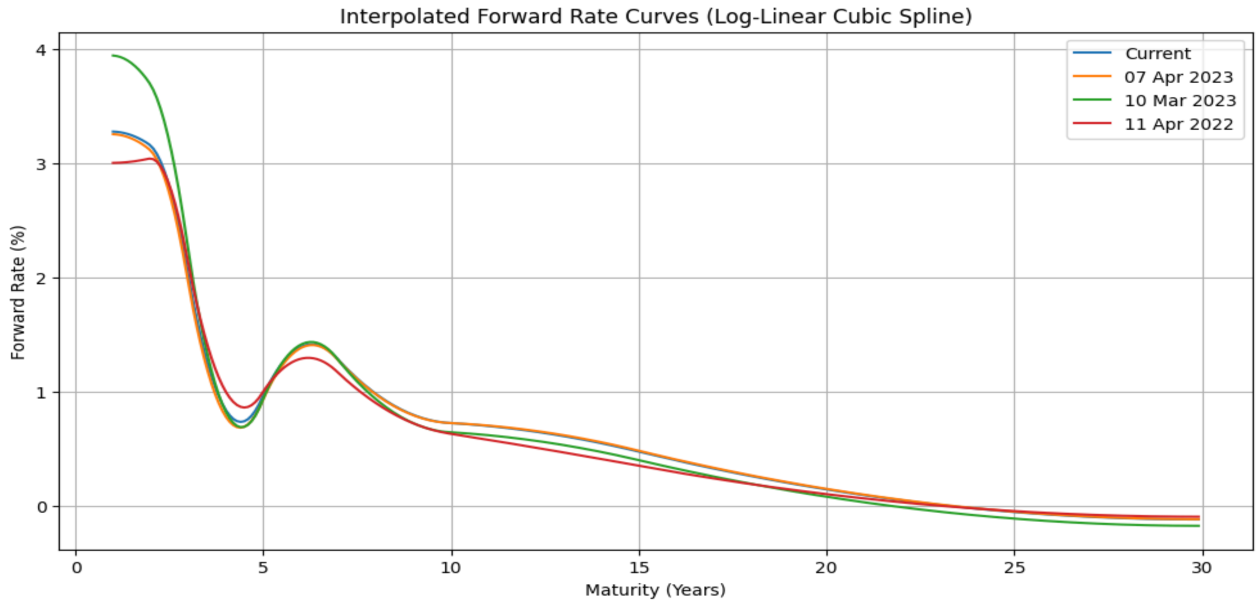
Key Observations

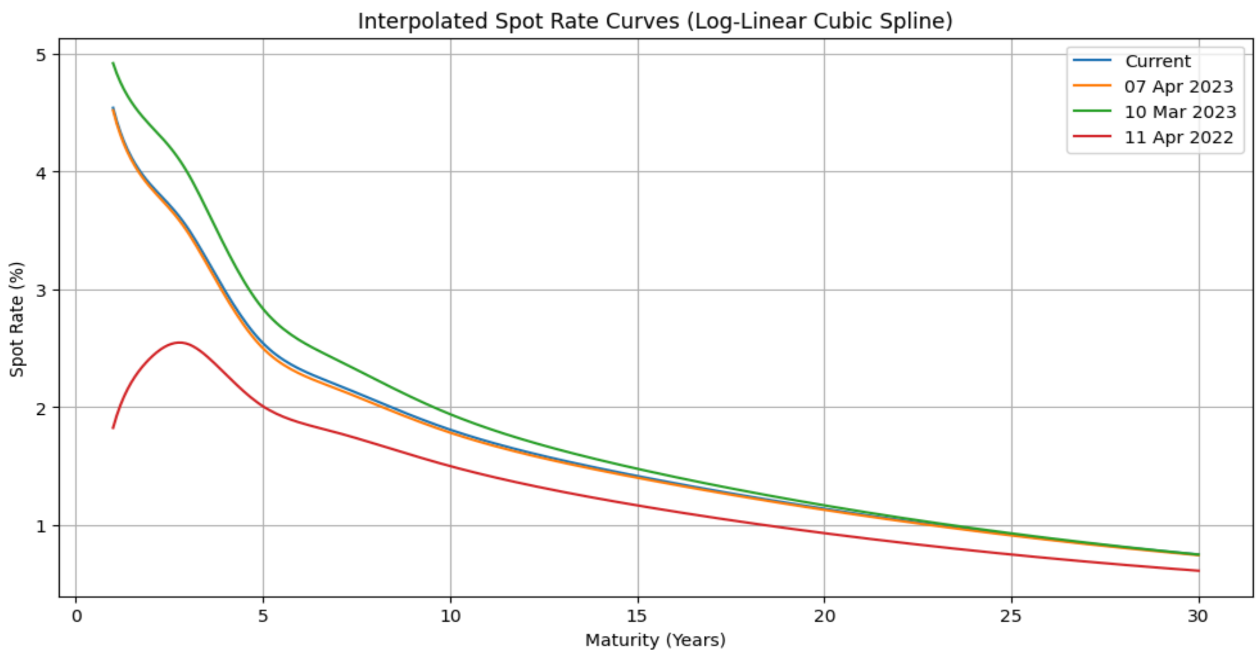
- Short-term forward rates (1Y–3Y) changed significantly across dates.
- In 2022 (April), rates were much lower, reflecting a lower interest rate environment.
- Current and March-April 2023 curves show higher short-term rates, reflecting rising interest rates or inflation expectations.
- After 5 years, the forward rates flatten and converge across dates - indicating market expectations of long-term stabilization.

(b)

- To obtain a smooth curve for discount factors (DFs) and to enable calculation of forward rates at arbitrary maturities, I applied a cubic spline interpolation on the logarithm of the discount factors (log-linear cubic spline).
- This approach is widely used in practice because it ensures monotonicity of the discount factors and produces a smooth, continuous yield and forward rate curve
- Why was log-linear cubic spline chosen as interpolation scheme:
    - Interpolating on log(DF) (rather than DF or rates directly) respects the exponential nature of discounting and helps avoid negative or non-monotonic DFs, which could otherwise lead to arbitrage
    - Cubic splines are preferred over linear interpolation because they produce smooth curves with continuous first and second derivatives, which is important for financial modeling and derivative pricing
    - This method is standard in the industry for constructing smooth, arbitrage-free curves suitable for both discounting and projecting forward rates

Limitations of this approach: The cubic spline log-linear interpolation method provides smooth, generally monotonic curves but sacrifices exact repricing accuracy at input maturities and may oversmooth sharp market features, though its balance of smoothness and practical utility makes it widely accepted for academic and non-critical applications.





Observed Results:

- The curve fits the 1Y–3Y maturities almost exactly, but shows significant repricing errors at longer maturities (5Y and beyond), with errors up to 200 basis points.
- This is a direct consequence of using a global cubic spline interpolation, which does not enforce exact fit at the nodes

(c) Parametric Curve Fitting with Nelson-Siegel-

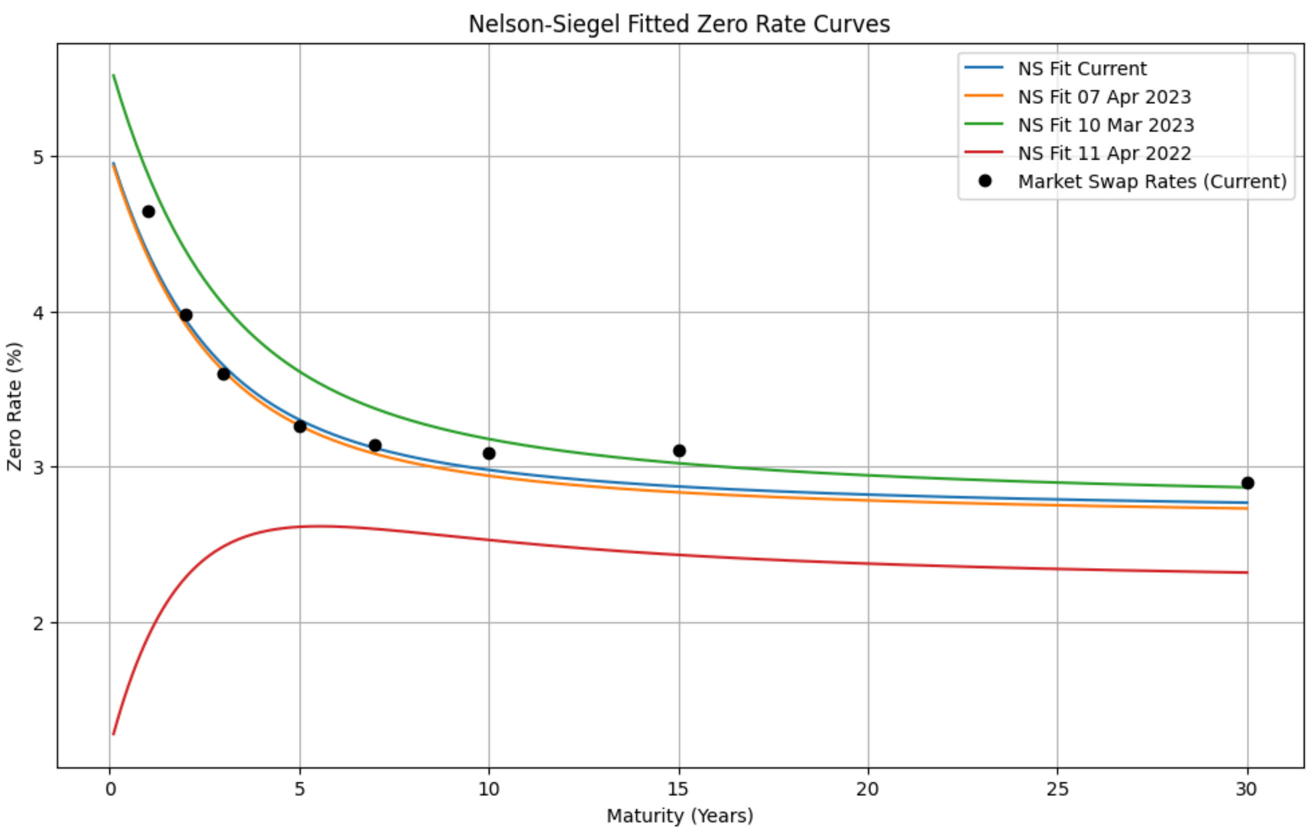
Instead of building the curve piece by piece (bootstrapping), this approach assumes the curve follows a specific mathematical formula. We then use an optimizer to find the formula's parameters that best fit the observed market swap rates

Explanation of the Method

The Nelson-Siegel model describes the zero-coupon yield curve with four parameters:

- `beta0`: The long-term interest rate (the level the curve converges to).
- `beta1`: The short-term slope (the difference between the long-term and very short-term rates).
- `beta2`: The curvature parameter (creates the "hump" or "trough" in the middle of the curve).
- `tau`: A decay factor that controls the position and shape of the hump.

Our goal is to find the set of `(beta0, beta1, beta2, tau)` that minimizes the difference between the swap rates implied by the model and the actual market rates.



Observations-

- 11 Apr 2022 : A "normal" upward-sloping curve. The market expected rates to rise or stay stable, which is typical of a healthy economic environment.
- 10 Mar 2023: Sharply inverted. Short-term rates are much higher than long-term rates. This shape often signals that the market expects an economic slowdown or recession,

anticipating that the central bank will have to cut rates in the future.