Assignment on Derivatives

Presenter: Ritwik Ganguly

Problem Statement 1:

The Black-Scholes model (BSM) is a differential equation widely used to price options contracts. In its standard form, the BSM is used to price European put and call options. The model requires five input variables: the strike price of an option K, the current stock price S_0, the time to expiration T, the risk-free rate r, and the volatility $\sigma_{K,T}$. Given these inputs, the model outputs the price of the call and put options.

$$Call\ price,\ c = S_0\ N(d_1) - K\ e^{-rT} N(d_2)$$

$$Put\ price,\ p = K\ e^{-rT}\ N(-d_2) - S_0\ N(-d_1)$$

where,

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}\ d_2 = \frac{\ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

N(*x*) is the probability that a normally distributed variable with a mean of zero and a standard deviation of 1 is less than *x.*

1. (a) Calculate the price of a European call option using the Black-Scholes formula. Assume the following parameters:
   Current stock price (S): $100
   Strike price (K): $105
   Time to expiration (T): 1 year
   Risk-free interest rate (*r*): 5% per annum
   Volatility (*σ*): 20% per annum
   (b) Calculate the price of European put option having the same parameters.
   (c) Verify the put call parity relationship using the above two prices and other parameters provided.
   (d) Calculate the delta of the call option and the put option. Find the relationship between the two.
   (e) Calculate the price of the above provided call option using the Black-Scholes model for volatilities of 0%, 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, 90% and 100%. Plot the graph and discuss how changes in volatility affect the option price.
   (f) Based on the Black Scholes formula, derive the formulae of the following Option Greeks: Gamma, Theta, Vega and Rho.

Problem Statement 2:

You are a trader who has entered into a long position in 10 futures contracts on crude oil. Each contract represents 1,000 barrels of oil. The initial margin requirement per contract is $5,000, and the maintenance margin is $4,000. The current futures price at the time of entering the position is $70 per barrel.

Scenario:

1. **Day 1:**
    - The futures price increases to $72 per barrel.
2. **Day 2:**
    - The futures price decreases to $68 per barrel.
3. **Day 3:**
    - The futures price decreases further to $66 per barrel.
4. **Day 4:**
    - The futures price rebounds to $69 per barrel.

Questions

1. **Initial Margin Calculation:**
    - Calculate the total initial margin required to enter the position.
2. **Daily Margin Balance:**
    - Calculate the daily margin balance at the end of each day, considering the price changes and any margin calls that may occur.
3. **Margin Call Determination:**
    - Determine on which days, if any, a margin call occurs. Calculate the amount required to meet the margin call.
4. **Final Margin Balance:**
    - Calculate the final margin balance at the end of Day 4.
5. **Profit or Loss Calculation:**
    - Calculate the total profit or loss on the position at the end of Day 4.