Gain Scheduling and Robust Control of a Lunar Lander

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***Abstract*— The ability to land a vehicle safely from orbit is required for the exploration of the moon other planets, and thus it is a problem that has garnered significant attention by governments and private companies alike. This problem is particularly interesting when it is solved using propulsive landing methods, as it requires a robust controller that can adapt to the changing system parameters, such as the loss of fuel mass as thrust is produced.**

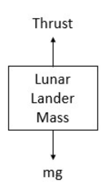
I. Objective

Designing a controller for propulsive landing of a lunar lander poses many challenges, one of which is to robustly control the system as it loses fuel mass on descent. Another important design challenge is the requirement to touch down on the surface softly, which means there is no room for system overshoot. The objective of this paper is to investigate LMI methods that can be used to develop a set of gains that are well-adapted for specific regions of the descent where parameters are somewhat similar and use them to safely control the descent of a lunar lander.

II. System Model

The model of a lunar lander can be very complex. This paper is concerned only with the controller that can be adapted for fuel mass loss, so the model can be simplified to focus on this design parameter. Thus, the model used is simplified from a 6-DOF model to a 1-DOF model as seen in Figure 1 with its state space representation.

Figure 1. Simplified 1DOF Model of Lunar Lander and State-Space Representation, where i = 1,2,…,n and j = 1,2 indicated the bounding conditions for the vehicle mass



$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad B_{i,j} = \begin{bmatrix} 0 \\ \frac{1}{m_{i,j}} \end{bmatrix}$$

$$C = [1 \quad 0], \qquad D = [0]$$

The bounding conditions for vehicle mass are designed to simulate the range of possible operating conditions in which the lunar lander will need to function properly, which are determined by maximum and minimum fuel consumption rates. The mass of fuel required for safely landing the lunar lander on the surface of the moon is determined for each case by simulating the descent of the lander under constant thrust. In the minimum fuel consumption case, the descent is simulated under ideal conditions starting with the maximum fuel possible while maximum fuel consumption case is determined by starting with fuel amount offset from the maximum case.

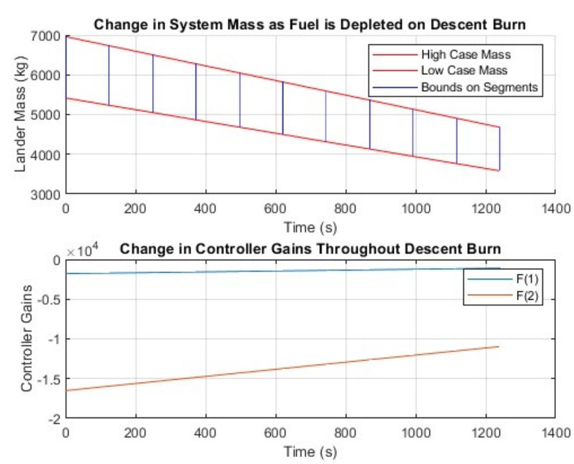
Each case is simulated using the discretized state space model for the system under constant thrust. The mass depletion is determined by the specific impulse equation, which takes the discretized form:

$$m_{i+1} = m_i - \frac{1}{I_{sp}g} F dt$$

Where m is the vehicle mass, I_{sp} is specific impulse, g is the value for acceleration due to gravity on the moon, and F is thrust. Thus, the B matrix for the system changes for each time step while the vehicle is under thrust. The value for thrust is found using the bisection method, iterating until the velocity when the lander touches down on the lunar surface is close to 0. This simulation gives the bounding conditions for vehicle mass to be used for calculating system gains.

It is somewhat computationally expensive to determine a new set of gains for each time step using the LMI method, so the total descent is divided into time segments with similar mass properties for which a controller can be found. The range of possible mass properties for each time segment are found and simplified into an interval form. Then using the LMI method described in the following section, the gains for each mass interval are found.

Figure 2. Bounding Conditions for System Mass and Corresponding Controller Gains Determined by LMI.



III. Optimization and LMI

The controller gains for the lunar lander system can be found using LMI methods. Ideally, the lunar lander will descend with low fuel expenditure and land softly on the surface of the moon. To achieve this, the system must be stabilized. Thus, the discrete-time stability method can potentially be used, but as this system has uncertainty conditions on the mass properties, this is not sufficient to fully stabilize the system. Therefore, robust stability conditions are used in addition to the stability conditions. on each mass interval i coupled with the closed-loop robust stability conditions for discrete-time systems with polytopic uncertainty [1,2,3].

$$\text{For each mass interval } i, \text{with bounds } j = 1,2$$

$$\text{There exist some } P > 0, Z, G \text{ such that}$$

$$\begin{bmatrix} P & A_d P + B_{dij} Z \\ *^T & P \end{bmatrix} > 0$$

$$\begin{bmatrix} P & A_d P - B_{dij} Z \\ *^T & G + G^T + P \end{bmatrix} > 0$$

$$\text{Then for the interval } i \text{ the controller gains are given by: } F_i = -ZG^{-1}$$

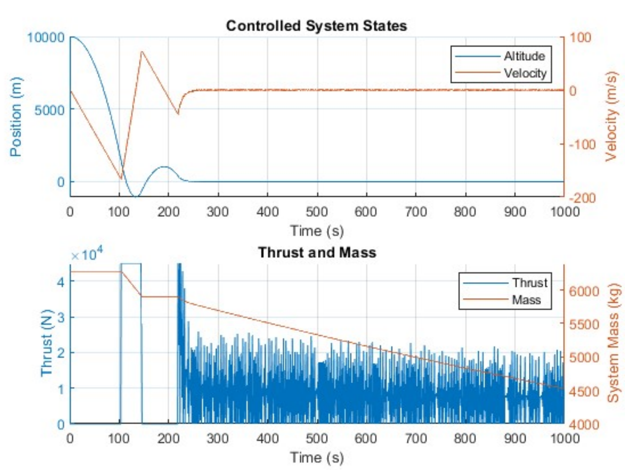
Using this method, the controller gains shown in Figure 2 are determined for each mass interval.

For comparison, a controller is also designed using the same LMI method for the parameters of the entire descent rather than on individual mass intervals. This should give a more conservative controller that has gains optimized for all operating points of the vehicle.

IV. Results

Once the controller gains are determined, the closed loop controller can be simulated to verify that the calculated gains satisfactorily achieve the design objectives. Thus, lunar lander descent with closed loop control is simulated using a method similar to the simulation method described in section II, except with closed loop feedback control to determine the thrust required rather than the simplified bisection method. In addition, the simulation includes a sequencer to switch gains at the time increments determined in section II. The simulation results with gain scheduling are shown in Figure 3. Note that this simulation lacks real-world constraints such as the surface of the moon providing a physical barrier to the motion of the lander, so the vehicle is able to pass through the surface of the moon and requires thrust to remain there as well. This is intended to demonstrate the performance of the controllers for comparison purposes.

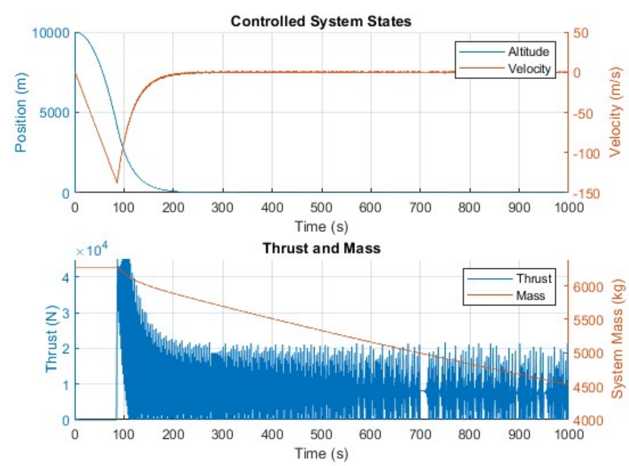
Figure 3. Simulation of Closed Loop Feedback Controller for Lunar Lander with Gain Scheduling



As can be seen in the figure, this controller allows the vehicle to freefall for a period before firing the thruster at maximum level to arrest its descent rate. This controller does not effectively meet the design objectives as it is optimized for minimum time to reach the objective, which causes the lander to impact the lunar surface with significant velocity, which is clearly undesirable.

The second controller which is designed for all operating points of the vehicle was also simulated using the same method as for the first controller, but no logical switching is required as the gains are static for the full duration of the descent, The performance of this closed loop feedback controller designed for all operating points of the lunar lander is shown in Figure 4.

Figure 4. Simulation of Closed Loop Feedback Controller for Lunar Lander at All Operating Points



V. Discussion

The gains found for the gain scheduling method were able to control the system, but produced unsatisfactory results. The lander impacted the surface of the moon with too great a velocity to be useful for either probe or human-rated missions because of the gains prioritizing response time rather than minimizing fuel consumption. This could be rectified by pursuing other LMI methods, such as mixed H_2/H_∞ optimization methods or including D-stability constraints to the uncertain system. Despite the drawbacks of this gain set, it can be seen in Figure 3 that the controller is able to hold its commanded position with good accuracy, even as the fuel mass is being depleted. With further investigation into other LMI methods better suited to this problem, the gain scheduling method could show some promise in providing a good control solution.

Finding a singular set of static controller gains for the system by concatenating all the bounding conditions into a single LMI results in a more conservative controller, but in this case, it provides a much better control solution. The system converges to the commanded position slower than the gain scheduled solution, but with no overshoot. This solution is also able to hold its commanded position with accuracy. Overall, the more conservative robust control solution is the ideal choice of controller for this problem.

REFERENCES

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