Problem2

2022-11-14

Problem 2 : Simulation Study to Understand Sampling Distribution

Part A Suppose $X_1, X_2, \cdots, X_n \overset{iid}{\sim} Gamma(\alpha, \sigma)$, with pdf as

$$f(x|\alpha,\sigma) = \frac{1}{\sigma^\alpha \Gamma(\alpha)} e^{-x/\sigma} x^{\alpha-1}, \quad 0 < x < \infty,$$

The mean and variance are $E(X) = \alpha\sigma$ and $Var(X) = \alpha\sigma^2$. Note that `shape = ` α and `scale = ` σ. 1. Write a `function` in `R` which will compute the MLE of $\theta = \log(\alpha)$ using `optim` function in `R`. You can name it `MyMLE`

```
My_MLE <- function(x){
      ll_gamma1 <- function(p){
        ll_gamma(p,x)
      }
      z <- optim(par=c(1,1),ll_gamma1)
      log(z$par[1])
}
```

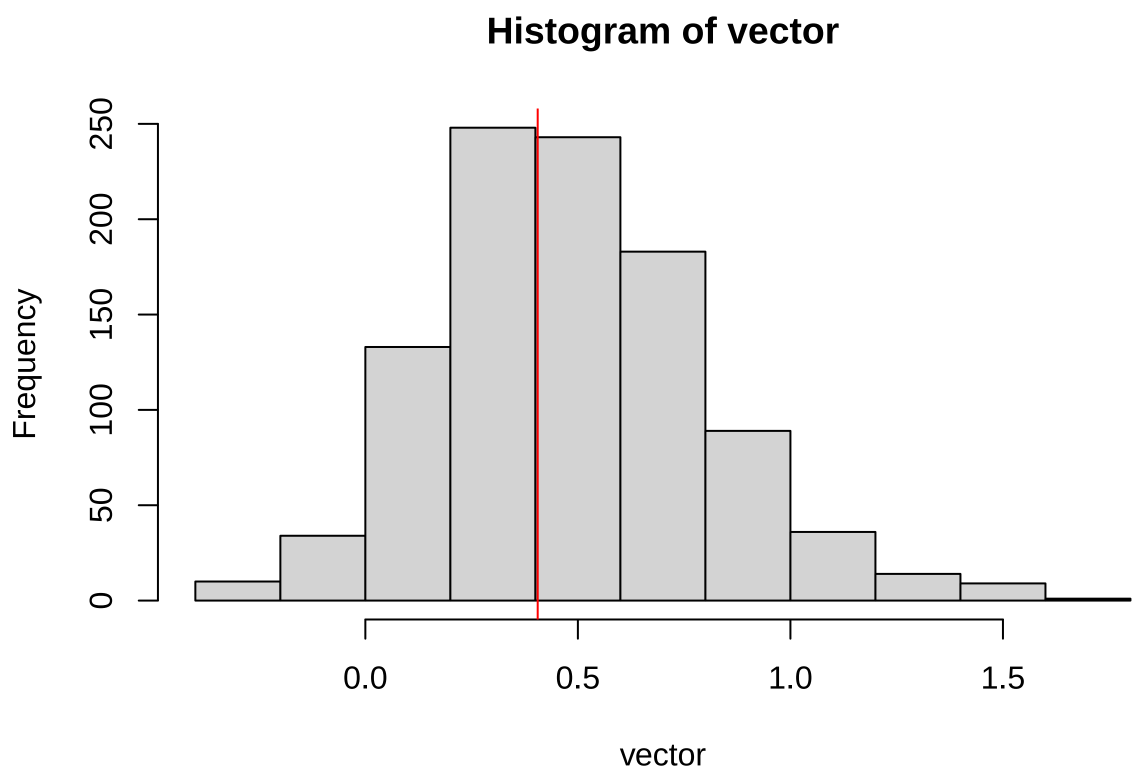
2. Choose `n=20`, and `alpha=1.5` and `sigma=2.2`
   (i) Simulate $\{X_1, X_2, \cdots, X_n\}$ from `rgamma(n=20,shape=1.5,scale=2.2)`
   (ii) Apply the `MyMLE` to estimate θ and append the value in a vector
   (iii) Repeat the step (i) and (ii) 1000 times
   (iv) Draw histogram of the estimated MLEs of θ.
   (v) Draw a vertical line using `abline` function at the true value of θ.
   (vi) Use `quantile` function on estimated θ's to find the 2.5 and 97.5-percentile points.

```
library(stats)
set.seed(69)
vector = c()
gamma_simulation_MLE <- function(N,sim_size,alpha,sigma){
  for (j in 1:sim_size)
  {
    x<-rgamma(N,shape=alpha,scale=sigma)

    ll_gamma <- function(p,x){
      alpha<-p[1]
      sigma<-p[2]
      N*alpha*log(sigma)+N*log(gamma(alpha))-(alpha-1)*sum(log(x))+sum(x)/sigma
    }

    My_MLE <- function(x){
      ll_gamma1 <- function(p){
```

```
        ll_gamma(p,x)
      }
      z <- optim(par=c(1,1),ll_gamma1)
      log(z$par[1])
    }
    vector <- c(vector, My_MLE(x))
  }
  hist(vector)
  abline(v=log(alpha),col='red')
  cat("The 2.5 and 97.5 percentile points are:")
  quantile(vector,probs=c(0.025,0.975))
}
gamma_simulation_MLE(20,1000,1.5,2.2)
```

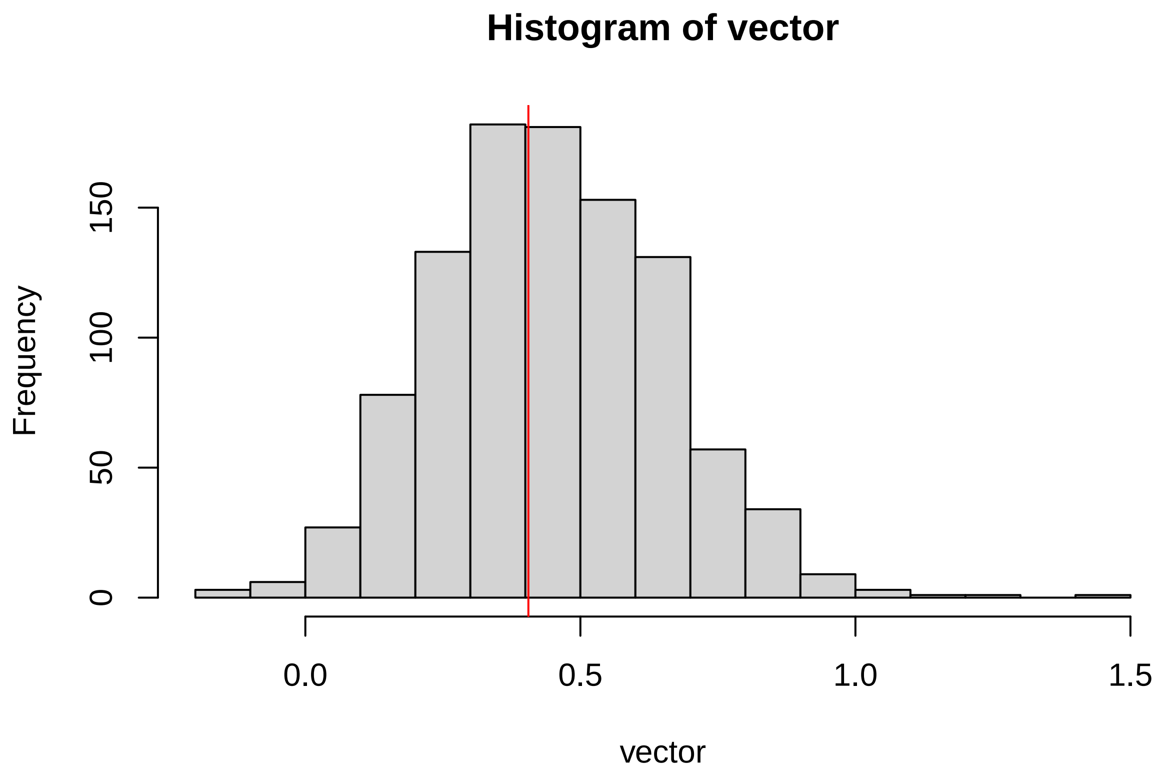



```
## The 2.5 and 97.5 percentile points are:

##        2.5%       97.5%
## -0.06701715  1.16944822
```

3. Choose `n=40`, and `alpha=1.5` and repeat the (2).

```
gamma_simulation_MLE(40,1000,1.5,2.2)
```

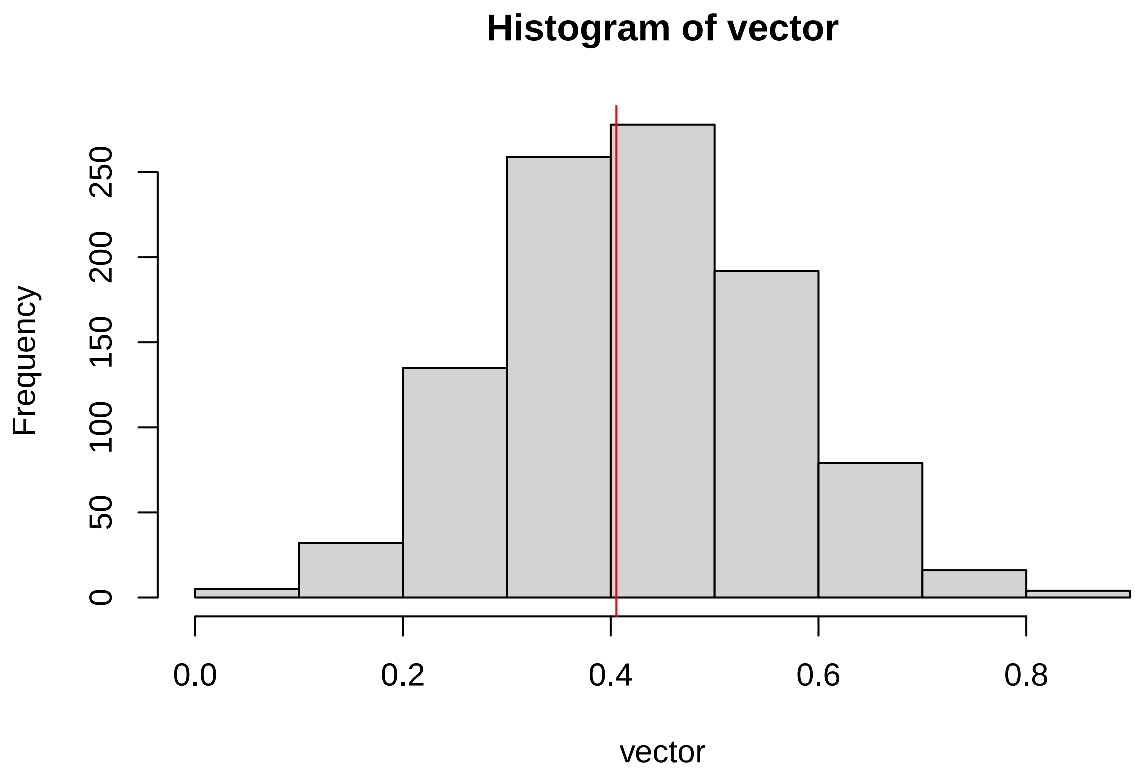



```
## The 2.5 and 97.5 percentile points are:

##       2.5%      97.5%
## 0.08500657 0.85448635
```

4. Choose `n=100`, and `alpha=1.5` and repeat the (2).

```
gamma_simulation_MLE(100,1000,1.5,2.2)
```




```
## The 2.5 and 97.5 percentile points are:

##      2.5%     97.5%
## 0.1709145 0.6791760
```

5. Check if the gap between 2.5 and 97.5-percentile points are shrinking as sample size `n` is increasing?

Yes the gap between the 2.5 and 97.5 percentile points gets smaller as n increases.