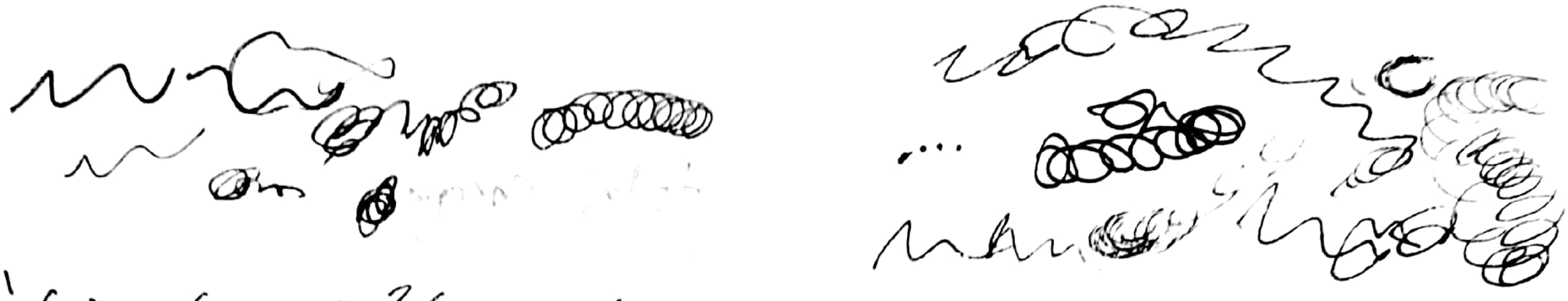

$$f^1(\vec{p}) = \left(\frac{1}{2\pi m k_B T}\right)^{3/2} \exp\left(\frac{-p^2}{2 m k_B T}\right)$$

$$f^2(\vec{p}_1, \vec{p}_2) = \left(\frac{1}{2\pi m_1 k_B T}\right)^{3/2} \left(\frac{1}{2\pi m_2 k_B T}\right)^{3/2} \exp\left(\frac{-p_1^2}{2 m k_B T}\right) \exp\left(\frac{-p_2^2}{2 m k_B T}\right)$$

$$\boxed{\vec{P}_c = \vec{p}_1 + \vec{p}_2} \Rightarrow \vec{p}_1 = \vec{P}_c - \vec{P}_2$$

$$\cancel{\vec{p}_1^2} \quad \vec{p}_1^{\,2} = \vec{p}_1 \cdot \vec{p}_1 = \vec{P}_c^{\,2} + p_2^2 - 2\vec{P}_c \cdot \vec{P}_2$$

$$\boxed{\vec{p}_r = \frac{\vec{p}_1 - \vec{p}_2}{2}} \Rightarrow \vec{p}_2 = \cancel{2\vec{p}_A} \; \vec{p}_1 - 2\vec{p}_r$$

$$\cancel{p_2^2} \quad p_2^2 = (\vec{p}_1 - 2\vec{p}_r) \cdot (\sim)$$

$$\cancel{p_1^2 = p_1 = P_c^2 + p_2^2} \qquad = p_1^2 + 4p_r^2 - 4\vec{p}_1 \cdot \vec{p}_r$$

$$p_2^2 = \left[P_c^2 + p_2^2 - 2\vec{P}_c \cdot \vec{P}_2\right]^2 + 4p_r^2 -$$

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$$\vec{p}_1 = P_c - \vec{P}_2 = 2\vec{p}_r + \vec{p}_2$$

Total energy must remain fixed

$M = 2m$

$\mu = m/2$

$$\frac{p_1^2}{2m} + \frac{p_2^2}{2m} = \frac{P_c^2}{2M} + \frac{p_r^2}{2\mu}$$

$$= \frac{P_c^2}{4m} + \frac{p_r^2}{m}$$

$$p_1^2 + p_2^2 = \frac{P_c^2}{2} + 2p_r^2$$

so

$$\left(\frac{1}{2\pi M k_B T}\right)^{3/2} \left(\frac{1}{2\pi \mu k_B T}\right)^{3/2} \exp\left(\frac{-P_c^2}{2M k_B T}\right) \exp\left(\frac{-p_r^2}{2\mu k_B T}\right)$$

total energy

integrate out $P_c$ to get the two-particle relative momentum distribution

$$= \left(\frac{1}{2\pi M k_B T}\right)^{3/2} \left(\frac{1}{2\pi \mu k_B T}\right)^{3/2} \int_{\infty} d^3\vec{P}_c \, e^{\sim_c} e^{\sim_r}$$

$\uparrow A$ $\uparrow C$ $\uparrow R$

convert to spherical (assuming distribution is isotropic and leads to alignment of coors along inter-particle axis)

$$= C^{3/2} R^{3/2} e^{\sim r} \, 4\pi \int_0^1 dx \underbrace{\int_0^\infty dP_c \, P_c^2 \, e^{\sim_c}}_{\int_0^\infty d\alpha \, x^2 e^{-x^2} = \frac{\sqrt{\pi}}{4}}$$

$$= \cancel{C^{3/2}} R^{3/2} e^{\sim r} \cancel{4\pi} \left(\frac{C^{3/2}}{\pi}\right) \frac{\cancel{\sqrt{\pi}}}{\cancel{4}} \frac{C^{3/2}}{\cancel{\pi^{3/2}}}$$

$$\boxed{f^2(\vec{p}_r) = \left(\frac{1}{2\pi \mu k_B T}\right)^{3/2} \exp\left(\frac{-p_r^2}{2\pi \mu k_B T}\right)}$$