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# Understanding cold atomic and molecular collisions
# 1. Basics

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Thanks to many colleagues in theory and experiment who have contributed to this work

http://www.jqi.umd.edu/

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# Cold Collisions

"Good" -- Essential interactions for control and measurement
"Bad" -- Source of trapped atom loss, heating, and decoherence

- Atom-atom collisions can be quantitatively understood and controlled--essential for quantum gas studies.
- Basic concepts, illustrated by examples.
- Part 1: What is a scattering length, and why is it significant? Why is the long range potential so important?
- Part 2: Magnetically tunable "Feshbach" resonances are a key to measurement and control.
- Part 3(?):What is special about molecules? Molecules are different from atoms. What about few-body phenomena?

# Some resources

PSJ, Ch. 6 of *Cold Molecules*, ed. by R. Krems et al.
also arXiv:0902.1727, threshold bound and scattering states

Chin, Grimm, PSJ, Tiesinga, Rev. Mod. Phys. 82, 1225 (2010)
review of Feshbach resonances

PSJ, Faraday Discussions 142, 361 (2009)
ultracold molecules: a case study with KRb

Kohler, Goral, PSJ, Rev. Mod. Phys. 78, 1311 (2006)
review of molecule formation through Feshbach res.

Jones, Lett, Tiesinga, PSJ, Rev. Mod. Phys. 78, 483 (2006)
review of cold atom photoassociation

PSJ and Mies, J. Opt. Soc. Am. B 6, 2257 (1989)
quantum defect theory for cold collisions

# Two kinds of collision

Elastic--do not change internal state a + b --> a + b

- Thermalization
- Evaporation
- Mean field of BEC
- BEC-BCS crossover in Fermi gases
- Equation of state
- Phase change--quantum logic gates

Inelastic--change internal state a + b --> a' + b' + $\Delta$E

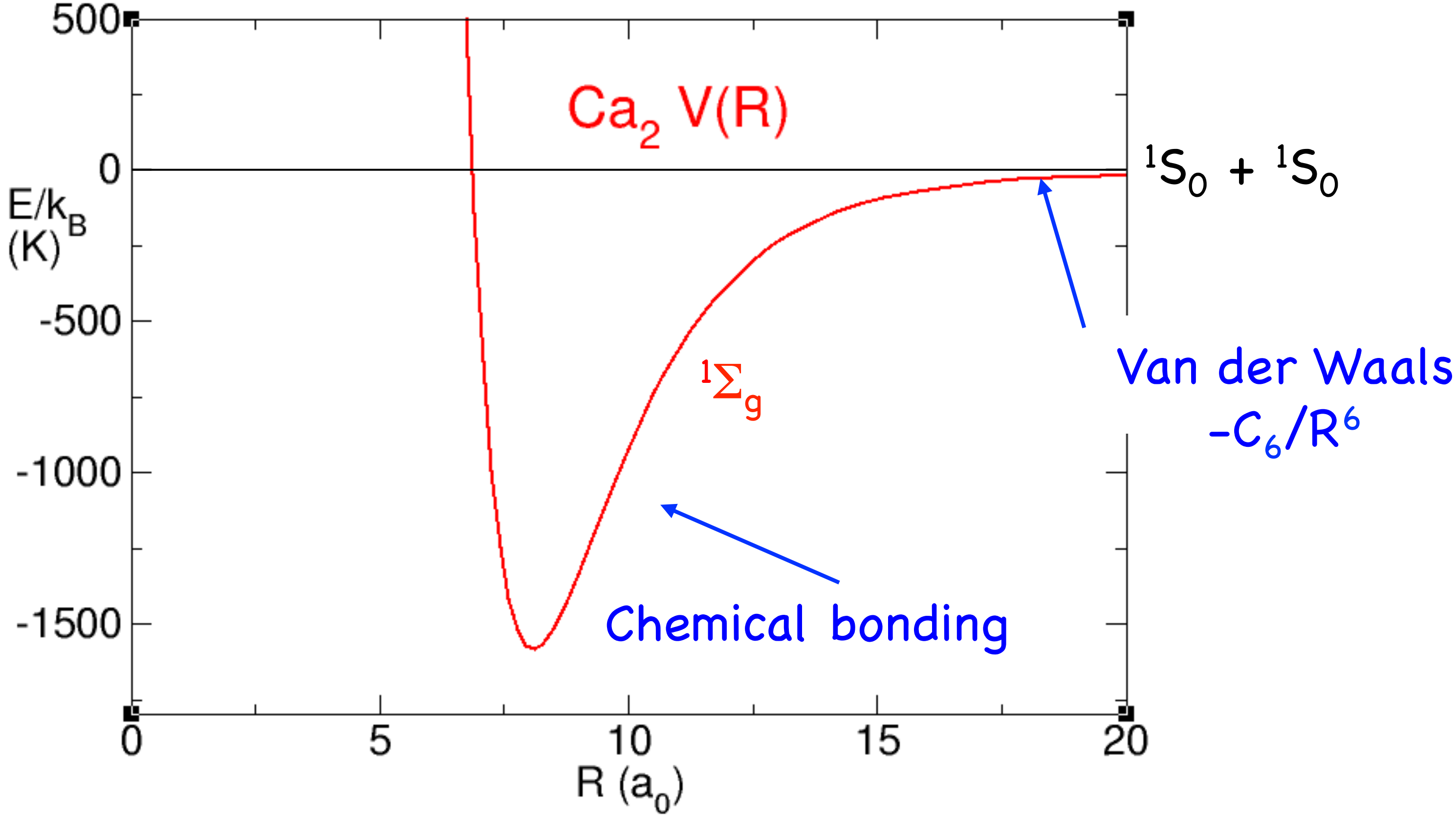
- Spin relaxation
- Loss of trapped atoms, gas lifetime
- Decoherence
- Spinor condensates ($\Delta E=0$)
- Three-body, detect resonances
- Photoassociation

Ca $4s^2\,{}^1S_0$ atom

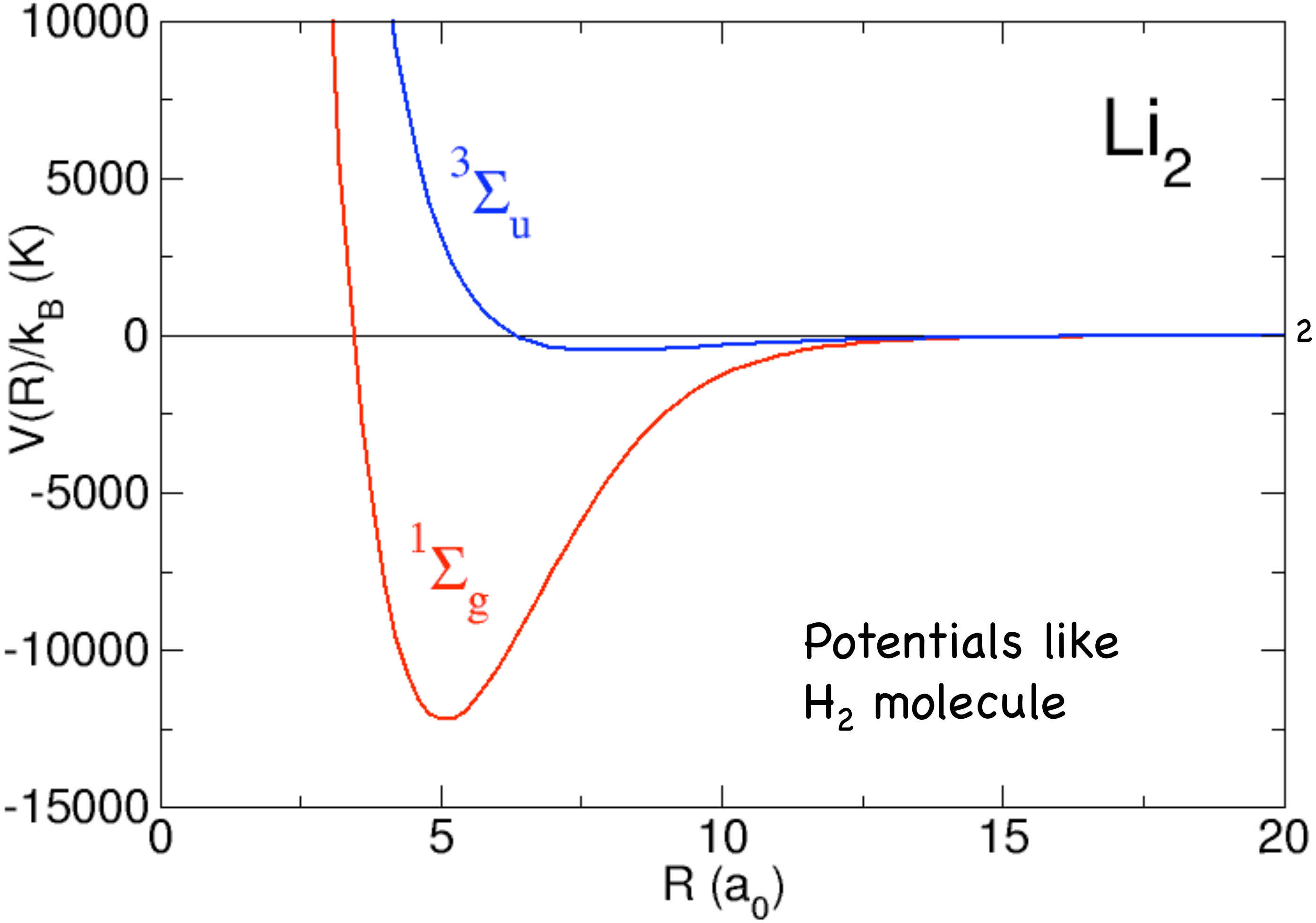
Spin degeneracy

Total angular momentum

Born-Oppenheimer approximation = potential energy curve

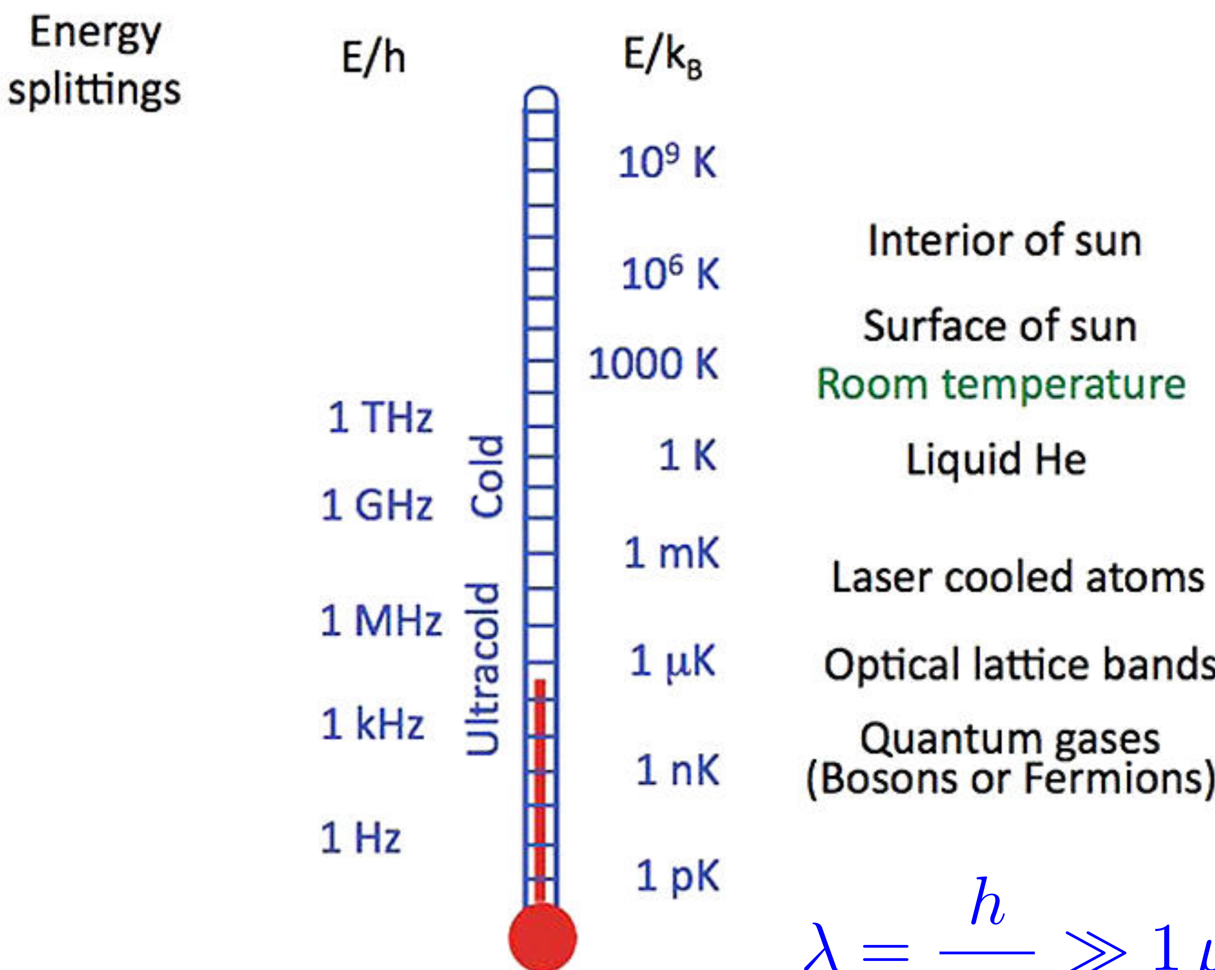


Li $1s^2 2s\ ^2S_{1/2}$ atom




$^2S_{1/2} + {}^2S_{1/2}$

Similar for
Na
K
Rb
Cs

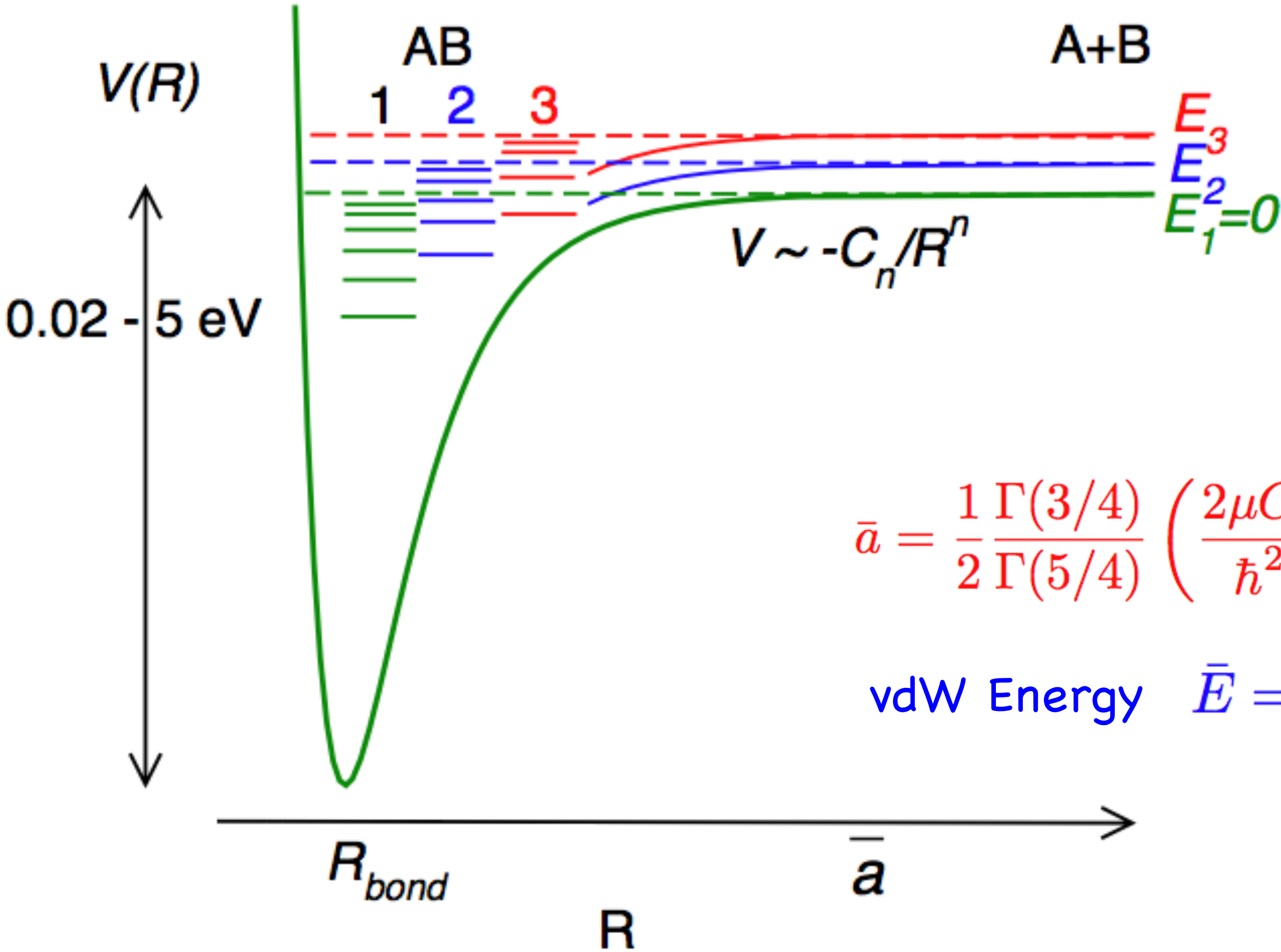



$$\lambda = \frac{h}{mv} \gg 1\,\mu m$$

From Quéméner and Julienne, Chemical Reviews 112, 4949 (2012) *Ultracold Molecules Under Control!*

1 eV 10$^{-4}$ eV 10$^{-10}$ eV (1 μK)

Short-range Long-range Separated




$$\bar{a} = \frac{1}{2}\frac{\Gamma(3/4)}{\Gamma(5/4)}\left(\frac{2\mu C_6}{\hbar^2}\right)^{\frac{1}{4}}$$

vdW Energy $\bar{E} = \frac{\hbar^2}{2\mu\bar{a}^2}$

Fig. 2, PSJ Faraday Disc 142, 361 (2009)

# Collision of two atoms

Separate center of mass $R_{CM}$ and relative $R$ motion with reduced mass $\mu$.

Expand $\Psi(R,E)$ in relative angular momentum basis *lm*.
$l$ = 0, 1, 2… *s*–, *p*–, *d*–waves, …

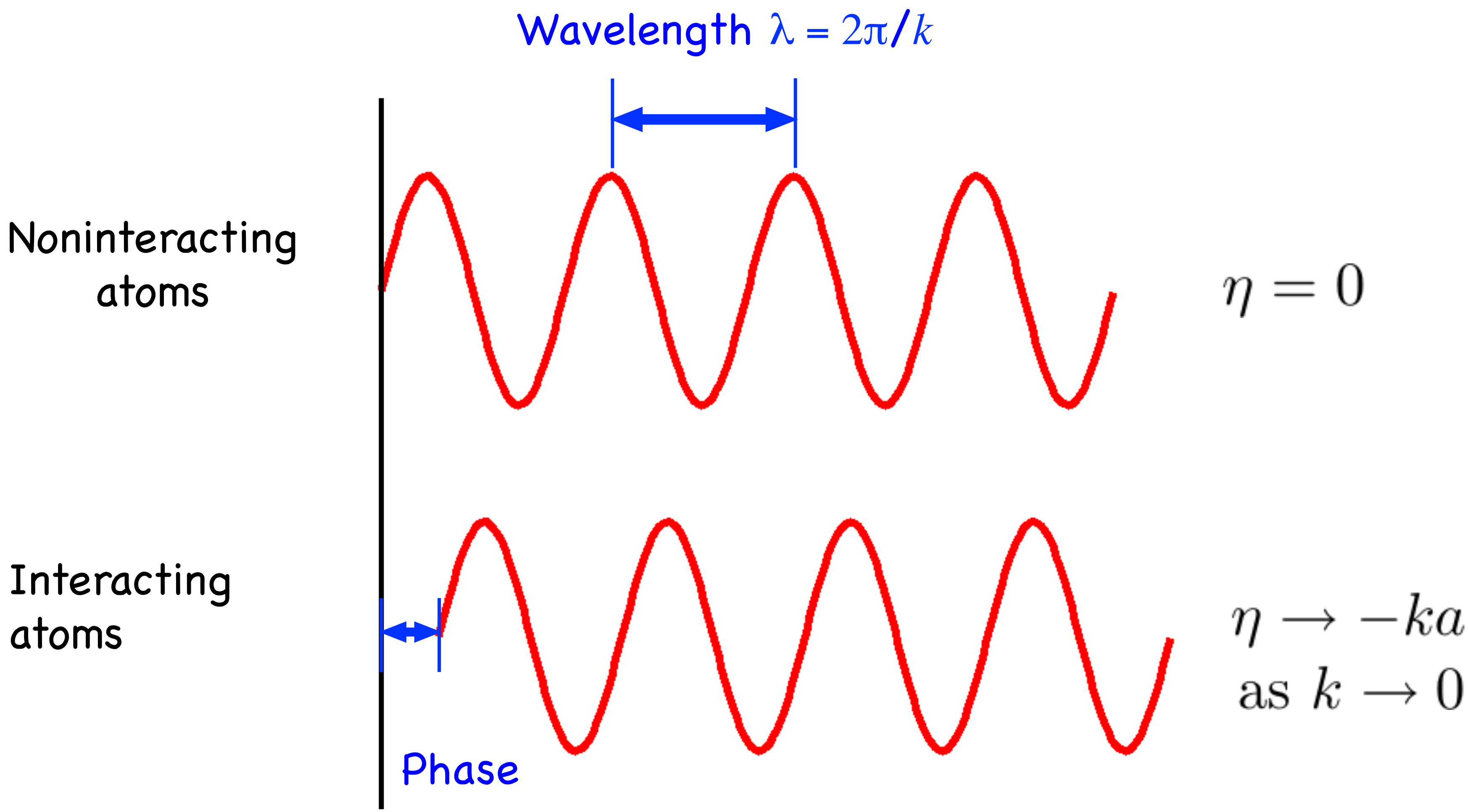
Potential energy: $V(R) + \frac{\hbar\ell(\ell+1)}{2\mu R^2}$ --> phase shift $\eta_\ell(E)$

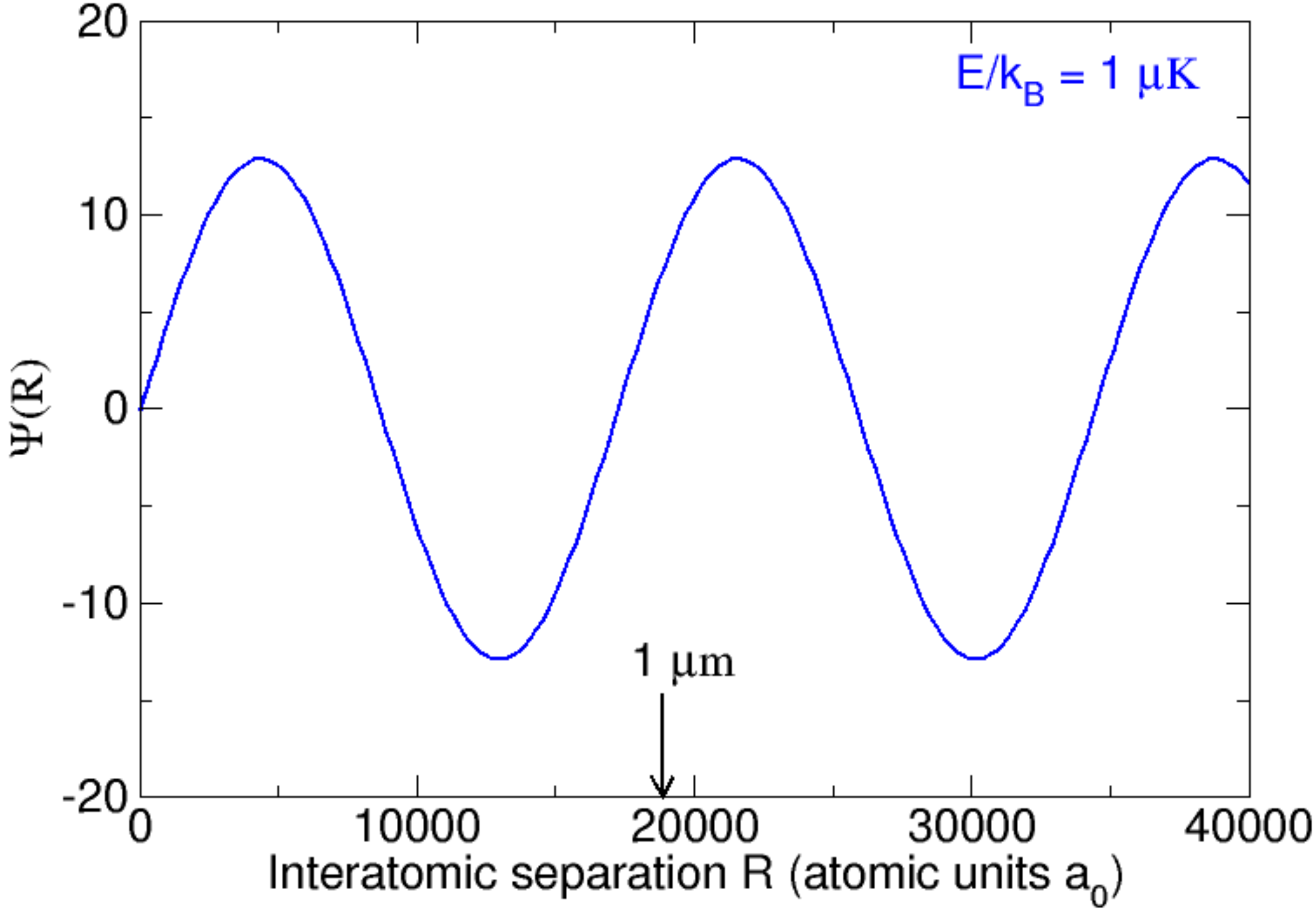
Neutral atoms (S-state): $V(R)$ --> $-C_6/R^6$ van der Waals

Solve Schrödinger equation for bound and scattering $\Psi(R,E)$
--> bound states $E_n$
--> scattering phases, amplitudes

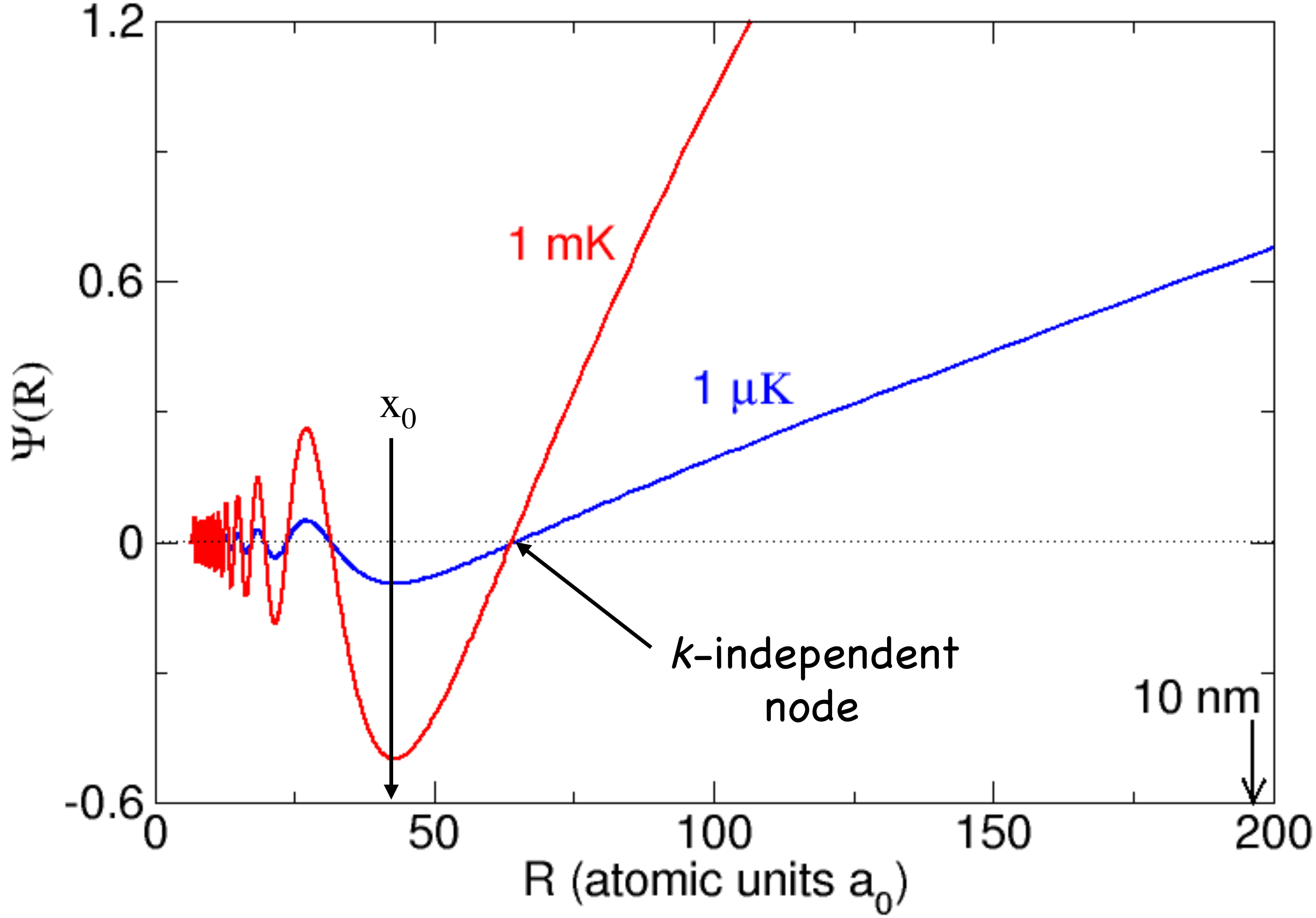
# S-wave scattering phase shift

$$\Psi(R) \to \sin(kR + \eta)$$





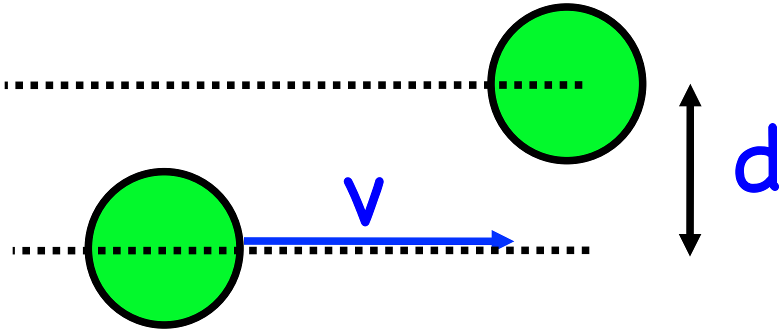
E/k_B = 1 μK
Ψ(R)
20
10
0
-10
-20
1 μm
0
10000
20000
30000
40000
Interatomic separation R (atomic units a_0)



1.2
0.6
0
-0.6
0
50
100
150
200
Ψ(R)
R (atomic units a0)
1 mK
1 μK
x0
k-independent
node
10 nm

# Cross section $\sigma$

Classical balls with distance of closest approach d (diameter)




define an area with $\sigma = \pi d^2$ ($10^{-12}$ cm$^2$)

and a collision rate $\Gamma = nv\sigma$ (typical: 1 $s^{-1}$, MOT

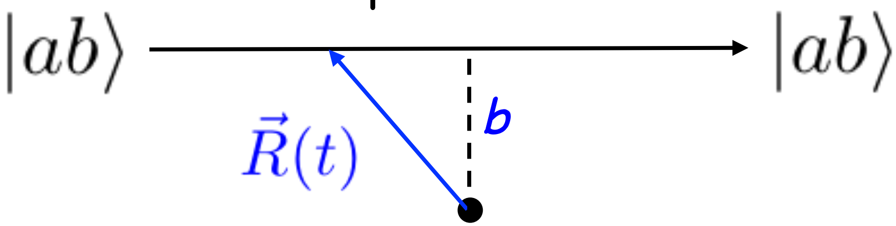
$10^4$ $s^{-1}$, BEC)

Rate constant $K = v\,\sigma$

Time between collisions = $1/\Gamma$ = $1/(Kn) \approx$ 1s (MOT), 100$\mu$s (BEC)

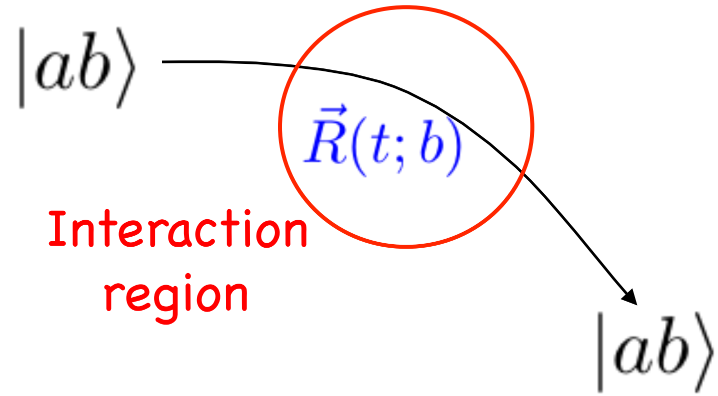
# Classical picture

## No potential

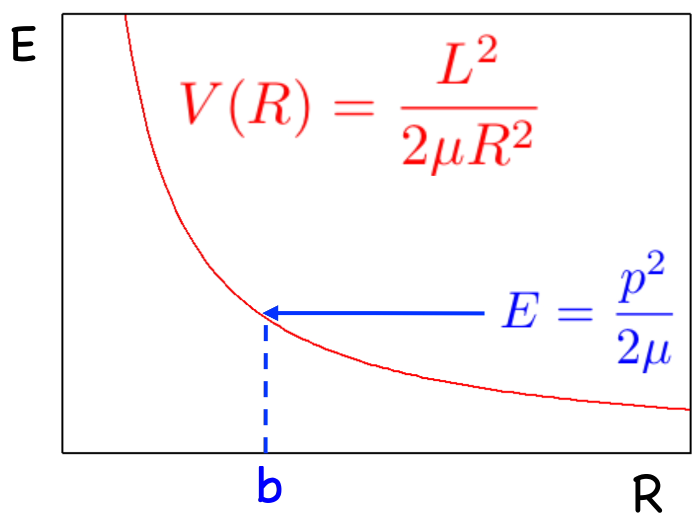



$$|\vec{L}| = |\vec{R} \times \vec{p}| = bp$$

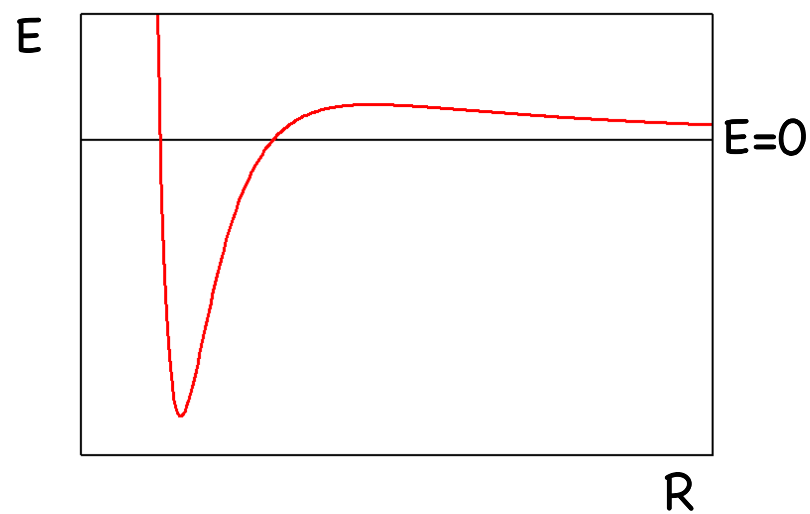
## With potential



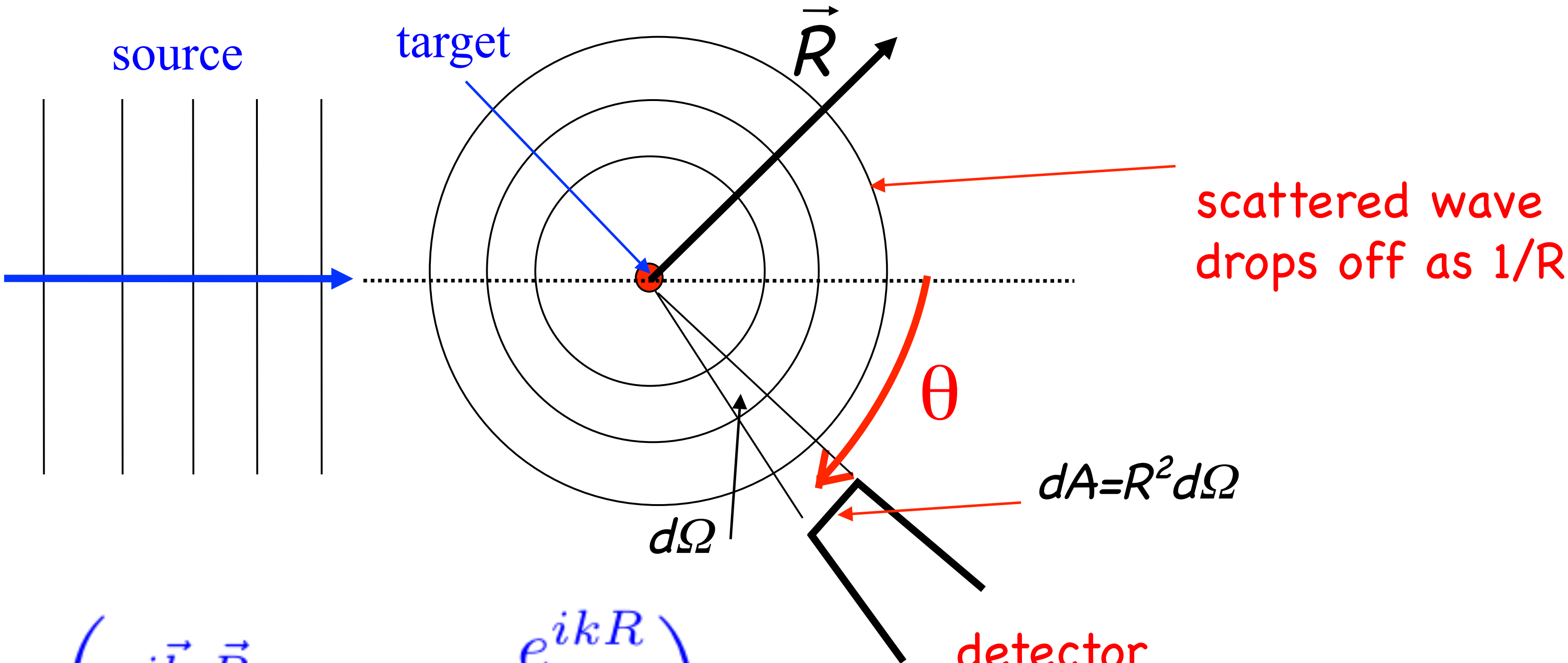

## Centrifugal potential




## Centrifugal barrier



# Quantum scattering theory




$$\left(e^{i\vec{k}\cdot\vec{R}} + f(\Omega)\frac{e^{ikR}}{R}\right)|ab\rangle$$

cross section

$$\frac{\text{atoms/s scattered flux into } d\Omega}{\text{plane wave flux (atoms/cm}^2\text{/s)}} = \frac{d\sigma}{d\Omega} = |f(\Omega)|^2$$

$$\sigma_{\text{tot}} = \int d\Omega \frac{d\sigma}{d\Omega}$$

# Partial wave expansion

Expansion of a plane wave (Messiah, Quantum Mechanics, Vol.1, Appendix B.III):

$$e^{i\vec{k}\cdot\vec{R}} = 4\pi \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} i^{\ell} Y^{*}_{\ell m}(\hat{k}) Y_{\ell,m}(\hat{R}) j_{\ell}(kR)$$

Geometric

Dynamic

solution to the radial Schrödinger equation
for the centrifugal potential

At large R: $\sin\left(kR - \pi\ell/2\right)$

When V(R) is present: $\sin\left(kR - \pi\ell/2 + \eta_{\ell}(E)\right)$

# How do we get the S-matrix, or bound states?

Coupled channels expansion:

$$\Psi_\alpha(R,E) = \sum_{\alpha'} \frac{\phi^{+}_{\alpha',\alpha}(R,E)}{R} |\alpha'\rangle$$

Solve matrix Schrödinger equation $\mathbf{H}\boldsymbol{\Psi}(R,E) = E\boldsymbol{\Psi}(R,E)$

Extract **S** from solution at large R >> $R_{vdW}$

Potential matrix $V_{\alpha\alpha'}(R)$

$M_{\text{tot}} = M_1 + M_2 + m_\ell$ conserved

Electronic (Born-Oppenheimer) V(R) does not change $\ell$

Anisotropic (spin-dependent) potential changes $\ell$

# Collision cross section

Solve Schrödinger equation for each $\ell$

Get phase shift $\eta_\ell(E)$

$$\sigma(E) = \frac{4\pi}{k^2} \sum_\ell (2\ell + 1) \sin^2 \eta_\ell(E)$$

Identical bosons: even $\ell$
Identical fermions: odd $\ell$
Nonidentical species: all $\ell$

van der Waals potential:

$$\eta_\ell(E) \to -Ak \quad s\text{−wave as } k \to 0$$

$$\eta_\ell(E) \to -(A_1 k)^3 \quad p\text{−wave as } k \to 0$$

$$\eta_\ell(E) \propto k^4 \quad d\text{−wave and higher as } k \to 0$$

# S-wave collision summary

If only a single s-wave channel, $S_{\alpha\alpha} = e^{2i\eta} \rightarrow e^{-2ika}$ as $k \rightarrow 0$

If inelastic channels $\alpha' \neq \alpha$ exist, unitarity ensures

$$|S_{\alpha\alpha}|^2 = 1 - \sum_{\alpha' \neq \alpha} |S_{\alpha\alpha'}|^2 = 1 - 4kb_\alpha$$

Thus, $$S_{\alpha\alpha} = e^{-2ik(a-ib)} \text{ as } k \rightarrow 0$$

Complex scattering length *a-ib*

$$\sigma_{\alpha\alpha} = 4\pi(a^2 + b^2)$$

$$K_{\text{loss}} = \sum_{\alpha' \neq \alpha} K_{\alpha\alpha'} = 2\frac{h}{\mu} b$$

# Collision cross sections and rate constants

## Scattering channels

Start in initial channel: $\{ab\}\ell m = \alpha$

Exit in final channel: $\{a'b'\}\ell' m' = \alpha'$

Transition amplitudes $T_{\alpha\alpha'} = \delta_{\alpha\alpha'} - S_{\alpha\alpha'}$ are expressed in terms of the elements of the unitary S-matrix

Elastic cross section $$\sigma_{\alpha}^{\mathrm{el}}(E) = \frac{\pi}{k^2}|1 - S_{\alpha\alpha}(E)|^2$$

Inelastic cross section $$\sigma_{\alpha}^{\mathrm{loss}}(E) = \frac{\pi}{k^2}\left(1 - |S_{\alpha\alpha}(E)|^2\right)$$

$$\sum_{\alpha' \neq \alpha} |S_{\alpha\alpha'}(E)|^2 = 1 - |S_{\alpha\alpha}(E)|^2$$ since S in unitary.

Sum over all contributing $\ell m$ for a given $\{ab\}$ to get the TOTAL (observable) cross section.

# Rate constants

Loss rate constant

$$K_\alpha^{\rm loss}(E) = v\sigma_\alpha^{\rm loss}(E) = 2\frac{h}{\mu} b_\alpha(E)$$

where $b_\alpha(E) = \dfrac{1-|S_{\alpha\alpha}(E)|^2}{k}$ has units of length.

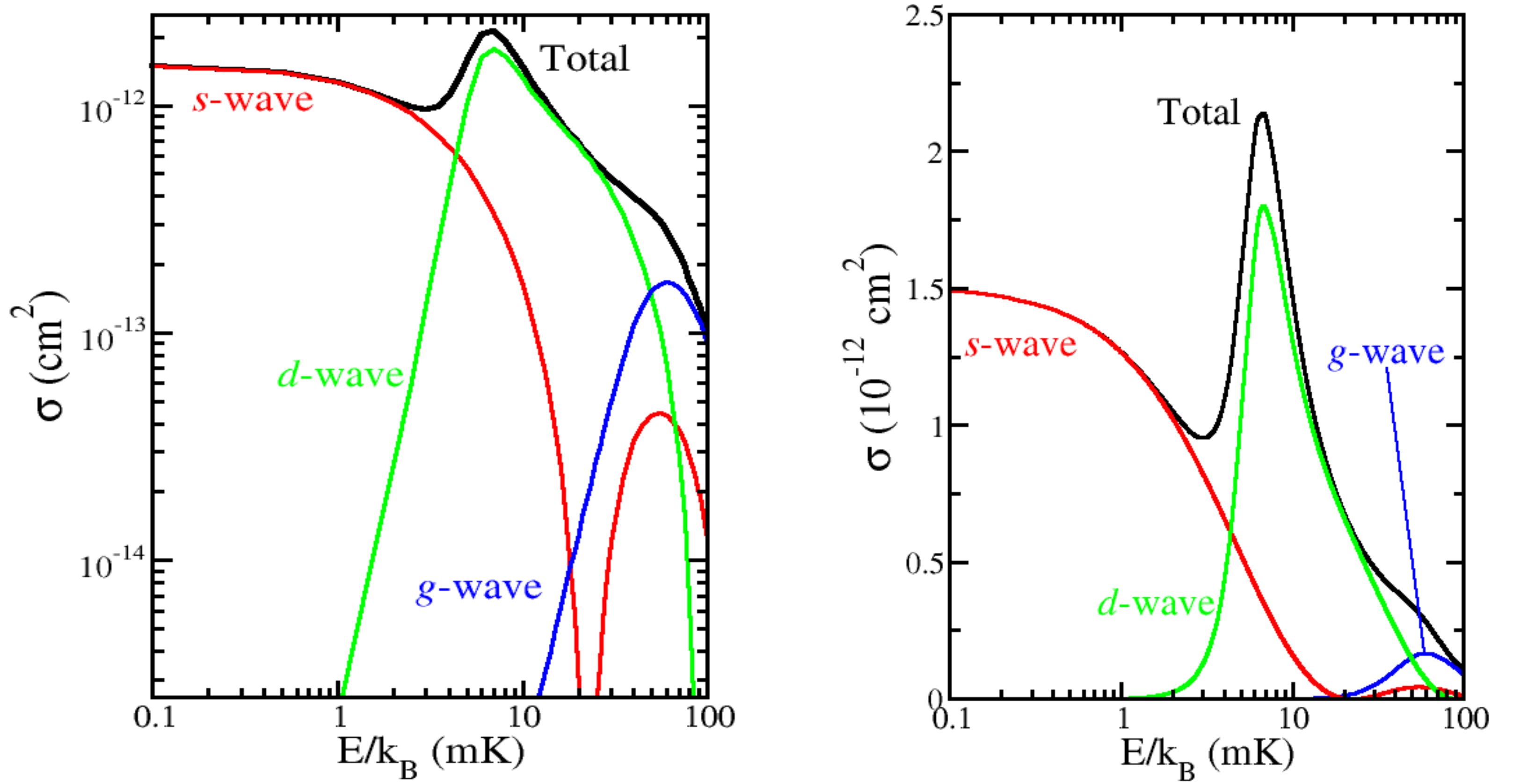
$$K_\alpha^{\rm loss}(E) = 2\frac{h}{\mu} b_\alpha(E) = 4.2\times 10^{-11}\,{\rm cm}^3/{\rm s}\frac{b[a_0]}{\mu[{\rm amu}]}$$

Typical values: "Allowed" b ~ 10-100 $a_0$

"Forbidden" b << 1 $a_0$

Upper bound $4b = k^{-1} = \lambda/2\pi$

# Elastic cross section σ for like Na atoms



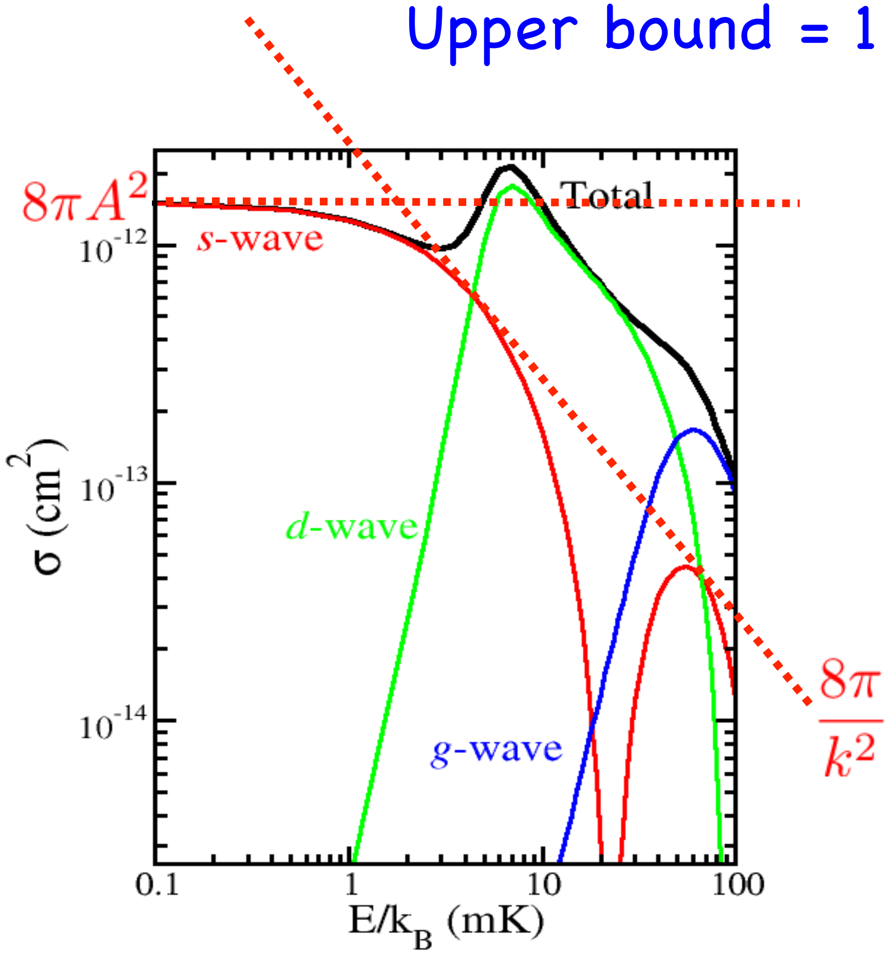

Many (even) partial waves

# Threshold properties (elastic only)

$$\sigma(E) = \frac{4\pi}{k^2}\sin^2(kA) = 4\pi A^2 \text{ as } k \to 0$$

$$(8\pi A^2 \text{ for identical bosons})$$

Upper bound = 1 (unitarity limit)




$$\sigma(E) = \frac{4\pi}{k^2} \propto \frac{1}{E}$$

$$= 4\pi\left(\frac{\lambda}{2\pi}\right)^2$$

# Atomic and molecular collision rates

Standard chemical form from Mies, J. Chem. Phys. 51, 787, 798 (1969); see arXiv:0812.1233

$$\frac{dn_a}{dt} = -K n_a n_b = -\frac{1}{\tau} n_a \quad \text{where} \quad \frac{1}{\tau} = K n_b$$

$$K = \frac{1}{Q_T}\frac{k_B T}{h} f_D \quad \text{where} \quad \frac{1}{Q_T} = \Lambda_T^3 = \left(\frac{h}{2\pi\mu k_B T}\right)^{\frac{3}{2}}$$

$Q_T$ = translational partition function

$\Lambda_T$ = thermal de Broglie wavelength of pair

$$f_D = \int_0^\infty \sum_{\ell m} \left(1 - |S(\ell m)|^2\right) e^{-E/(k_B T)} dE/(k_B T)$$

$$\frac{1}{\tau} = K n_b = (n\Lambda_T^3)\frac{k_B T}{h} f_D$$

Phase Space density | Time scale | Dynamics

Replace $1 - |S(\ell m)|^2$ by $|1 - S(\ell m)|^2$ for elastic collisions

# Van der Waals potentials

# Van der Waals potential

Write the Schrödinger equation in length and energy units of

$$R_{\rm vdw} = \frac{1}{2}\left(\frac{2\mu C_6}{\hbar^2}\right)^{\frac{1}{4}}$$

or

$$\bar{a} = \frac{\Gamma(3/4)}{\Gamma(5/4)} R_{\rm vdW} = 0.956 R_{\rm vdW}$$

Gribakin and Flambaum, Phys. Rev. A 48, 546 (1993)

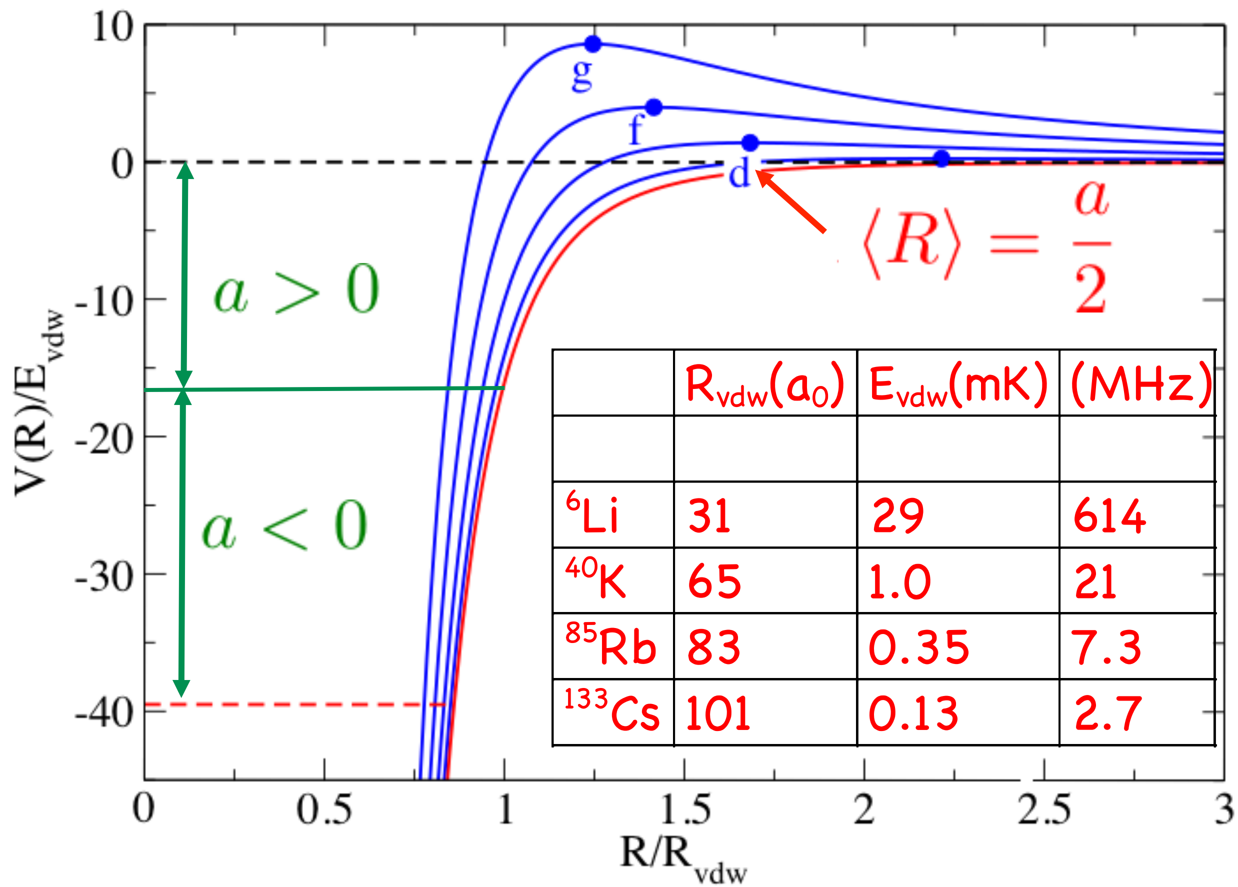
$$E_{\rm vdw} = \frac{\hbar^2}{2\mu R_{\rm vdw}^2}$$

The potential becomes $-\frac{16}{r^6} + \frac{\ell(\ell+1)}{r^2}$ in vdw units.

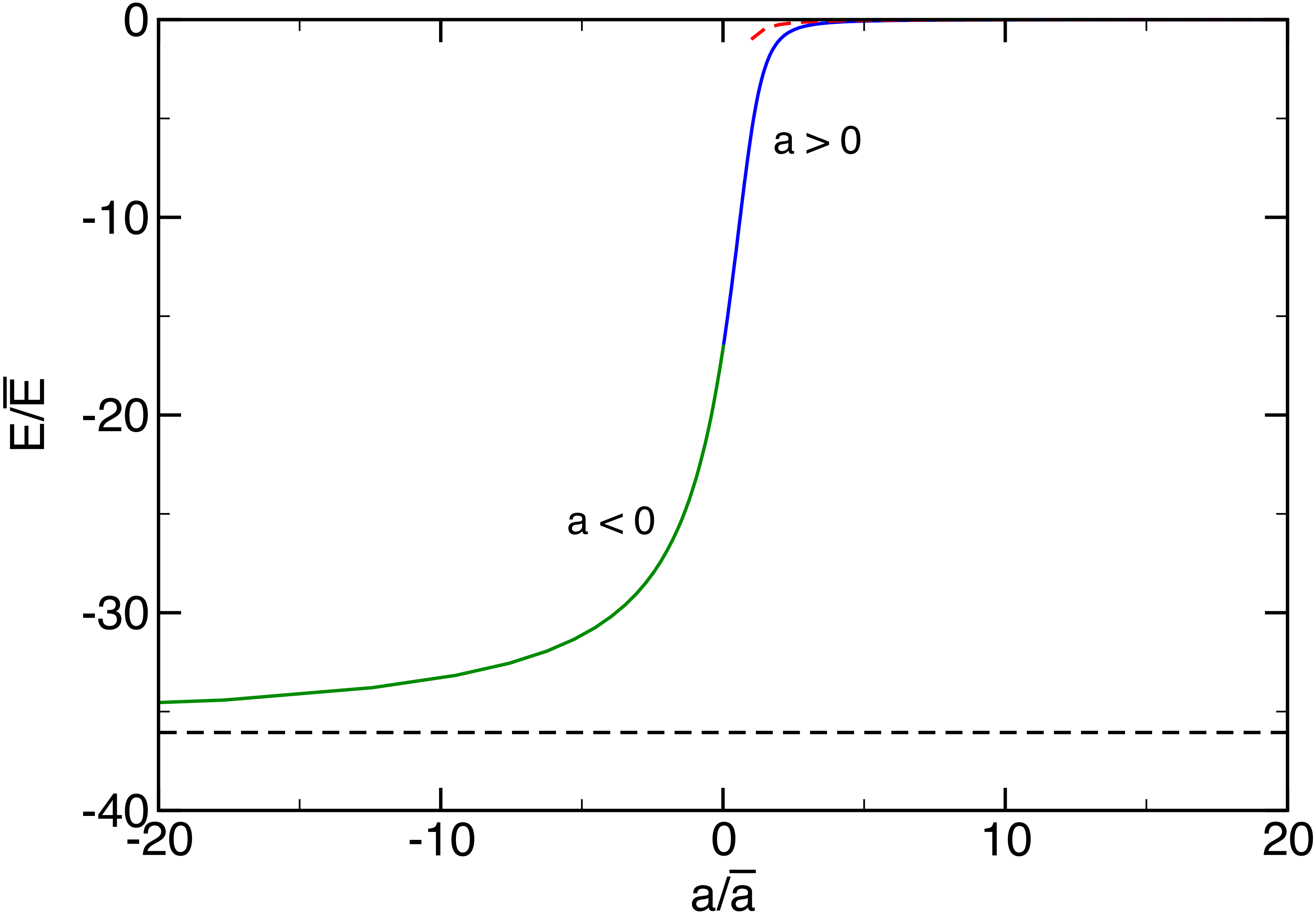
This potential has exact analytic solutions and many useful properties.

B. Gao, Phys. Rev. A 58, 1728, 4222 (1998) + series of papers.
See Jones, et al., Rev. Mod. Phys. 78, 483 (2006)
and Chin et al., Feshbach review

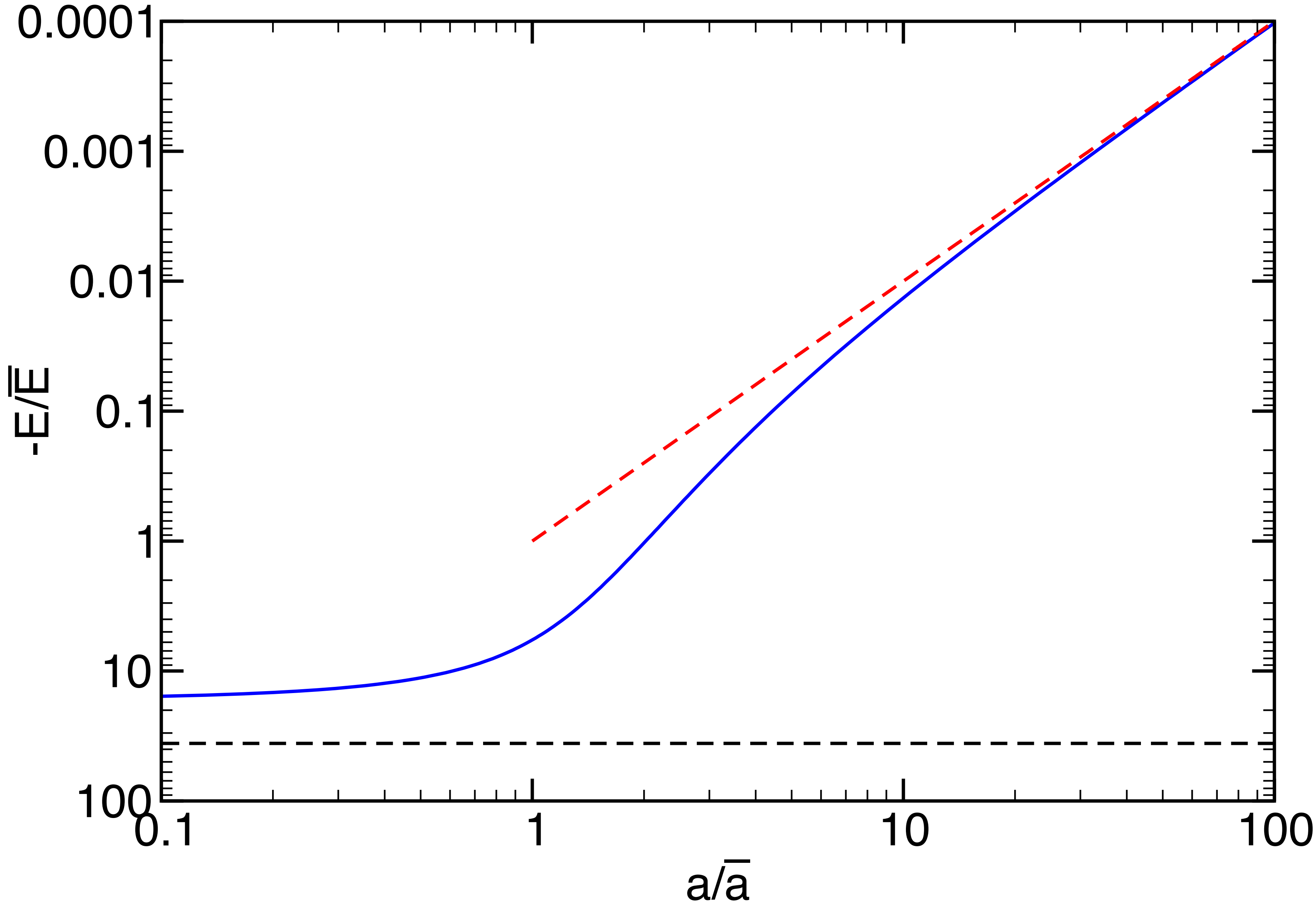
# “Size” of vdW potential

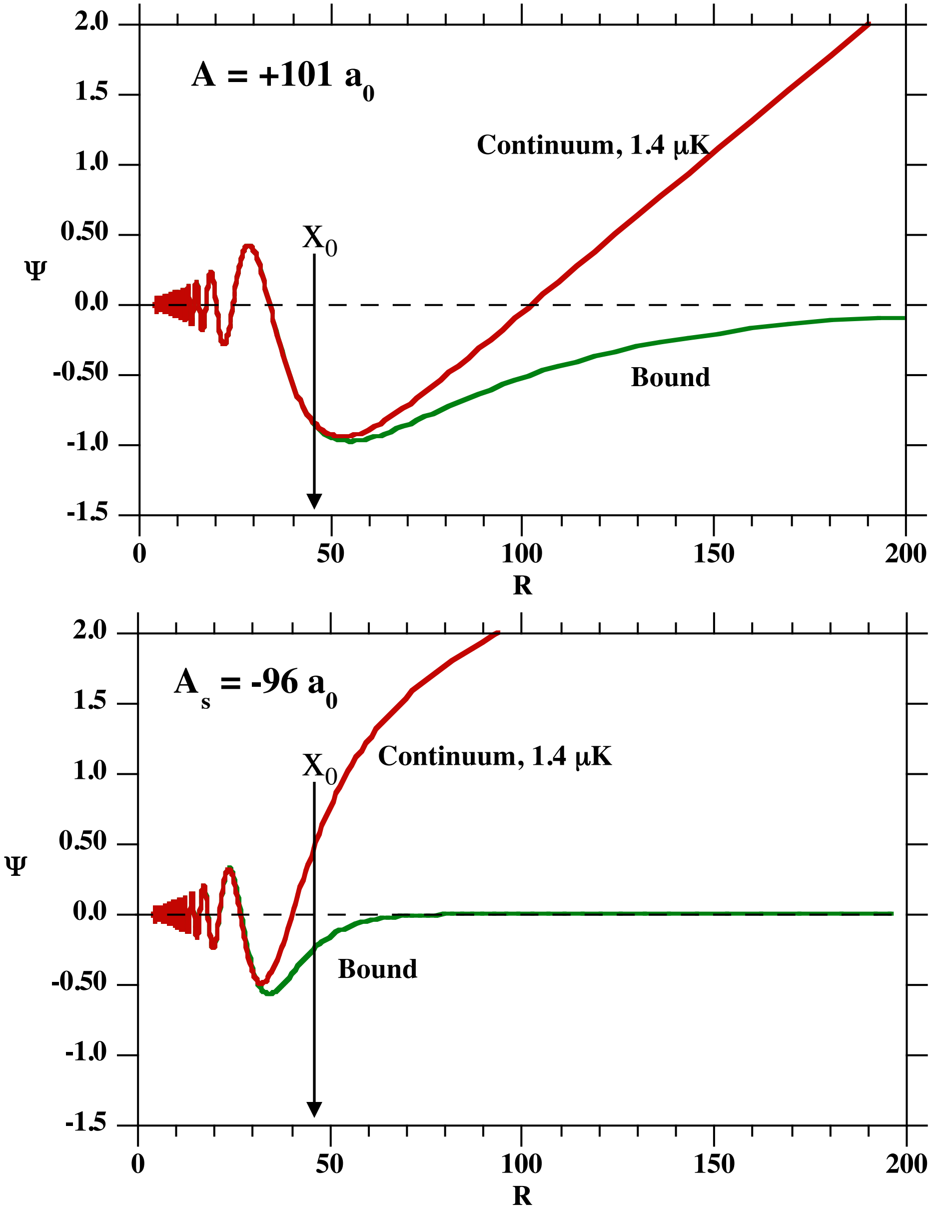



| | $R_{vdw}(a_0)$ | $E_{vdw}$(mK) | (MHz) |
|---|---|---|---|
| | | | |
| $^{6}$Li | 31 | 29 | 614 |
| $^{40}$K | 65 | 1.0 | 21 |
| $^{85}$Rb | 83 | 0.35 | 7.3 |
| $^{133}$Cs | 101 | 0.13 | 2.7 |



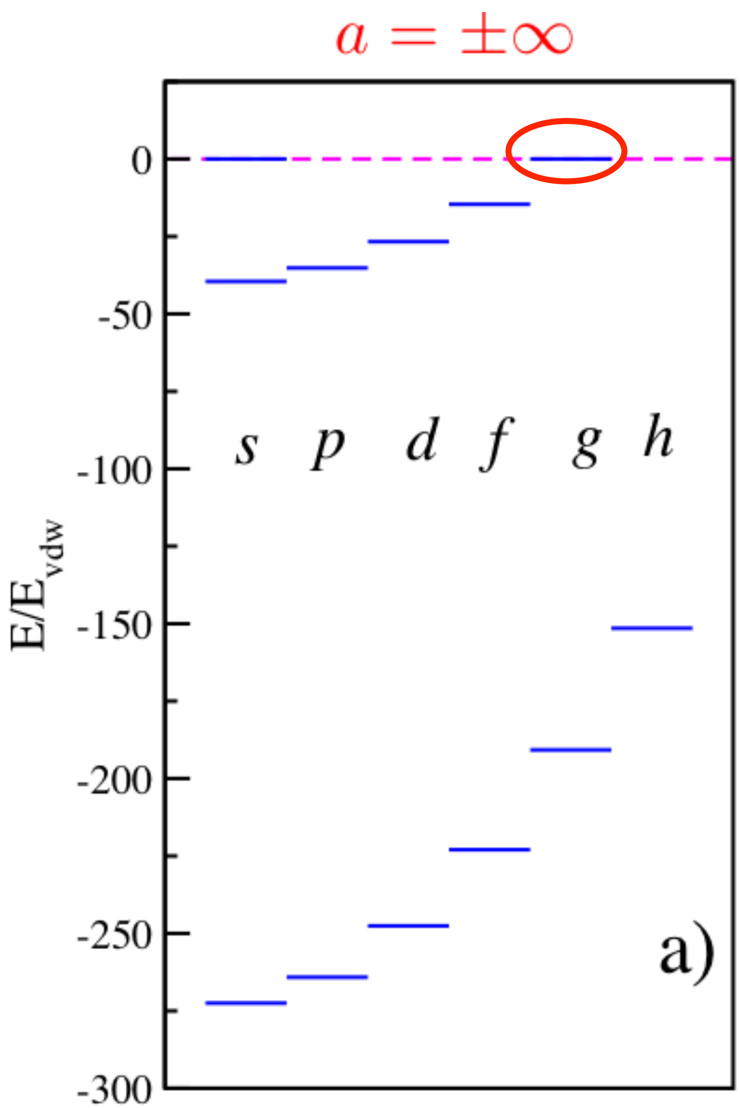
0
-10
-20
-30
-40
-20
-10
0
10
20
E/E̅
a/a̅
a > 0
a < 0


0.0001
0.001
0.01
0.1
1
10
100
-E/Ē
0.1
1
10
100
a/ā




Scattering and last bound state near threshold (normalized to same value at small R)

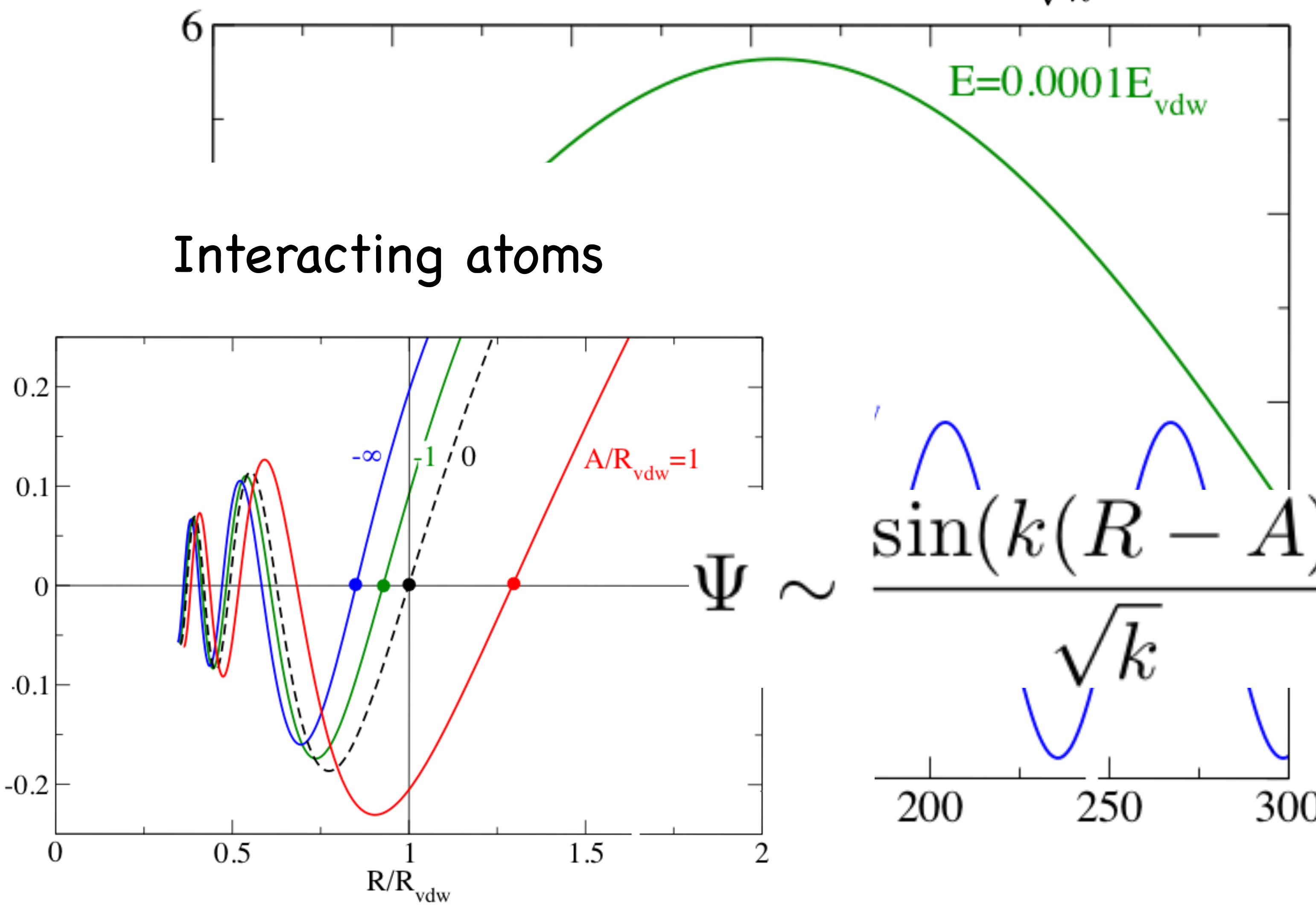
# Bound states from van der Waals theory




Adapted from Gao, Phys. Rev. A 62, 050702 (2000); Figure from Chin et al., review

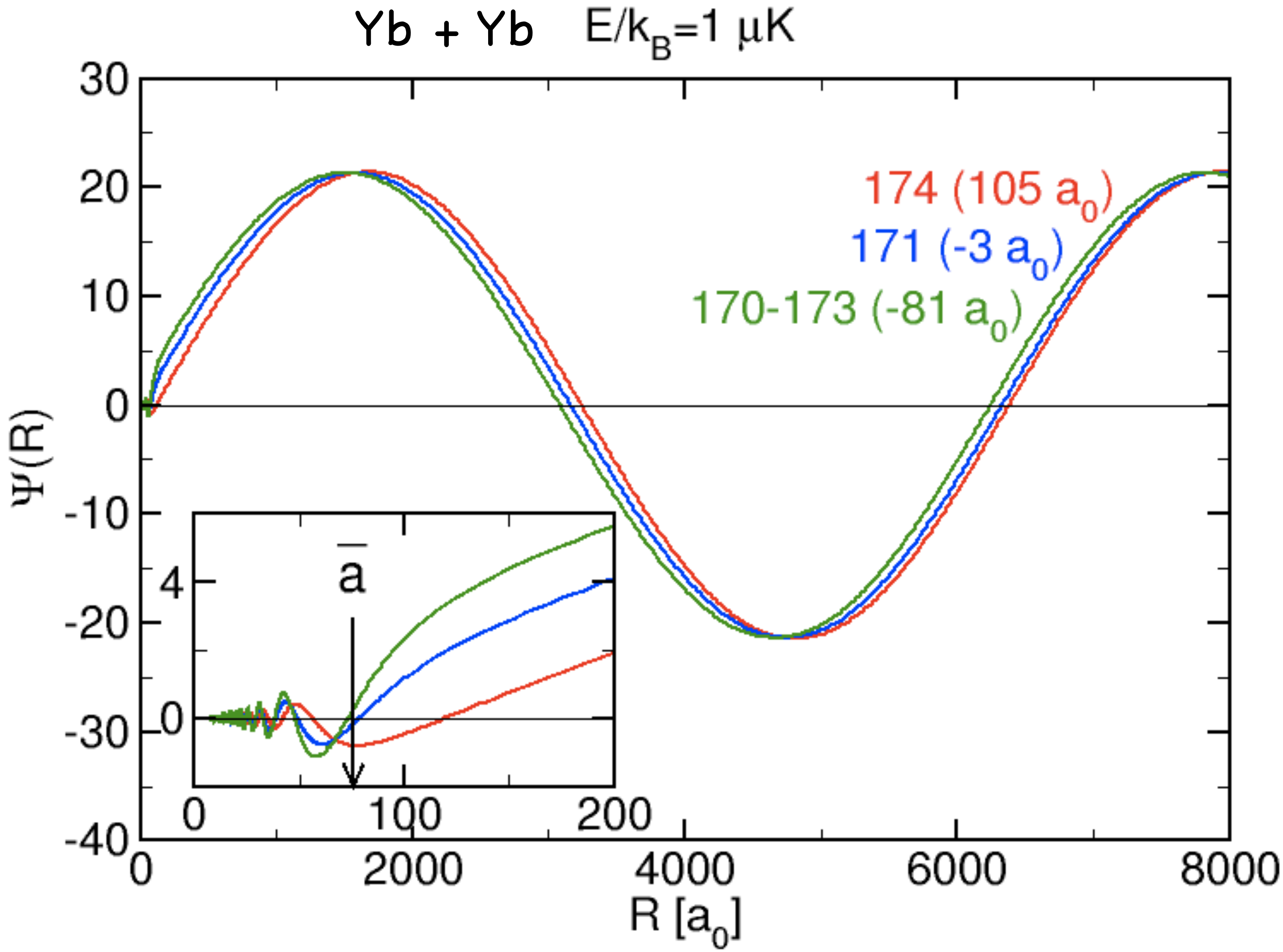
Noninteracting atoms $\Psi \sim \frac{\sin(kR)}{\sqrt{k}}$

Interacting atoms




$$\Psi \sim \frac{\sin(k(R-A))}{\sqrt{k}}$$

From Jones, Lett, Tiesinga, Julienne, Rev. Mod. Phys. 78, 483 (2006).



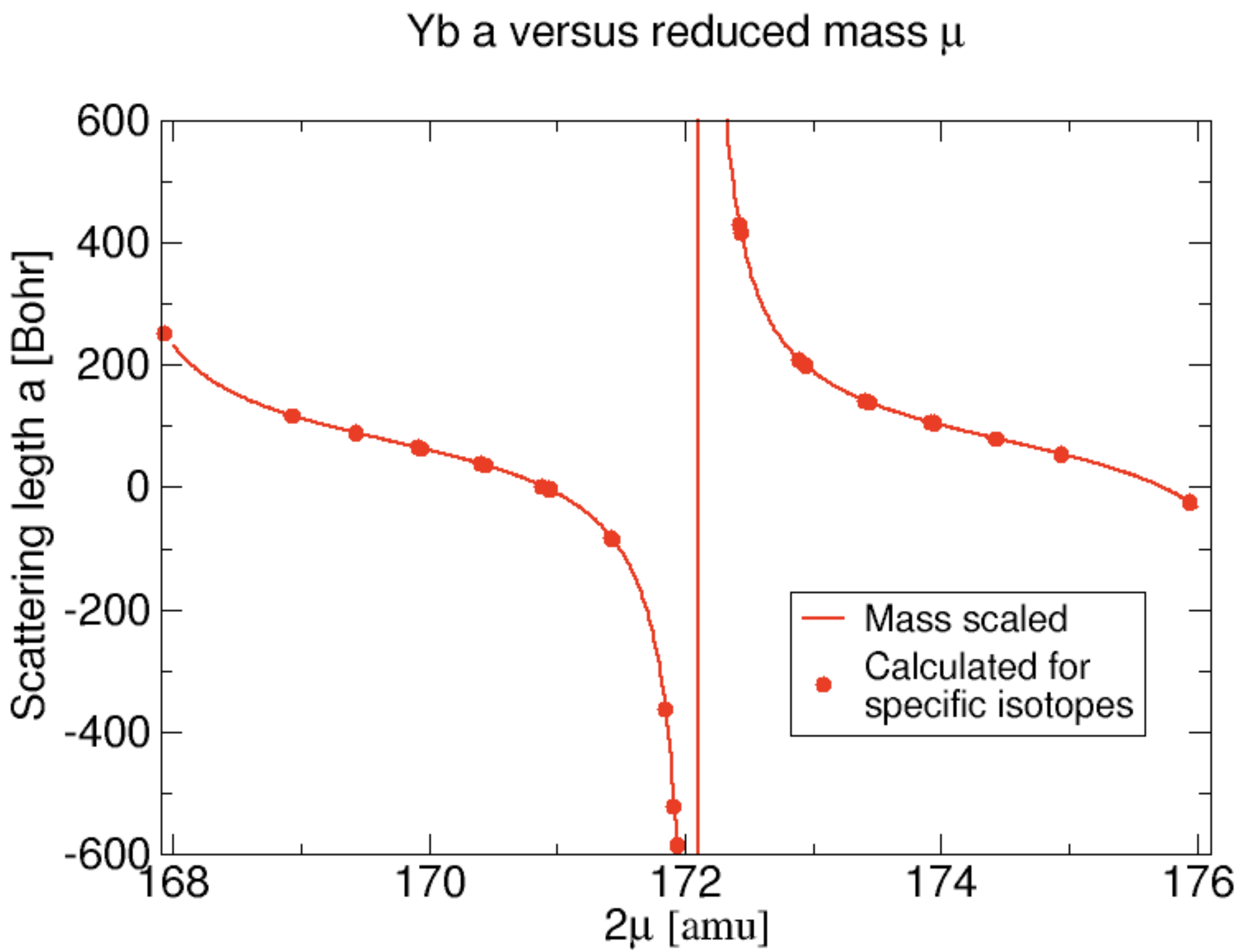

100 nm 200 nm 300 nm

From arXiv:0902.1727

## Scattering lengths for Yb ground state model (in $a_0$ units)

| | 168 | 170 | 171 | 172 | 173 | 174 | 176 |
|---|---|---|---|---|---|---|---|
| 168 | 252(6) | 117(1) | 89(1) | 65(1) | 39(1) | 2(2) | −360(30) |
| 170 | 117 | 64(1) | 37(1) | −2(2) | −81(4) | −520(50) | 209(4) |
| 171 | 89 | 37 | −3(2) | −84(5) | −580(60) | 430(20) | 142(2) |
| 172 | 65 | −2 | −84 | −600(60) | 420(20) | 201(3) | 106(1) |
| 173 | 39 | −81 | −580 | 420 | 199(3) | 139(2) | 80(1) |
| 174 | 2 | −520 | 430 | 201 | 139 | 105(1) | 55(1) |
| 176 | −360 | 209 | 142 | 106 | 80 | 55 | −24(2) |

M. Kitagawa, K. Enomoto, K. Kasa, Y. Takahashi, R. Ciurylo, P. Naidon, P. Julienne, Phys. Rev. A 77, 012719 (2008)


Yb a versus reduced mass μ
Scattering legth a [Bohr]
600
400
200
0
-200
-400
-600
168
170
172
174
176
2μ [amu]
Mass scaled
Calculated for specific isotopes

# Gribakin and Flambaum

Phys. Rev. A 48, 546 (1993)

$$a = \bar{a}\left(1 - \tan\left(\Phi - \frac{\pi}{8}\right)\right)$$

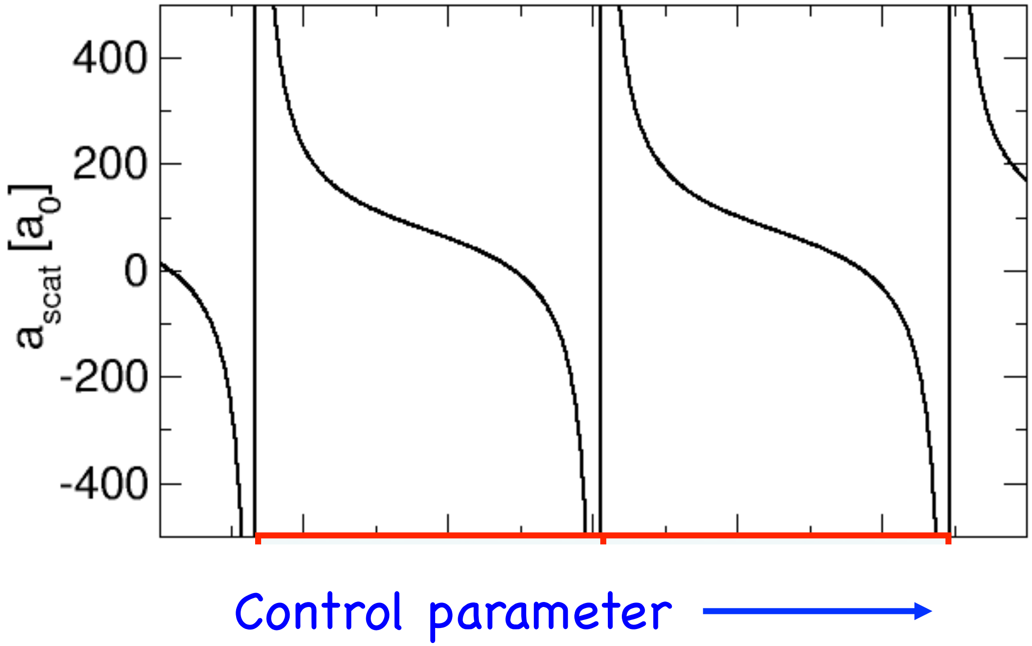
$$\Phi = \int_{r_{in}}^{\infty} \left(\frac{2\mu}{\hbar^2}(-V(R)\right)^{1/2} dR$$

Number of bound states in V(R) = $\mathrm{Int}\left[\frac{\Phi}{\pi} - \frac{5}{8}\right] + 1$

## Simplest model:

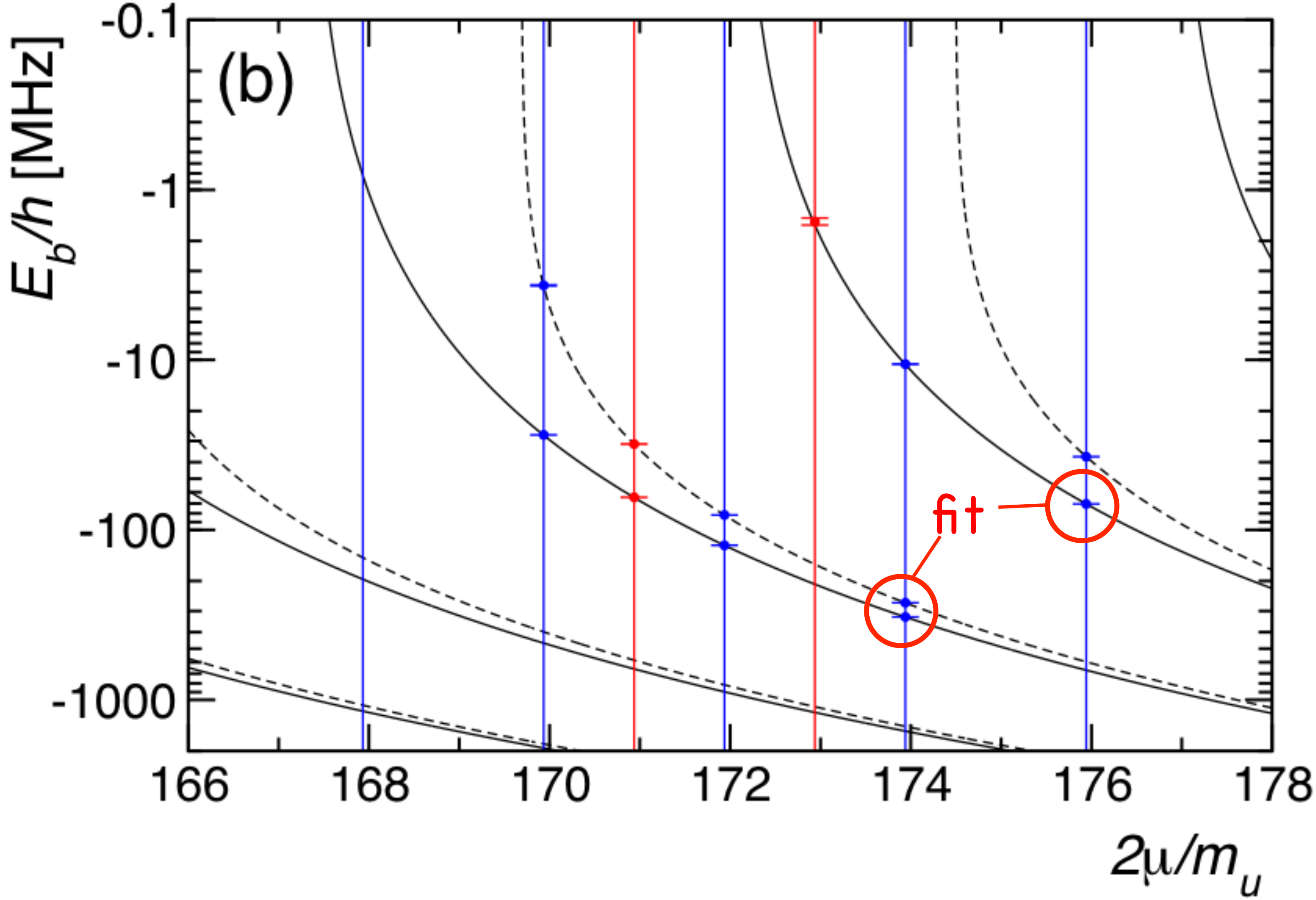
$$V(R) = -\frac{C_6}{R^6} \text{ for } R_{in} < R \leq \infty$$

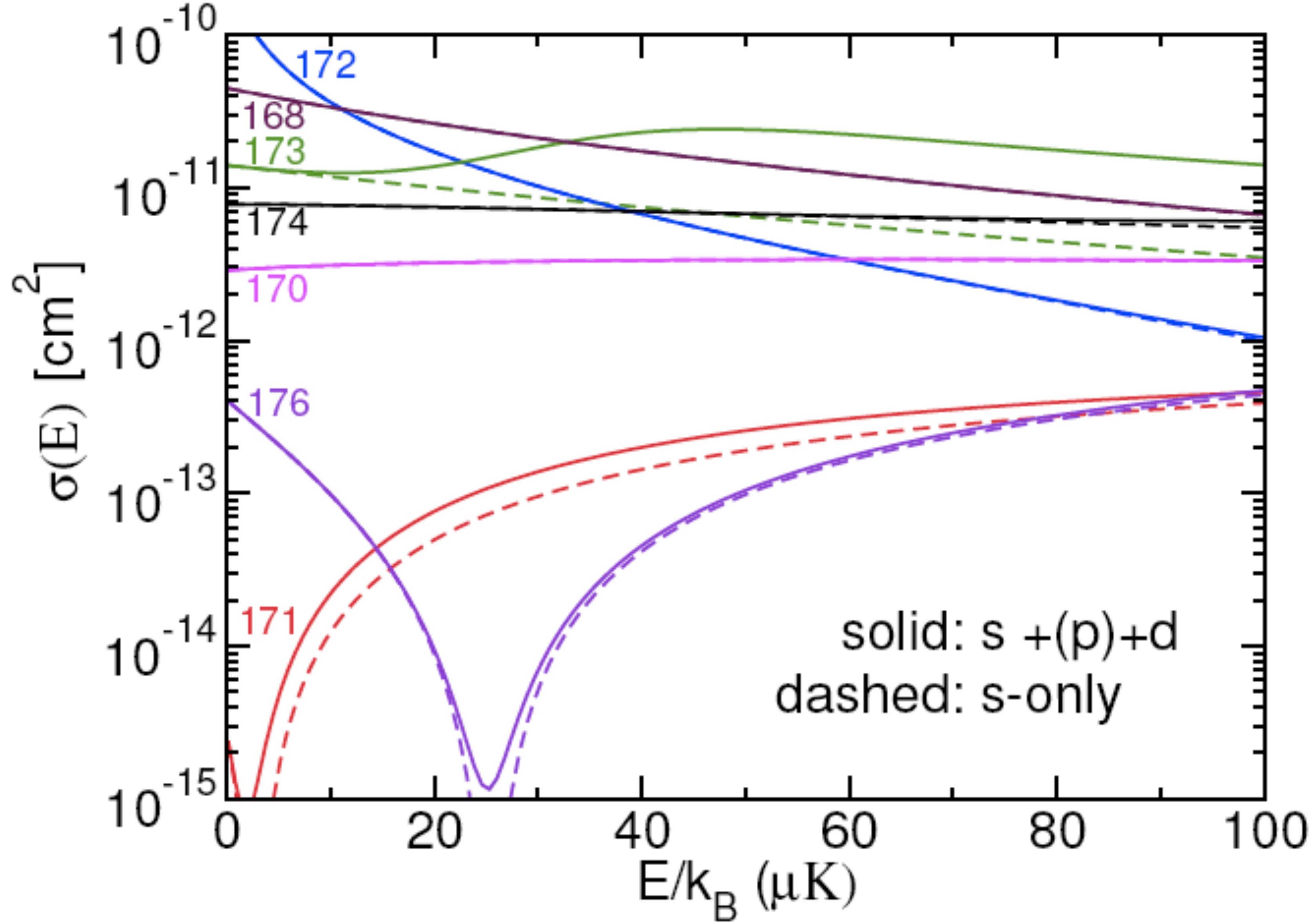
$$V(R) = \infty \text{ for } 0 < R \leq R_{in}$$



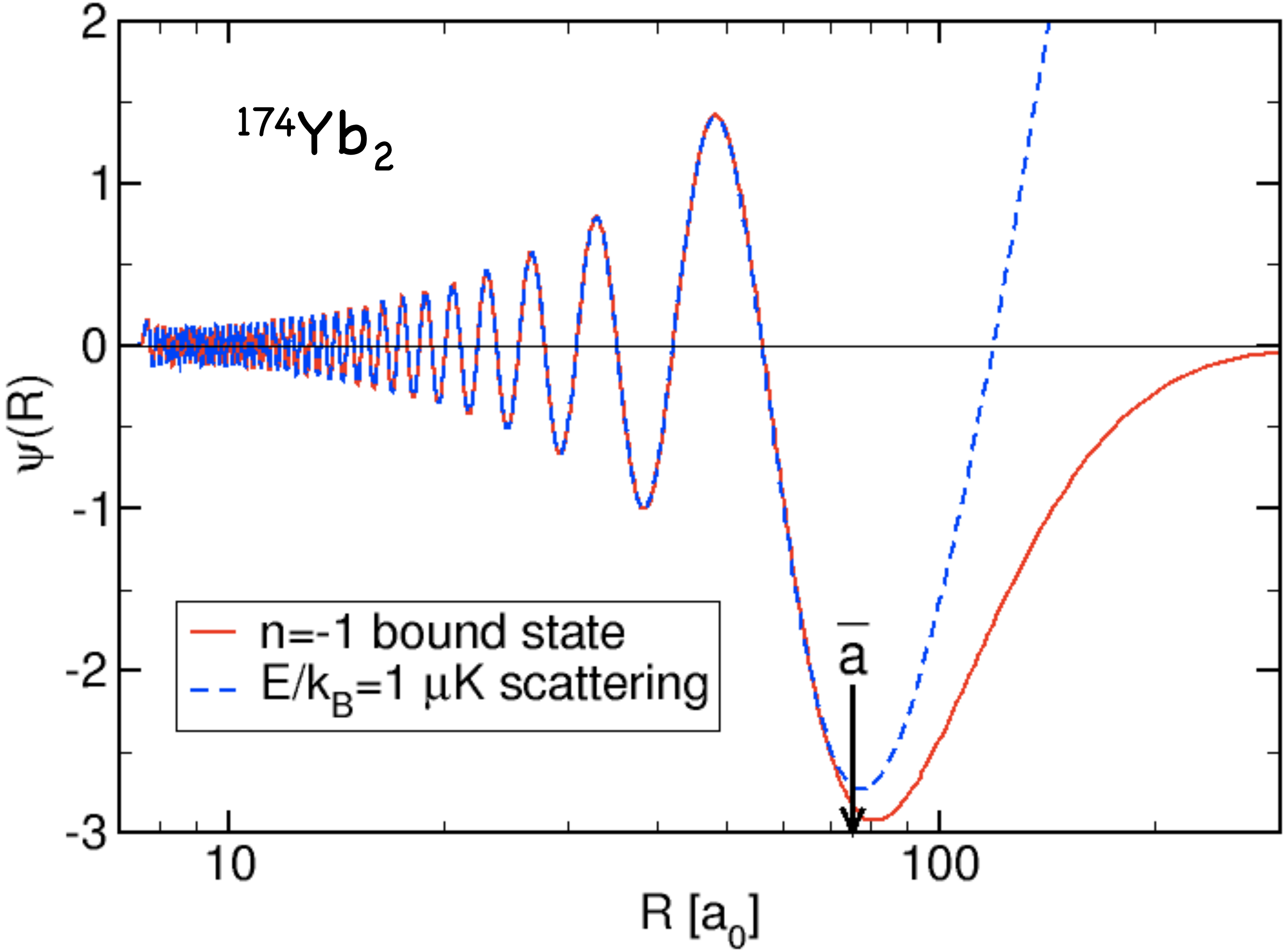
400
200
0
-200
-400
$a_{scat}$ [$a_0$]
Control parameter

Last bound state energies versus mass
Solid: J=0
Dashed: J=2





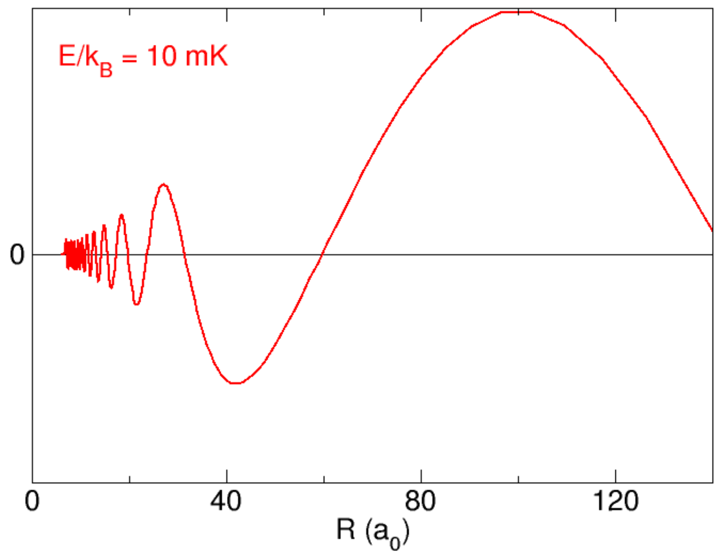
$10^{-10}$
$10^{-11}$
$10^{-12}$
$10^{-13}$
$10^{-14}$
$10^{-15}$
0
20
40
60
80
100
$E/k_B$ ($\mu$K)
$\sigma(E)$ [$cm^2$]
172
168
173
174
170
176
171
solid: s +(p)+d
dashed: s-only



$^{174}Yb_2$
n=-1 bound state
E/k$_B$=1 μK scattering
$\bar{a}$
ψ(R)
R [a$_0$]
2
1
0
-1
-2
-3
10
100

# Semiclassical considerations

WKB phase-amplitude form: $$\phi^{\mathrm{WKB}}(R,E) = \alpha(R,E)\,\sin\beta(R,E)$$




$$\alpha(R,E) = \frac{1}{k(R,E)^{\frac{1}{2}}}$$

$$\beta(R,E) = \int_{R_t}^{R} k(R',E)\,dR' + \frac{\pi}{4}$$

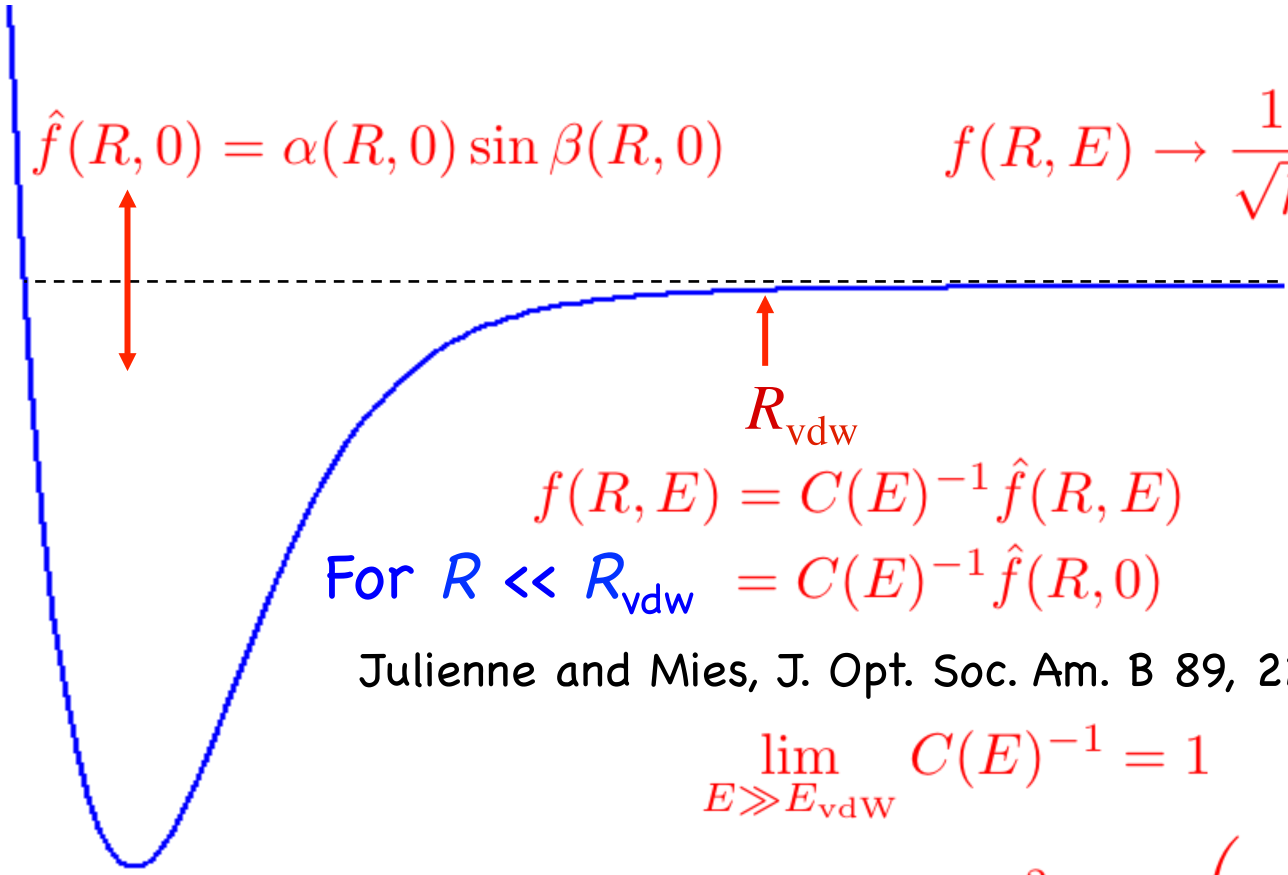
$$k(R,E) = \left(\frac{2\mu}{\hbar^2}(E - V(R))\right)^{\frac{1}{2}} = \frac{2\pi}{\lambda(R,E)}$$

Validity criterion: $$\frac{d\lambda(R,E)}{dR} \ll 1$$

# Semiclassical considerations continued

$$\hat{f}(R,0) = \alpha(R,0)\sin\beta(R,0)$$

$$f(R,E) \to \frac{1}{\sqrt{k}}\sin(kR+\eta(E))$$




$$f(R,E) = C(E)^{-1}\hat{f}(R,E)$$

For $R \ll R_{\mathrm{vdw}}$ $$= C(E)^{-1}\hat{f}(R,0)$$

Julienne and Mies, J. Opt. Soc. Am. B 89, 2257 (1989)

$$\lim_{E \gg E_{\mathrm{vdW}}} C(E)^{-1} = 1$$

$$\lim_{E\to 0} C(E)^{-2} = k\bar{a}\left(1+\left(\frac{a}{\bar{a}}-1\right)^2\right)$$

$$\alpha(R,E) = \frac{1}{k(R,E)^{\frac{1}{2}}}$$

$$\beta(R,E) = \int_{R_t}^{R} k(R',E)dR' + \frac{\pi}{4}$$

The End

ICAP 2014 Summer School, July 31, Williamsburg

# Understanding cold atomic and molecular collisions
# 1. Feshbach resonances

Paul S. Julienne

Joint Quantum Institute
NIST and The University of Maryland

Thanks to many colleagues in theory and experiment who have contributed to this work

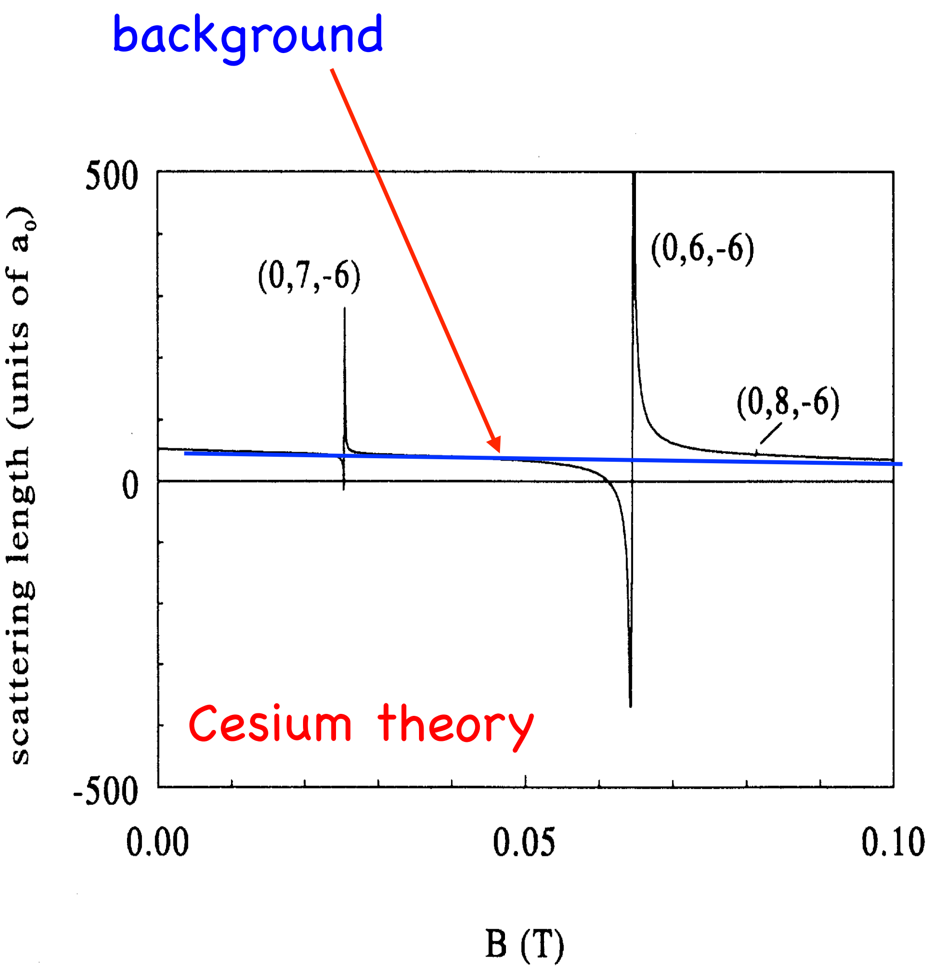
http://www.jqi.umd.edu/

Supported by an AFOSR MURI

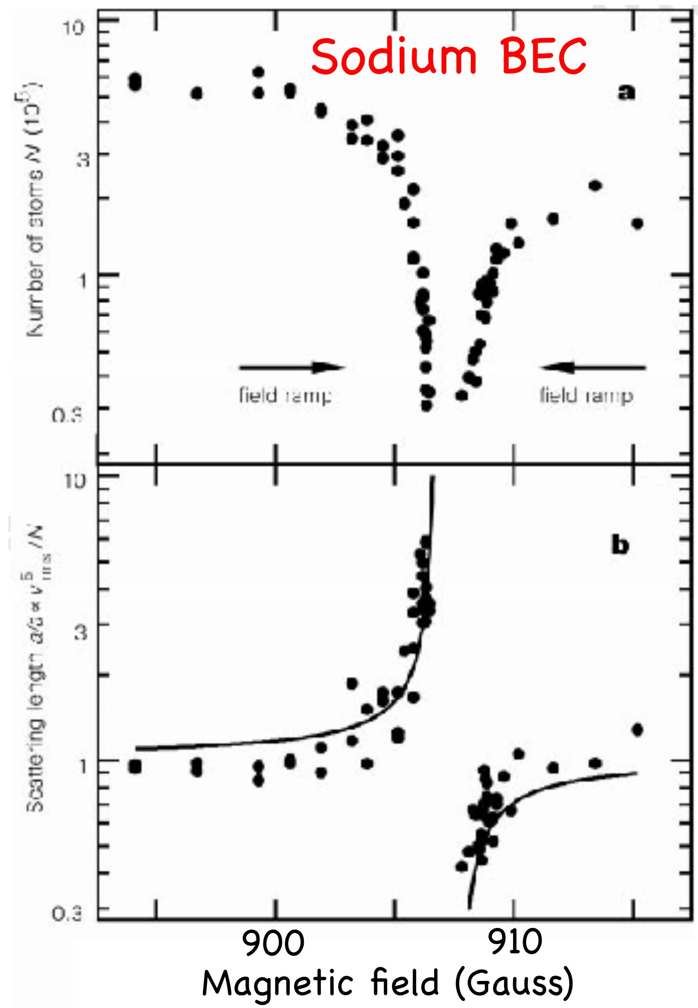
NIST

Joint Quantum Institute

# Some examples of Feshbach resonances




E. Tiesinga *et al.*, Phys. Rev. A **47**, 4114 (1993)

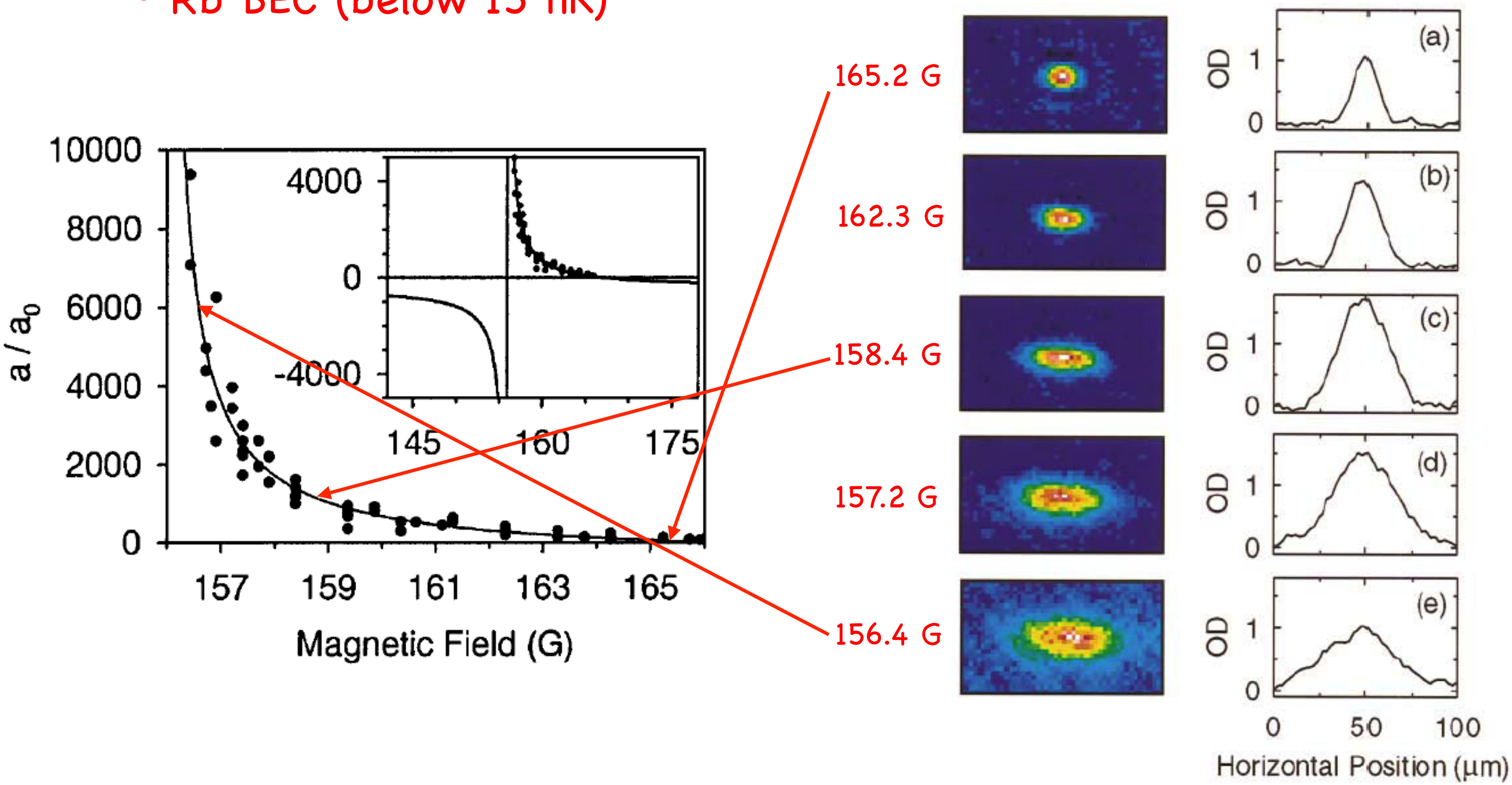



S. Inouye *et al*
Nature **392**, 141 (1998)

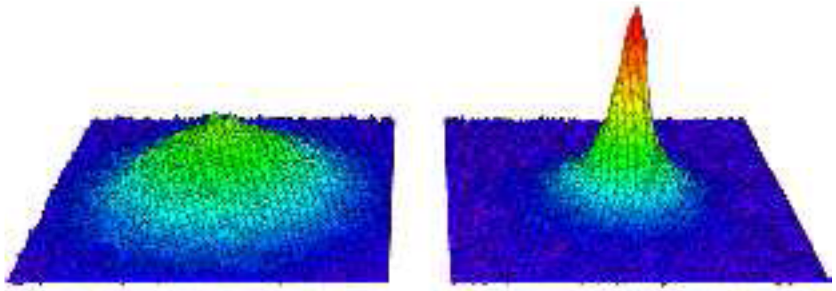
# An example for E→ 0

Cornish, Claussen, Roberts, Cornell, Wieman,
Phys. Rev. Lett. 85, 1795 (2000)

$^{85}$Rb BEC (below 15 nK)



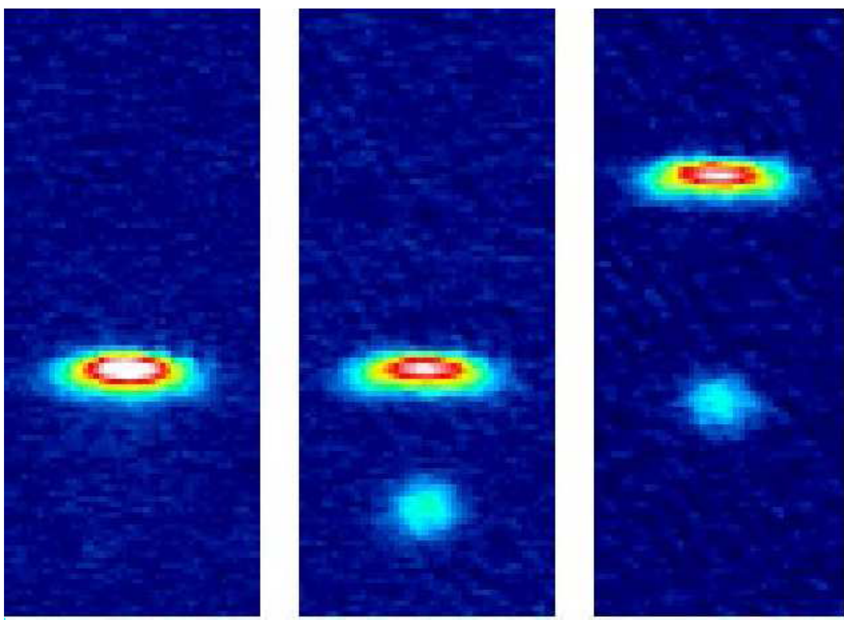
# Tunable scattering resonances used for



Thermal (250 nK) BEC (79 nK)

## Making $^{40}K_2$ molecules

Greiner, M., C. A. Regal, and D. S. Jin, 2003, Nature (London) 426, 537.

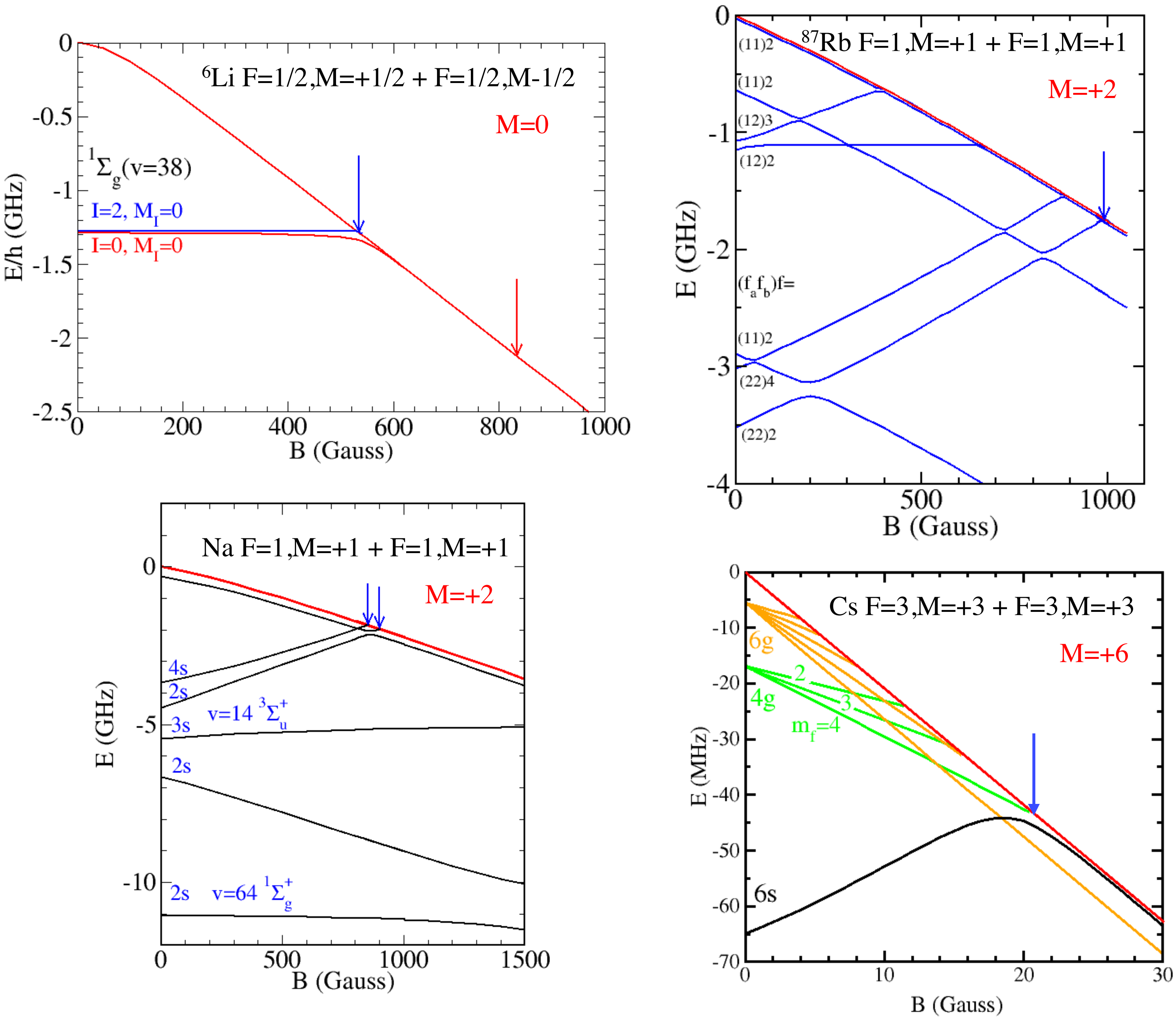


## Making $^{133}Cs_2$ molecules

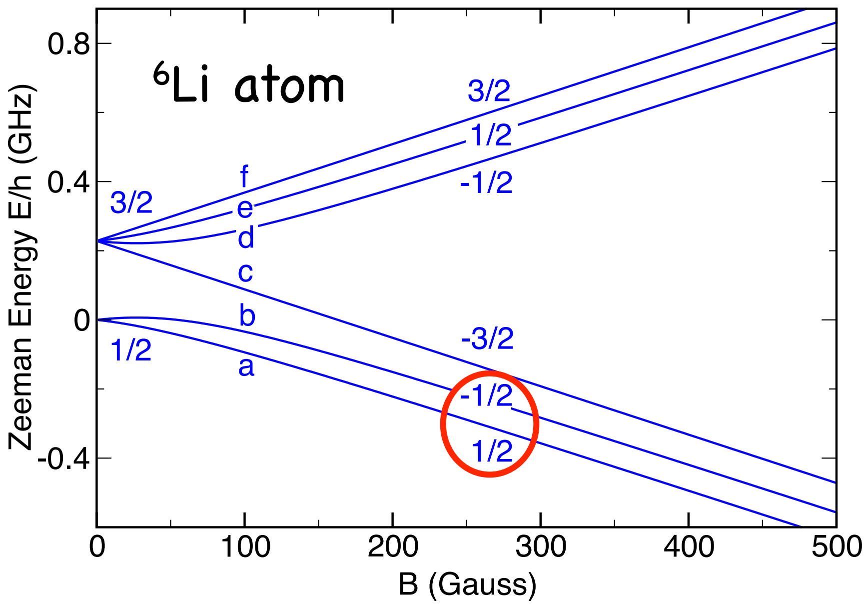
$^{133}Cs$ atom cloud

$^{133}Cs_2$ molecule cloud

Herbig, J., T. Kraemer, M. Mark, T. Weber, C. Chin, H.-C. Nagerl, and R. Grimm, 2003, Science 301, 1510.



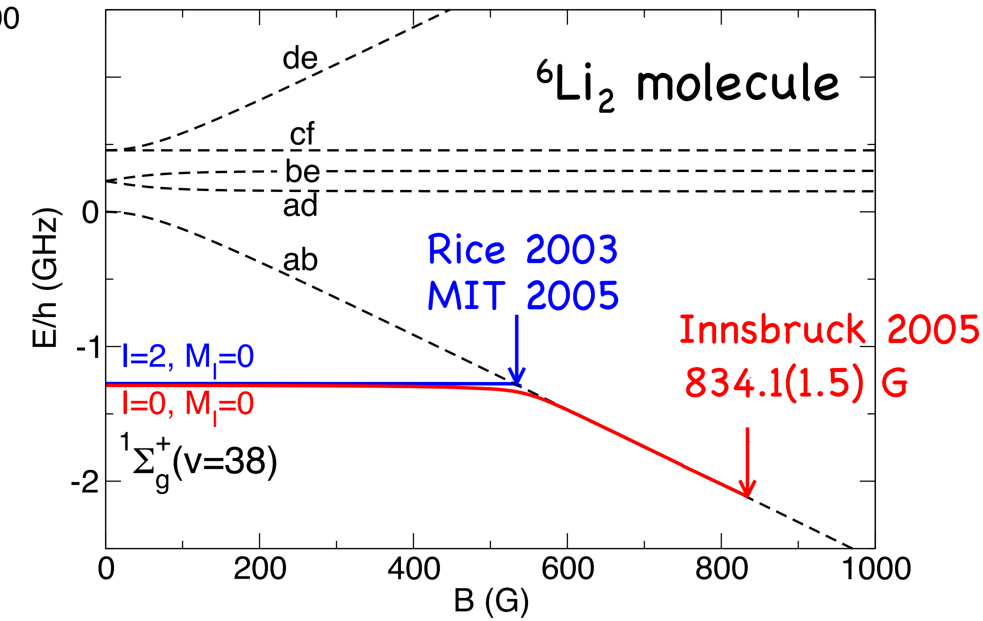
6Li F=1/2,M=+1/2 + F=1/2,M-1/2
M=0
1Σg(v=38)
I=2, MI=0
I=0, MI=0
E/h (GHz)
B (Gauss)
0
-0.5
-1
-1.5
-2
-2.5
0
200
400
600
800
1000
87Rb F=1,M=+1 + F=1,M=+1
M=+2
(11)2
(11)2
(12)3
(12)2
(fafb)f=
(11)2
(22)4
(22)2
E (GHz)
B (Gauss)
0
-1
-2
-3
-4
0
500
1000
Na F=1,M=+1 + F=1,M=+1
M=+2
4s
2s
v=14 3Σu+
3s
2s
2s
v=64 1Σg+
E (GHz)
B (Gauss)
0
-5
-10
0
500
1000
1500
Cs F=3,M=+3 + F=3,M=+3
M=+6
6g
2
4g
3
mf=4
6s
E (MHz)
B (Gauss)
0
-10
-20
-30
-40
-50
-60
-70
0
10
20
30




Quantum degenerate Fermi mixtures
BEC-BCS crossover
Strongly interacting gas
Equation of state

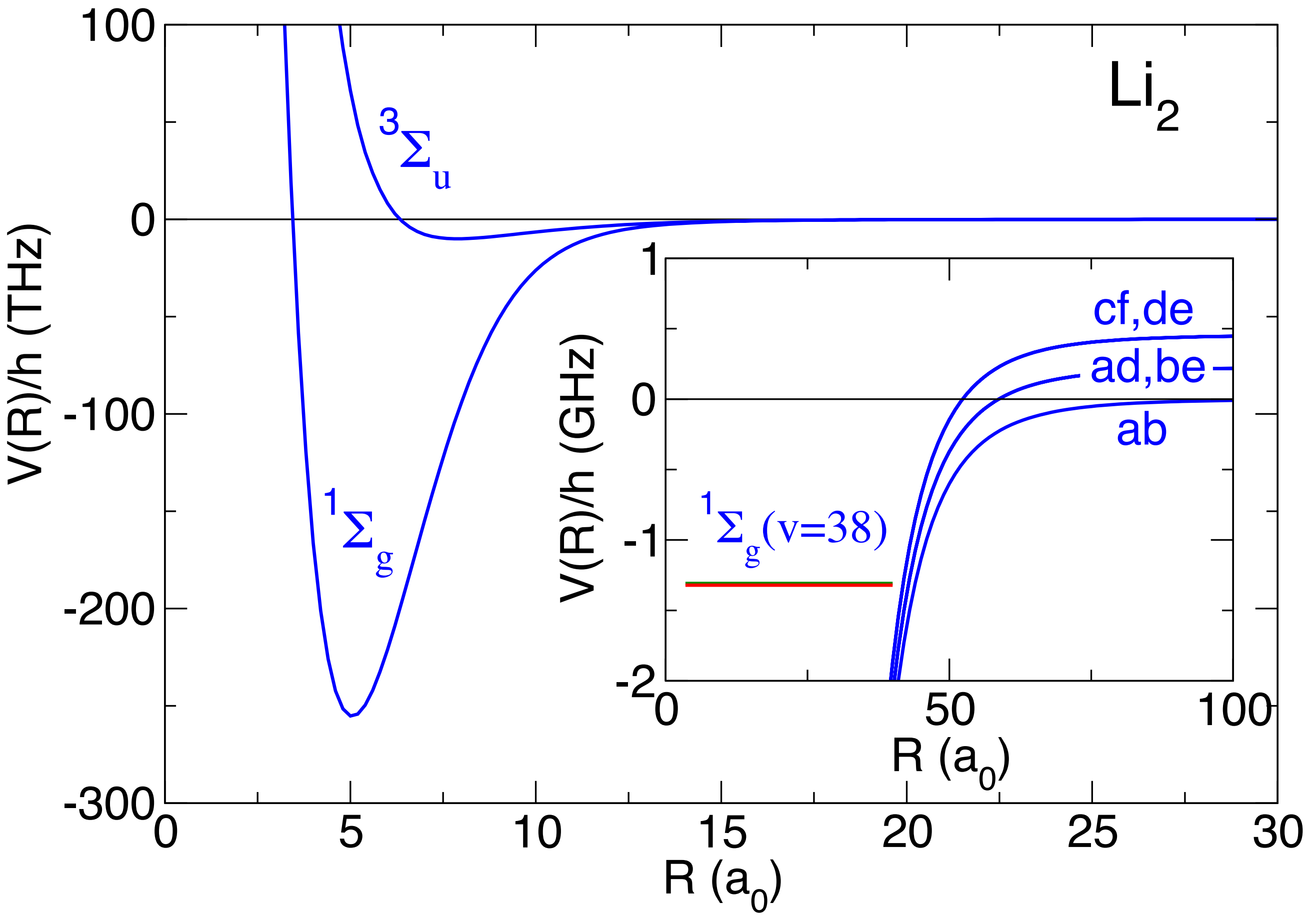
Molecular physics of Li+Li
is well-known

a+b s-wave

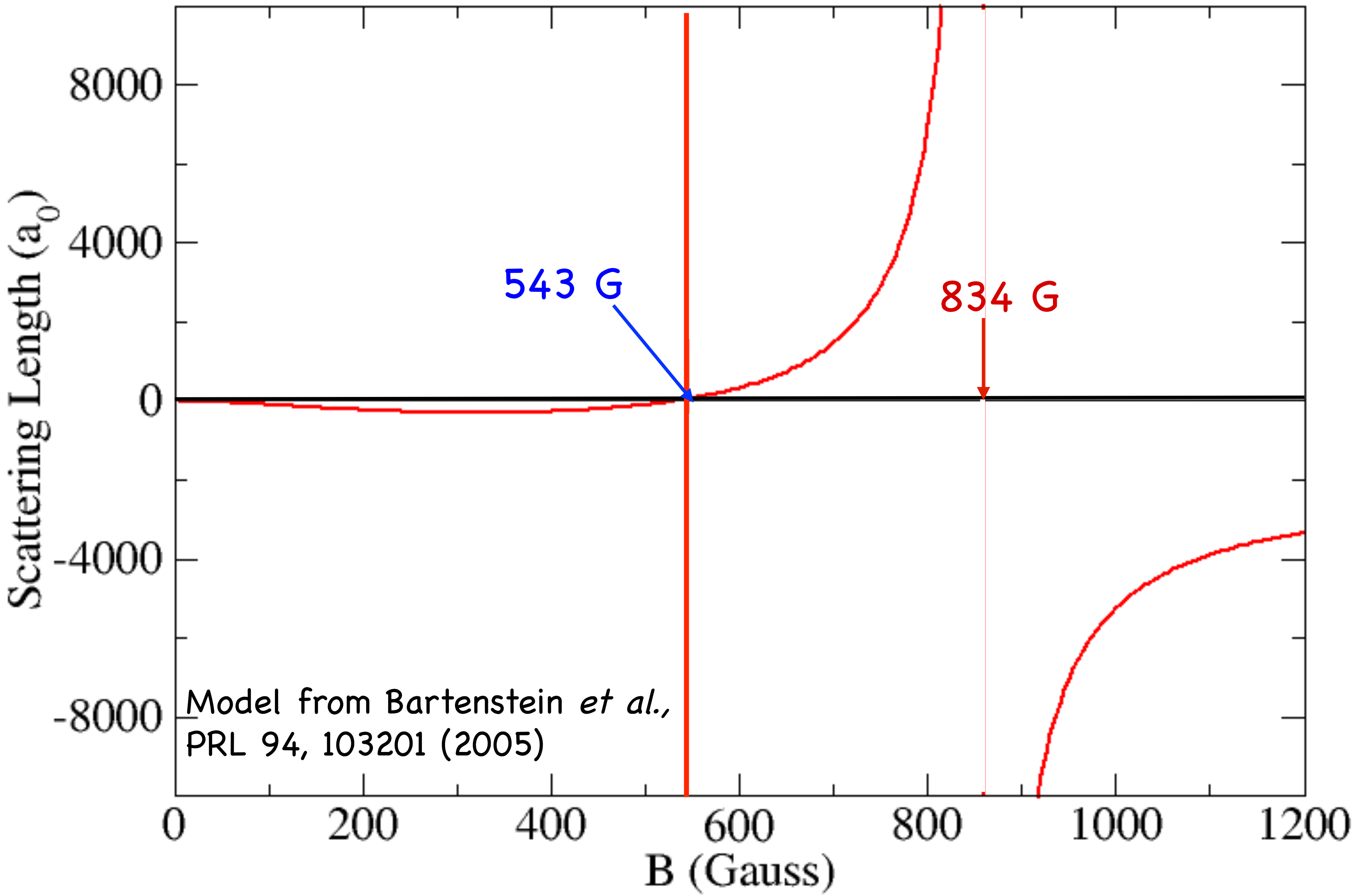


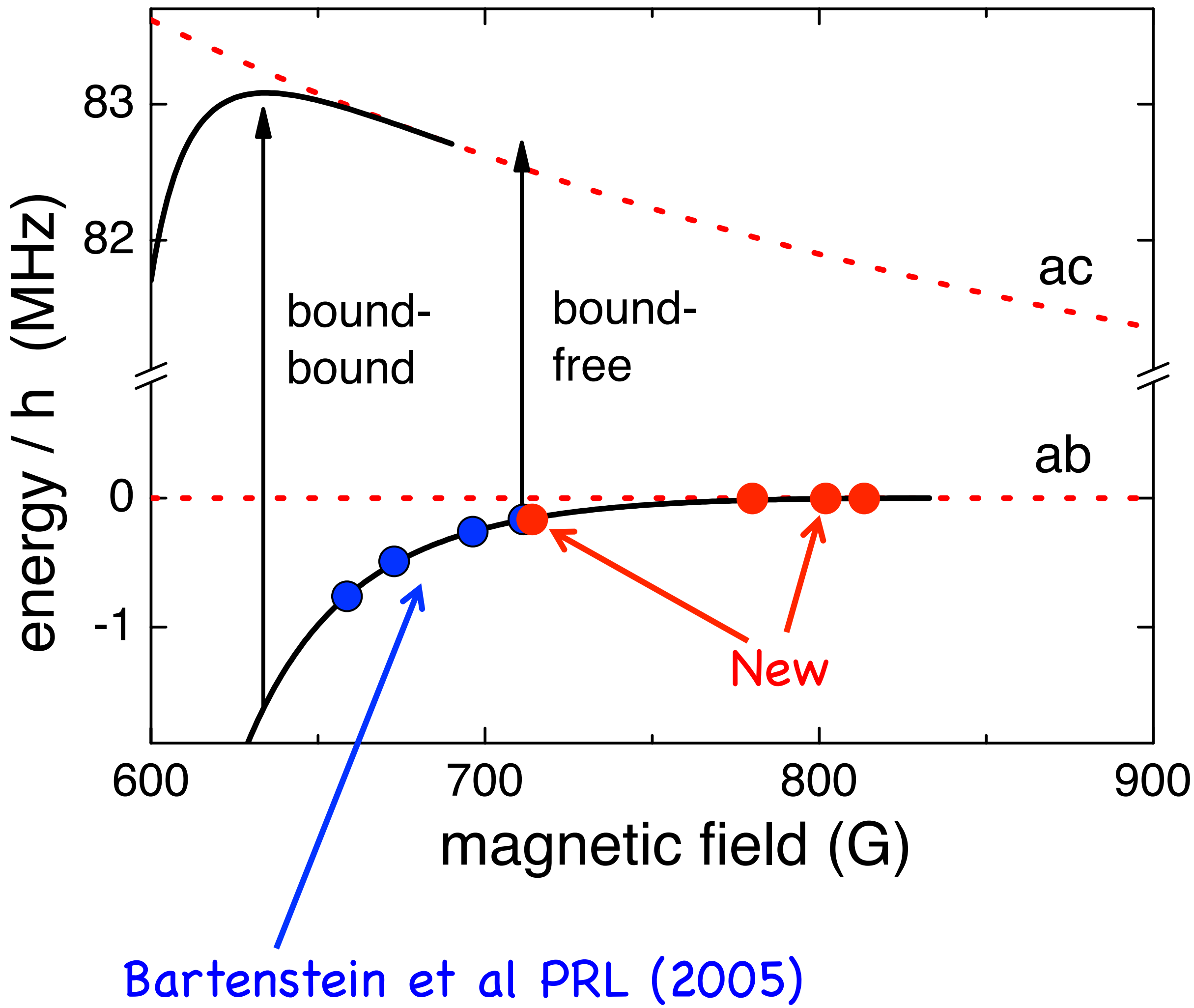

M. Bartenstein, *et al.*, *PRL* 94, 103201(2005)

Coupled channels calculations based on accurately known potentials
All spin-dependent interactions treated in Hamiltonian
2 free parameters: S and T scattering lengths

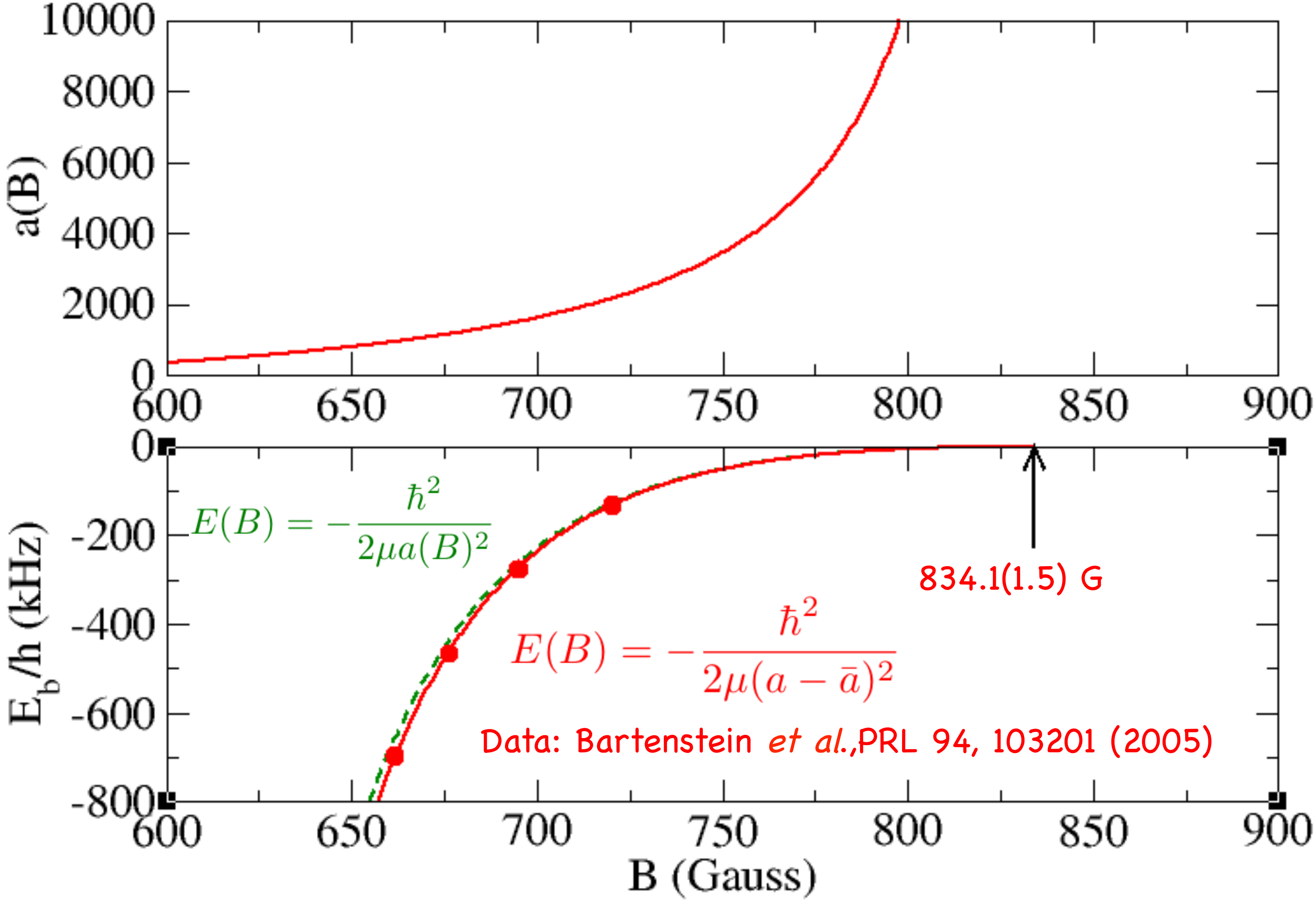


# $^6$Li *a+b* Scattering Length vs. B

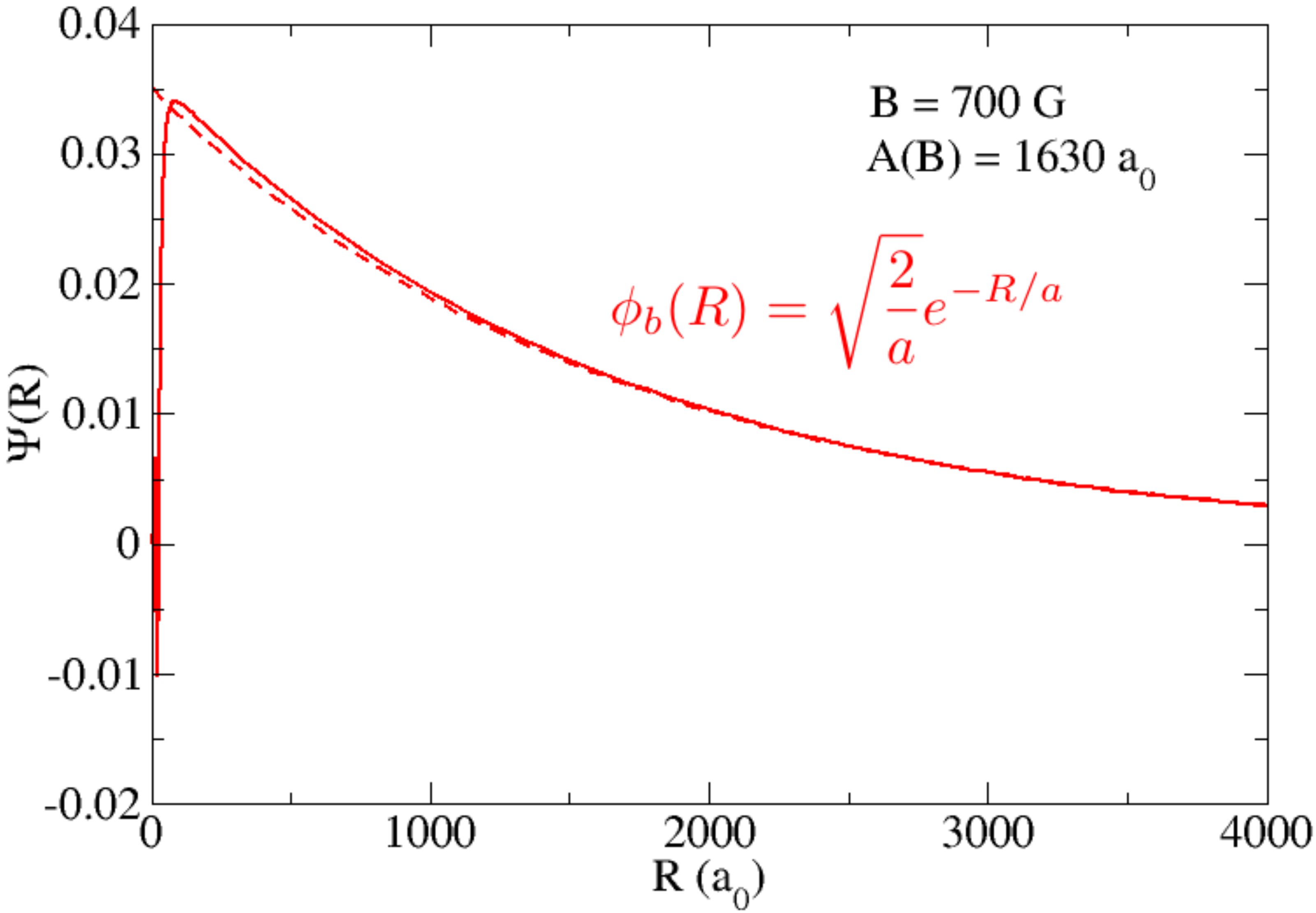




energy / h (MHz)
83
82
0
-1
bound-
bound
bound-
free
ac
ab
New
600
700
800
900
magnetic field (G)
Bartenstein et al PRL (2005)



10000
8000
6000
4000
2000
0
a(B)
600
650
700
750
800
850
900
0
-200
-400
-600
-800
$E_b/h$ (kHz)
$E(B) = -\frac{\hbar^2}{2\mu a(B)^2}$
834.1(1.5) G
$E(B) = -\frac{\hbar^2}{2\mu(a-\bar{a})^2}$
Data: Bartenstein *et al.*,PRL 94, 103201 (2005)
B (Gauss)



0.04
0.03
0.02
0.01
0
-0.01
-0.02
0
1000
2000
3000
4000
R ($a_0$)
Ψ(R)
B = 700 G
A(B) = 1630 $a_0$
$\phi_b(R) = \sqrt{\frac{2}{a}} e^{-R/a}$

# Long history of resonance scattering

O. K. Rice, J. Chem. Phys. 1, 375 (1933)

U. Fano, Nuovo Cimento 12, 154 (1935)

J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics (1952)

H. Feshbach, Ann. Phys. (NY) 5, 357 (1958); 19, 287 (1962)

U. Fano, Phys. Rev. 124, 1866 (1961)

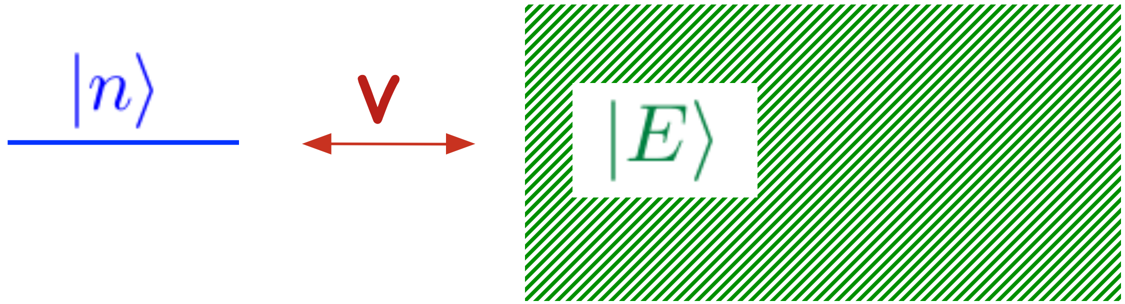
Separation of system into:

An (approximate) bound state
A scattering continuum
with some coupling between them

# Resonant Scattering Picture

(following U. Fano, Phys. Rev. 124, 1866 (1961); see arXiv:0812.1486)

**Bound state** | **Continuum**




**Closed channel (Resonance)** | **Open channel (Background)**

$$\eta(E) = \eta_{\rm bg} + \eta_{\rm res}(E)$$

width

$$\Gamma_n = 2\pi |\langle n|V|E\rangle|^2$$

$$\eta_{\rm res} = -\tan^{-1} \frac{\frac{1}{2}\Gamma_n}{E - E_n - \delta E_n}$$

$$\delta E_n = \int \frac{|\langle n|V|E'\rangle|^2}{E_n - E'} dE'$$

shift

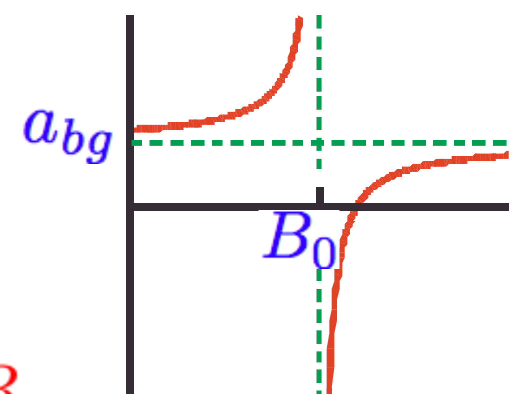
# Threshold Resonant Scattering

$$E_n = \delta\mu(B - B_n)$$


$$\eta(E,B) = \eta_{\text{bg}}(E) - \tan^{-1}\frac{\frac{1}{2}\Gamma(E)}{E - E_n - \delta E_n(E)}$$

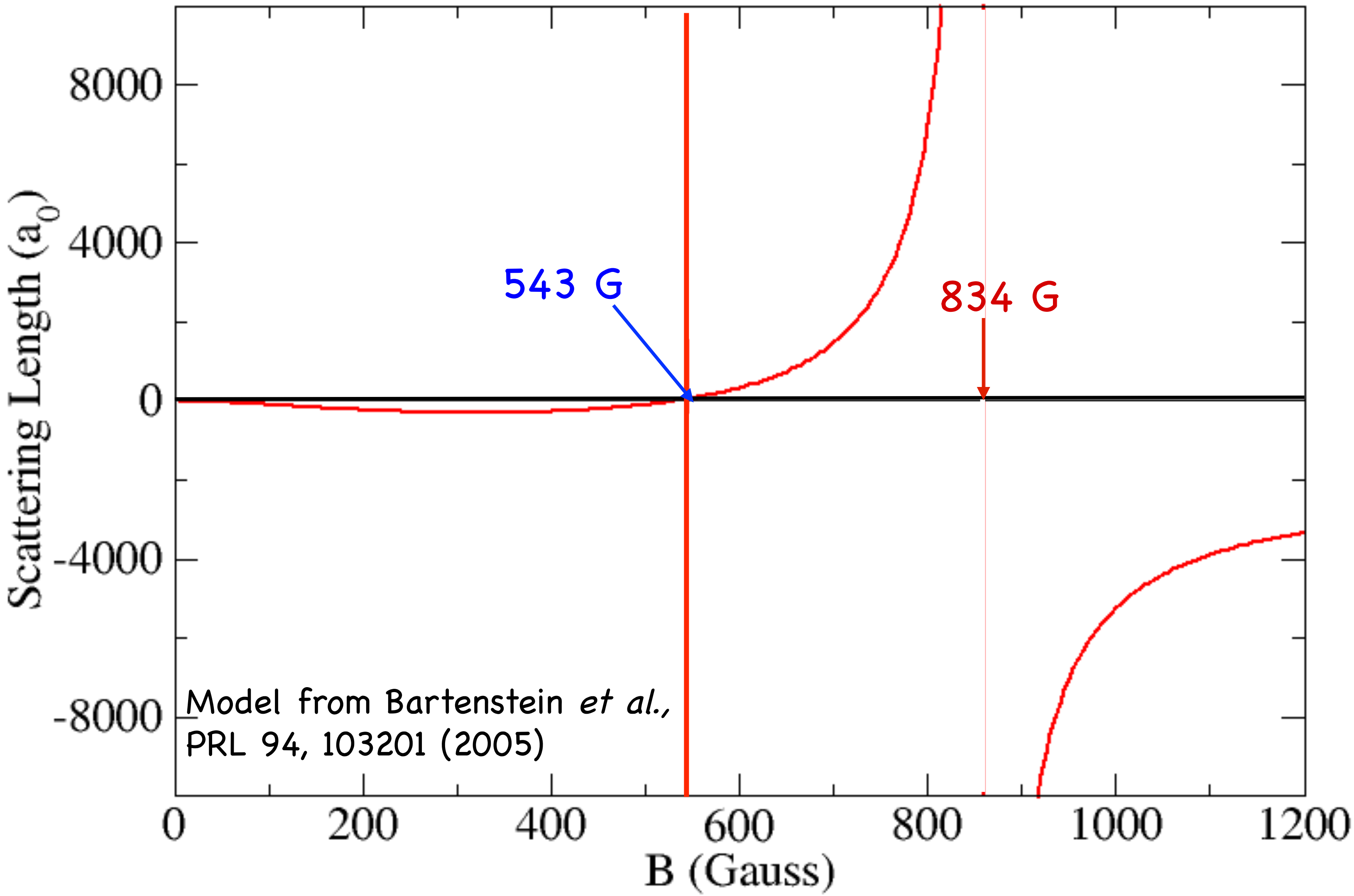
As E → 0 $\eta_{\text{bg}} = -ka_{\text{bg}}$ $\frac{1}{2}\Gamma_n(E) = (ka_{\text{bg}})\,\delta\mu\,\Delta_n$

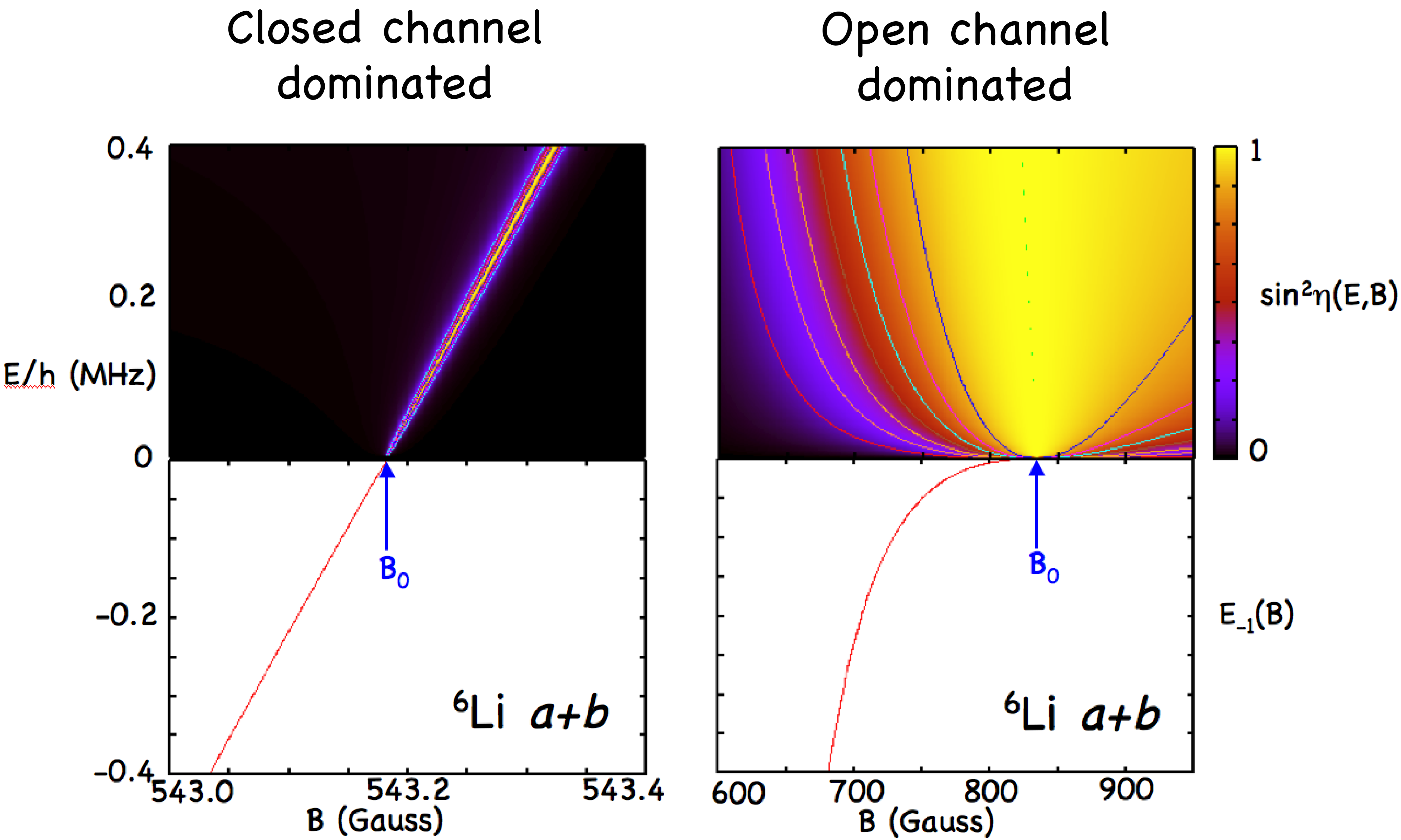
$$a(B) = a_{bg}\left(1 - \frac{\Delta_n}{B - B_0}\right)$$




Shifted $B_0 = B_n + \delta B_n$

# $^6$Li *a+b* Scattering Length vs. B






From arXiv:0812.1486

# Classification of resonances by strength

Resonance strength $s_{\mathrm{res}} = \dfrac{a_{bg}}{\bar{a}} \dfrac{\delta\mu\Delta}{\bar{E}}$

See Kohler et al, Rev. Mod. Phys. 78, 1311 (2006)
And Chin, Grimm, Julienne, Tiesinga, arXiv: 0812.1486

$$a(B) = a_{\mathrm{bg}} \left(1 - \frac{\Delta}{B - B_0}\right)$$

For magnetically tunable resonances: $E_c = \delta\mu(B - B_c)$

Bound state norm Z as E $\rightarrow$ 0
and B $\rightarrow$ $B_0$

$$Z = \zeta^{-1} \left| \frac{B - B_0}{\Delta} \right|$$

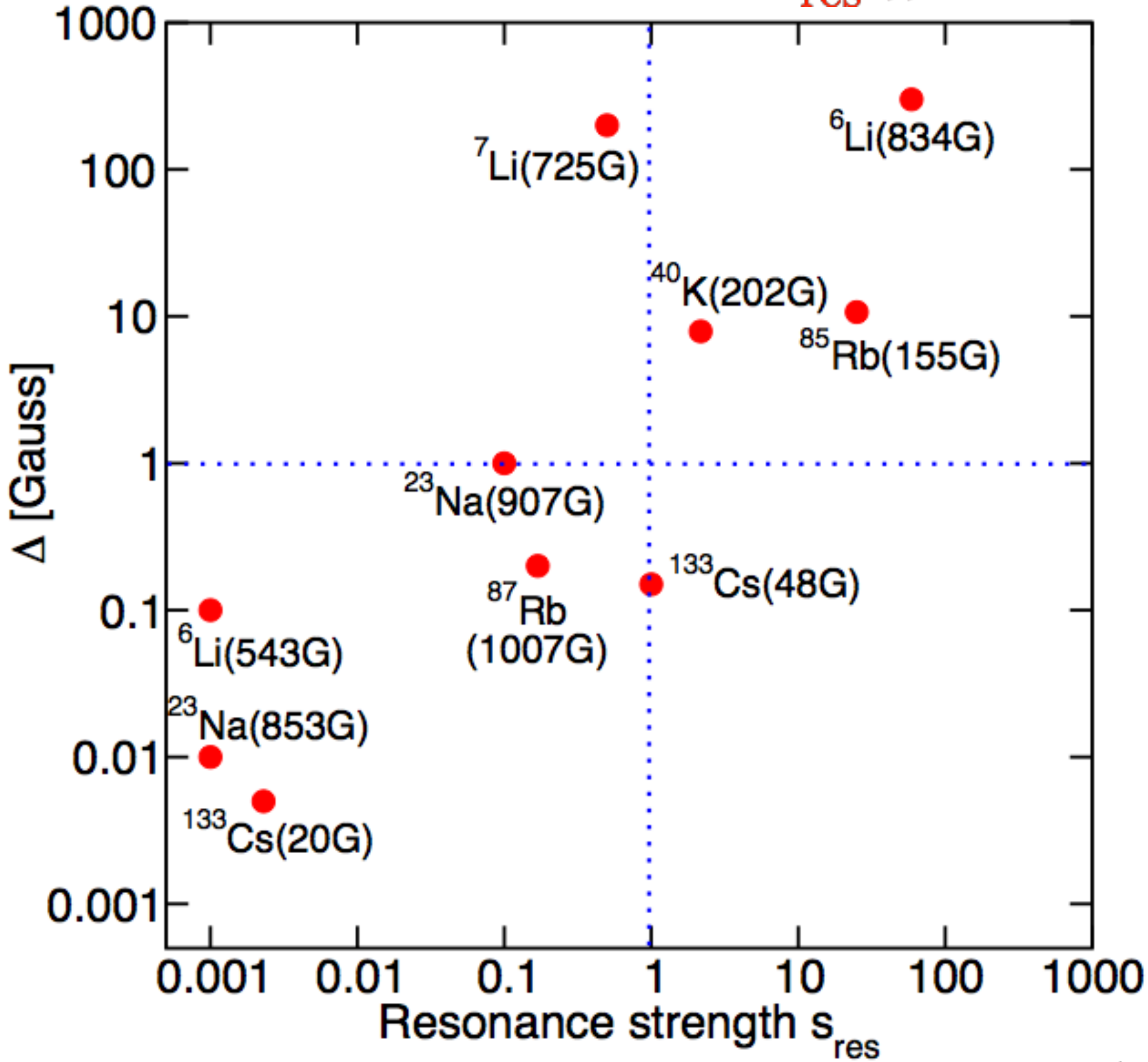
$$\zeta = \frac{1}{2} s_{\mathrm{res}} \frac{a_{bg}}{\bar{a}}$$

Closed channel dominated

$s_{\mathrm{res}} \ll 1$

Entrance channel dominated

$s_{\mathrm{res}} \gg 1$




"Broad"

"Narrow"

From arXiv:0812.1486

$s_{\mathrm{res}} = 59$

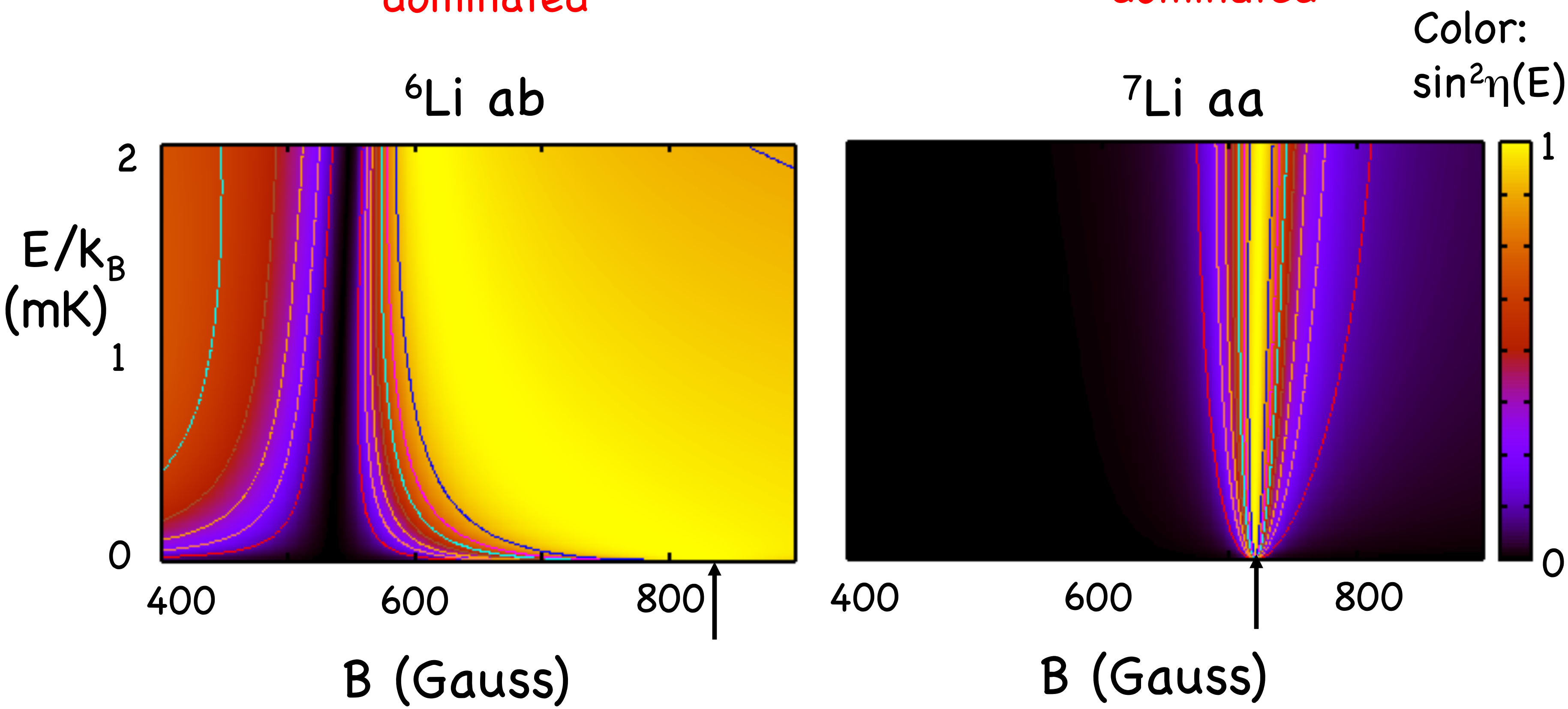
$\Delta = 300\,\mathrm{G}$

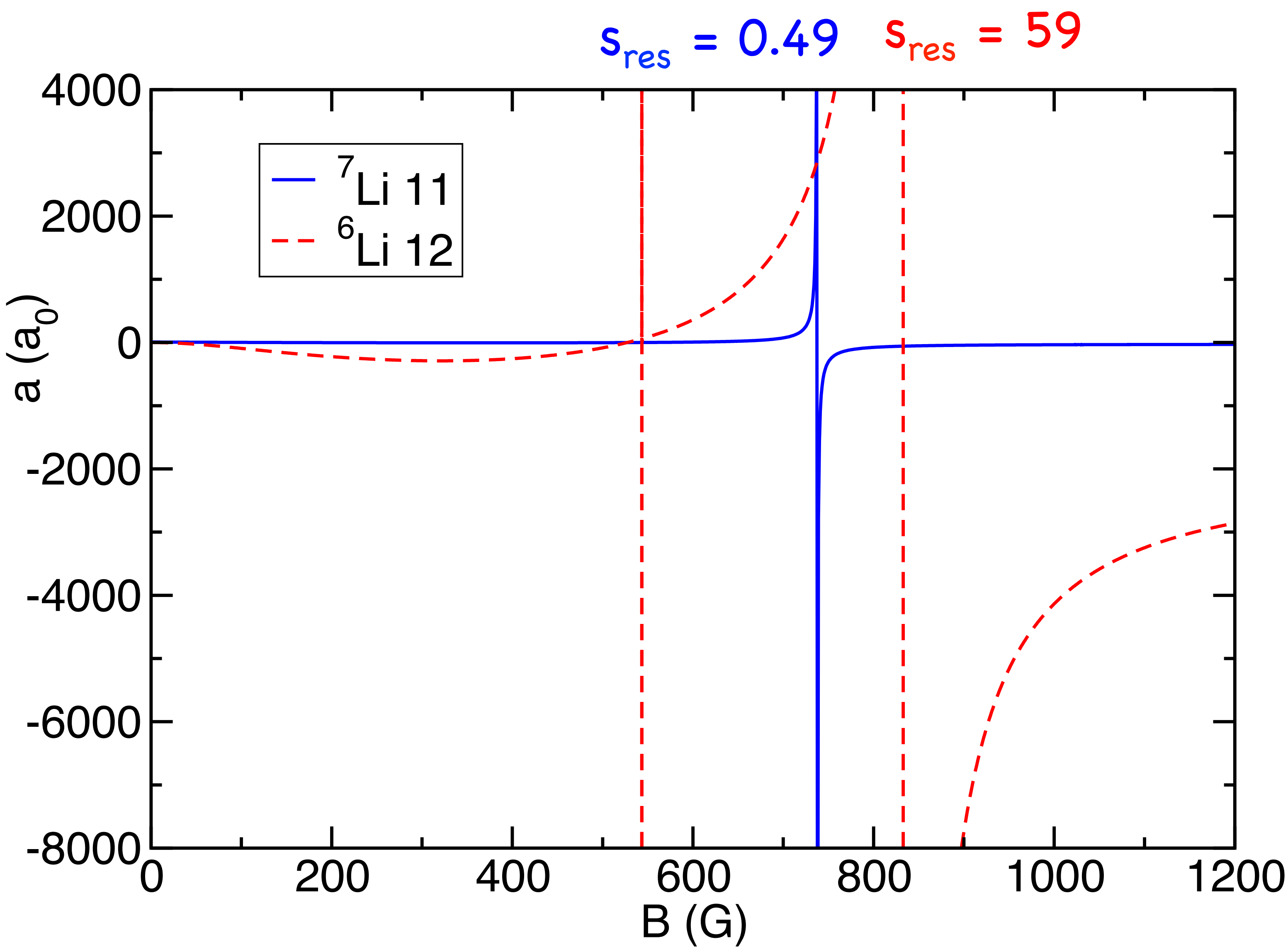
Entrance channel dominated

$s_{\mathrm{res}} = 0.61$

$\Delta = 180\,\mathrm{G}$

Closed channel dominated





$s_{res}$ = 0.49
$s_{res}$ = 59
$^7$Li 11
$^6$Li 12
4000
2000
0
-2000
-4000
-6000
-8000
a ($a_0$)
0
200
400
600
800
1000
1200
B (G)

$s_{\mathrm{res}} = 59$

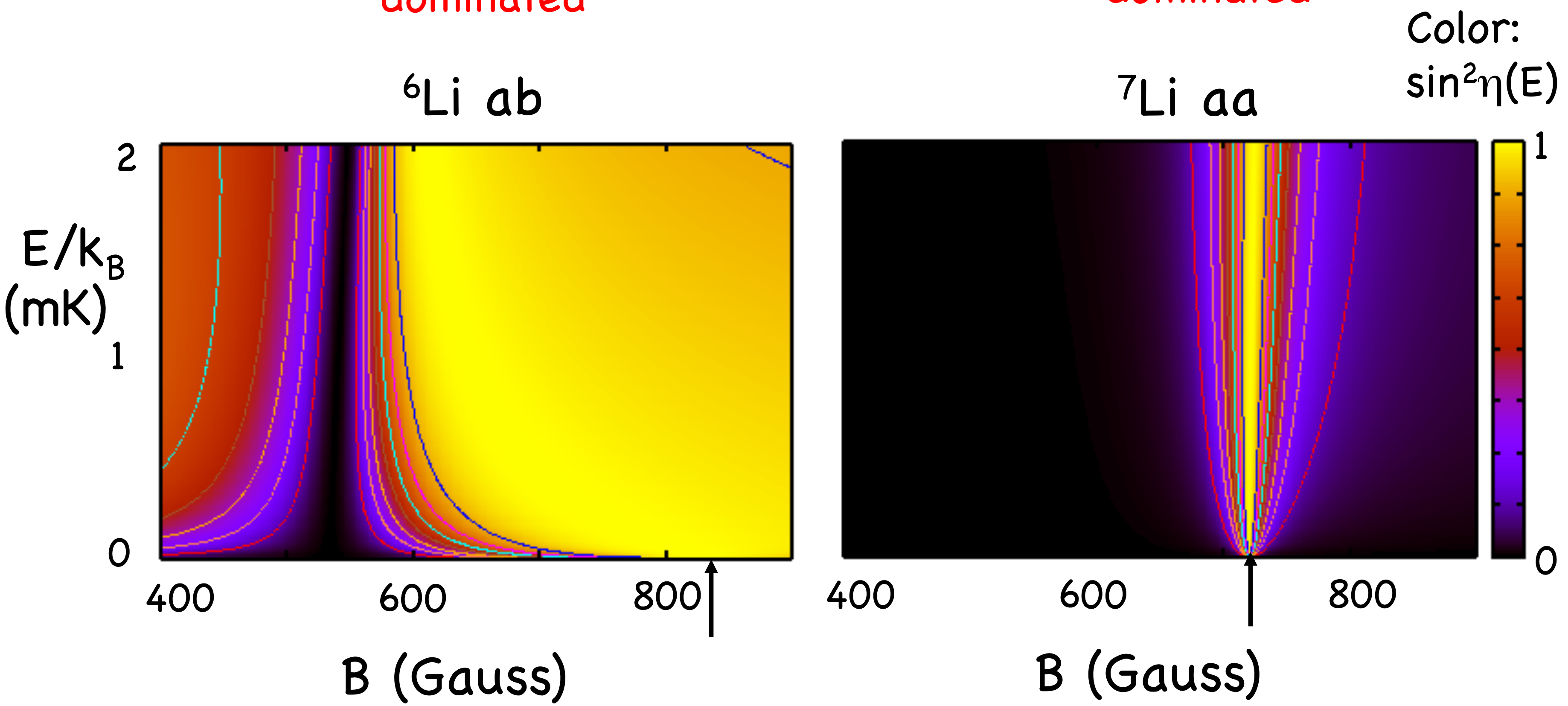
$\Delta = 300\,\mathrm{G}$

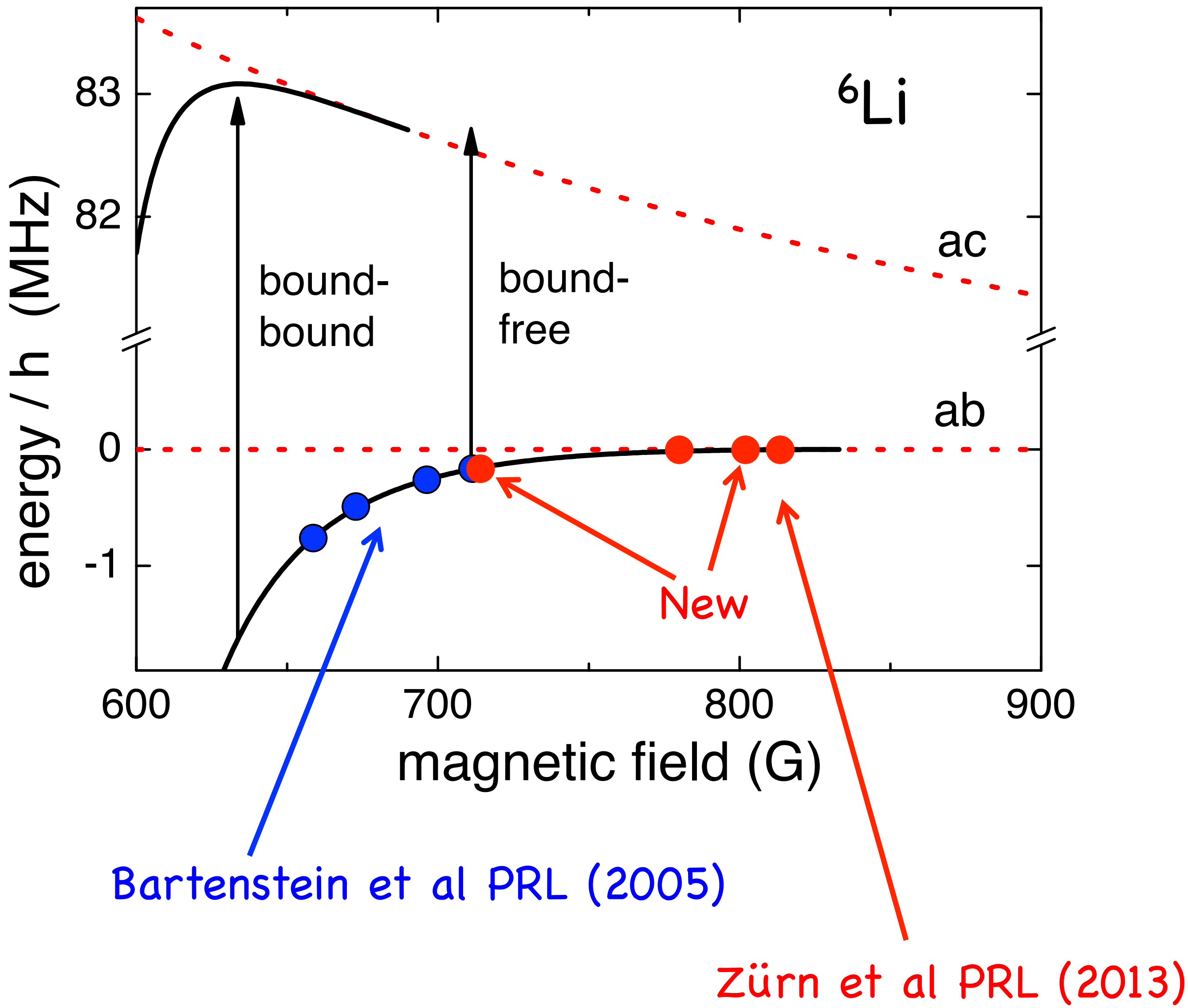
Entrance channel dominated

$s_{\mathrm{res}} = 0.61$

$\Delta = 180\,\mathrm{G}$

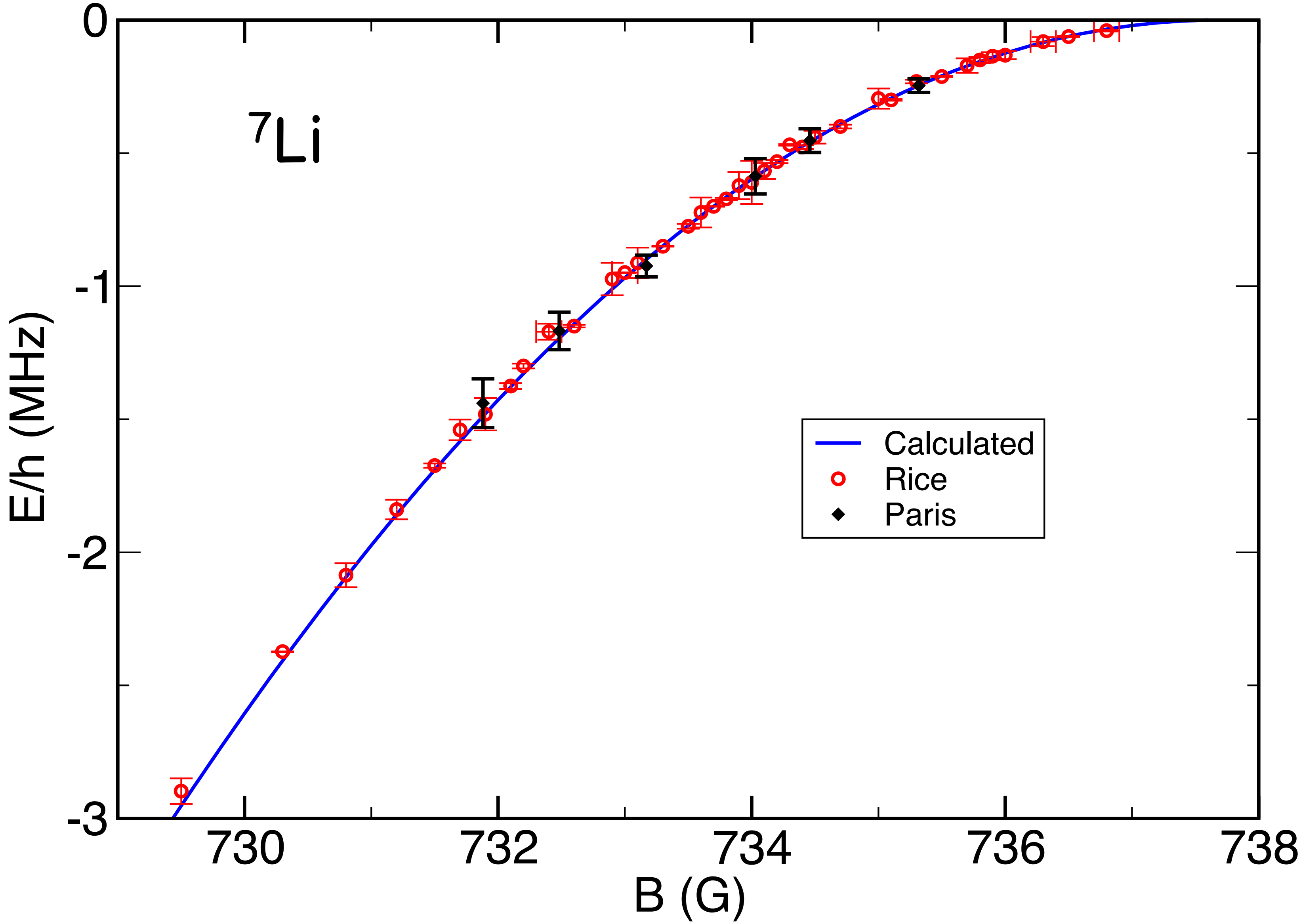
Closed channel dominated





$^6$Li
83
82
0
-1
energy / h (MHz)
ac
ab
bound-
bound
bound-
free
600
700
800
900
magnetic field (G)
New
Bartenstein et al PRL (2005)
Zürn et al PRL (2013)

Coupled channels fit, PSJ & J. Hutson, arxiv:1404.2623
(full Hamiltonian)



Universal energy: $$E^{\mathrm{U}} = -\frac{\hbar^2}{2\mu a^2}$$

Reduced E and length: $\epsilon = E/\bar{E}$ and $r = a/\bar{a}$

$$\epsilon^{\mathrm{U}} = -\frac{1}{r^2}$$

$$\epsilon^{\mathrm{U}} r^2 = -1$$

$$\epsilon^{\mathrm{U}} = -\frac{1}{r^2}$$

Universal

$$\epsilon^{\mathrm{GF}} = -\frac{1}{(r-1)^2}$$

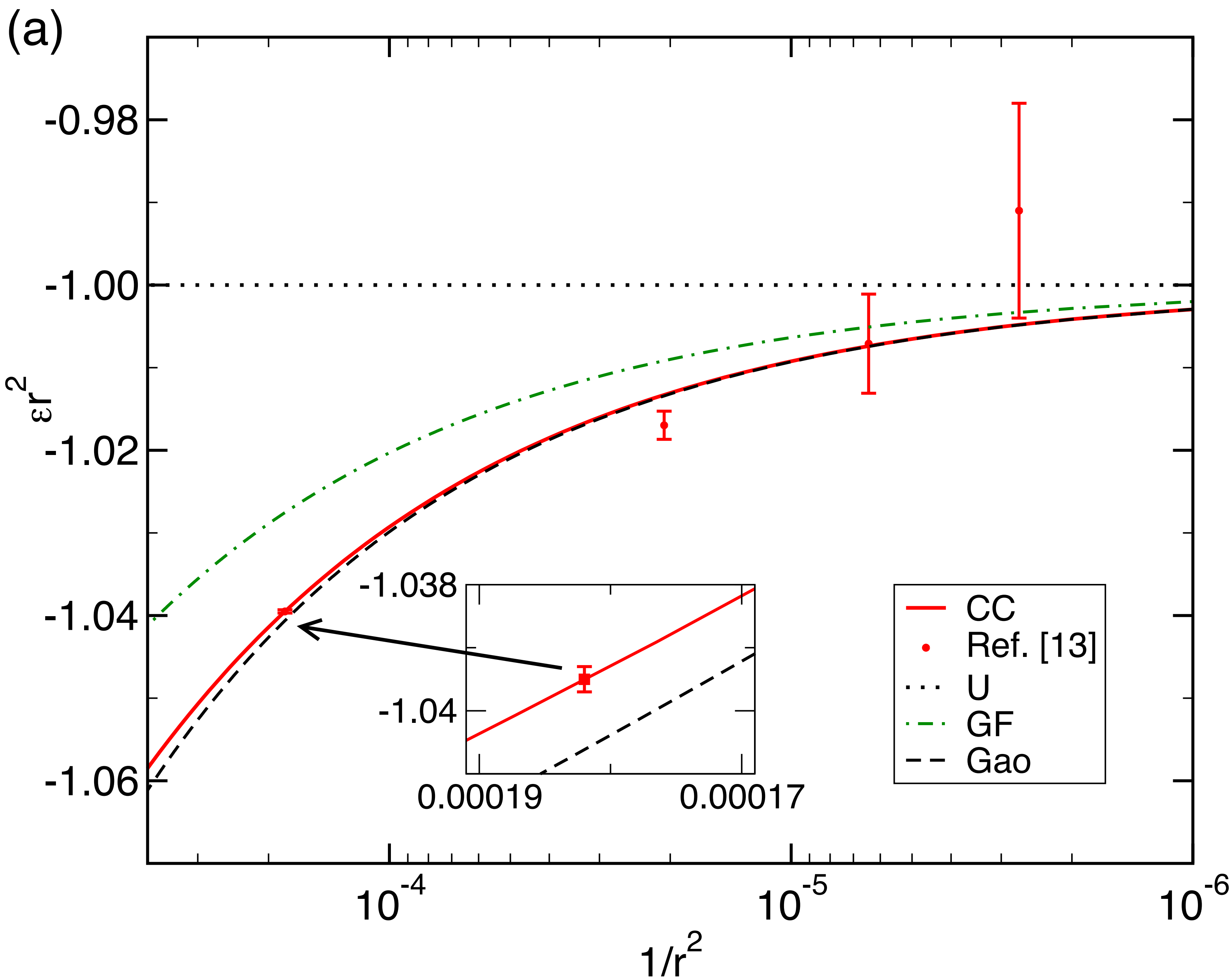
Gribakin and Flambaum, PRA 48, 546 (1993)

$$\epsilon^{\mathrm{G}} = \epsilon^{\mathrm{GF}} \left( 1 + \frac{g_1}{r-1} + \frac{g_2}{(r-1)^2} \right)$$

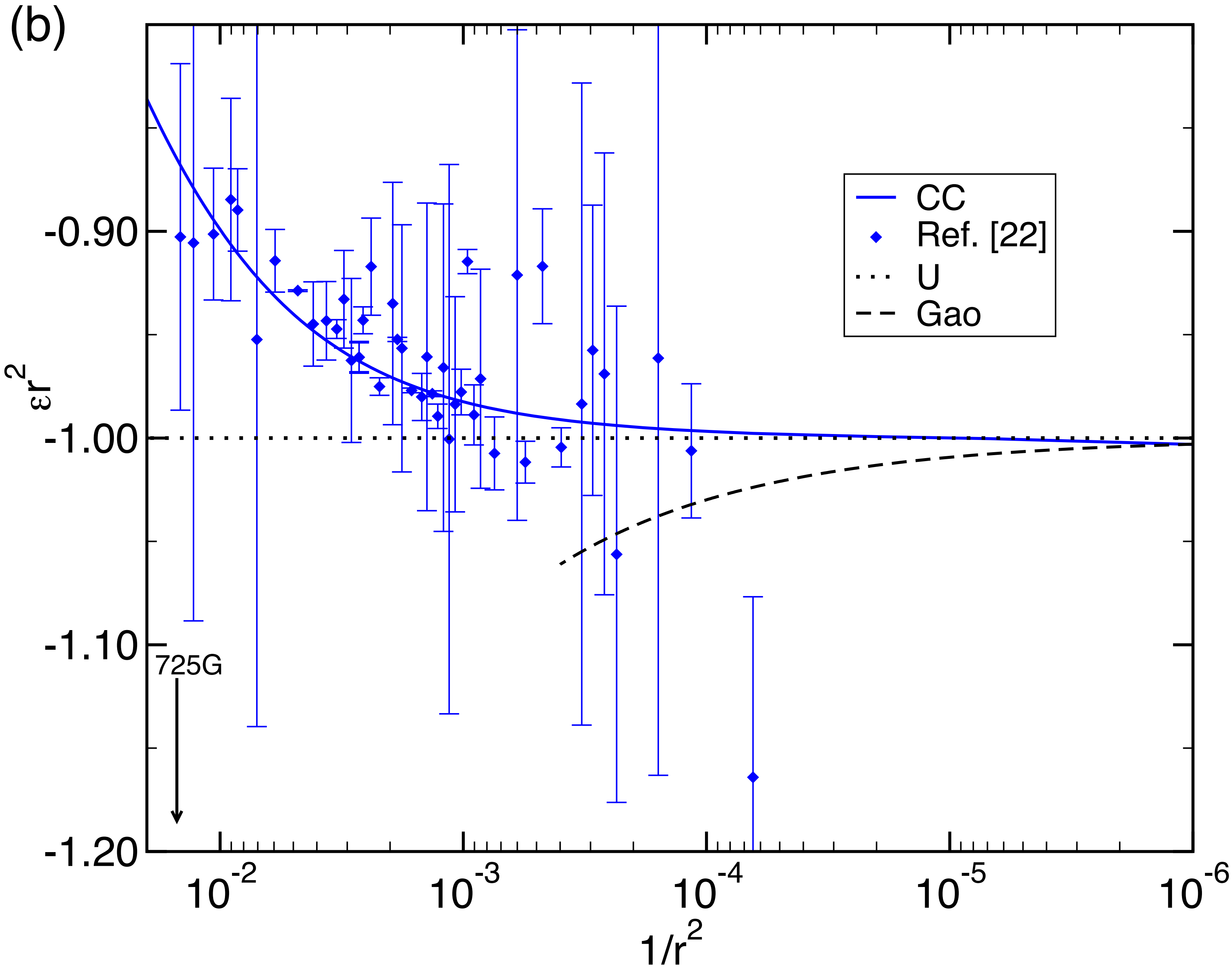
$$g_1 = \Gamma(\frac{1}{4})^4/(6\pi^2) - 2 \approx 0.9179$$

$$g_2 = (5/4)g_1^2 - 2 \approx -0.9468$$

Gao, J. Phys. B 37, 4273 (2004)

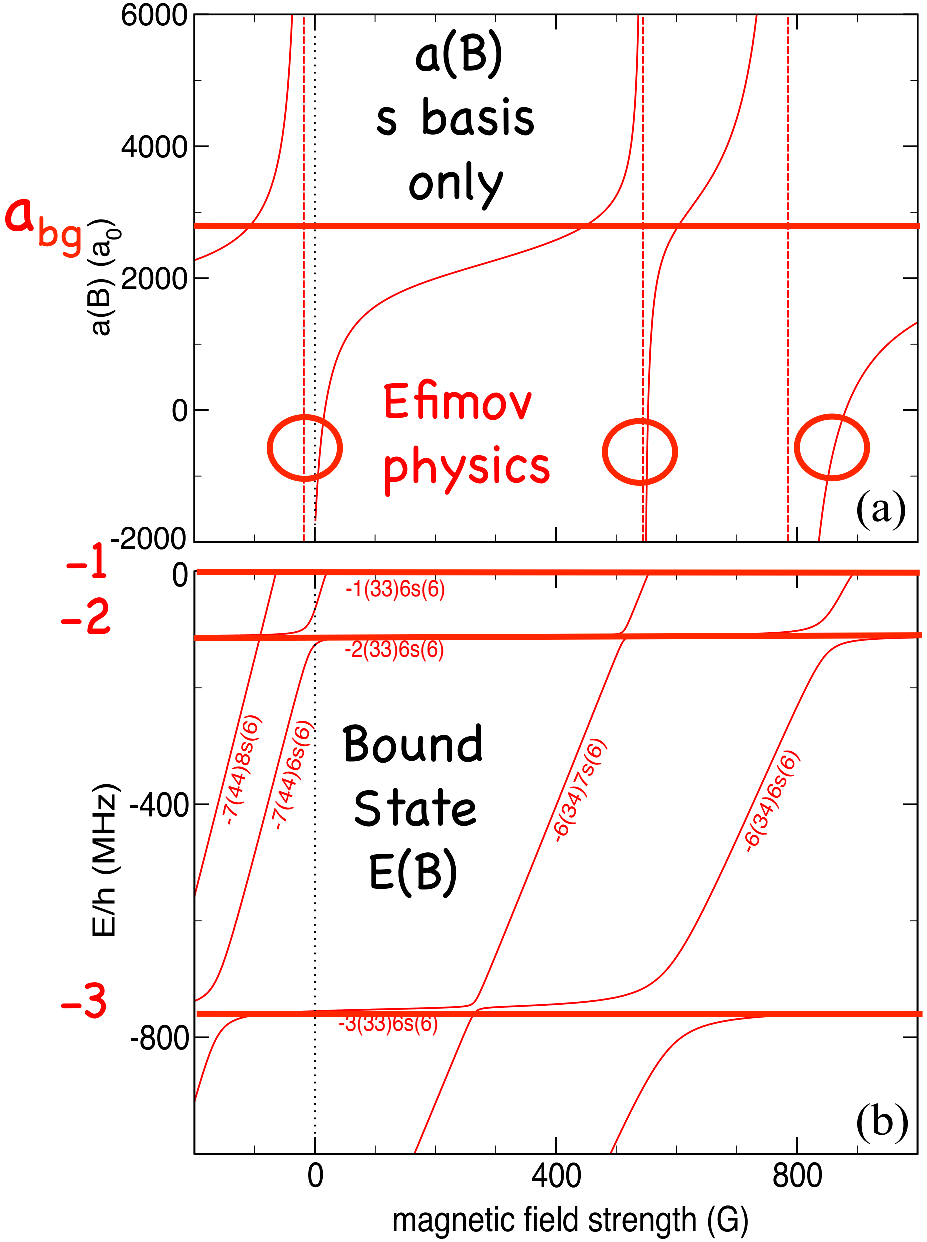



PSJ & J. Hutson, arxiv:1404.2623




PSJ & J. Hutson, arxiv:1404.2623

Cs+Cs Innsbruck/Hutson/PSJ Phys. Rev. A 87, 032517 (2013)



The End