Problem 1.

Derive the completeness relation for a massive particle of spin 1 (see Problem 9.27 for the massless analog.) [Hint: Let the z axis point along **p**. First construct three mutually orthogonal polarization vectors ($\epsilon_u^{(1)}$,$\epsilon_u^{(2)}$,$\epsilon_u^{(3)}$) that satisfy $p^u\epsilon_u = 0$ $\epsilon_u\epsilon^u =$ -1.]

Solution

First, considering the Lorentz condition : $\partial_u A^u = 0$ (7.82)
For photon's case, which has spin 1 but massless,

$$A^\mu = ae^{-(i/\hbar)p\cdot x}\epsilon^\mu(p) \qquad (7.89)$$

This should be the same. but p(momentum) will be different, because of the mass.
1) The polarization vectors $\epsilon_\mu^{(s)}$ should satisfy the momentum space Lorentz and othorgonal conditions (7.101) $\sim$ (7.103):
$p^\mu\epsilon_\mu = 0, \quad \epsilon_\mu^{(1)}\epsilon^{(2)\mu} = 0, \quad \epsilon_\mu^{(1)}\epsilon^{(1)\mu} = -1$
2) Using the Lorentz condition : $\epsilon^\mu p_\mu = 0$ (9.2), and set the z axis as a polarization, $\epsilon_\mu^{(1)}$ = (1, 0, 0) and $\epsilon_\mu^{(2)}$ = (0, 1, 0),