

Problem 1.

- (a) Use Equation 10.132 to determine the mass of the W, in terms of $v = \frac{\mu}{\lambda}$ and $q = g_w \sqrt{\frac{\hbar c}{4\pi}}$. Thus, confirm Equation (12.1)
- (b) Use Problem 10.21 and Equation 10.130 to determine the vertex factor for the coupling of the Higgs to a quark or lepton.
- (c) Use Equation 10.136 to determine the vertex factors for the couplings hWW, hZZ, and hhh.

Solution

$$(a) m_A = 2\sqrt{\pi} \left(\frac{q\mu}{\lambda c^2} \right) \quad (10.132)$$

To get a mass term from a lagrangian, we expand L in powers of ϕ and pick out the term proportional to ϕ^2 . (second order in the field).

$$L = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda^2\phi^4 \quad (10.108)$$

And we can assume that $L = T - U$ (10.109)

$$\text{and } U(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda^2\phi^4 \quad (10.110)$$

$\phi = \frac{\mu}{\lambda}$ (10.111) we assume this as a ground state.

Now, starts from (10.132), $m_A = 2\sqrt{\pi} \left(\frac{q\mu}{\lambda c^2} \right)$,

This is the mass term for a free gauge field lagrangian(10.131) and was derived from the lagrangian of the spontaneously broken continuous symmetry(chapter 10.8).

$$m_A = 2\sqrt{\pi} \left(\frac{g_w \sqrt{\hbar c / 4\pi v}}{c^2} \right)$$

$$\frac{m_A c^2}{g_w} = 2\sqrt{\pi \hbar c / 4\pi} = \frac{1}{2}\sqrt{\hbar c v} \rightarrow \sqrt{\hbar c v} = \frac{2m_A c^2}{g_w} = 246 \text{ GeV}$$

the vertex factor is $2iM_m^2 c^2 g^{\mu\nu} / \hbar^2 v$: one higs + 2 Ws or 2 Zs.

Problem 2.

- (a) Calculate the decay rate for $h \rightarrow f + \bar{f}$ (where f is a quark or lepton), in the MSM.

$$\left[\text{Answer} : \frac{\alpha_w}{8\hbar} m_h c^2 \left(\frac{2m_f}{M_W} \right)^2 \left[1 - \left(\frac{2m_f}{m_h} \right)^2 \right]^{3/2} \right]$$

- (b) If $m_h = 120 \text{ GeV}/c^2$, what are the branching ratios $\Gamma(b\bar{b})/\Gamma(c\bar{c})$ and $\Gamma(b\bar{b})/\Gamma(\tau^+\tau^-)$? [Include a factor of 3, for color, in the case of quarks.]