Problem 1.

Prove that I, R_+, R_-, R_a, R_b, and R_c are all the symmetries of the equilateral triangle. [Hint: One way to do this is to label the three corners, as in Figure 4.2 A given symmetry operation carries A into the position formerly occupied by A, B, or C. If A $\to$ A, then either B $\to$ B and C $\to$ C, or else B $\to$ C and C $\to$ B. Take it from there.]

Solution

Figure 1 equilateral triangle.

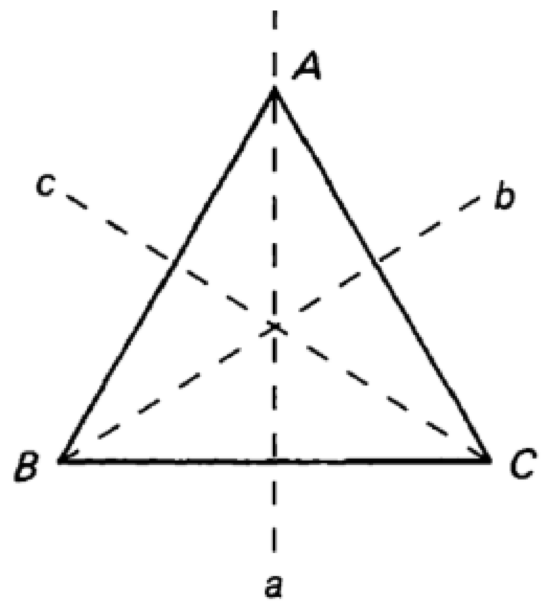

Fig. 4.2 Symmetries of the equilateral triangle.

Figure 1: equilateral triangle.

Where,

R_+ : A clockwise rotation through 120°

R_- : A counterclockwise rotation through 120°

R_a : by flipping it about the vertical axis a.

R_b : by flipping it about the vertical axis b.

I : Doing nothing.

Q. How to prove the symmetries?

A. By symmetry properties.

Symmetry properties

1. Closure : $R_i R_j \rightarrow$ first perform R_j and then $R_i \rightarrow$ and there exists R_k $= R_i R_j$.
2. Identity : $\mathrm{I}R_i = R_i\mathrm{I} = R_i$.
3. Inverse : $R_i R_i^{-1} = R_i^{-1} R_i = \mathrm{I}$.
4. Associativity : $R_i(R_j R_k) = (R_i R_j)R_k$.

1. $R_{-}R_{+} = \mathrm{I}$
$R_a R_b R_c = \mathrm{I}$
$R_a R_b = R_c$
2. Yes
3. Yes
4. Yes
(Not sure this is proper ?)