Problem 1.

Prove that I, R_+, R_-, R_a, R_b, and R_c are all the symmetries of the equilateral triangle. [Hint: One way to do this is to label the three corners, as in Figure 4.2 A given symmetry operation carries A into the position formerly occupied by A, B, or C. If A $\rightarrow$ A, then either B $\rightarrow$ B and C $\rightarrow$ C, or else B $\rightarrow$ C and C $\rightarrow$ B. Take it from there.]

Solution
Figure 1 equilateral triangle.

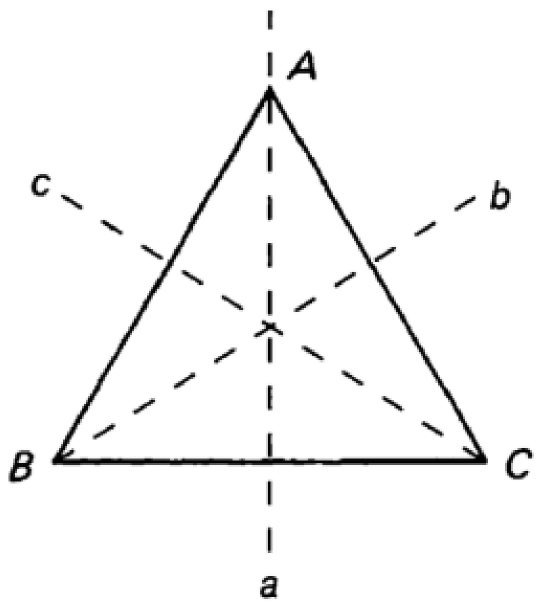

Fig. 4.2 Symmetries of the equilateral triangle.

Figure 1: equilateral triangle.

Where,
R_+ : A clockwise rotation through 120°
R_- : A counterclockwise rotation through 120°
R_a : by flipping it about the vertical axis a.
R_b : by flipping it about the vertical axis b.
I : Doing nothing.

Q. How to prove the symmetries?

A. By symmetry properties.

Symmetry properties

1. Closure : R_iR_j $\rightarrow$ first perform R_j and then R_i $\rightarrow$ and there exists R_k $= R_iR_j$.
2. Identity : $\mathrm{I}R_i = R_i\mathrm{I} = R_i$.
3. Inverse : $R_iR_i^{-1} = R_i^{-1}R_i = \mathrm{I}$.
4. Associativity : $R_i(R_jR_k) = (R_iR_j)R_k$.

1. $R_-R_+ = \mathrm{I}$
$R_aR_bR_c = \mathrm{I}$
$R_aR_b = R_c$
2. Yes
3. Yes
4. Yes
(Not sure this is proper ?)

Problem 2.
Construct a 'multiplication table' for the triangle group, filling in the blanks on the following diagram:

	I	R_+	R_-	R_a	R_b	R_c
I						
R_+						
R_-						
R_a						
R_b						
R_c						

In row i, column j, put the product R_iR_j. Is this an Abelian group? How can you tell, just by looking at the multiplication table?

Solution

	I	R_+	R_-	R_a	R_b	R_c
I	I	R_+	R_-	R_a	R_b	R_c
R_+	R_+	R_-	I	R_b	R_c	R_a
R_-	R_-	I	R_+	R_c	R_a	R_b
R_a	R_a	R_c	R_b	I	R_-	R_+
R_b	R_b	R_a	R_c	R_+	I	R_-
R_c	R_c	R_b	R_a	R_-	R_+	I

Q. Abelian group?

A. a commutative group.

This group is not commutative. so it is not an abelian group.