Problem 1.

Show that $\partial\phi/\partial x^\mu$ is a covariant four-vector (ϕ is a scalar function of x, y, z, and t).
Hint: First determine (from Equation 3.8) how covariant four-vectors transform; then use $\partial\phi \ / \ \partial x^{\mu'} = (\partial\phi \ / \ \partial x^\nu)(\partial x^\nu \ / \ \partial x^{\mu'})$ transforms.

Solution

The four-vectors in x, y, z, t :
$x^0 = ct$, $x^1 = x$, $x^2 = y$, $x^3 = z$. Equation (3.7)

Equation (3.8)

$x^{0'} = \gamma(x^0$ - $\beta x^1)$
$x^{1'} = \gamma(x^1$ - $\beta x^0)$
$x^{2'} = x^2$
$x^{3'} = x^3$
$x^{\mu'} = \sum_{\nu=0}^{3} \Lambda^\mu_\nu x^\nu \ (\nu = 0, 1, 2, 3) = \Lambda^\mu_\nu x^\nu$

This is a contravariant tensor.

A contravariant tensor: axes changes $\rightarrow$ the components **inversly** change. And the vector does not change.

This is why it is called contravariant tensor.

A covariant : axes changes $\rightarrow$ the components also changes in the same way : vector $\rightarrow$ scalar.

A covariant vectors' components change the same as coordinates' change.

From Equation 3.8,

$x^{0'} = \gamma(x^0$ - $\beta x^1) \rightarrow x'_0 = \gamma(x^0 + \beta x^1)$

(The details are in Equation 3.1 and 3.3 , 3.3 is the inverse transform of 3.1. using the same inverse transform here.)

$x^{1'} = \gamma(x^1$ - $\beta x^0) \rightarrow x'_1 = \gamma(x^1 + \beta x^0)$

$x'_2 = x^2$

$x'_3 = x^3$

These are the covariant transform.

Q. What is the scalar function ϕ?

The quantity I (in Equation 3.13) and $r^2 = x^2 + y^2 + z^2$ can be scalar function. (not sure)

So, using, I = $(x^{0'})^2$ -$(x^{1'})^2$-$(x^{2'})^2$-$(x^{3'})^2$,

Since $I = x_\mu x^\mu$,

$\partial\phi$ / $\partial x^{\mu'}$ = x_μ, covariant tensor.

$https://en.wikipedia.org/wiki/Covariance_and_contravariance_of_vectors$