Problem 1.

Show that $\partial\phi/\partial x^\mu$ is a covariant four-vector (ϕ is a scalar function of x, y, z, and t).
Hint: First determine (from Equation 3.8) how covariant four-vectors transform; then use $\partial\phi \ / \ \partial x^{\mu'} = (\partial\phi \ / \ \partial x^\nu)(\partial x^\nu \ / \ \partial x^{\mu'})$ transforms.

Solution

Equation (3.8)

$x^{0'} = \gamma(x^0 - \beta x^1)$
$x^{1'} = \gamma(x^1 - \beta x^0)$
$x^{2'} = x^2$
$x^{3'} = x^3$
$x^{\mu'} = \sum_{\nu=0}^{3} \Lambda^\mu_\nu x^\nu \ (\nu = 0, 1, 2, 3) = \Lambda^\mu_\nu x^\nu$

$\partial\phi \ / \ \partial x^{\mu'} = (\partial\phi \ / \ \partial(\Lambda^\mu_\nu x^\nu))$
$\Lambda =$

(will continue)