

## Chapter 1

Quantum Electrodynamics (QED) : Maxwell's equation , Dirac equation .

Feynman diagrams, Quantum mechanics, Relativity

Physical intuition  $\rightarrow$  bottom-up approach  $\rightarrow$  many gaps

Goal is the top-down approach

Cross section calculation

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 E_{cm}^2 \cdot |M|^2}$$

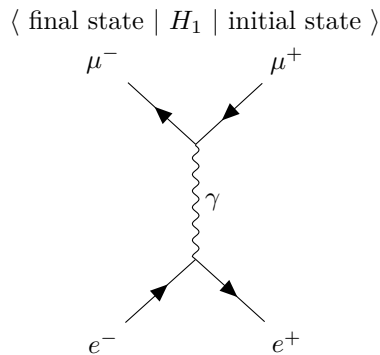
( CM scattering )

For the QED, The 'M' is not known.

The best we can do : Set M as a perturbation series of QED, and evaluate the first term.

The Feynman diagram  $\rightarrow$  visualize the perturbation.

In QM perturbation theory, to first order, the amplitude is,



This is the first order, but the hamiltonian can not mediate the two state, but gamma does it.

So, expand this equation to the next order with  $\gamma$ .

For  $(e^- + e^+ \rightarrow \mu^- + \mu^+)$ ,

$$M \sim \langle \mu^+ \mu^- | H_1 | \gamma \rangle^\mu \langle \gamma | H_1 | e^+ e^- \rangle_\mu$$

1. External electron lines :  $|e^+ e^- \rangle$
2. External muon lines :  $\langle \mu^+ \mu^- |$
3. The vertices :  $H_1$
4. Internal photon line :  $|\gamma \rangle \langle \gamma |$
5. The amplitude  $M$  will be a Lorentz-invariant scalar as long as each states are 4-vectors.

$\langle \gamma | H_1 | e^+ e^- \rangle_\mu \rightarrow H_1$  is related to  $\gamma$  and electrons  $\rightarrow$  the elements of the matrix is proportional to  $e$ .

Spin Orientations :

1. The electron and Muon  $\rightarrow$  parallel spins.
2. The electron and Muon : "right handed", The positron and positive Muon : "left handed".
3. The electron and positron spins add up to one angular momentum in the  $+z$  direction.

The  $H_1$  should conserve the angular momentum.

$\rightarrow$  photon's polarization vector :  $\epsilon^\mu = (0, 1, i, 0)$

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<u>Electrons</u>	<u>Positrons</u>	
$\psi(x) = a e^{-i(i/\hbar)p \cdot x} u^{(s)}(p)$	$\psi(x) = a e^{i(i/\hbar)p \cdot x} v^{(s)}(p)$	(7.94)

where  $s = 1, 2$  for the two spin states. The spinors  $u^{(s)}$  and  $v^{(s)}$  satisfy the momentum space Dirac equation(s):

$$(\gamma^\mu p_\mu - mc)u = 0 \qquad (\gamma^\mu p_\mu + mc)v = 0 \qquad (7.95)$$

Finally, we must put in the appropriate photon polarization vectors. Recall that for 'spin up' ( $m_s = +1$ ) we have (see footnote to Equation 7.94)

$$\epsilon_+ = -(1/\sqrt{2})(1, i, 0) \qquad (7.159)$$

whereas for 'spin down' ( $m_s = -1$ )

$$\epsilon_- = (1/\sqrt{2})(1, -i, 0) \qquad (7.160)$$

Figure 1: Grifitth chapter7