Problem 1.

Derive Equation 6.3. [Hint: What fraction of the original sample decays between t and t + dt? What, is the (initial) probability, p(t)dt, of any given particle decaying between t and t + dt? The average lifetime is $\int_0^\infty tp(t)\,dt$.]

Solution

$\tau = \frac{1}{\Gamma}$, Equation (6.3)

Q. What fraction of the original sample decays between t and t + dt?

A. dN = -ΓNdt, Equation (6.1)

Q. What is the initial probability, p(t)dt?

A. N(t) = N(0)$e^{-\Gamma t}$, Equation (6.2)

Using probability density function, (from wiki),

1 = C $\int_0^\infty p(t)\,dt$ = C $\int_0^\infty N(0)e^{-\Gamma t}\,dt$, where C is normalizing coefficient.

= C $\cdot \frac{N_0}{\Gamma}$

C = $\frac{\Gamma}{N_0}$

Q. The average lifetime?

A. $\tau = \langle t \rangle = \int_0^\infty tp(t)\,dt = \int_0^\infty t\frac{\Gamma}{N_0}N(0)e^{-\Gamma t}\,dt = \Gamma\int_0^\infty te^{-\Gamma t}\,dt$

$\Gamma\int_0^\infty te^{-\Gamma t}\,dt = (integration\ by\ parts) = \frac{1}{\Gamma} = \tau$

reference [wiki - Exponential decay, Probability Density function, integration by parts]