Problem 1.

If a charged particle is undeflected in passing through uniform crossed electric and magnetic fields E and B(mutually perpendicular and both perpendicular to the direction of motion), what is its velocity? if we now turn off the electric field, and the particle moves in an arc or radius R, what is its charge-to-mass ratio?

solution

1. lorentz force

$$F = q(E + v \times B) \tag{1}$$

2. poynting vector (not used)

$$S = E \times H \tag{2}$$

from the lorentz force, the velocity is derived by

$$qE = -v \times B \tag{3}$$

because the force is zero. (The E and B are perpendicular and the particle is undeflected.)

Turn off E, and then moving in radius R, what is its charge-to-mass ratio?

motion of circle

$$F = mv^2/R = qvBq/m = (v(vel))^2/(v(vol)BR) \tag{4}$$

Problem 2.

The mass of Yukawa's meson can be estimated as follows. When two protons in a nucleus exchange a meson (mass m), they must temporarily violate the conservation of energy by an amount mc^2 (the rest energy of the meson). The Heisenberg uncertainty principle says that you may 'borrow' an energy $\triangle E$, provide you 'pay it back' in a time $\triangle t$ given by $\triangle E \triangle t = \frac{\hbar}{2}$ (where $\hbar \equiv \frac{h}{\pi}$). In this case, we need to borrow $\triangle E = mc^2$ long enough for the meson to make it from one proton to the other. It has to cross the nucleus (size r_0), and it travels, presumably, at some substantial fraction of the speed of light, so, roughly speaking, $\triangle t = r_0/c$. Putting all this together, we have

$$m = \frac{\hbar}{2r_0 c}$$

Using $r_0 = 10^{-13}$cm (the size of a typical nucleus), calculate the mass of Yukawa's meson. Express your answer in MeV/c^2, and compare the observed

mass of the pion. [Comment: If you find that argument compelling. I can only say that you're pretty gullible. Try it for an atom, and you'll conclude that the mass of the photon is about 7 X 10^{-30}g, which is nonsense. Nevertheless, it is a useful device for 'back-of-the-envelope' calculations, and it does very well for the pi meson. Unfortunately, many books present it as through it were a rigorous derivation, which is certainly is not. The uncertainty principle does not license violation of conservation of energy (nor does any such violation occur in this process; we shall see later on how this comes about). Moreover, it's an inequality, $\triangle E \triangle t >= \frac{\hbar}{2}$], which at most could give you a lower bound on m. it is typically true that the range of a force is inversely proportional to the mass of the mediator, but the size of a bound state is not always a good measure of the range. (That's why the argument fails for the photon:the range of the electromagnetic force is infinite, but the size of an atom is not.) In general, when you hear a physicist invoke the uncertainty principle, keep a hand on your wallet.]

$\hbar = 6.58 \times 10^{-22}$ MeV·s

$r_0 = 10^{-15}$m

c $= 3 \times 10^8$m/s

$$m = \frac{6.58 \times 10^{-22} MeV \cdot s}{2 \times 10^{-15} m \cdot 3 \times 10^8 m/s}$$

$$m = \frac{6.58 \times 10^{-22} MeV \cdot s \cdot 3 \times 10^8 m/s}{2 \times 10^{-15} m \cdot 3 \times 10^8 m/s \cdot 3 \times 10^8 m/s}$$

$$m = 98.7 \ MeV/c^2$$

Problem 3.

In the period before the discovery of the neutron, many people thought that the nucleus consisted of protons and electrons, with the atomic number equal to the excess number of protons. Beta decay seemed to support this idea - after all,
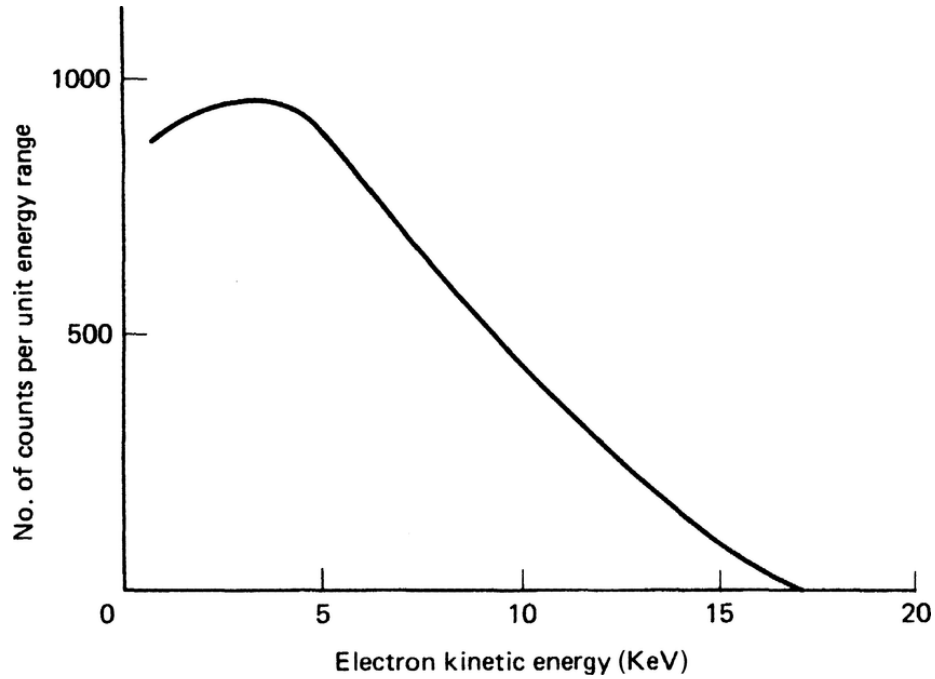
Figure 1: Beta decay.

electrons come popping out; doesn't that imply that there were electron inside? Use the position-momentum uncertainty relation, $\triangle x \triangle p \geq \hbar/2$, to estimate the minimum momentum of an electron confined to a nucleus (radius 10^{-13} cm). From the relativistic energy-momentum relation, E^2 - $\mathbf{p^2} \geq m^2 c^4$, determine the corresponding energy and compare it with that of an electron emitted in, say, the beta decay of tritium (Figure 1, textbook-Figure 1.5). (This result convinced some people that the beta decay electron could not have been rattling around inside the nucleus, but must be produced in the disintegration itself.)

Solution

$\triangle p \geq \frac{\hbar}{2 \triangle x}$,

$2 \triangle x = 10^{-15}$m,

$$\triangle p = \frac{6.58 \times 10^{-22} MeV \cdot s}{2 \times 10^{-15} m}$$

$E^2 = \mathbf{p^2} c^2 + m^2 c^4$

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$$m_e c^2 = 0.511 MeV$$

$$m_e^2 c^4 = 0.261 MeV^2$$

$$p^2 c^2 = (\frac{6.58 \times 10^{-22} MeV \cdot s \cdot 3 \times 10^8 m/s}{2 \times 10^{-15} m})^2 = 9740 MeV^2$$

$$E = \sqrt{\mathbf{p^2} c^2 + m^2 c^4} = 98.68 MeV$$

This is too large compared to the beta decay energy spectrum.

Problem 4.

The Gell-Mann/Okubo mass formula relates the masses of members of the baryon octet(ignoring small differences between p and n; Σ^+, Σ^0, and Σ^-; and Ξ^0 and Ξ^-):

$$2(m_N + m_\Xi) = 3m_\Lambda + m_\Sigma$$

Using this formula, together with the known masses of the nucleon N (use the average of p and n), Σ (again, use the average), and Ξ (ditto), 'predict' the mass of the Λ. How close do you come to the observed value?

Solution

Baryon masses:
$m_N(m_p$ or $m_n) = (938.272+939.565)/2 = 938.919$
$m_\Xi = (1314.8+1321.3)/2 = 1318.05$
$m_\Sigma = (1189.37+1189.37+1192.64)/3 = 1190.46$

$3m_\Lambda = 2m_N + 2m_\Xi$ - $m_\Sigma = 3323.478$

Therefore, $m_\Lambda = 1107.826$.
The experimental mass of the $\Lambda = 1115.68$.

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So, it is 7.854 MeV smaller than experimental value.

Problem 5.

The same formula applies to the mesons (with $\Sigma \to \pi, \Lambda \to \eta, etc$), except that in this case, for reasons that remain something of a mystery, you must use the squares of the masses. Use this to 'predict' the mass of the η. How close do you come?

Solution
In short, in the octet geometry, ($2 \times$ (the average of the top two + the average of the bottom two) = Middle one(+,-,0) + $3 \times$ Middle (one neutral only one))

$$(m_{K^0} + m_{K^+} + m_{K^-} + m_{\bar{K}^0}) = 3m_\eta + m_\pi$$

$\pi^0 = u\bar{u}, \ \pi^+ = u\bar{d}, \ \pi^- = d\bar{u}, \ \eta = d\bar{d}$
$2(493.677 + 497.611) = 3 \times \eta + (\frac{2 \times 139.57 + 134.98}{3})$
$\eta = 614.845 MeV$, but the experimental value is $547.862 \ MeV$.
This is a large discrepancy. Using Mass square instead of just a Mass as indicated in the book,
$2(493.677^2 + 497.611^2) = 3 \times \eta^2 + 138.04^2$,
$\eta = 566.748 \ MeV$, the experimental value is $547.862 \ MeV$.
This is a much closer value.

Problem 6.
The mass formula for decuplets is much simpler - equal spacing between the rows:

$$m_\triangle - m_{\Sigma^*} = m_{\Sigma^*} - m_{\Xi^*} = m_{\Xi^*} - m_\Omega$$

Use this formula (as Gell-Mann did) to predict the mass of the Ω^-. (Use the average of the first two spacings to estimate the third.) How close is your prediction to the observed value?

Solution
Using the mass table in this book,
$m_\triangle - m_{\Sigma^*} = 1232 - 1385 = -153 MeV/c^2$
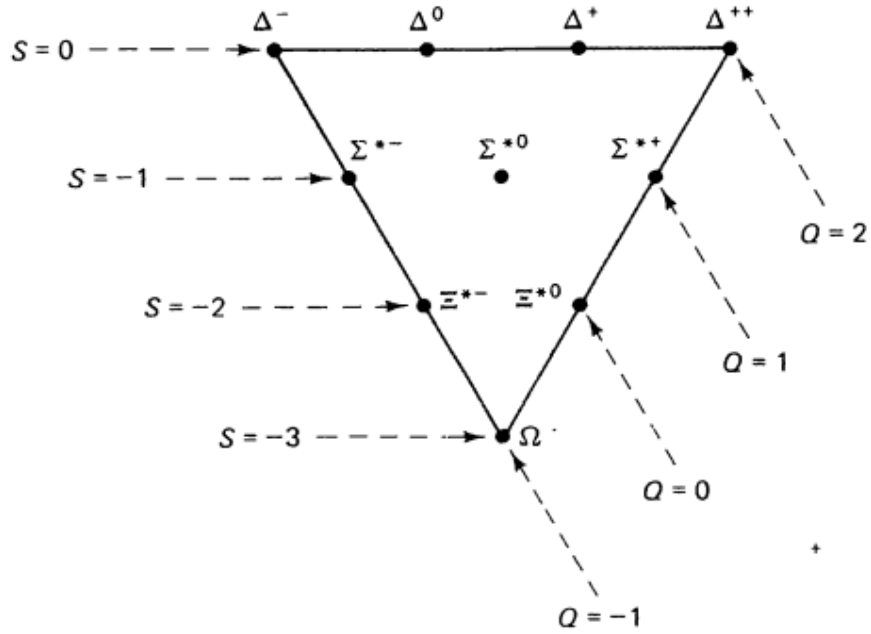$m_{\Sigma^*} - m_{\Xi^*} = 1385 - 1533 = -148 MeV/c^2$

Figure 2: decuplet

Using the average of the first two gaps $= \frac{-153-148}{2} = -150.5 MeV/c^2$

$m_{\Xi^*} - m_\Omega = -150.5 MeV/c^2$, So, the $m_{\Omega^-} = 1533 + 150.5 = 1683.5$ MeV. The experimental value, $1672\ MeV/c^2$ is very close from the expected value, $1683.5\ MeV/c^2$

Baryons (spin 1/2)

Baryon	Quark Content	Charge	Mass	Lifetime	Principal Decays
$N\begin{cases} p \\ n \end{cases}$	uud	1	938.272	∞	–
	udd	0	939.565	885.7	$pe\bar{\nu}_e$
Λ	uds	0	1115.68	2.63×10^{-10}	$p\pi^-,\, n\pi^0$
Σ^+	uus	1	1189.37	8.02×10^{-11}	$p\pi^0,\, n\pi^+$
Σ^0	uds	0	1192.64	7.4×10^{-20}	$\Lambda\gamma$
Σ^-	dds	-1	1197.45	1.48×10^{-10}	$n\pi^-$
Ξ^0	uss	0	1314.8	2.90×10^{-10}	$\Lambda\pi^0$
Ξ^-	dss	-1	1321.3	1.64×10^{-10}	$\Lambda\pi^-$
Λ_c^+	udc	1	2286.5	2.00×10^{-13}	$pK\pi,\, \Lambda\pi\pi,\, \Sigma\pi\pi$

Baryons (spin 3/2)

Baryon	Quark Content	Charge	Mass	Lifetime	Principal Decays
Δ	$uuu,\, uud,\, udd,\, ddd$	$2,1,0,-1$	1232	5.6×10^{-24}	$N\pi$
Σ^*	$uus,\, uds,\, dds$	$1,0,-1$	1385	1.8×10^{-23}	$\Lambda\pi,\, \Sigma\pi$
Ξ^*	$uss,\, dss$	$0,-1$	1533	6.9×10^{-23}	$\Xi\pi$
Ω^-	sss	-1	1672	8.2×10^{-11}	$\Lambda K^-,\, \Xi\pi$

Figure 3: decuplet2