

Chapter 1

Quantum Electrodynamics (QED) : Maxwell's equation , Dirac equation .

Feynman diagrams, Quantum mechanics, Relativity

Physical intuition → bottom-up approach → many gaps

Goal is the top-down approach

Cross section calculation

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 E_{cm}^2 \cdot |M|^2}$$

(CM scattering)

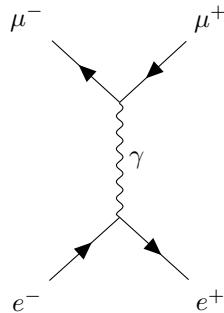
For the QED, The 'M' is not known.

The best we can do : Set M as a perturbation series of QED, and evaluate the first term.

The Feynman diagram → visualize the perturbation.

In QM perturbation theory, to first order, the amplitude is,

$$\langle \text{ final state} | H_1 | \text{ initial state} \rangle$$



This is the first order, but the hamiltonian can not mediate the two state, but gamma does it.

So, expand this equation to the next order with γ .

For $(e^- + e^+ \rightarrow \mu^- + \mu^+)$,

$$M \sim \langle \mu^+ \mu^- | H_1 | \gamma \rangle^\mu \langle \gamma | H_1 | e^+ e^- \rangle_\mu$$

1. External electron lines : $|e^+ e^- \rangle$
2. External muon lines : $\langle \mu^+ \mu^- |$
3. The vertices : H_1
4. Internal photon line : $|\gamma\rangle\langle\gamma|$
5. The amplitude M will be a Lorentz-invariant scalar as long as each states are 4-vectors.

$\langle \gamma | H_1 | e^+ e^- \rangle_\mu \rightarrow H_1$ is related to γ and electrons \rightarrow the elements of the matrix is proportional to e .

Spin Orientations :

1. The electron and Muon \rightarrow parallel spins.
2. The electron and Muon : "right handed", The positron and positive Muon : "left handed".
3. The electron and positron spins add up to one angular momentum in the +z direction.

The H_1 should conserve the angular momentum.

\rightarrow photon's polarization vector : $\epsilon^\mu = (0, 1, i, 0)$

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Electrons

Positrons

$$\psi(x) = a e^{-(i/\hbar)p \cdot x} u^{(s)}(p) \quad \psi(x) = a e^{(i/\hbar)p \cdot x} v^{(s)}(p) \quad (7.94)$$

where $s = 1, 2$ for the two spin states. The spinors $u^{(s)}$ and $v^{(s)}$ satisfy the momentum space Dirac equation(s):

$$(\gamma^\mu p_\mu - mc)u = 0 \quad (\gamma^\mu p_\mu + mc)v = 0 \quad (7.95)$$

Finally, we must put in the appropriate photon polarization vectors. Recall that for 'spin up' ($m_s = +1$) we have (see footnote to Equation 7.94)

$$\epsilon_+ = -(1/\sqrt{2})(1, i, 0) \quad (7.159)$$

whereas for 'spin down' ($m_s = -1$)

$$\epsilon_- = (1/\sqrt{2})(1, -i, 0) \quad (7.160)$$

Figure 1: Griffith chapter7