Problem 1.

(a) The deutron's mass is 1875.6 MeV/c^2. What is its binding energy? is this a relativistic system?
(b) If you take the up- and down-quark masses to be those given in Table 4.4, what is the binding energy of a pion? is this a relativistic system?

Solution
(a) One neutron + One proton - 1875.6 MeV/c^2
Neutron : 939.565 MeV/c^2
Proton : 938.272 MeV/c^2
Mass difference = 2.2 MeV/c^2

Q. A relativistic system?
A. mass difference $\times$ c^2 = binding energy = 2.2 MeV (in wikipedia, it says 2.2 MeV, yes)

(b) Figure 1 Quark masses.

$$M(meson) = m_1 + m_2 + A\frac{(S_1 \cdot S_2)}{m_1 m_2} \quad (5.48)$$

This is because of the fact that the pseudoscalr and vector mesons have different masses even though they have the same quark contents while their spin orientations are different.

$$S_1 \cdot S_2 = \frac{1}{2}(S^2 - S_1^2 - S_2^2) = \begin{cases} \frac{1}{4}\hbar^2, \text{ for } s = 1 \text{ (vector mesons)} \\ -\frac{3}{4}\hbar^2, \text{ for } s = 0 \text{ (pseudoscalars)} \end{cases} \tag{1}$$

$m_u = m_d = 308$ MeV/c^2, $m_s = 483$ MeV/c^2, A = 159 MeV/c^2

$$\begin{cases} |1\ 1\rangle = -u\bar{d} \\ |1\ 0\rangle = (u\bar{u} - d\bar{d})/\sqrt{2} \\ |1\ -1\rangle = d\bar{u} \end{cases} \tag{2}$$

$\pi^+ = u + \bar{d} = 2MeV/c^2 + 5MeV/c^2 + 159MeV/c^2\frac{(197\times197)MeV^2fm^2}{10MeV^2/c^4} = 7MeV/c^2 + 159 \times 197 \times 197 \times 10^{-30} \times 10^{16} \times \frac{1}{10}MeV/c^2 = 7MeV/c^2 + 6.17 \times 10^{-9}MeV/c^2$
The mass of π^+ is 139 MeV/c^2 .(wiki-pion)
Therefore the binding energy is approximately 139 - 7 = 132 MeV/c^2.

Table 4.4 Quark masses (MeV/c^2)

Quark flavor	Bare mass	Effective mass
u	2	336
d	5	340
s	95	486
c	1300	1550
b	4200	4730
t	174 000	177 000

Warning: These numbers are somewhat speculative and model dependent [12].

Figure 1: Table4.4.