

Introduction to Hardware Design

Basic Digital Logic: Solutions

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1. *Propagation delay:* A signal x has time-value change points

$$(10, 0), (20, 4), (30, 2)$$

where the time is in nanoseconds. The signal is passed through a circuit that produces $y = 2x + 1$ with a propagation delay of 3 ns. What are the time-value change points of y ? Use x for unknown values.

2. *Critical path:* Consider the following SystemVerilog code:

```
always_ff @(posedge clk) begin
    x1 <= 3*x1>>2 - 5*x>>3 + 7;
    x2 <= 3*x1>>2 + 2;
end
```

Here $3*x>>2$ denotes multiplication by 3 followed by a right shift by 2 bits (division by 4). Assume that:

- Each multiplication with shift takes 4 ns.
- Each addition or subtraction takes 2 ns.
- Registers have zero delay.

What is the critical path delay of this circuit?

3. *Sequential updates:* Consider the following SystemVerilog code:

```
always_ff @(posedge clk) begin
    x <= x + v;
    if (x > 30) begin
        v <= -10;
    end else if (x < 0) begin
        v <= 10;
    end
end
```

Starting from $(x, v) = (15, 10)$, what are the values of (x, v) for the next 5 clock cycles?

4. *Splitting paths over multiple cycles:* Consider the following SystemVerilog code:

```
always_comb begin
    act_in = w1*xreg+b1;
    if (act_in > 0) begin
        a = act_in;
    end else begin
        a = 0;
```

```

    end
    xsq = xreg*xreg>>4;
    y = xsq + w2*a + b2;
end
always_ff @(posedge clk) begin
    xreg <= x;
end

```

where $>>4$ denotes a right shift by 4 bits (division by 16). So, the code registers the input x into $xreg$ on each clock cycle, and produces the output y in a single clock cycle based on the registered value. The circuit is to be synthesized with components:

- Multiplication with shift: 4 ns
- Addition or subtraction: 2 ns
- Logic to compute $\max\{0, u\}$: 1 ns
- Registers: zero delay

Assume that w_1 , b_1 , w_2 , and b_2 are constants.

- Draw a block diagram of the circuit showing an implementation of the circuit. If this is being auto-graded, you do not need to submit the diagram. But, I will include it in the solutions.
- What is the critical path delay of this circuit?
- Rewrite the code to operate over two clock cycles instead of one to minimize the critical path delay. What is the critical path delay of the new circuit?

5. *Exponent* Suppose we wish to implement the function

$$y = x^i,$$

with an integer exponent $i \in \{0, 1, 2, 3\}$. The input x and output y are signed short integers – do not worry about overflow. Write a SystemVerilog module that computes y . The output should always be 2 cycles after the input, even if $i = 0, 1$ or 2 . Use only one multiplication in each clock cycle.

Hint: This problem is a bit harder. You will need to use delay lines to store the input x and exponent i .