Cooperative Manipulation of an Unknown Payload with Concurrent Mass and Drag Force Estimation

Sandesh Thapa[1], Ryan V. Self[1], Rushikesh L. Kamalapurkar[1] and He Bai[1]

Abstract— **In this paper, we consider a team of robots that cooperatively transport a payload with an unknown mass in the presence of unknown drag forces. We develop a concurrent learning based adaptive control algorithm that estimates the drag forces and the unknown mass and drives the agents and the payload to a common desired velocity. The algorithm also regulates the contact forces on the payload. We prove that the estimated parameters, including the mass of the payload, converge to their true values. We validate the effectiveness of the proposed algorithm using a simulation example.**

I. INTRODUCTION

Cooperative manipulation has been an active area of research for decades due to the complexity of the problem involved (see e.g., [1]–[9]). Multiple robots can be employed together to increase the capability of each individual robot, which enables better manipulation and control of heavier payloads for various applications. Possible applications of such multi-robot systems include disaster response, transport, manufacturing, search and rescue operations, and construction, among others.

Literature on cooperative manipulation is extensive, ranging from cooperative assembly robots in manufacturing [7], [10], ground manipulation with mobile robots [11], [12], multi-rotor based aerial manipulation [13]–[16], and marine applications using autonomous tugboats [17]. Recently, the authors in [16] develop a framework for control and estimation of an unknown payload and obstacle avoidance for cooperative aerial manipulation. The authors in [8] develop a decentralized model reference adaptive controller to transport a rigid payload in $\mathbb{R}^2$ or $\mathbb{R}^3$ by controlling the spatial velocity of the payload. Reference [6] develop a decentralized passive force control strategy for collaborative manipulation using micro-aerial vehicles based on master-slave methodology. The authors in [12] propose a distributed approach to estimate the internal properties of the payload (possibly large) using noisy measurements of velocity and contact forces applied to the payload. Reference [17] develop an adaptive force/torque control strategy that allows a swarm of autonomous tugboats to cooperatively move a heavier object on water to compensate for model uncertainties associated with the drag coefficient.

In this paper, we extend our previous work [18] to address the problem of force control in cooperative manipulation in the presence of unknown drag forces and unknown payload mass in either $\mathbb{R}^2$ or $\mathbb{R}^3$. Our control objective is that all the agents and the payload converge to a constant velocity and the contact force is regulated to a desired set-point. Since there are multiple unknown parameters, such as the mass and the drag coefficients, and the desired velocity does not possess the property of persistency of excitation (PE), we employ the recently developed concurrent learning (CL) technique [19]–[21] to address the parameter estimation problem. CL-based algorithms leverage transient data to estimate unknown parameters with a relaxed excitation condition in the regressor. Specifically, we develop an adaptive control law that consists of a CL update law and a stabilizing update law. The proposed algorithm simultaneously estimates the coefficients of the drag forces acting on the robots and the payload and the unknown mass of the payload online. We establish stability of the closed-loop system using Lyapunov theory. Using a numerical simulation, we demonstrate that the developed algorithm achieves the control objective and parameter convergence without PE. The contribution of this paper is the development, application, and stability analysis of a CL based controller for cooperative manipulation.

The rest of this paper is organized as follows. We formulate the cooperative payload transport problem in Section II. In Section III, we develop a CL based adaptive control law to achieve the force regulation of the payload and velocity convergence of the agents. In Section IV, we analyze the stability of the system using Lyapunov theory. A numerical example is presented in Section V to demonstrate the performance of our control law. Conclusions and future work are discussed in Section VI.

II. PROBLEM FORMULATION

A. Dynamic Model

Consider N agents holding a common load as shown in Fig. 1 ($N = 3$). Each agent is a robot with a rigid link extension. Agent i is attached to the load at the point a_i. Let $x_i \in \mathbb{R}^2$ or $\mathbb{R}^3$ be the position of the end-effector of agent i in the inertial frame and $r_i \in \mathbb{R}^3$ be a fixed vector in the body frame of the load. Initially, $x_i(0) = a_i(0) = x_c(0) + r_i$, where $x_c \in \mathbb{R}^2$ or $\mathbb{R}^3$ is the position of the center of mass of the load in the inertial frame. Fig. 1 also shows the coordinate system defined to derive the kinematics of the system. Σ_I is the world fixed inertial frame and $O_{c,i}$ is the body-fixed frame attached to each agent i.

We assume that the payload is a rigid object surrounded by elastic or deformable materials, such as bumpers and springs. When there is relative motion between an agent and the payload, the surrounding material will be deformed, causing

[1]The authors are with School of Mechanical & Aerospace Engineering, Oklahoma State University, Stillwater, OK, 74078, USA, {sandesh.thapa, rself, rushikesh.kamalapurkar, he.bai}@okstate.edu.

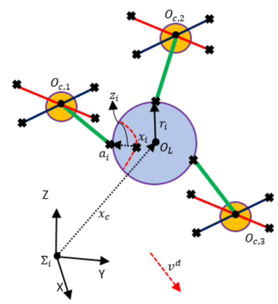

Fig. 1. Three robots transport a common load. Note that a_i is the initial position of agent i. As the agents move, the payload is deformed, the new position of the agent is x_i and the deformation is approximated by $z_i = x_i - a_i$.

the payload to experience tension or compression. We use a mass spring model to model the contact force generated due to the deformation. Specifically, we define a_i as

$$a_i(t) := x_c(t) + r_i, \tag{1}$$

which is the position of the ith agent if there is no deformation. Note that a_i satisfies

$$\dot{a}_i = \dot{x}_c. \tag{2}$$

We approximate the deformation as

$$z_i = x_i - a_i, \quad \forall i = 1, \cdots, N, \tag{3}$$

and assume that the contact force f_i between agent i and the payload is given by the gradient of a positive-definite potential function $P_i : \mathbb{R}^3 \to \mathbb{R}_{\geq 0}$, i.e.,

$$f_i(z_i) = \nabla P_i(z_i). \tag{4}$$

We further assume that P_i satisfies the following constraints:

$$P_i(z_i) = 0 \iff z_i = 0, \tag{5}$$

$$\nabla P_i(z_i) = 0 \iff z_i = 0. \tag{6}$$

The dynamics of the load is given by

$$M_c \ddot{x}_c = \sum_{i=1}^{N} f_i(z_i) - M_c g e_3 - F_d, \tag{7}$$

where M_c is the mass of the load, g is the gravitational constant, e_3 is the unit vector $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$, and F_d is a constant disturbance acting on the payload. In (7), we approximate the payload dynamics as a rigid body. This is valid when the deformations are small.

The translational dynamics of the N agents are given by

$$m_i \ddot{x}_i = F_i - f_i(z_i) - m_i g e_3 - C_i \|\dot{x}_i\| \dot{x}_i, \quad \forall i = 1, ..., N, \tag{8}$$

where m_i is the mass of the ith agent, F_i is the force applied to agent i, f_i is the contact force to agent i and $C_i \in \mathbb{R}^{3\times 3}$ is the drag coefficient matrix for each agent i. We assume that C_i is a diagonal matrix given by $\text{diag}\{C_i^x, C_i^y, C_i^z\}$, where

$$\text{diag}\{C_i^x, C_i^y, C_i^z\} = \begin{bmatrix} C_i^x & 0 & 0 \\ 0 & C_i^y & 0 \\ 0 & 0 & C_i^z \end{bmatrix}.$$

Note that the $C_i \|\dot{x}_i\| \dot{x}_i$ term is a standard second order drag force model [22].

B. Control Objective

In our previous work [18] and [23], the disturbances F_d and $C_i \|\dot{x}_i\| \dot{x}_i$ were neglected. In this paper, we assume that the load has an unknown mass M_c and that unknown disturbances F_d and $C_i \|\dot{x}_i\| \dot{x}_i$ act on the payload and agent i, $\forall i = 1, \ldots, N$, respectively. Our control objective is to design F_i in (8) such that all the agents and the payload converge to a constant velocity v^d and the contact force f_i is regulated to a set-point f_i^d. Motivated by results in [19]–[21], [24], [25], a concurrent learning adaptive controller is developed to achieve the stated objective.

III. Control Design

If the velocity of the payload converges to v^d, the sum of the contact forces f_i would satisfy

$$\sum_{i=1}^{N} f_i = \underbrace{M_c g e_3 + F_d}_{F_c}. \tag{9}$$

Assuming each agent experiences an equal contact force, we choose the set-points for the individual contact forces as

$$f_i^d = \frac{F_c}{N}. \tag{10}$$

We assume that for a given f_i^d there exists a locally unique constant deformation z_i^d, such that

$$f_i^d = \nabla P_i(z_i^d) \quad \text{and} \quad \nabla^2 P_i(z_i^d) > 0. \tag{11}$$

Assumption (11) is satisfied by linear spring-force model as well as certain classes of nonlinear models, such as $f_i = b_i |z_i|^2 z_i$, with $b_i > 0$. Using the estimates $\hat{M}_c$ and $\hat{F}_d$ for the mass and the constant drag, respectively, the set-points can be estimated as

$$\hat{f}_i^d = \frac{1}{N} Y_f \hat{\theta}_c, \tag{12}$$

where $\hat{\theta}_c = \begin{bmatrix} \hat{F}_d^x, & \hat{F}_d^y, & \hat{F}_d^z, & \hat{M}_c \end{bmatrix}^T$ and $Y_f = \begin{bmatrix} I_{3\times 3}, & g e_3 \end{bmatrix}$.

We propose to design the controller F_i as

$$F_i = -K_i(\dot{x}_i - v^d) + \hat{f}_i^d + m_i g e_3 + \text{diag}\{\hat{C}_i\} \|\dot{x}_i\| \dot{x}_i, \tag{13}$$

where $K_i = K_i^T > 0$. The first term in (13) is the feedback term to ensure $\dot{x}_i \to v^d$, and the second term is the feed-forward term that regulates $f_i \to \hat{f}_i^d$. The last two terms are introduced to cancel the gravity and drag force acting on the agents. We next design a drag force estimation law for each agent to drive $\hat{C}_i \to C_i$.

A. Drag Coefficient Estimation for the Agents

We rewrite the dynamics for each agent as:

$$\ddot{x}_i = \frac{1}{m_i}\left(F_i - f_i(z_i)\right) - Y_i\theta_i, \tag{14}$$

where

$$Y_i = \left[\operatorname{diag}\left\{\frac{1}{m_i}\|\dot{x}_i\|\dot{x}_i\right\}\right] \quad \text{and} \quad \theta_i := \left[C_i^x, C_i^y, C_i^z\right]^T.$$

We propose the following update law to learn the drag force coefficient θ_i

$$\dot{\hat{\theta}}_i = -\lambda_i \Gamma_i \Phi_i + \dot{\hat{\theta}}_i^{cl}, \tag{15}$$

where $\lambda_i > 0$, $\Gamma_i \in \mathbb{R}^{3\times 3}$ is the learning gain computed using (22), $\dot{\hat{\theta}}_i^{cl}$ is based on the concurrent learning update law in (21) and

$$\Phi_i = \|\dot{x}_i\|\left[\xi_i^1\dot{x}_i^1 \quad \xi_i^2\dot{x}_i^2 \quad \xi_i^3\dot{x}_i^3\right]^T, \tag{16}$$

where ξ_i^j is the jth element of ξ_i and

$$\xi_i := \dot{x}_i - v^d. \tag{17}$$

We next briefly explain the construction of the concurrent learning update law $\dot{\hat{\theta}}_i^{cl}$. Interested readers are referred to [26] for more details. Integrating (14) over the interval $[t-\tau_1, t]$ for some constant $\tau_1 \in \mathbb{R}_{>0}$,

$$\underbrace{x_i(t) - x_i(t-\tau_1)}_{P_i(t)} = \underbrace{\int_{t-\tau_1}^{t} f_o\left(F_i(\gamma), z_i(\gamma)\right)\mathrm{d}\gamma}_{H_i(t)} + \underbrace{\int_{t-\tau_1}^{t} Y_i(\gamma)\,\mathrm{d}\gamma}_{G_i(t)}\,\theta_i, \tag{18}$$

where $f_o(F_i, z_i) = \frac{1}{m_i}\left(F_i - f_i(z_i)\right)$.

For ease of exposition, it is assumed that a history stack, i.e., a set of ordered pairs $\{(P_{k,i}, H_{k,i}, G_{k,i})\}_{k=1}^{M}$ such that

$$P_{k,i} = H_{k,i} + \theta_i^T G_{k,i},\ \forall k \in \{1, \cdots, M\}, \tag{19}$$

is available a priori. A history stack $\{(P_{k,i}, F_{k,i}, G_{k,i})\}_{k=1}^{M}$ is called *full rank* if there exists a constant $\underline{c_i} \in \mathbb{R}$ such that

$$0 < \underline{c_i} < \lambda_{\min}\{\mathscr{G}_i\}, \tag{20}$$

where the matrix $\mathscr{G}_i \in \mathbb{R}^{3\times 3}$ is defined as $\mathscr{G}_i := \sum_{k=1}^{M} G_{k,i}G_{k,i}^T$. To select the data points in $\mathscr{G}_i$, a singular value maximization algorithm can be used [26]. If the condition in (20) is not satisfied, meaning that the matrix $\mathscr{G}_i$ is not full rank, then data is added to the history stack $\mathscr{G}_i$ until $\underline{c_i} > 0$. Once (20) is satisfied, then a data point is added to $\mathscr{G}_i$ only if a predefined amount of time has passed since the last change and if it increases the minimum singular value of $\mathscr{G}_i$. Therefore, although $\mathscr{G}_i$ can be discontinuous, it is always piecewise continuous with a lower-bounded dwell time.

The concurrent learning update law is then given by

$$\dot{\hat{\theta}}_i^{cl} = k_i\Gamma_i \sum_{k=1}^{M} G_{k,i}\left(P_{k,i} - H_{k,i} - \hat{\theta}_i^T G_{k,i}\right)^T, \tag{21}$$

where $k_i \in \mathbb{R}_{>0}$ is a constant adaptation gain, and $\Gamma_i \in \mathbb{R}^{3\times 3}$ is the least-squares gain updated using the update law

$$\dot{\Gamma}_i = \beta_i\Gamma_i - k_i\Gamma_i\frac{1}{1+\alpha_i\|\mathscr{G}_i\|}\mathscr{G}_i\Gamma_i. \tag{22}$$

in which $\alpha_i,\ \beta_i \in \mathbb{R}_{>0}$. Using arguments similar to [27, Corollary 4.3.2], it can be shown that provided $\lambda_{\min}\left\{\Gamma_i^{-1}(0)\right\} > 0$, the least squares gain matrix satisfies

$$\underline{\Gamma_i}\mathrm{I}_3 \le \Gamma_i(t) \le \overline{\Gamma_i}\mathrm{I}_3, \tag{23}$$

where $\underline{\Gamma_i}$ and $\overline{\Gamma_i}$ are positive constants and I_n denotes an $n \times n$ identity matrix.

B. Mass and Disturbance Estimation for the Payload

We rewrite the payload dynamics (7) as:

$$M_c\ddot{x}_c = \begin{bmatrix}\sum_{i=1}^{N} f_i^x \\ \sum_{i=1}^{N} f_i^y \\ \sum_{i=1}^{N} f_i^z\end{bmatrix} - \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}F_d^x \\ F_d^y \\ F_d^z\end{bmatrix}, \tag{24}$$

where $F_c^z = M_c g + F_d^z$, which leads to

$$\begin{bmatrix}\sum_{i=1}^{N} f_i^x \\ \sum_{i=1}^{N} f_i^y \\ \sum_{i=1}^{N} f_i^z\end{bmatrix} = \underbrace{\begin{bmatrix}1 & 0 & 0 & \ddot{x}_{2,x}\\ 0 & 1 & 0 & \ddot{x}_{2,y}\\ 0 & 0 & 1 & \ddot{x}_{2,z}+g\end{bmatrix}}_{Y_C}\underbrace{\begin{bmatrix}F_d^x \\ F_d^y \\ F_d^z \\ M_c\end{bmatrix}}_{\theta_c}. \tag{25}$$

We propose the following update for the payload parameters $\hat{\theta}_c$

$$\dot{\hat{\theta}}_c = -\lambda_c\Gamma_c\left(\frac{1}{N}Y_f^T\sum_{i=1}^{N}\xi_i\right) + \dot{\hat{\theta}}_c^{cl} \tag{26}$$

where $\lambda_c > 0$, $\Gamma_c \in \mathbb{R}^{4\times 4}$ is the learning gain computed using (29), and $\dot{\hat{\theta}}_c^{cl}$ is based on the concurrent learning update law developed in (28).

The construction of $\dot{\hat{\theta}}_c^{cl}$ is similar to that of $\dot{\hat{\theta}}_i^{cl}$. Integrating (25) over the interval $[t-\tau_1, t]$ for some constant $\tau_1 \in \mathbb{R}_{>0}$ yields

$$\underbrace{\int_{t-\tau_1}^{t}\sum_{i=1}^{N} f_i(\gamma)\,\mathrm{d}\gamma}_{P_c(t)} = \underbrace{\int_{t-\tau_1}^{t} Y_c(\gamma)\,\mathrm{d}\gamma}_{G_c(t)}\,\theta_c. \tag{27}$$

The concurrent learning update law to estimate the unknown parameters for the payload is then given by

$$\dot{\hat{\theta}}_c^{cl} = k_c\Gamma_c\sum_{k=1}^{M} G_{k,c}\left(P_{k,c} - \hat{\theta}_c^T G_{k,c}\right)^T, \tag{28}$$

where $k_c \in \mathbb{R}_{>0}$ is a constant adaptation gain, $\Gamma_c \in \mathbb{R}^{4\times 4}$ is the least-squares gain updated using the update law

$$\dot{\Gamma}_c = \beta_c\Gamma_c - k_c\Gamma_c\frac{1}{1+\alpha_c\|\mathscr{G}_c\|}\mathscr{G}_c\Gamma_c, \tag{29}$$

in which $\alpha_c, \beta_c \in \mathbb{R}_{>0}$, and the matrix $\mathscr{G}_c \in \mathbb{R}^{4\times 4}$ is defined as $\mathscr{G}_c := \sum_{k=1}^{M} G_{k,c}G_{k,c}^T$.

Remark 1: Using similar arguments as in Section III-A, it can be shown that

$$\underline{\Gamma_c}\mathrm{I}_4 \leq \Gamma_c(t) \leq \overline{\Gamma}_c\mathrm{I}_4, \tag{30}$$

where $\underline{\Gamma_c}$ and $\overline{\Gamma_c}$ are positive constants. ■

IV. Stability Analysis

The dynamics (8) with the proposed control (13) takes the following form:

$$m_i\dot{\xi}_i = -K_i\xi_i + \frac{1}{N}Y_f\hat{\theta}_c - f_i - \mathrm{diag}(\tilde{\theta}_i)\|\dot{x}_i\|\dot{x}_i, \tag{31}$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$. We let $\xi_c = \dot{x}_c - v^d$ and obtain the payload dynamics as

$$M_c\dot{\xi}_c = \sum_{i=1}^{N} f_i - M_c g e_3 - F_d. \tag{32}$$

We further let $\tilde{z}_i = z_i - z_i^d, \forall i = 1, \cdots, N$ and $\tilde{\theta}_c = \theta_c - \hat{\theta}_c$. The desired equilibrium of the closed-loop system (31) and (32) with the update laws (15) and (26) is given by

$$\begin{aligned}\mathcal{E}^* = \Bigg\{ &\left(\{\xi_i\}_{i=1}^N, \left\{\tilde{\theta}_i\right\}_{i=1}^N, \tilde{\theta}_c, \{\tilde{z}_i\}_{i=1}^N, \xi_c\right) \mid \\ &\xi_i = 0, \tilde{\theta}_i = 0, \tilde{z}_i = 0, \quad \forall i = 1, \cdots, N, \\ &\tilde{\theta}_c = 0, \quad \text{and} \quad \xi_c = 0\Bigg\}.\end{aligned} \tag{33}$$

Theorem 1 below establishes the asymptotic stability of $\mathcal{E}^*$. Note that convergence to $\mathcal{E}^*$ means that $\xi_i \to 0$ and $\xi_c \to 0$, which indicate the velocities of the payload and the agents converge to v^d. Similarly $\tilde{\theta}_i \to 0$ and $\tilde{\theta}_c \to 0$ indicate the estimates of the drag coefficients, the payload mass and the disturbance converge to the true values. Also $\tilde{z}_i \to 0$ ensures that $f_i \to f_i^d$ which means that the contact forces are regulated. The proof of Theorem 1 relies on the assumption that $\mathscr{G}_i$ and $\mathscr{G}_c$ are both full rank to achieve parameter convergence. Such an assumption is typically satisfied during the transient.

Theorem 1: The control law (13), with the update laws (15) and (26), ensures that the desired equilibrium $\mathcal{E}^*$ in (33) is asymptotically stable. ■

Proof: Consider the energy-motivated positive definite candidate Lyapunov function $\left(\{\xi_i\}_{i=1}^N, \xi_c, \{\tilde{z}_i\}_{i=1}^N\right) \mapsto V_1\left(\{\xi_i\}_{i=1}^N, \xi_c, \{\tilde{z}_i\}_{i=1}^N\right)$:

$$\begin{aligned}V_1 = &\sum_{i=1}^N \left[P_i(z_i) - P_i(z_i^d) - \left(f_i^d\right)^T(z_i - z_i^d)\right] \\ &+ \frac{1}{2}\left(\sum_{i=1}^N \xi_i^T m_i \xi_i + \xi_c^T M_c \xi_c\right).\end{aligned} \tag{34}$$

From (2) and (3), the kinematics of z_i is given by

$$\dot{z}_i = \dot{x}_i - \dot{a}_i = \dot{x}_i - \dot{x}_c = \xi_i - \xi_c. \tag{35}$$

The time derivative of V_1 yields

$$\dot{V}_1 = \sum_{i=1}^N \left(f_i - f_i^d\right)^T \dot{z}_i + \sum_{i=1}^N \xi_i^T m_i \ddot{x}_i + \xi_c^T M_c \ddot{x}_c. \tag{36}$$

We rewrite (36) from (7), (8), (9), (35) and (13) as

$$\begin{aligned}\dot{V}_1 = &\sum_{i=1}^N \left(f_i - f_i^d\right)^T (\dot{x}_i - \dot{x}_c) \\ &+ \sum_{i=1}^N \xi_i^T \left(-K_i\xi_i + \frac{1}{N}Y_f\hat{\theta}_c - f_i - \|\dot{x}_i\|\mathrm{diag}(\tilde{\theta}_i)\dot{x}_i\right) \\ &+ \xi_c^T \sum_{i=1}^N f_i - \xi_c^T(M_c g e_3 + F_d), \\ = &\sum_{i=1}^N \left(f_i - f_i^d\right)^T \xi_i - \sum_{i=1}^N \left(f_i - f_i^d\right)^T \xi_c \\ &- \sum_{i=1}^N \xi_i^T K_i \xi_i + \sum_{i=1}^N \xi_i^T\left(\frac{1}{N}Y_f\hat{\theta}_c - f_i\right) + \xi_c^T\sum_{i=1}^N f_i \\ &- \xi_c^T \sum_{i=1}^N f_i^d - \sum_{i=1}^N \|\dot{x}_i\| \sum_{j=1}^3 \xi_i^j \tilde{\theta}_i^j \dot{x}_i^j, \\ = &- \sum_{i=1}^N \xi_i^T K_i \xi_i - \sum_{i=1}^N \xi_i^T \frac{1}{N} Y_f \tilde{\theta}_c - \sum_{i=1}^N \Phi_i^T \tilde{\theta}_i.\end{aligned} \tag{37}$$

Consider another positive definite candidate Lyapunov function $\left(\left\{\tilde{\theta}_i\right\}_{i=1}^N, \tilde{\theta}_c, t\right) \mapsto V_2\left(\left\{\tilde{\theta}_i\right\}_{i=1}^N, \tilde{\theta}_c, t\right)$:

$$V_2 = \frac{1}{2\lambda_c}\tilde{\theta}_c^T\Gamma_c^{-1}\tilde{\theta}_c + \sum_{i=1}^N \frac{1}{2\lambda_i}\tilde{\theta}_i^T\Gamma_i^{-1}\tilde{\theta}_i. \tag{38}$$

Using (15) and (26) together with (21) and (28), we obtain $\dot{V}_2$ as:

$$\begin{aligned}\dot{V}_2 = &\frac{1}{\lambda_c}\tilde{\theta}_c^T\Gamma_c^{-1}\Big(\lambda_c\Gamma_c\frac{1}{N}Y_f^T\sum_{i=1}^N \xi_i - \dot{\hat{\theta}}_c^{cl}\Big) \\ &+ \frac{1}{2\lambda_c}\tilde{\theta}_c^T\dot{\Gamma}_c^{-1}\tilde{\theta}_c + \sum_{i=1}^N \frac{1}{2\lambda_i}\tilde{\theta}_i^T\dot{\Gamma}_i^{-1}\tilde{\theta}_i \\ &+ \sum_{i=1}^N \frac{1}{\lambda_i}\tilde{\theta}_i^T\Gamma_i^{-1}\Big(\lambda_i\Gamma_i\Phi_i - \dot{\hat{\theta}}_i^{cl}\Big), \\ = &\tilde{\theta}_c^T\left(\frac{1}{N}Y_f^T\sum_{i=1}^N \xi_i\right) - \frac{k_c}{2\lambda_c}\tilde{\theta}_c^T\mathscr{G}_c\tilde{\theta}_c - \frac{\beta_c}{2\lambda_c}\tilde{\theta}_c^T\Gamma_c^{-1}\tilde{\theta}_c \\ &+ \sum_{i=1}^N \tilde{\theta}_i^T\Phi_i - \sum_{i=1}^N \frac{k_i}{2\lambda_i}\tilde{\theta}_i^T\mathscr{G}_i\tilde{\theta}_i - \sum_{i=1}^N \frac{\beta_i}{2\lambda_i}\tilde{\theta}_i^T\Gamma_i^{-1}\tilde{\theta}_i,\end{aligned} \tag{39}$$

where we have used

$$\dot{\Gamma}_i^{-1} = \Gamma_i^{-1}\Big[\beta_i\Gamma_i - k_i\Gamma_i\mathscr{G}_i\Big]\Gamma_i^{-1},$$

and

$$\dot{\Gamma}_c^{-1} = \Gamma_c^{-1}\Big[\beta_c\Gamma_c - k_c\Gamma_c\mathscr{G}_c\Big]\Gamma_c^{-1}.$$

Let $V = V_1 + V_2$. The time derivative of V is given by

$$\begin{aligned}\dot{V} = &-\sum_{i=1}^{N}\xi_i^T K_i \xi_i - \frac{k_c}{2\lambda_c}\tilde{\theta}_c^T \mathscr{G}_c \tilde{\theta}_c - \frac{\beta_c}{2\lambda_c}\tilde{\theta}_c^T \Gamma_c^{-1}\tilde{\theta}_c \\ &-\sum_{i=1}^{N}\frac{k_i}{2\lambda_i}\tilde{\theta}_i^T \mathscr{G}_i \tilde{\theta}_i - \sum_{i=1}^{N}\frac{\beta_i}{2\lambda_i}\tilde{\theta}_i^T \Gamma_i^{-1}\tilde{\theta}_i.\end{aligned} \tag{40}$$

Using the bound in (23) and (30), we rewrite $\dot{V}$ as

$$\begin{aligned}\dot{V} \leq &-\sum_{i=1}^{N}\xi_i^T K_i \xi_i \\ &- \frac{k_c}{2\lambda_c}\tilde{\theta}_c^T \lambda_{min}\{\mathscr{G}_c\}\tilde{\theta}_c - \frac{\beta_c}{2\lambda_c}\tilde{\theta}_c^T \overline{\Gamma_c}\tilde{\theta}_c \\ &-\sum_{i=1}^{N}\frac{k_i}{2\lambda_i}\tilde{\theta}_i^T \lambda_{min}\{\mathscr{G}_i\}\tilde{\theta}_i - \sum_{i=1}^{N}\frac{\beta_i}{2\lambda_i}\tilde{\theta}_i^T \overline{\Gamma_i}\tilde{\theta}_i \leq 0.\end{aligned} \tag{41}$$

It follows from [28, Theorem 4.8] that ξ_i, $\tilde{\theta}_i$, $\tilde{\theta}_c$, $\tilde{z}_i$ and ξ_c are uniformly bounded and the desired equilibrium $\mathcal{E}^*$ is uniformly stable. Given the states are bounded, we further conclude that $\mathscr{G}_i$ and $\mathscr{G}_c$ are bounded. Applying the Barbalat's Lemma [28, Theorem 8.4], we conclude that $\xi_i \to 0$, $\tilde{\theta}_i \to 0$ and $\tilde{\theta}_c \to 0$ as $t \to \infty$, which further implies that $\dot{x}_i \to v^d$ and $\hat{\theta}_i \to \theta_i$ and $\hat{\theta}_c \to \theta_c$.

We next prove $\dot{\xi}_i \to 0$ using [29, Lemma 1]. From (31), the time derivative of $\dot{\xi}_i$, whenever it exists, is given by

$$\begin{aligned}m_i\ddot{\xi}_i = &-\Big(K_i + 2[\mathrm{diag}(\tilde{\theta}_i)]\|\dot{x}_i\|\Big)\frac{1}{m_i}\Big(-K_i\xi_i \\ &+\frac{1}{N}Y_f\hat{\theta}_c - f_i - \mathrm{diag}(\tilde{\theta}_i)\|\dot{x}_i\|\dot{x}_i\Big) \\ &+\frac{1}{N}Y_f\dot{\hat{\theta}}_c - \dot{f}_i - \mathrm{diag}(\dot{\tilde{\theta}}_i)\|\dot{x}_i\|\dot{x}_i.\end{aligned} \tag{42}$$

From (42), we note that $\ddot{\xi}_i$ is bounded. The points of non-differentiability of $\dot{\xi}_i$ coincide with the points of discontinuity of $G_{k,i}$. By introducing a dwell time in the singular value maximization algorithm, it can be easily ensured that the points of discontinuity of $G_{k,i}$ do not accumulate. Direct application of [29, Lemma 1] then leads to $\dot{\xi}_i \to 0$ as $t \to \infty$.

Since $\dot{\xi}_i \to 0$, (31) indicates that $f_i \to \frac{1}{N}Y_f\hat{\theta}_c$ which further implies that $\sum_{i=1}^{N} f_i \to Y_f\theta_c = M_c g e_3 + F_d$. Therefore, f_i and z_i converge to f_i^d and z_i^d, respectively. We can similarly use [29, Lemma 1] to prove that $\dot{z}_i \to 0$. It then follows from (35) that $\xi_c \to 0$ which leads to $\dot{x}_c \to v^d$. ■

Remark 2: Without concurrent learning, the bound on $\dot{V}$ in (41) becomes $-\sum \xi_i^T K_i \xi_i$, which is negative semi-definite, and as a result, guarantees stability and boundedness of all signals. However, parameter convergence may not be achieved due to the lack of PE. As proven in Theorem 1 and validated in simulation, if $\mathscr{G}_i$ and $\mathscr{G}_c$ are full rank, the combined update law ensures both stability and parameter convergence. ■

V. Numerical Simulations: Cooperative Aerial Manipulation

A. Simulation Environment

In this section, we present a simulation using two quadcopters transporting a load. The mass of each of the quadcopters is $m_i = 0.75$ kg. The mass of the payload is $M_c = 1.5$ kg and with a radius of 15 cm. We use the same linear spring-force model to compute the contact force f_i in the simulation as our previous paper [18], with the spring constant, $k = 2.5 \times 10^4$ N/m. We set $r_1 = [0, 0.15, 0]^T$ and $r_2 = [0, -0.15, 0]^T$. We set $v^d(t) = (1 - e^{-t})\begin{bmatrix}5.0 & 2.0 & 0\end{bmatrix}^T$, $k_i = k_c = 0.0033, \beta_i = \beta_c = 1$, $\lambda_i = \lambda_c = 10^{-6}$, and $K_i = \mathrm{diag}(10, 10, 25)$. The actual drag force coefficients for the quadcopters and the disturbance for the payload are given by $C_i = \begin{bmatrix}0.2061, 0.2061, 0.2061\end{bmatrix}^T$ and $F_d = \begin{bmatrix}6.7315 & 2.6926 & 0\end{bmatrix}^T$N, respectively.

B. Implementation on a quadcopter

Once F_i is designed from (13), we can compute the desired thrust T_i^{des} and desired attitude angles θ_i^{des}, ϕ_i^{des}, ψ_i^{des} required for each quadcopter from equations (17-21) in [18] and low-level attitude and thrust tracking controllers (e.g., a PD controller) can be implemented to track these desire commands for the ith quadcopter.

Fig. 2 shows that the velocities of the agents and the payload converge to v^d. Fig. 3 shows the convergence of the drag coefficients for quadcopter 1. Fig. 4 demonstrates that all the estimation errors for the drag forces and the mass of the payload converge to zero. We observe from Fig. 5 that the contact forces converges to the desired set points.

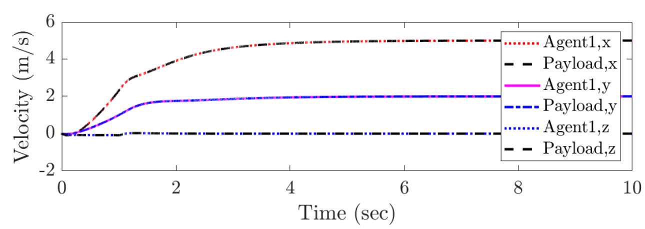

Fig. 2. Linear velocities for both quadcopter 1 and the payload. The x and y components of the velocity converges to 5.0 and 2.0 m/s respectively and the z component converge to zero. Quadcopter 2 has similar velocity convergence.

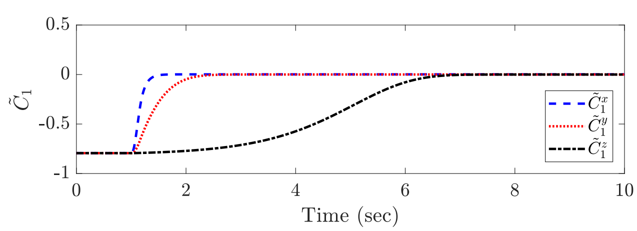

Fig. 3. Estimation errors for drag force coefficient for quadcopter 1 in all 3 directions. Quadrotor 2 has similar convergence.

The CL algorithm uses the first second of time to collect data. During that time period, the CL update is turned off. As

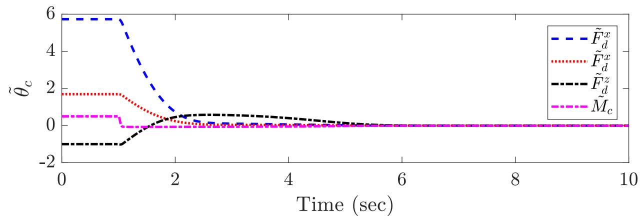

Fig. 4. Estimation errors for drag forces acting on the payload and the mass of the payload.

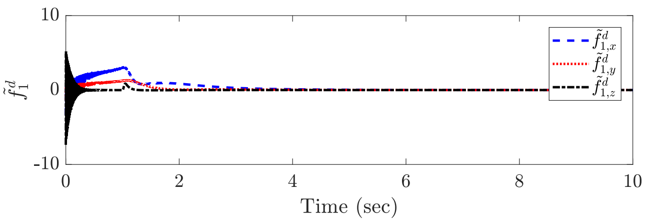

Fig. 5. Contact force error acting on the payload for quadcopter 1.

discussed in Remark 2, the system remains stable. Once the CL update is turned on, the parameters quickly converge. Note that due to the very small scaling factors λ_i and λ_c in the stabilizing control, the learning estimates do not vary significantly during the first one second.

VI. Conclusion and Future Work

In this paper, we address the problem of cooperative manipulation of a payload with an unknown mass in the presence of unknown drag forces. We develop a CL based adaptive controller and analyze its stability and convergence properties. We show that the controller guarantees parameter convergence, velocity convergence of the payload and the agents, and contact force regulation. We validate the performance of the controller using a simulation example. Future work will involve experimental validation of the control law designed in this paper with a group of robots. We also plan to design force control laws coupled with state estimation and address time-varying drag forces on the payload.

References

[1] K. Sreenath and V. Kumar, "Dynamics, control and planning for cooperative manipulation of payloads suspended by cables from multiple quadrotor robots," in *Robotics: Science and Systems (RSS)*, 2013.

[2] I. Maza, K. Kondak, M. Bernard, and A. Ollero, "Multi-UAV cooperation and control for load transportation and deployment," *Journal of Intelligent and Robotic Systems*, vol. 57, no. 1, p. 417, Aug 2009.

[3] H. Lee, H. Kim, and H. J. Kim, "Path planning and control of multiple aerial manipulators for a cooperative transportation," in *Intelligent Robots and Systems (IROS), IEEE/RSJ International Conference on*, Sept 2015, pp. 2386–2391.

[4] T. Lee, "Geometric control of quadrotor UAVs transporting a cable-suspended rigid body," *IEEE Transactions on Control Systems Technology*, vol. 26, no. 1, pp. 255–264, 2018.

[5] Z. Wang, G. Yang, X. Su, and M. Schwager, "Ouijabots: Omnidirectional robots for cooperative object transport with rotation control using no communication," in *Distributed Autonomous Robotic Systems*. Springer, 2018, pp. 117–131.

[6] A. Tagliabue, M. Kamel, S. Verling, R. Siegwart, and J. Nieto, "Collaborative transportation using mavs via passive force control," in *2017 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2017, pp. 5766–5773.

[7] H. Bai and J. T. Wen, "Cooperative load transport: A formation-control perspective," *IEEE Transactions on Robotics*, vol. 26, no. 4, pp. 742–750, 2010.

[8] P. Culbertson and M. Schwager, "Decentralized adaptive control for collaborative manipulation," in *2018 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2018, pp. 278–285.

[9] R. Ritz and R. D'Andrea, "Carrying a flexible payload with multiple flying vehicles," in *Intelligent Robots and Systems (IROS), 2013 IEEE/RSJ International Conference on*. IEEE, 2013, pp. 3465–3471.

[10] G. Montemayor and J. T. Wen, "Decentralized collaborative load transport by multiple robots," in *Proceedings of the 2005 IEEE International Conference on Robotics and Automation*. IEEE, 2005, pp. 372–377.

[11] D. Rus, B. Donald, and J. Jennings, "Moving furniture with teams of autonomous robots," in *Proceedings 1995 IEEE/RSJ International Conference on Intelligent Robots and Systems. Human Robot Interaction and Cooperative Robots*, vol. 1. IEEE, 1995, pp. 235–242.

[12] A. Franchi, A. Petitti, and A. Rizzo, "Decentralized parameter estimation and observation for cooperative mobile manipulation of an unknown load using noisy measurements," in *2015 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2015, pp. 5517–5522.

[13] N. Michael, J. Fink, and V. Kumar, "Cooperative manipulation and transportation with aerial robots," *Autonomous Robots*, vol. 30, no. 1, pp. 73–86, Sept 2011.

[14] T. Lee, K. Sreenath, and V. Kumar, "Geometric control of cooperating multiple quadrotor UAVs with a suspended payload," in *Decision and Control (CDC), 2013 IEEE 52nd Annual Conference on*, Dec 2013, pp. 5510–5515.

[15] H. Lee and H. J. Kim, "Estimation, control, and planning for autonomous aerial transportation," *IEEE Transactions on Industrial Electronics*, vol. 64, no. 4, pp. 3369–3379, 2017.

[16] H. Lee, H. Kim, W. Kim, and H. J. Kim, "An integrated framework for cooperative aerial manipulators in unknown environments," *IEEE Robotics and Automation Letters*, vol. 3, no. 3, pp. 2307–2314, 2018.

[17] J. Esposito, M. Feemster, and E. Smith, "Cooperative manipulation on the water using a swarm of autonomous tugboats," in *2008 IEEE International Conference on Robotics and Automation*. IEEE, 2008, pp. 1501–1506.

[18] S. Thapa, H. Bai, and J. Acosta, "Force control in cooperative aerial manipulation," in *2018 International Conference on Unmanned Aircraft Systems (ICUAS)*. IEEE, 2018, pp. 1302–1309.

[19] R. Kamalapurkar, "Simultaneous state and parameter estimation for second-order nonlinear systems," in *Decision and Control (CDC), 2017 IEEE 56th Annual Conference on*. IEEE, 2017, pp. 2164–2169.

[20] S. Kersting and M. Buss, "Concurrent learning adaptive identification of piecewise affine systems," in *53rd IEEE Conference on Decision and Control*. IEEE, 2014, pp. 3930–3935.

[21] G. V. Chowdhary, "Concurrent learning for convergence in adaptive control without persistency of excitation," Ph.D. dissertation, Georgia Institute of Technology, 2010.

[22] G. J. Leishman, *Principles of helicopter aerodynamics with CD extra*. Cambridge university press, 2006.

[23] S. Thapa, H. Bai, and J. Acosta, "Cooperative aerial load transport with force control," *IFAC-PapersOnLine*, vol. 51, no. 12, pp. 38–43, 2018.

[24] G. Chowdhary and E. Johnson, "A singular value maximizing data recording algorithm for concurrent learning," in *Proceedings of the 2011 American Control Conference*. IEEE, 2011, pp. 3547–3552.

[25] G. Chowdhary, T. Yucelen, M. Mühlegg, and E. N. Johnson, "Concurrent learning adaptive control of linear systems with exponentially convergent bounds," *International Journal of Adaptive Control and Signal Processing*, vol. 27, no. 4, pp. 280–301, 2013.

[26] A. Parikh, R. Kamalapurkar, and W. E. Dixon, "Integral concurrent learning: Adaptive control with parameter convergence using finite excitation," *International Journal of Adaptive Control and Signal Processing*.

[27] P. A. Ioannou and J. Sun, *Robust adaptive control*. PTR Prentice-Hall Upper Saddle River, NJ, 1996, vol. 1.

[28] H. K. Khalil, *Nonlinear Systems*. Prentice Hall, 1996.

[29] J. Yuan and W. Wonham, "Probing signals for model reference identification," *IEEE Transactions on Automatic Control*, vol. 22, no. 4, pp. 530–538, 1977.