ECS 170: Homework 2

January 19, 2017

Your answers should be succinct - our solutions for each problem are no more than a couple sentences.

Your submission should be a PDF. We make no guarantees we will grade submissions in other formats.

1. (R&N 3.18) Describe a search space in which iterative deepening search performs much worse than depth-first search. You do not need to give a specific example of such a space - just a short description is what we want.

2. (R&N 3.28) Show that if a heuristic h never overestimates by more than some constant c, A* using h returns a solution whose cost exceeds that of the optimal solution by no more than c.

3. For a graph $G = (V, E)$ where V is the set of nodes and E is the set of edges, a vertex cover is some set $S \subset V$ such that for all nodes $n \in V$ either $n \in S$ or there is an edge in E between n and some node $s \in S$. Let the k-vertex set cover problem be to find a vertex cover S for G such that $|S| \leq k$. This can be modeled as a graph search problem as follows: the states describe which nodes are in the partially constructed set cover; the actions are adding a node (not currently in the partial set cover) to the set cover; and the goal test checks if the set S is a vertex cover of size no more than k.

   (a) The number of nodes that are not currently "covered" (i.e. not in the set or having an edge to a node in the set) is not an admissible heuristic. Why?

   (b) $k - |S|$ (i.e. the maximum number of nodes we can still add to the set) is not an admissible heuristic. Why?

   (c) Construct a non-trivial admissible heuristic for this problem description. Why is it admissible?

4. Given a set of admissible heuristics $h_1, h_2, ...h_n$ one can define a new heuristic h_{max} such that for any node n:

$$h_{max}(n) = \max_i h_i(n)$$

(a) Show that h_{max} is an admissible heuristic.

(b) Show that h_{max} dominates all other h_i.

(c) What are the practical benefits of knowing that a heuristic h_1 dominates another heuristic h_2 (assume we know both are admissible)?

5. Recall the "number of misplaced tiles" heuristic for the 8-puzzle problem. Ian showed this heuristic is admissible in class. Consider a modified 9-puzzle problem where the blank space is replaced with a 9 tile and now the legal moves are swapping the location of pairs of adjacent tiles.

   (a) Show this heuristic is no longer admissible.

   (b) Give a nontrivial admissible heuristic for this problem. You will likely need to relax the game rules (you may also give a lower bound on the cost of the relaxed version).

6. (R&N 3.30) The traveling salesperson problem (TSP) looks for a tour of all the nodes in a connected graph such that the tour starts and ends at the same node and each node is visited *exactly* once (except start/end node). A minimum-spanning-tree (MST) of a connected graph is a subset of the edges which connect all the nodes with the minimum possible total edge costs. TSP can be posed as a search problem (from a given start node) and can be solved using MST as a heuristic, which estimates the cost of completing a tour, given that a partial tour has already been constructed.

   (a) Show this MST heuristic is admissible (Think about what condition is being relaxed by MST than TSP).

   (b) Show that the MST heuristic dominates straight-line distance (assume these nodes are the cities in a geographical region).

7. Suppose you have already computed an optimal path P using A* on a graph G with edge weights $w_{ij} \geq 0$ (where i and j are nodes in G). You learn that the weights of the edges have changed to $\hat{w}_{ij} = w_{ij} + \epsilon_{ij} >= 0$

   (a) If $\epsilon_{ij} = \epsilon > 0$ for all i, j, can we guarantee P will still be an optimal path? Explain.

   (b) If $\epsilon_{ij} = \epsilon < 0$ for all i, j, can we guarantee P will still be an optimal path? Explain.

   (c) If $\epsilon_{ij} = w_{ij}$ for all i, j, can we guarantee P will still be an optimal path? Explain.