



# **Time-frequency decomposition**

## **Theory and Practice**

2022 EEGLAB Workshop San Diego



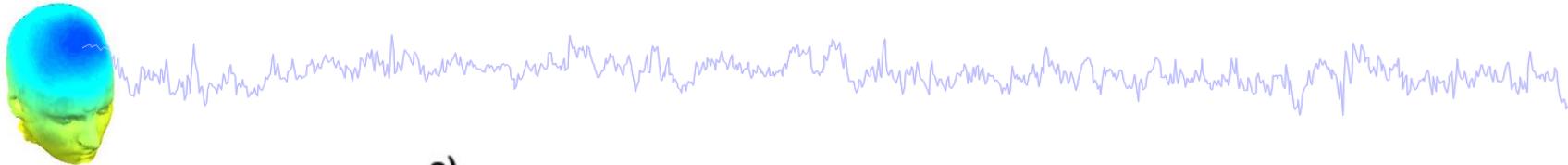
# N170



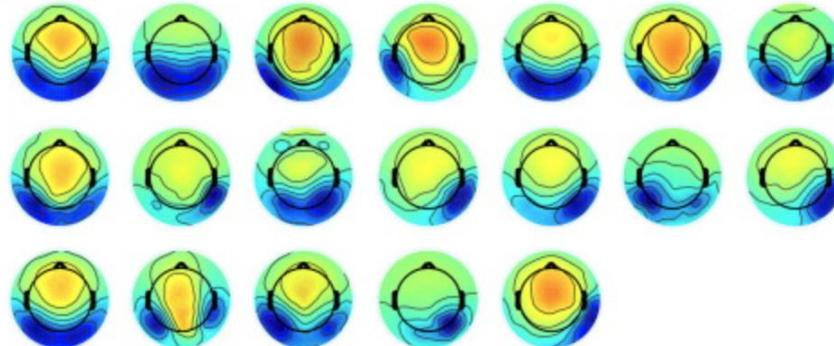
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# N170

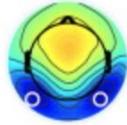


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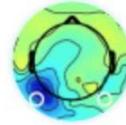


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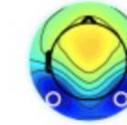
Faces



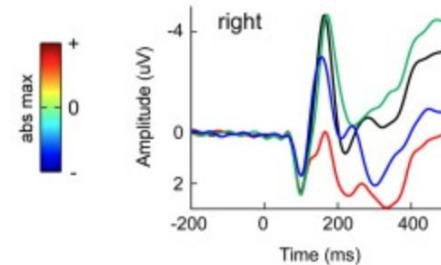
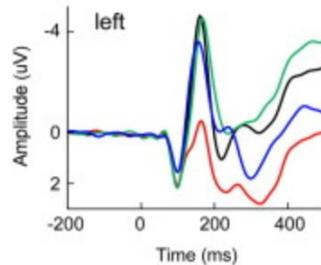
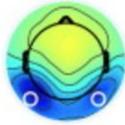
Houses



inv. Faces

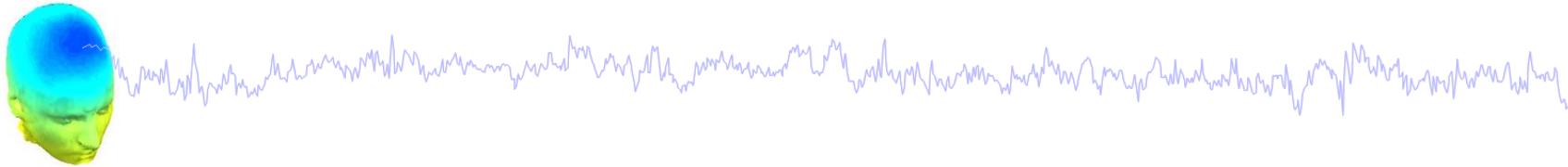


Words



De Vos, M., Thorne, J. D., Yovel, G., & Debener, S. (2012). Let's face it, from trial to trial: comparing procedures for N170 single-trial estimation. *Neuroimage*, 63(3), 1196-1202.

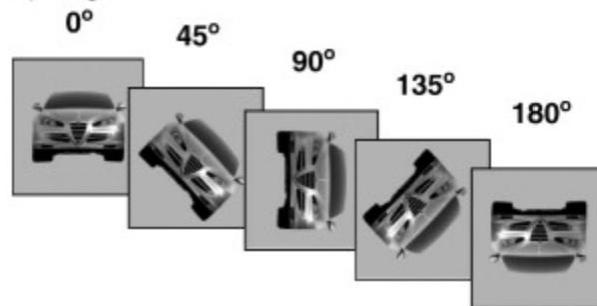
# N170



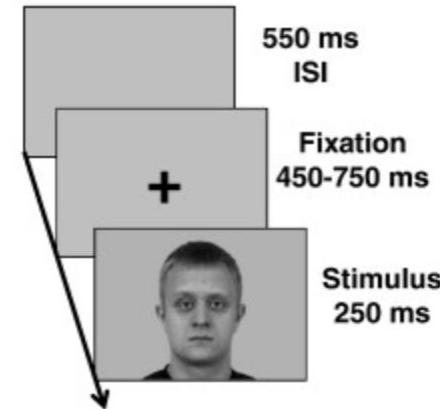
a) Stimulus category



b) Angles of rotation

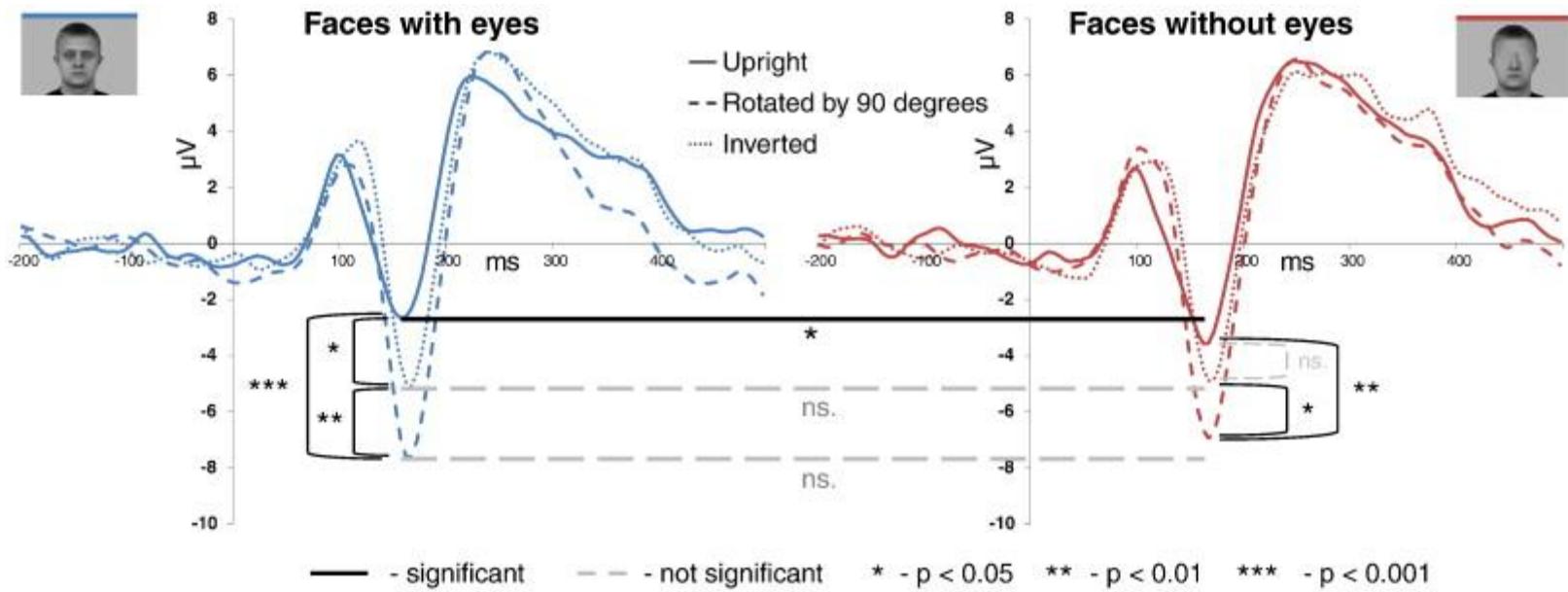


c) Trial design



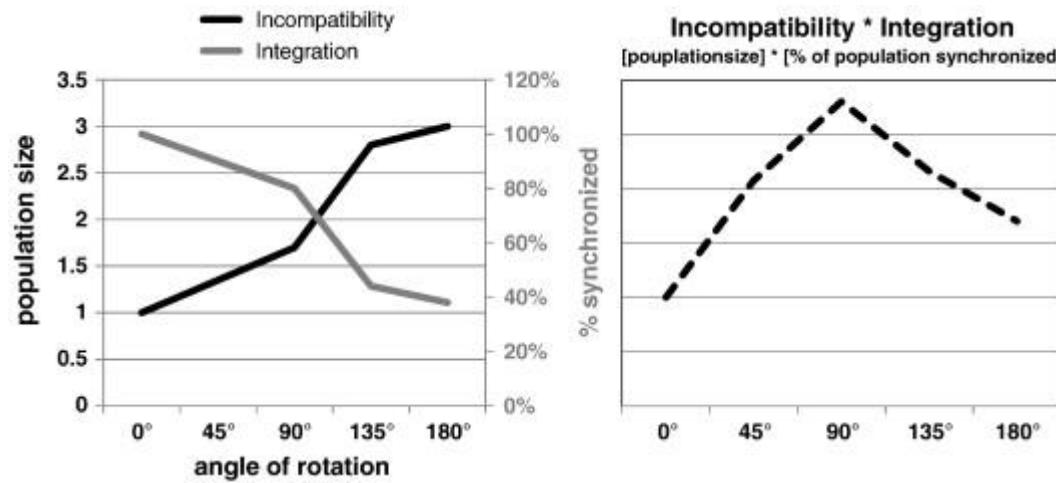
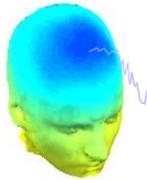
Magnuski, M., & Gola, M. (2013). It's not only in the eyes: Nonlinear relationship between face orientation and N170 amplitude irrespective of eye presence. *International Journal of Psychophysiology*, 89(3), 358-365.

# N170

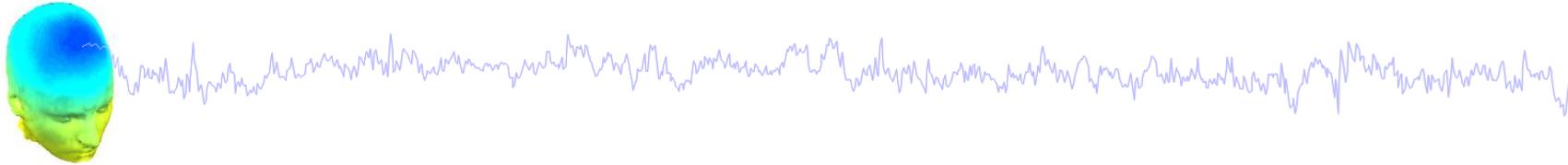


Magnuski, M., & Gola, M. (2013). It's not only in the eyes: Nonlinear relationship between face orientation and N170 amplitude irrespective of eye presence. *International Journal of Psychophysiology*, 89(3), 358-365.

# N170



Magnuski, M., & Gola, M. (2013). It's not only in the eyes: Nonlinear relationship between face orientation and N170 amplitude irrespective of eye presence. *International Journal of Psychophysiology*, 89(3), 358-365.



- Signals – EEG
- Goals
  - Describe dynamic characteristics of brain activity
- Approaches
  - Time domain
  - Frequency domain
  - Time/Frequency



# Introduction: Spectral analysis of the EEG signal



## EEG Signal

EEG

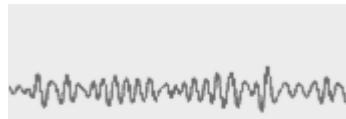


δ



1-4 Hz

θ



4-7 Hz

α



8-12 Hz

β



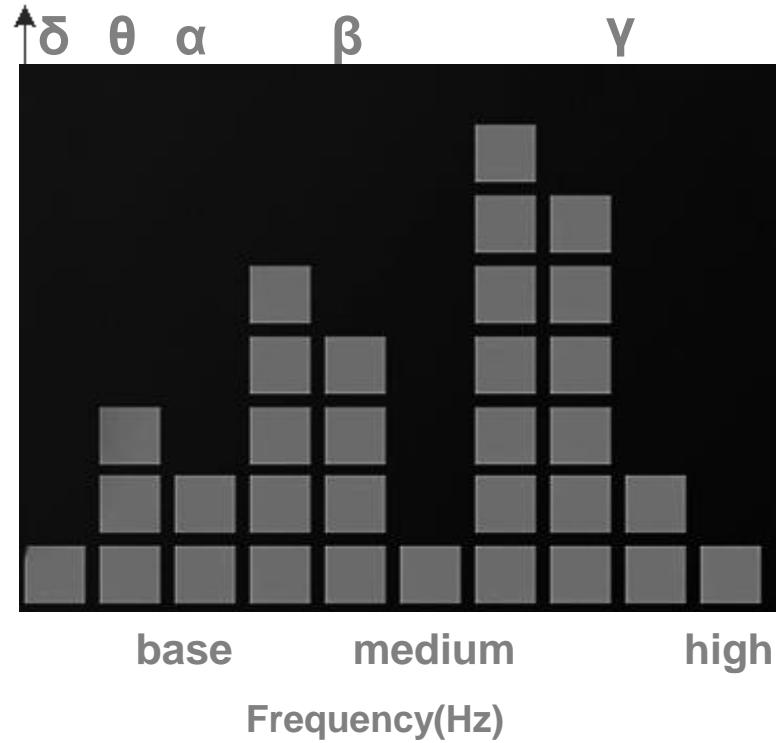
13-30 Hz

γ



30 + Hz

Power ( $10^{\ast}\log_{10}(\mu\text{V}^2/\text{Hz})$ )



# Different meanings traditionally given to different frequency bands



## Beta 15-30 Hz

Awake, normal alert consciousness

## Alpha 9-14 Hz

Relaxed, calm, meditation, creative visualisation

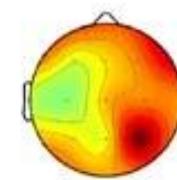
## Theta 4-8 Hz

Deep relaxation and meditation, problem solving

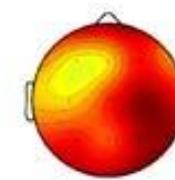
## Delta 1-3 Hz

Deep, dreamless sleep

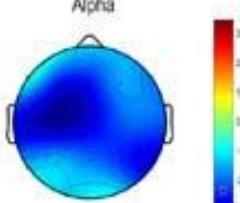
## Delta



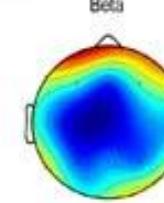
## Theta



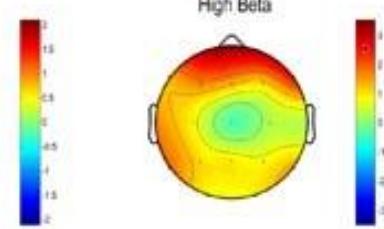
## Alpha



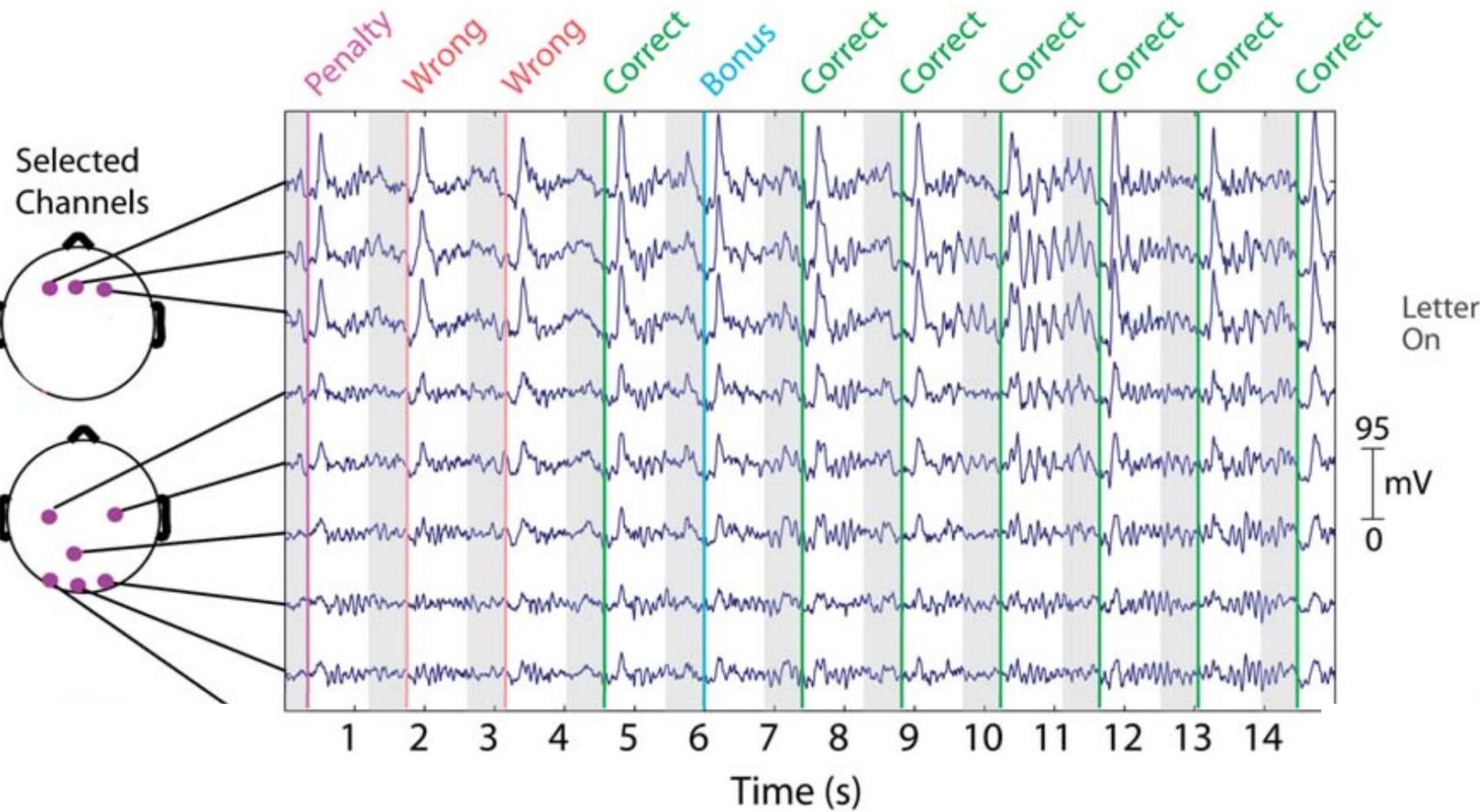
## Beta



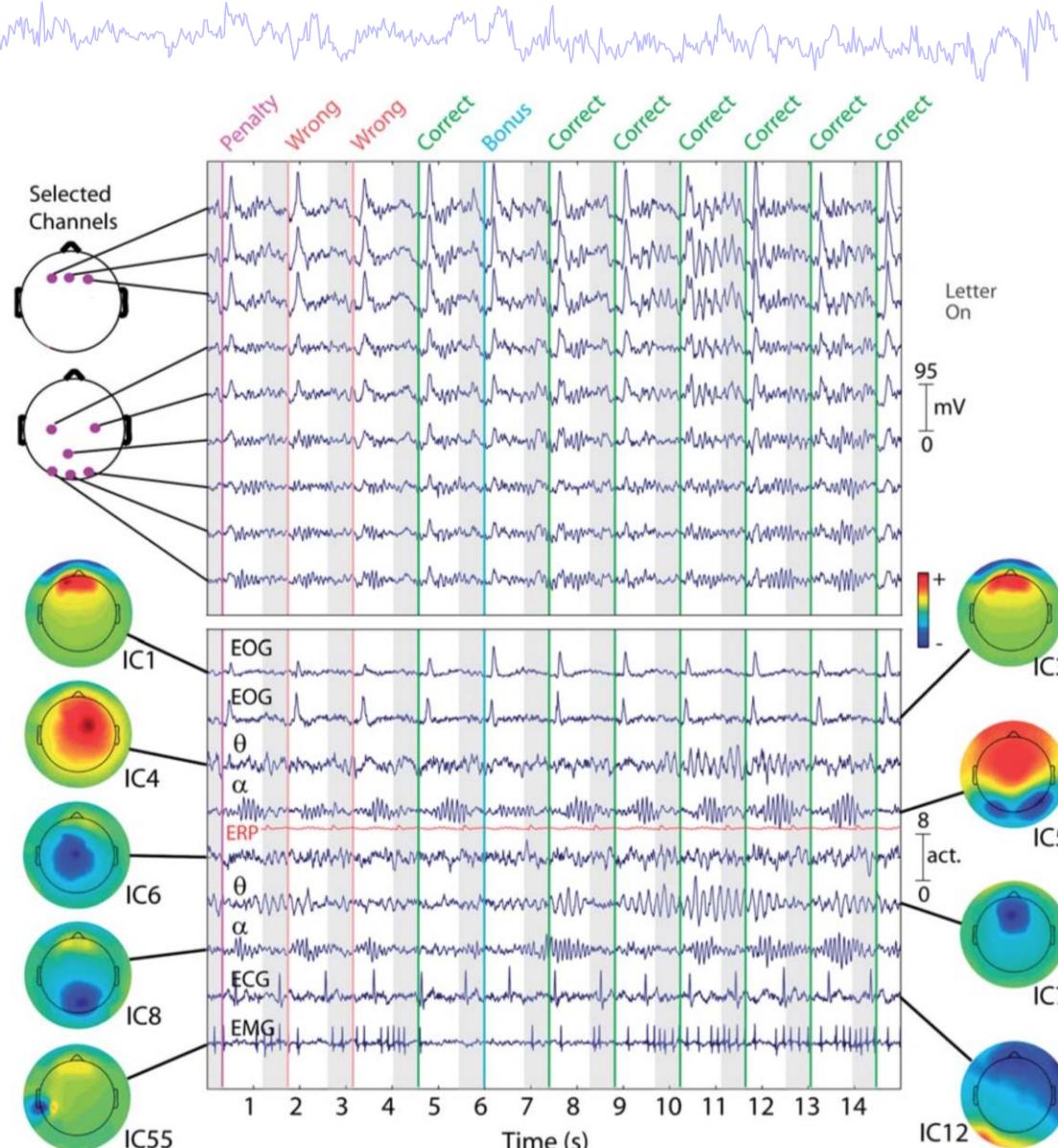
## High Beta



# Time varying frequency content

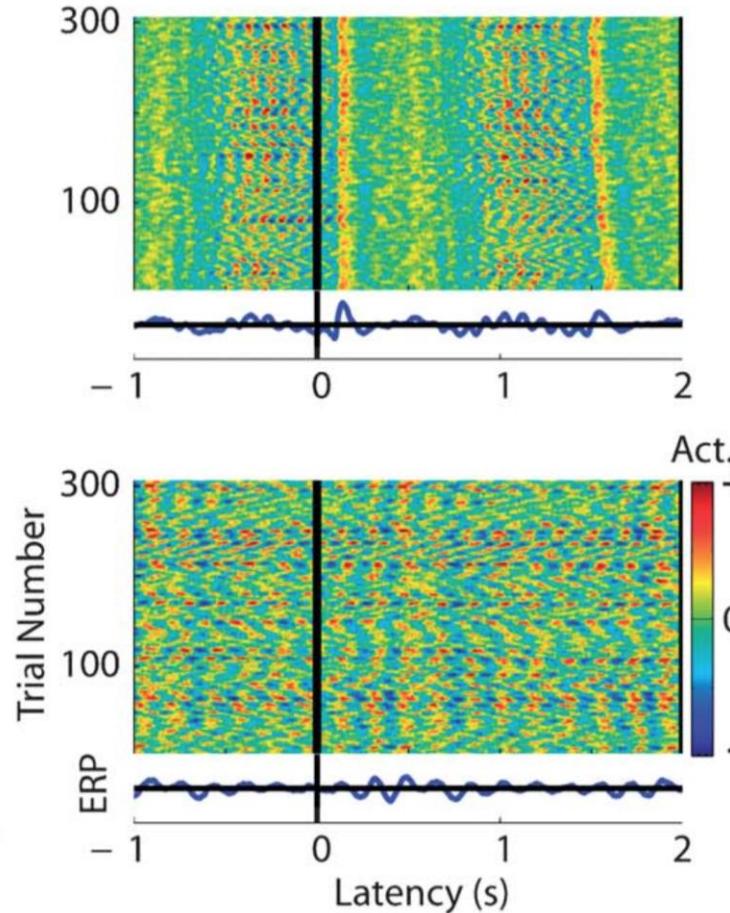
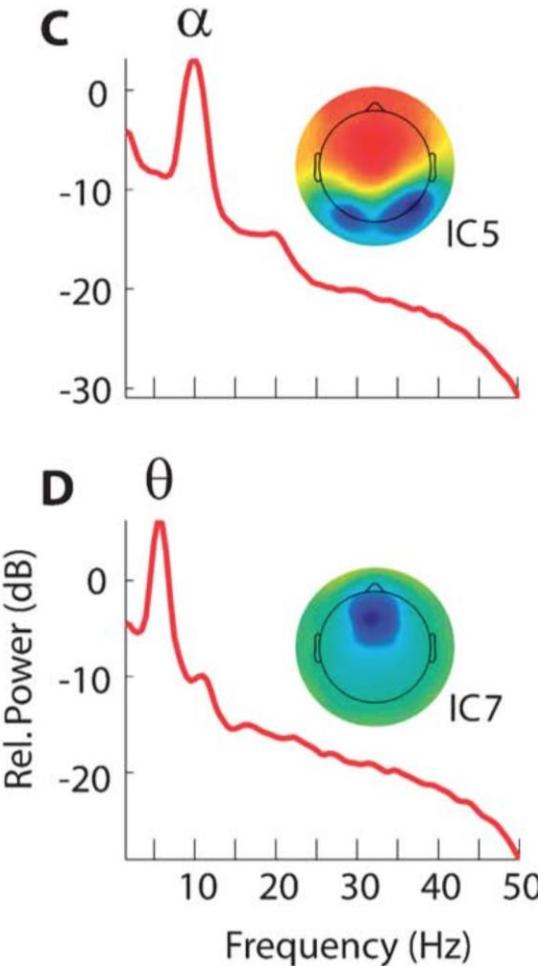
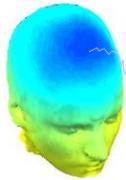


# Time-varying frequency content

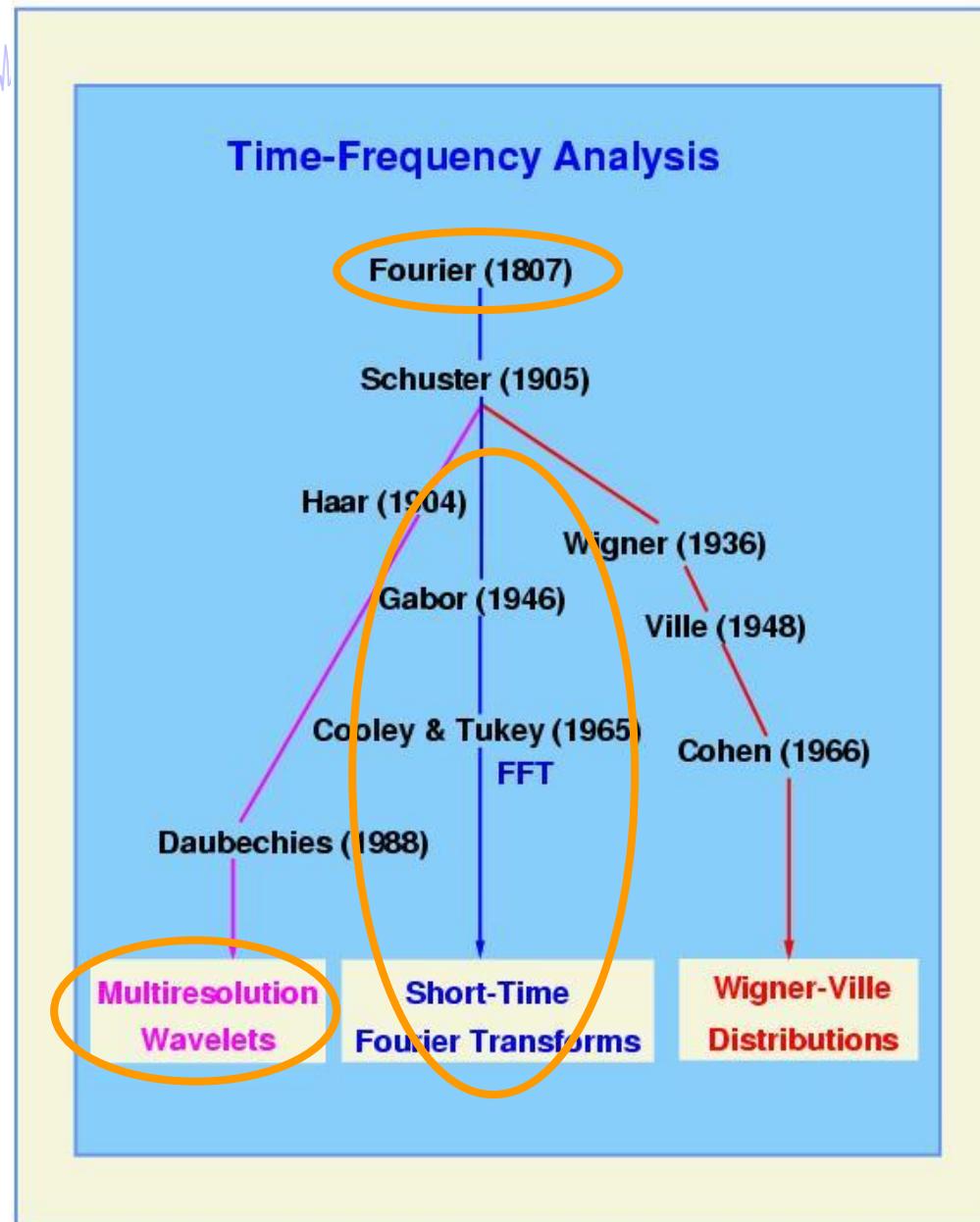


Onton & Makeig, 2006

# Power Spectrum does not describe temporal variation



Onton & Makeig, 2006



S. Makeig, 2005

# Plan



- **Part 1: Frequency Analysis**
  - Power Spectrum
    - Approaches
      - FFT
      - Welch's Method
    - Windowing
- **Part 2: Time-Frequency Analysis**
  - Short Time Fourier Transform
  - Wavelet Transform
  - ERSP
- **Part 3: Coherence Analysis**
  - Inter-Trial Coherence
  - Event-Related Coherence
- **Part 4: Other Applications**

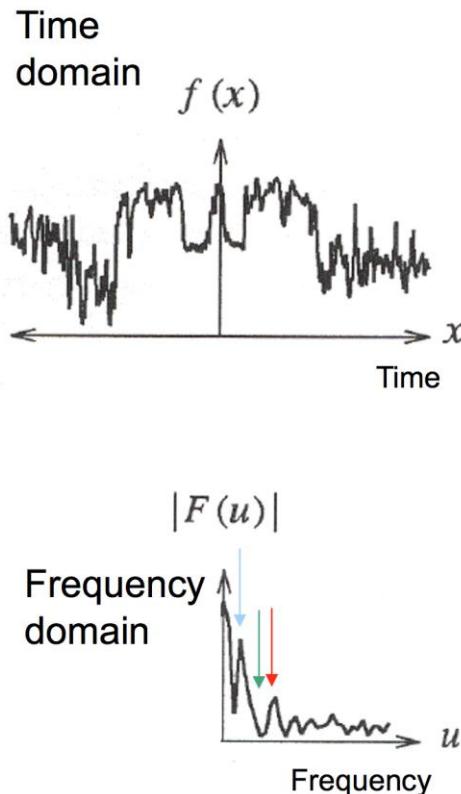
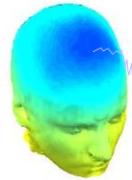
# Part 1: Frequency Analysis



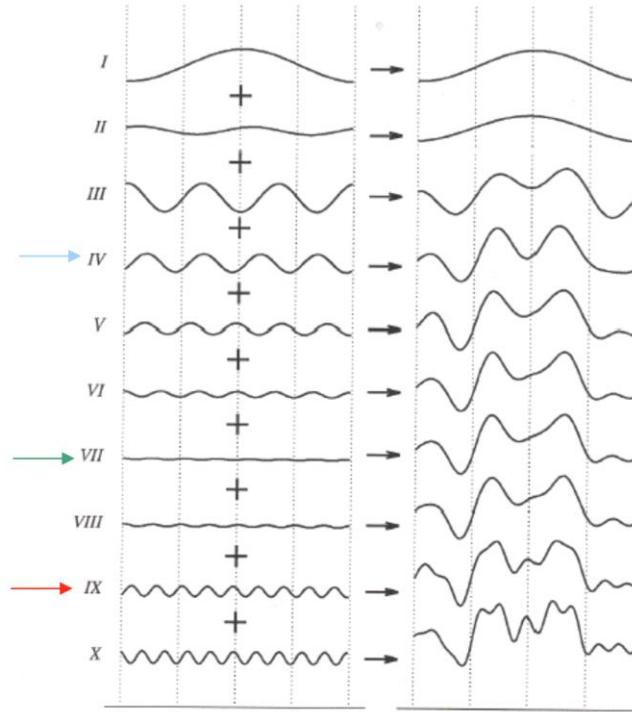
- Goal: What frequencies are present in signal?
- What is power at each frequency?
- Principle: Fourier Analysis



# Fourier Analysis



Freq. decomp.      Sum of freq.



Forward transform 
$$F(u) = \int_{-\infty}^{+\infty} f(x)e^{-2\pi i ux} dx$$

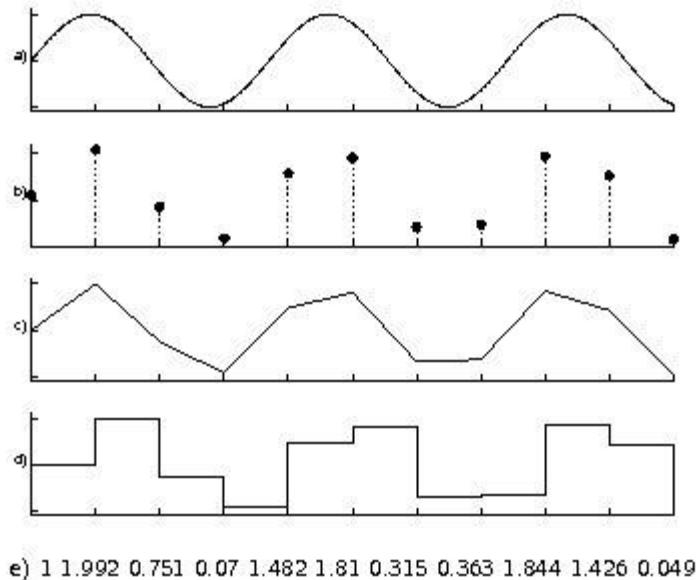
Inverse transform 
$$f(x) = \int_{-\infty}^{+\infty} F(u)e^{2\pi i ux} du$$

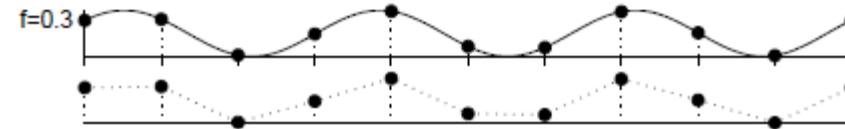
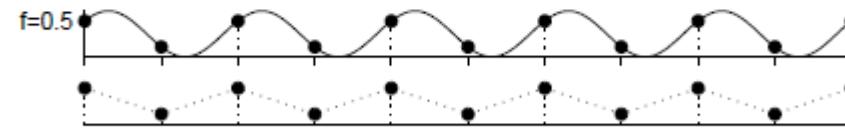
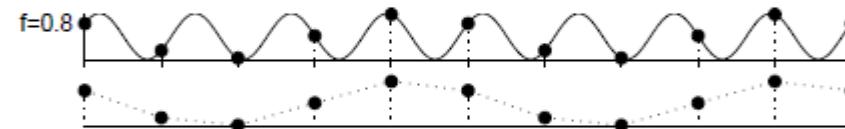
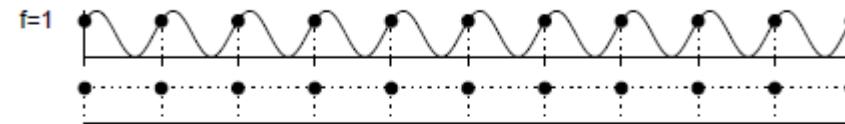
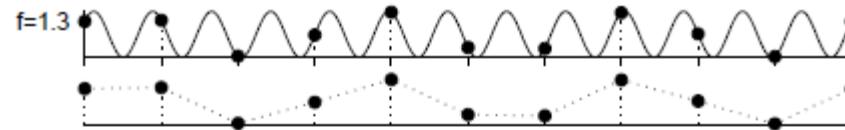
Figure, courtesy of Ravi Ramamoorthi & Wolberg

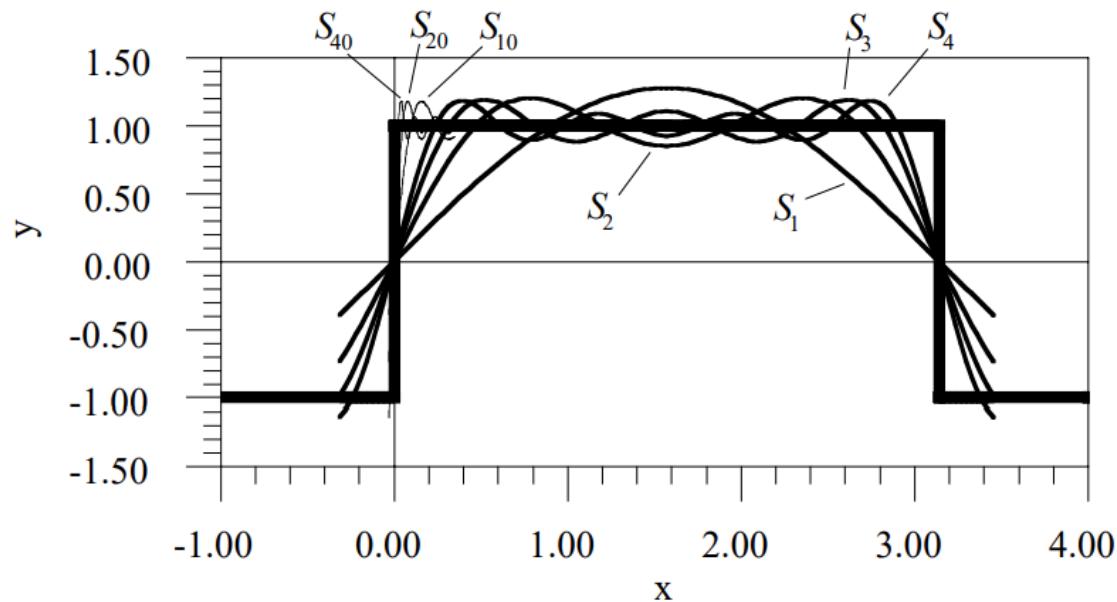


# Sampling

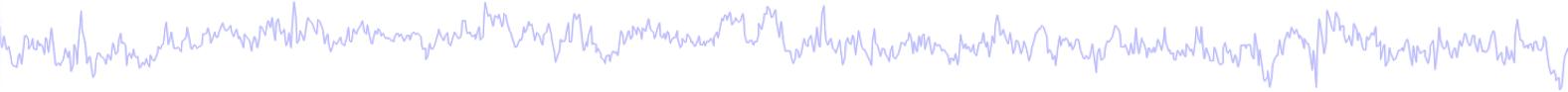








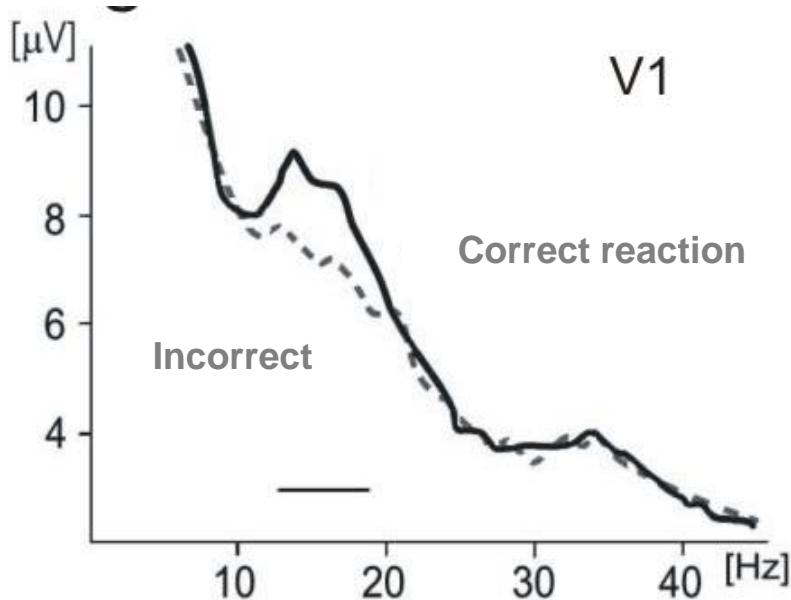
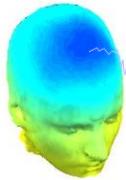
# Power Spectrum. Approach 1: FFT



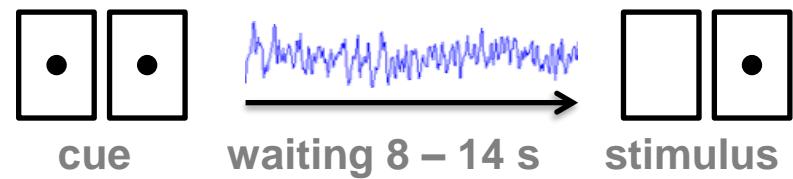
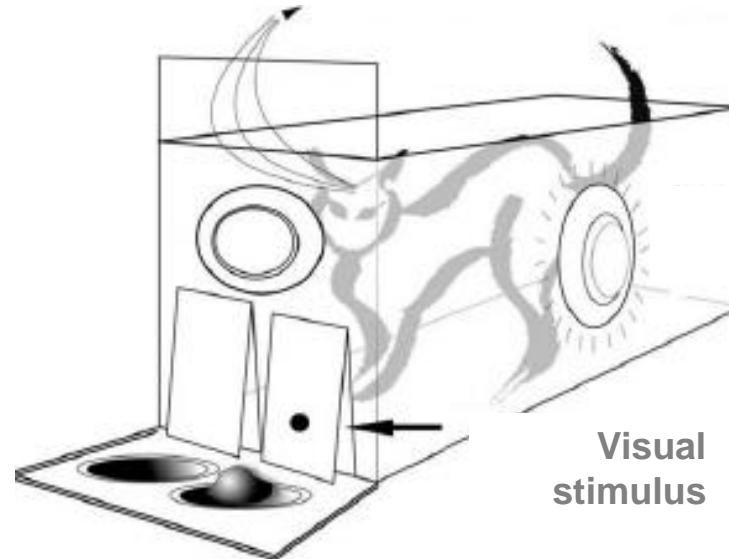
- Why not just take an FFT of our entire signal of interest?
- Advantage – fine frequency resolution
  - $\Delta F = 1 / \text{signal duration (s)}$
  - E.g. 100s signal has 0.01 Hz resolution
  - But, do we really need this?
- Disadvantage 1 – high variance
  - Solution: e.g. Welch's method
- Disadvantage 2 – no temporal resolution
  - Solution 1: Short-Time Fourier Transform



# Beta-band activity & attention



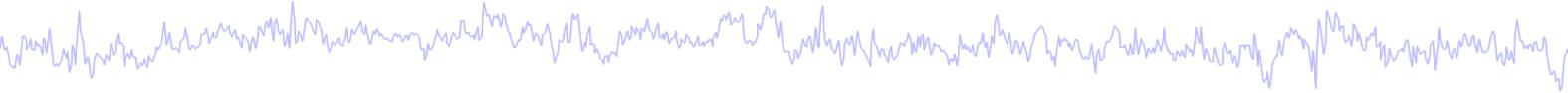
Wróbel et al., 2007



sustain attention or alertness – top down process  
of preparation for stimulus perception.

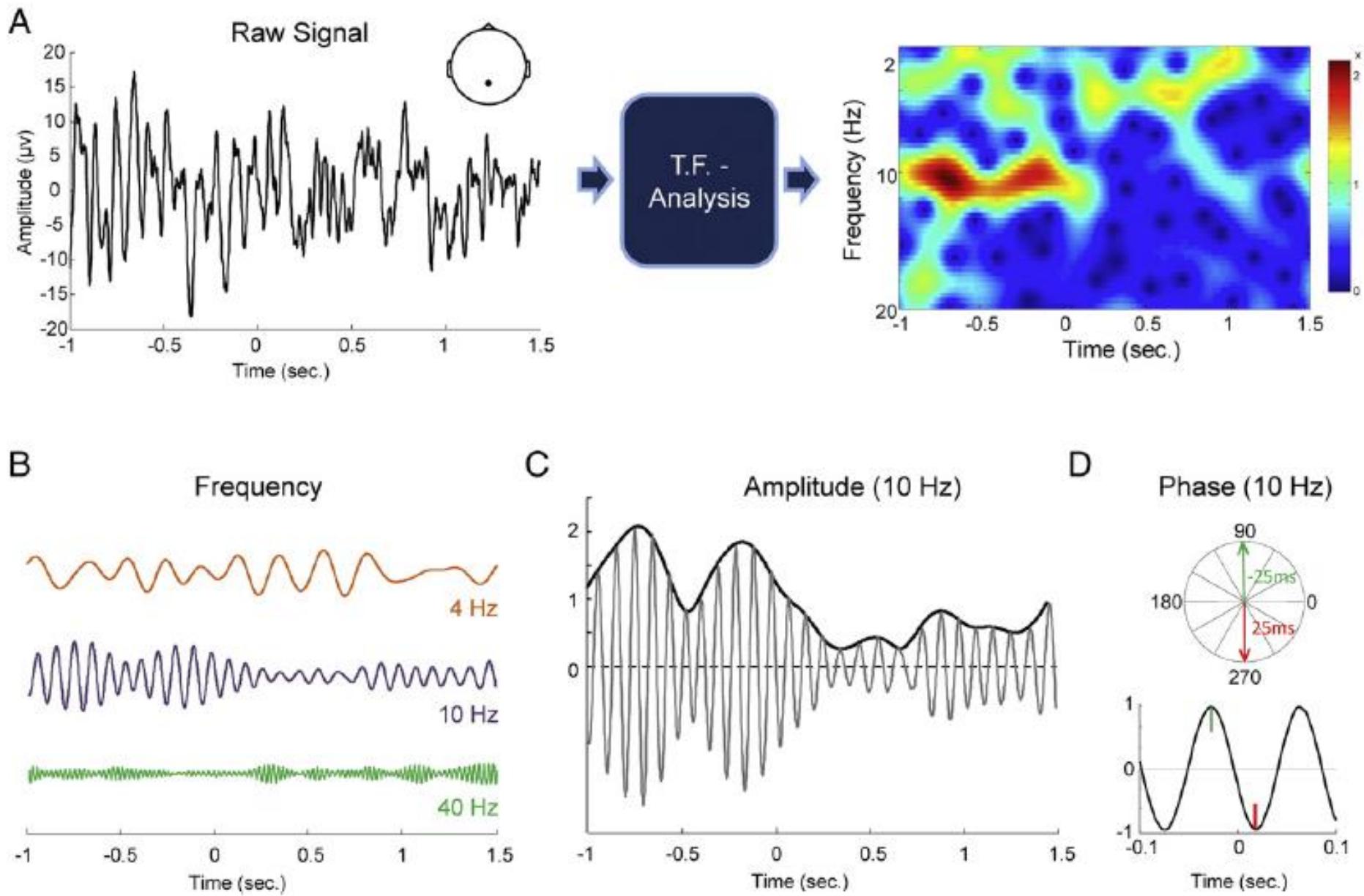


# Amplitude and phase

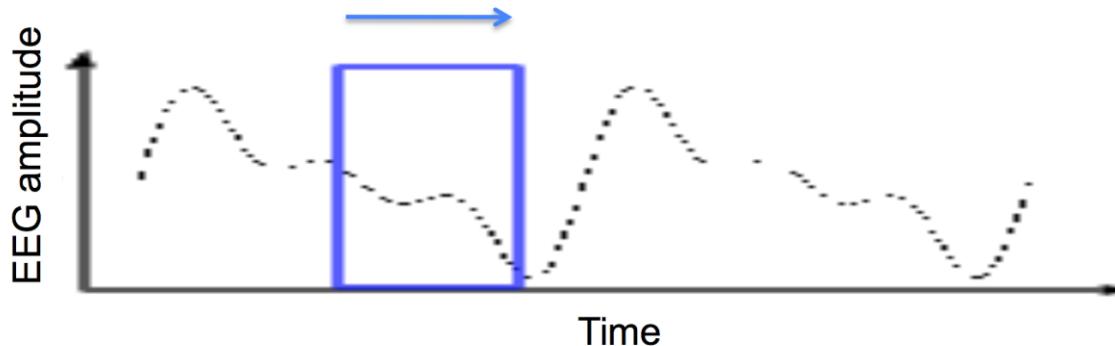


- Power spectra describe the *amount* of a given frequency present. Often expressed in **dB** [ $10^*\log_{10}(\text{Power})$ ]
- Power is NOT a complete description of a signal: We also must know the *phase* at each frequency
- FFT/STFT/Wavelet return an amplitude and phase at each time and frequency (represented as complex #).
- To find power, we compute the magnitude, which discards phase.





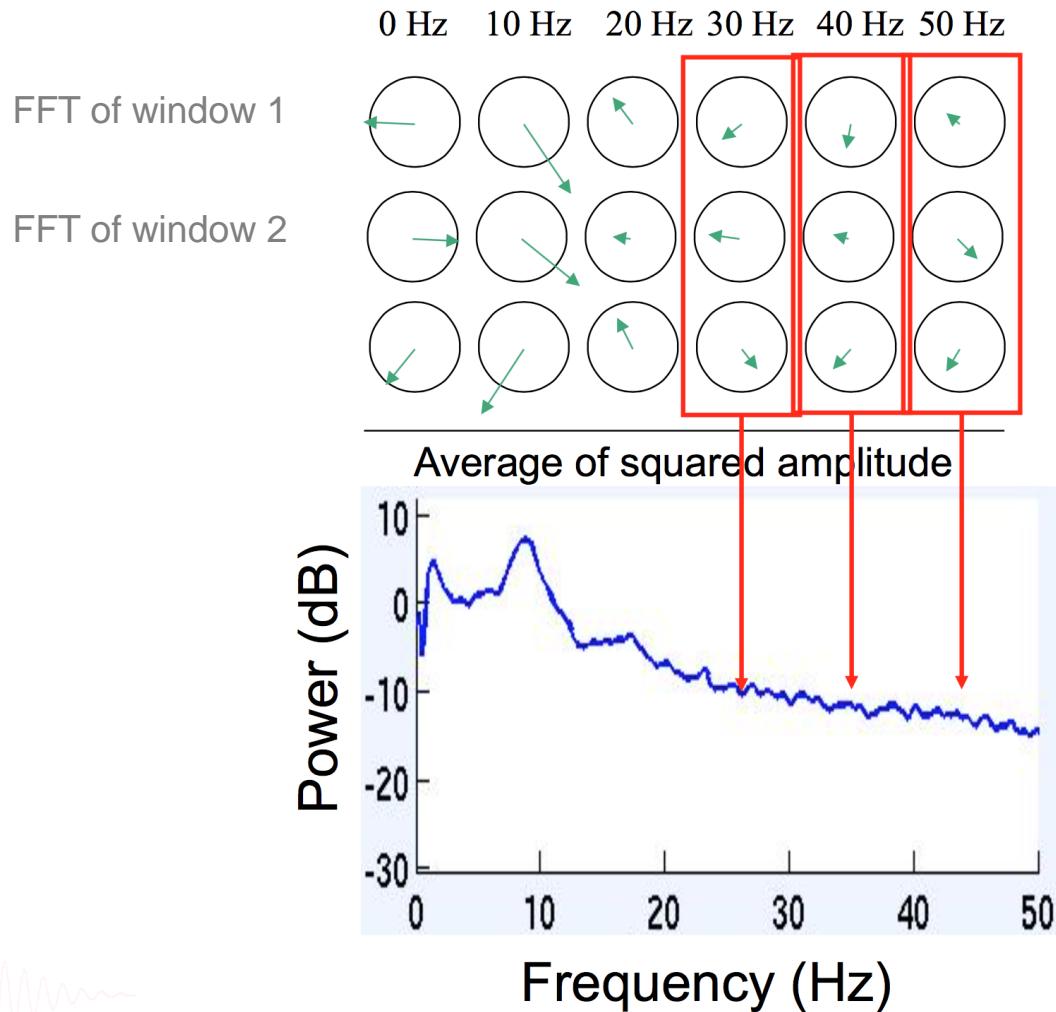
# Approach 2: Welch's Method



Calculate power spectrum of short signal windows, average.  
Advantage: Smoother estimate of power spectrum

Frequency resolution now set by *window length*  
e.g. 1s window -> 1 Hz resolution

In practice: *taper*, don't use rectangular window

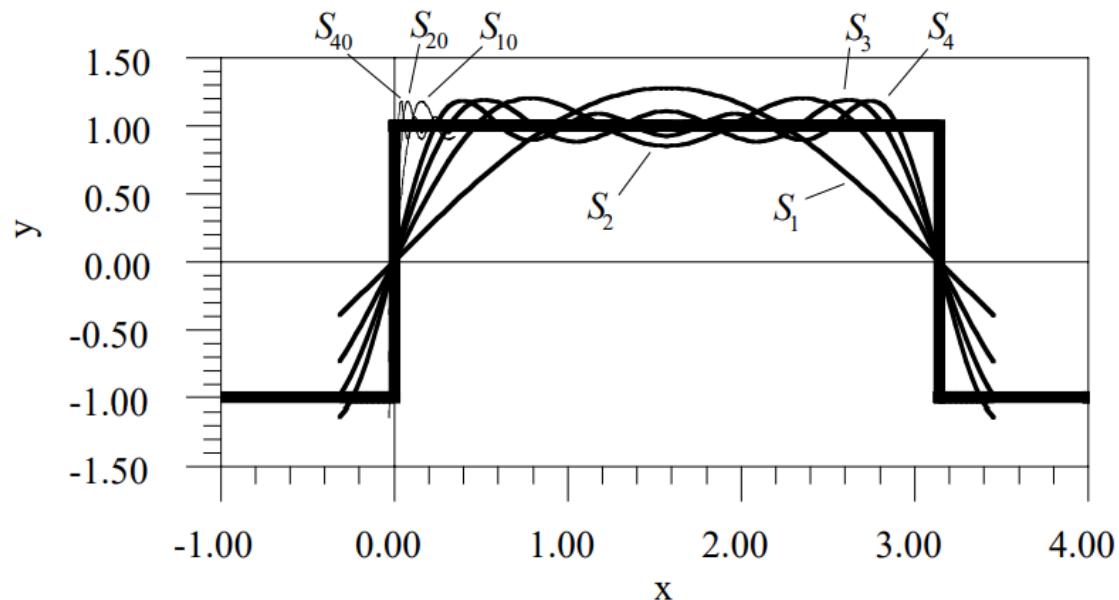


# Windowing

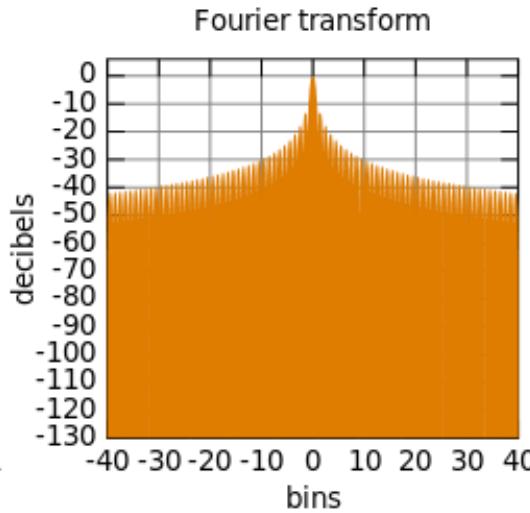
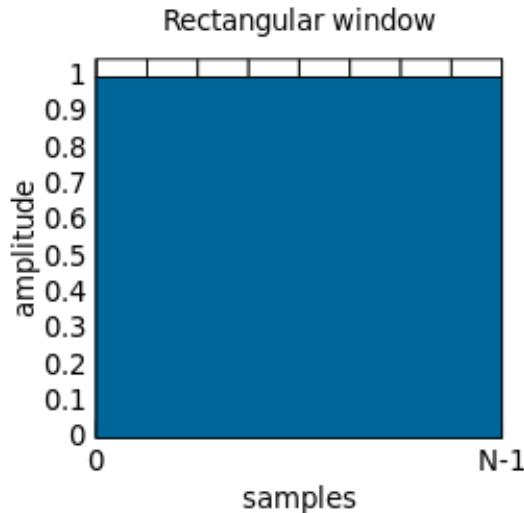


- When we pick a short segment of signal, we typically window it with a smooth function (taper).
- Windowing in time = convolving (filtering) the spectrum with the Fourier transform of the window
- No window (=rectangular window) results in the most smearing of the spectrum
- There are many other windows optimized for different purposes: Hamming, Gaussian...

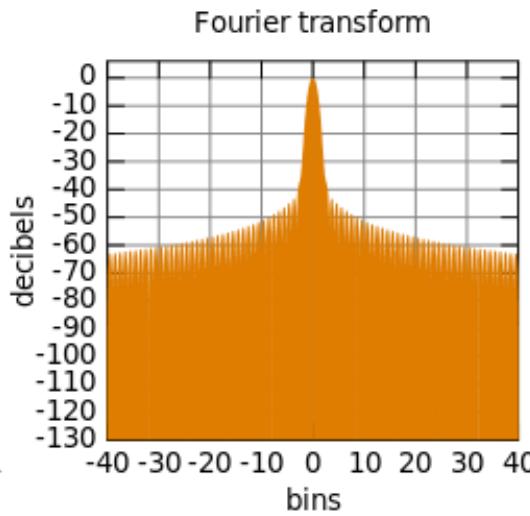
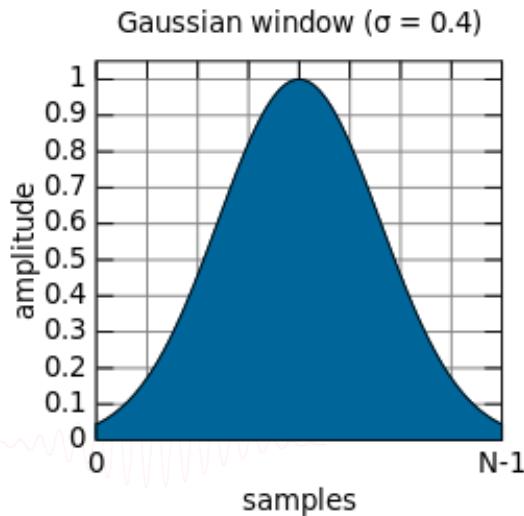




# Windows and their Fourier transforms



Narrowest main peak, but  
Highest side-lobes  
Most spectral 'smearing'

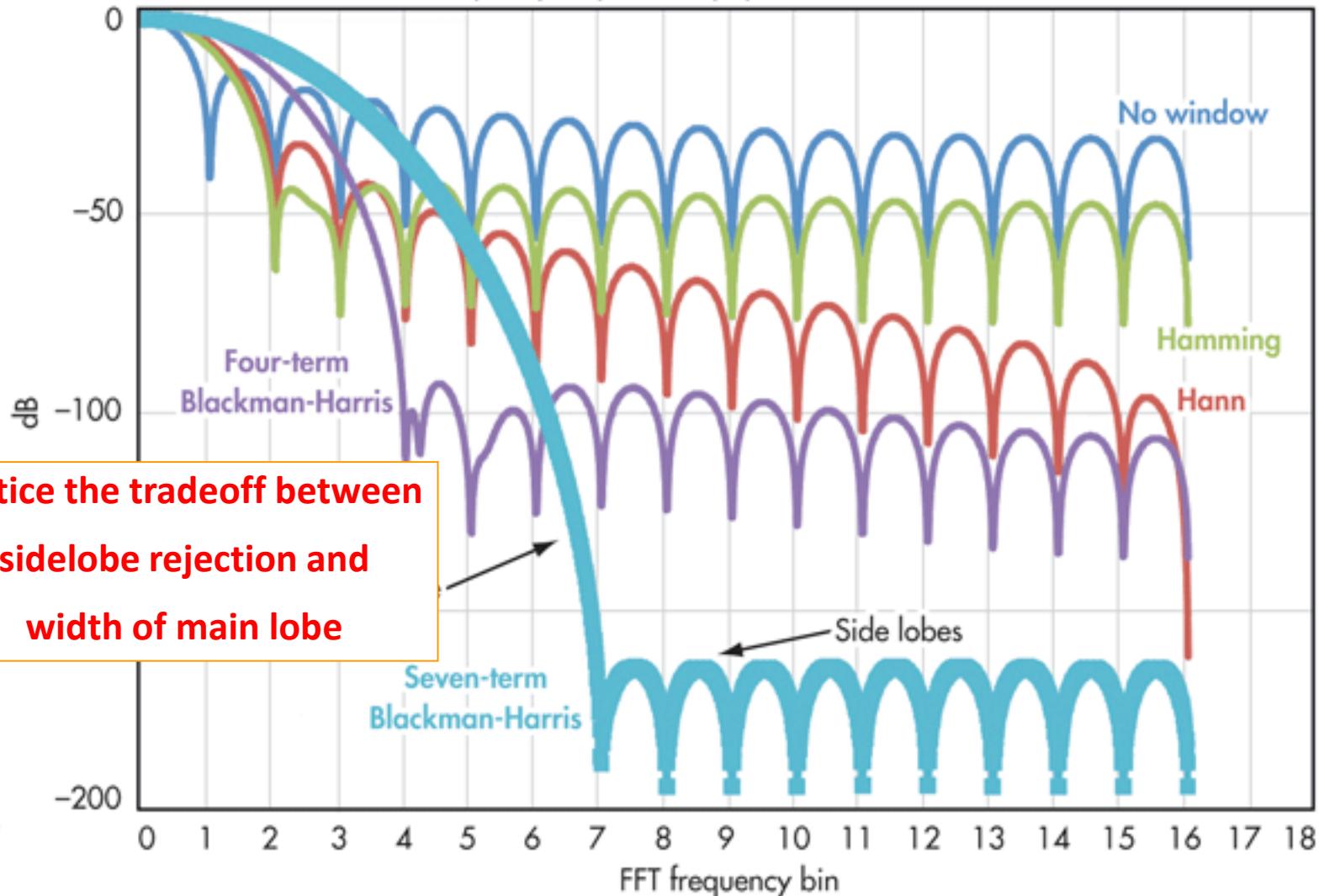


Wider main peak, but  
much lower side-lobes

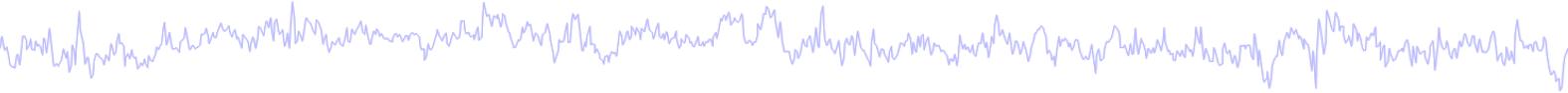
# Close-up view



Frequency response of popular window functions



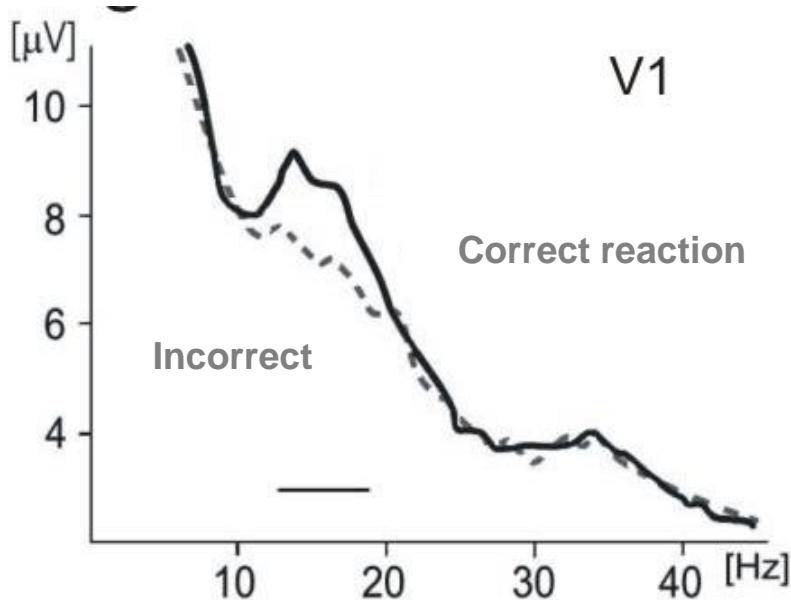
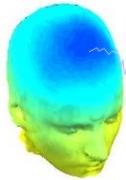
# Part 2: Time-Frequency Analysis



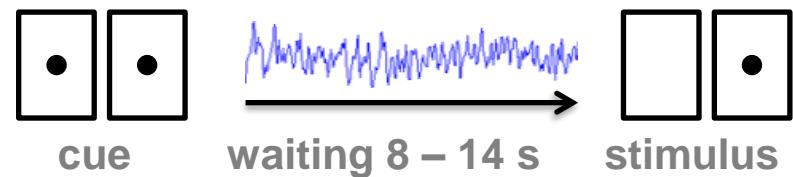
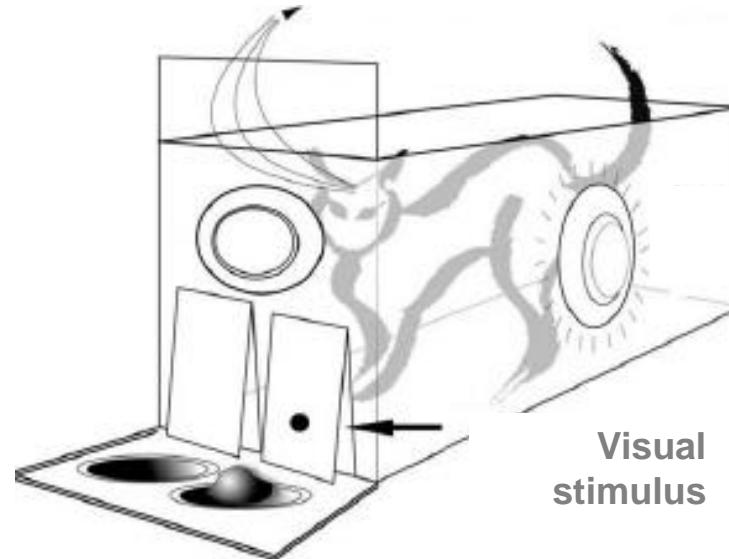
- Short-Time Fourier Transform
  - Find power spectrum of short windows
  - “Spectrogram”
- Advantage: Can visualize time-varying frequency content
- Disadvantage: Fixed temporal resolution is not optimal



# Beta-band activity & attention



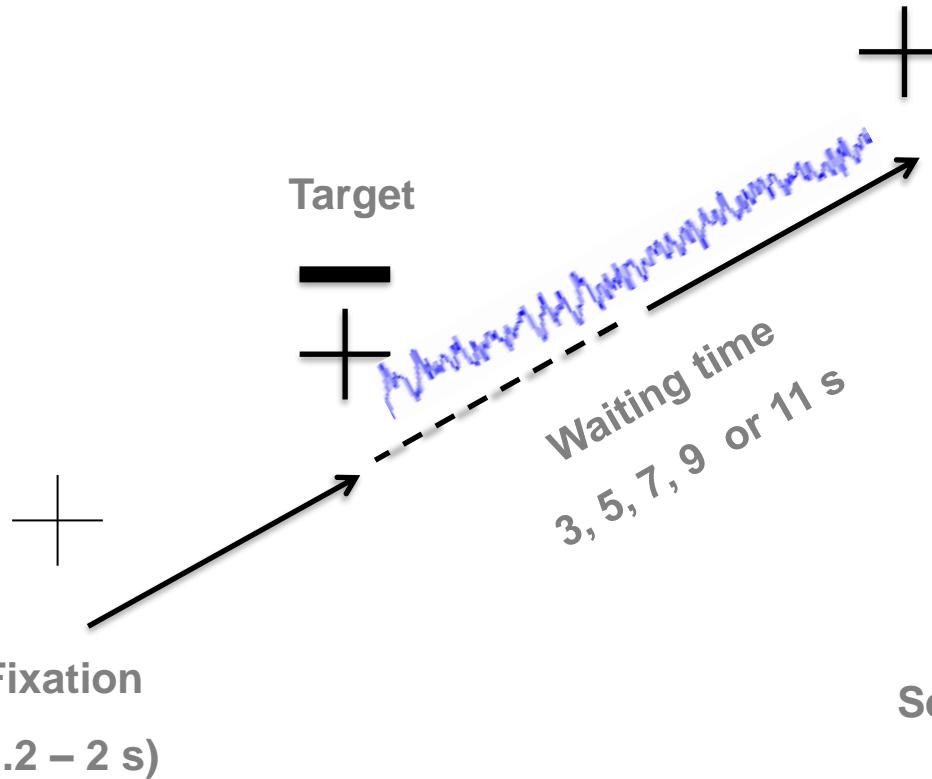
Wróbel et al., 2007



sustain attention or alertness – top down process  
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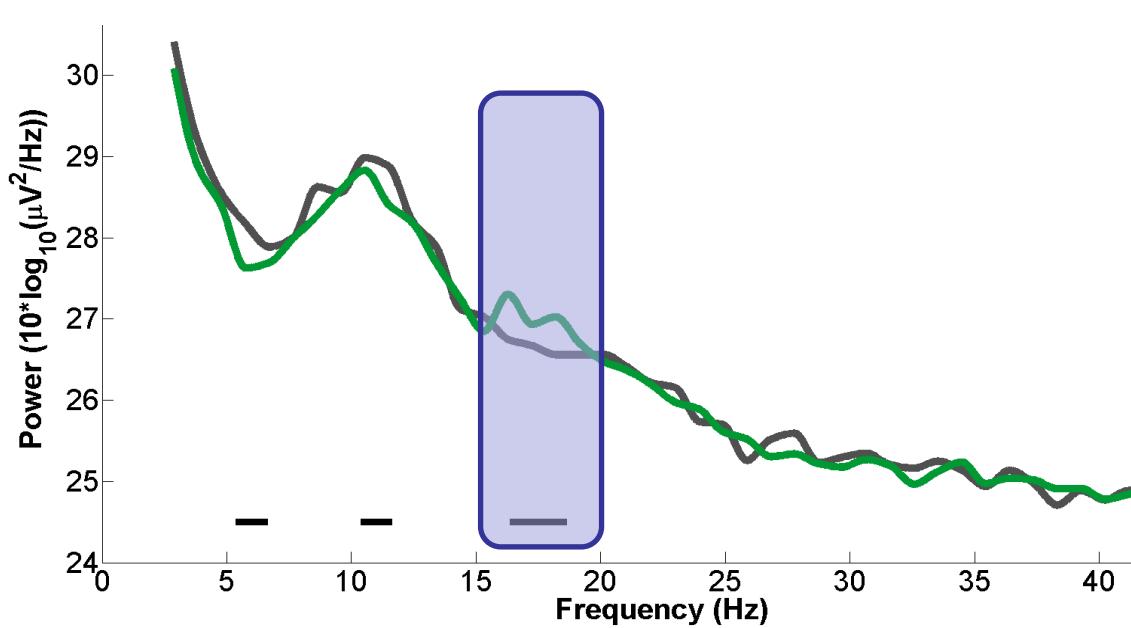


# Procedure

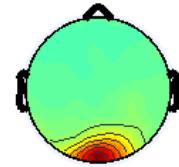


- With or without cue (50%)  
Sey if the target was preset in the target matrix or not.  
If you are not sure do not response.
- Self adapting time of presentation:**
- 5% after correct response
  - + 10% after incorrect response

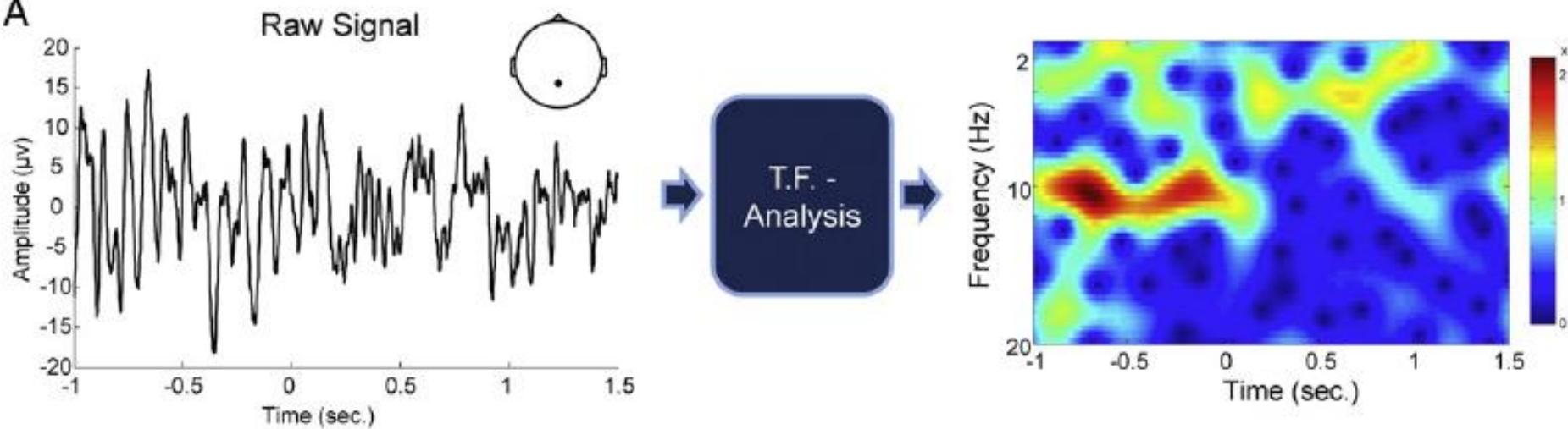
Gola et al., 2013



Cls 4 (23 Ss, 63 ICs)



Gola et al., 2013

**A**



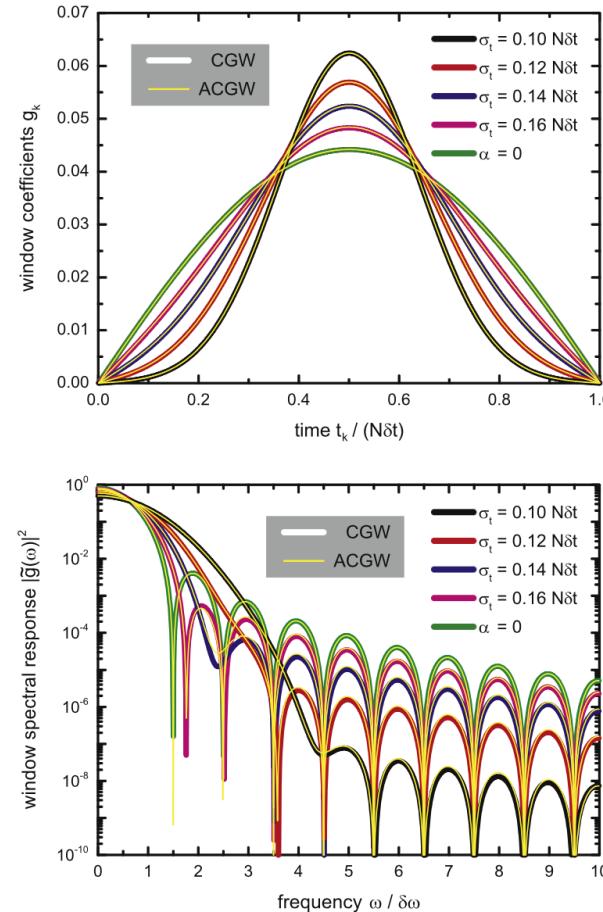
# Haisenberg Uncertainty



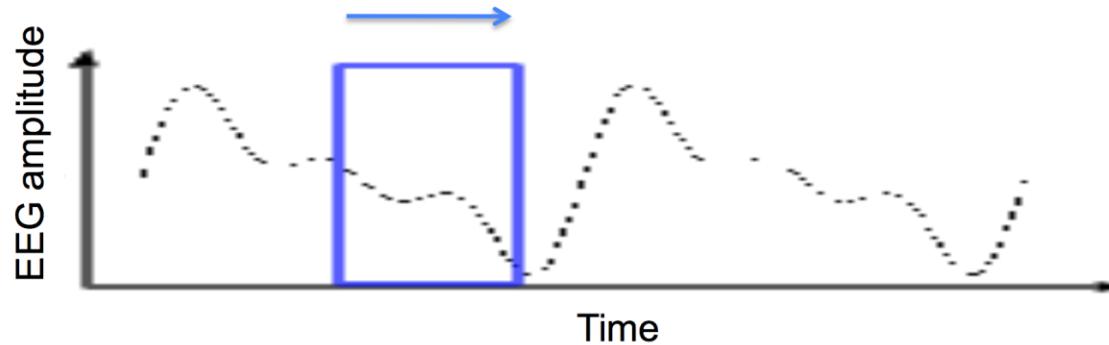
# Time-Frequency Uncertainty



- You cannot have both arbitrarily good temporal and frequency resolution!
  - $\sigma_t * \sigma_f \geq 1/2$
- If you want sharper temporal resolution, you will sacrifice frequency resolution, and vice versa.
- (Optimal: Confined Gaussian)



Starosielec S, Hägele D (2014) Discrete-time windows with minimal RMS bandwidth for given RMS temporal width. *Signal Processing* 102:240–6.



Sinusoid



\*

Gaussian



Tapered  
sinusoid



For each time point

Analyze signal using the wavelets  
for different frequencies.



# Consequence for STFT

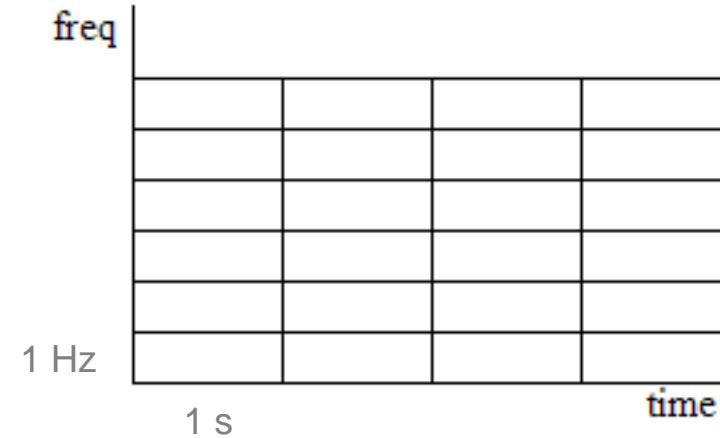
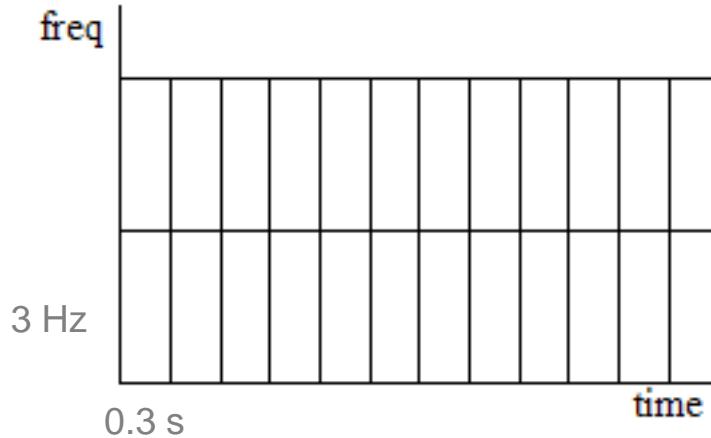


**Shorter Windows**

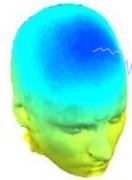
**poorer frequency resolution**

**Longer Windows**

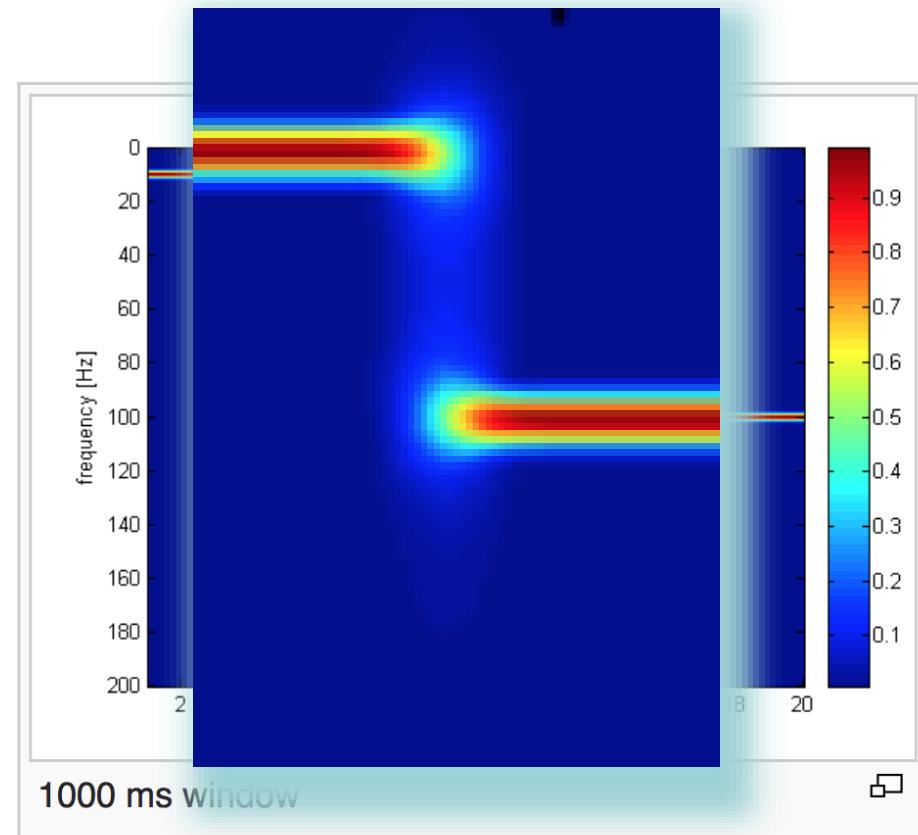
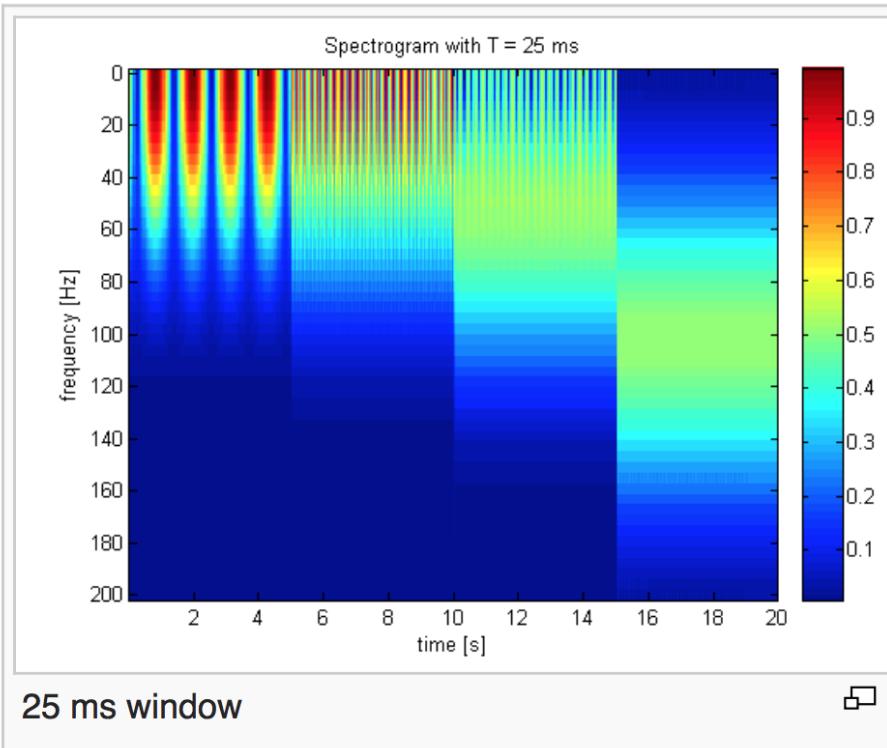
**finer frequency resolution**



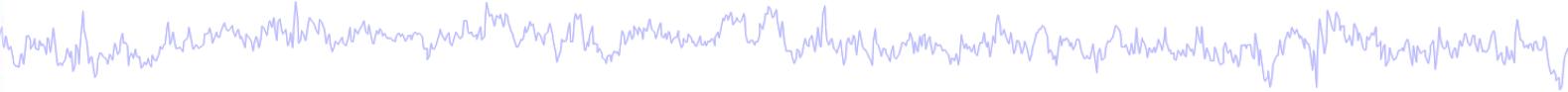
# Time-Frequency Tradeoff



Signal: 10, 25, 50, 100 Hz



# One better way: Wavelet transform



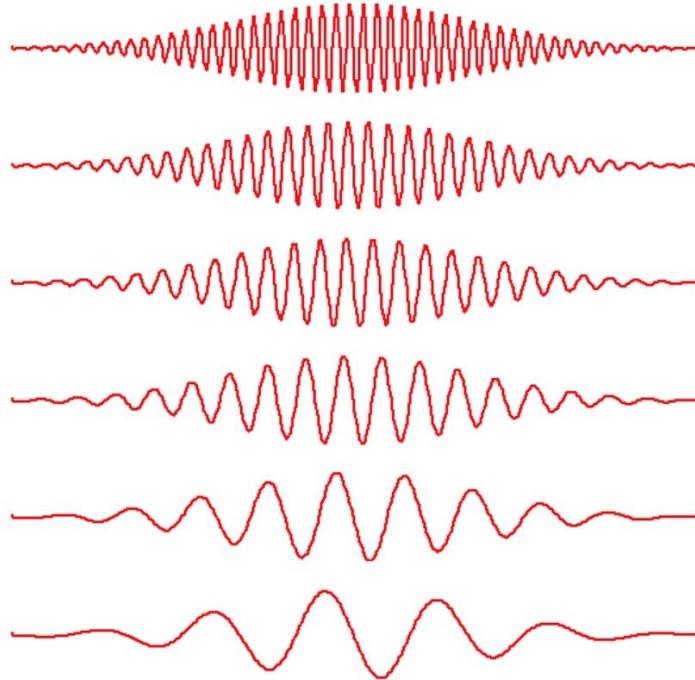
- Wavelet transform is a ‘multi-resolution’ time-frequency decomposition.
- Intuition: Higher frequency signals have a faster time scale
- So, vary window length with frequency!
  - longer window at lower frequencies
  - shorter window at higher frequencies



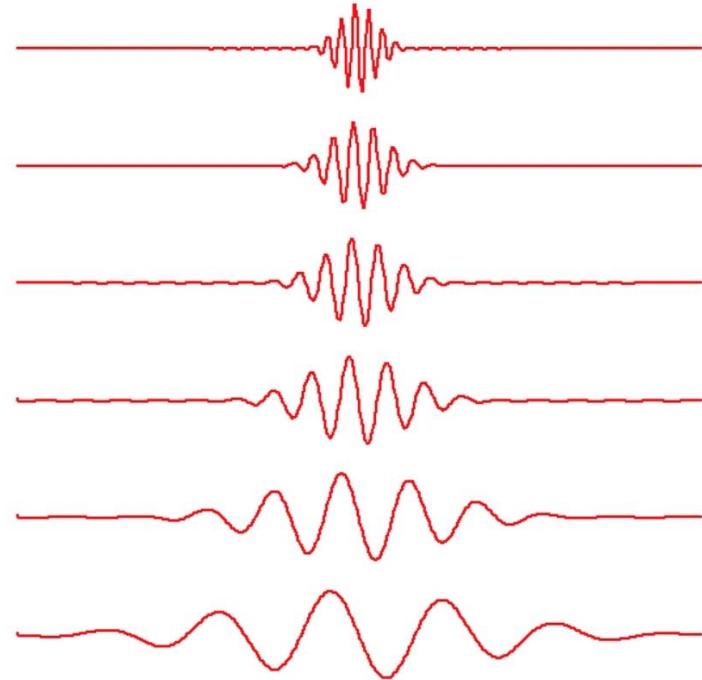
# Comparison of FFT & Wavelet



FFT

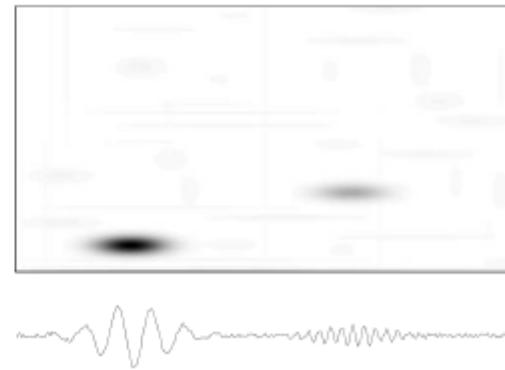
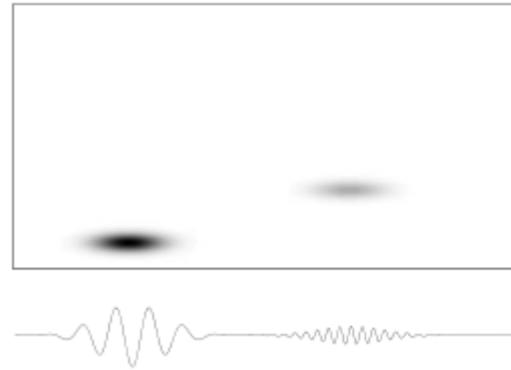
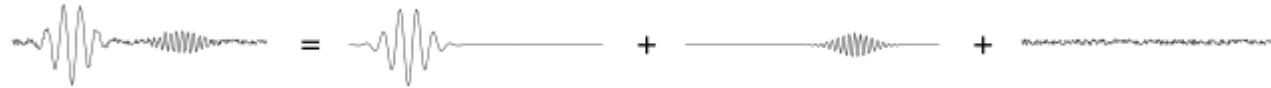


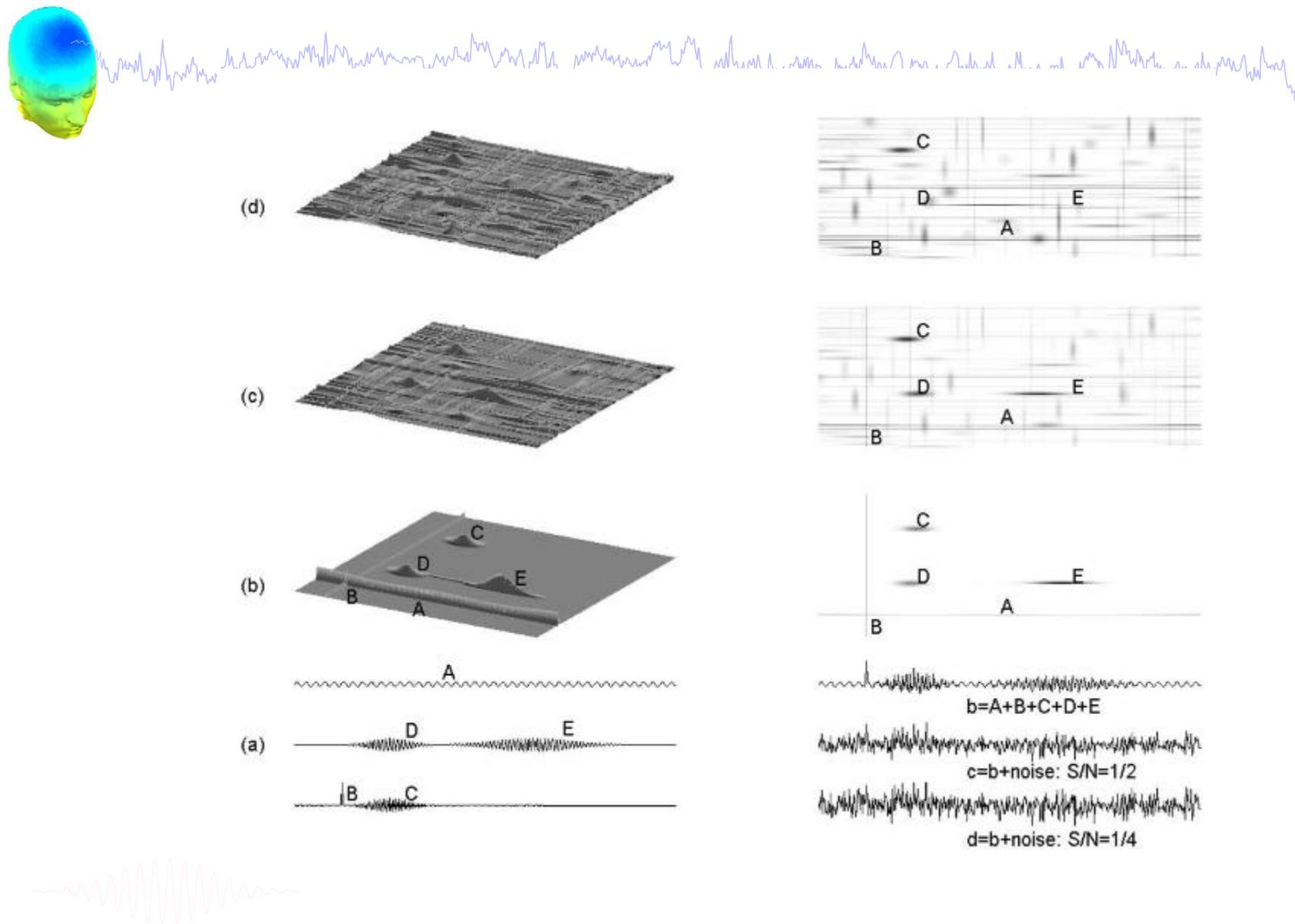
Wavelet

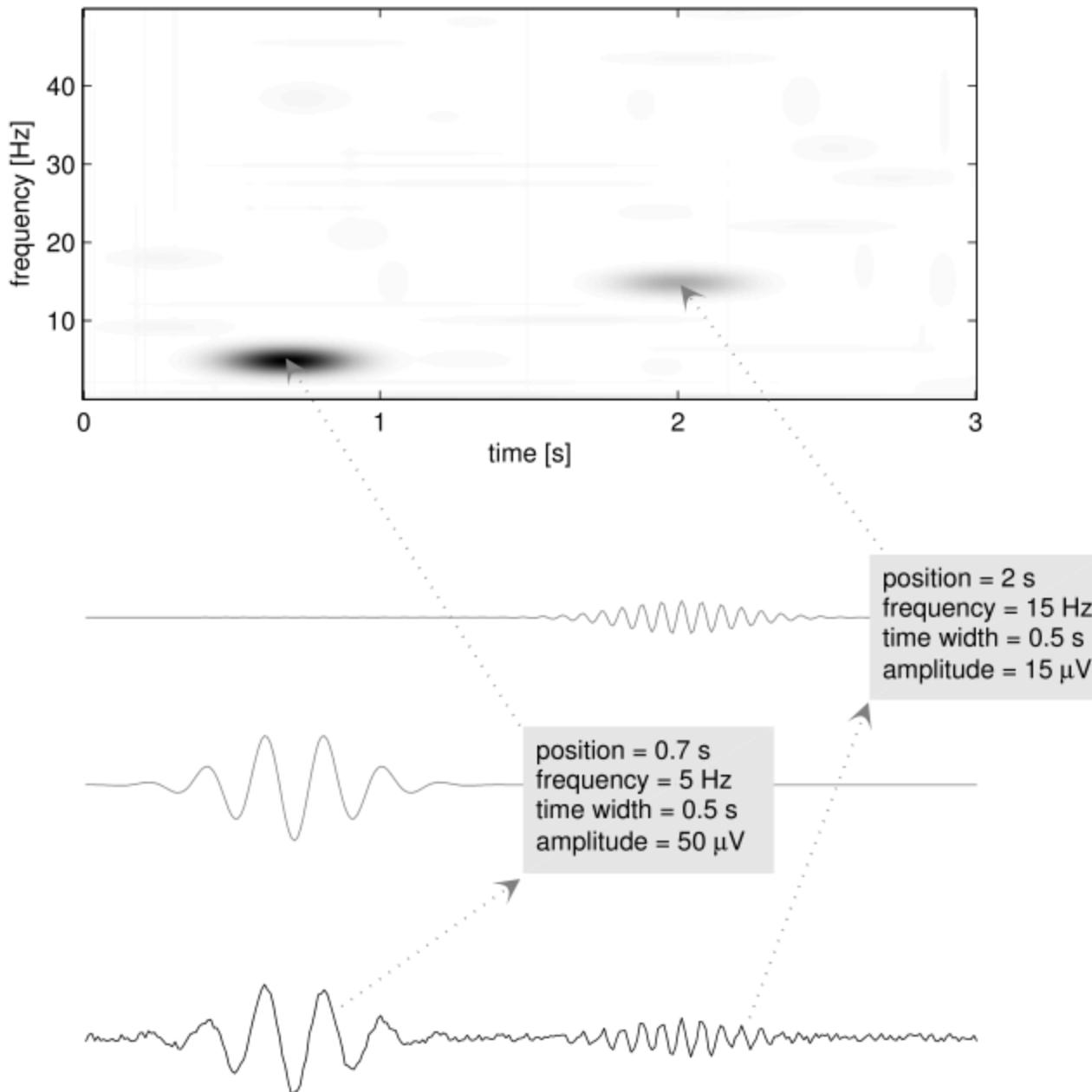


Scaled versions of one shape

Constant number of cycles



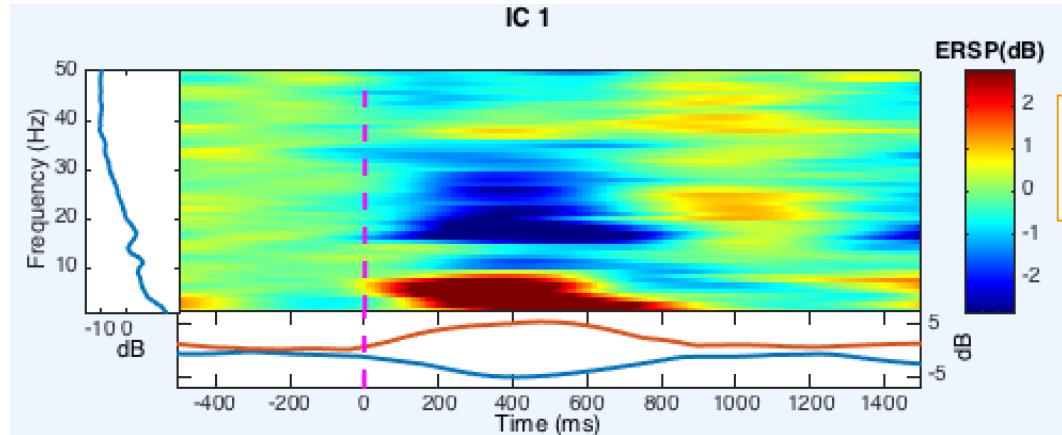




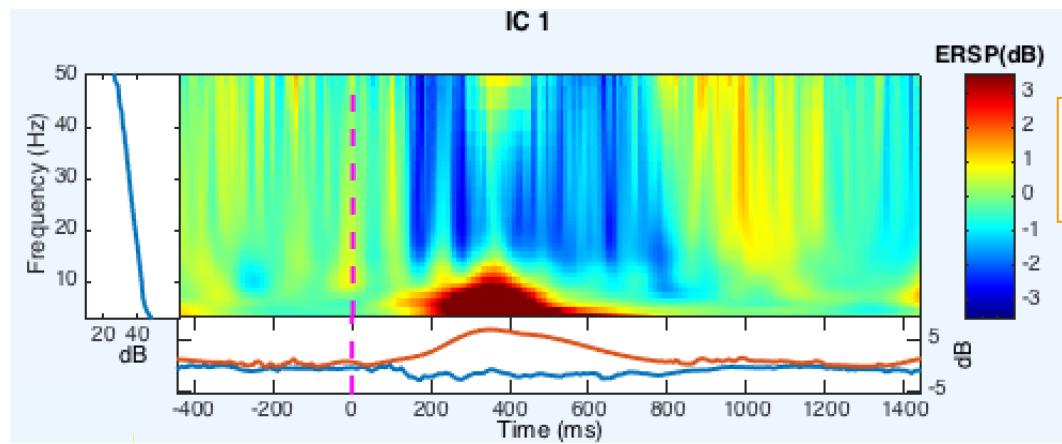
# Comparison of FFT & Wavelet



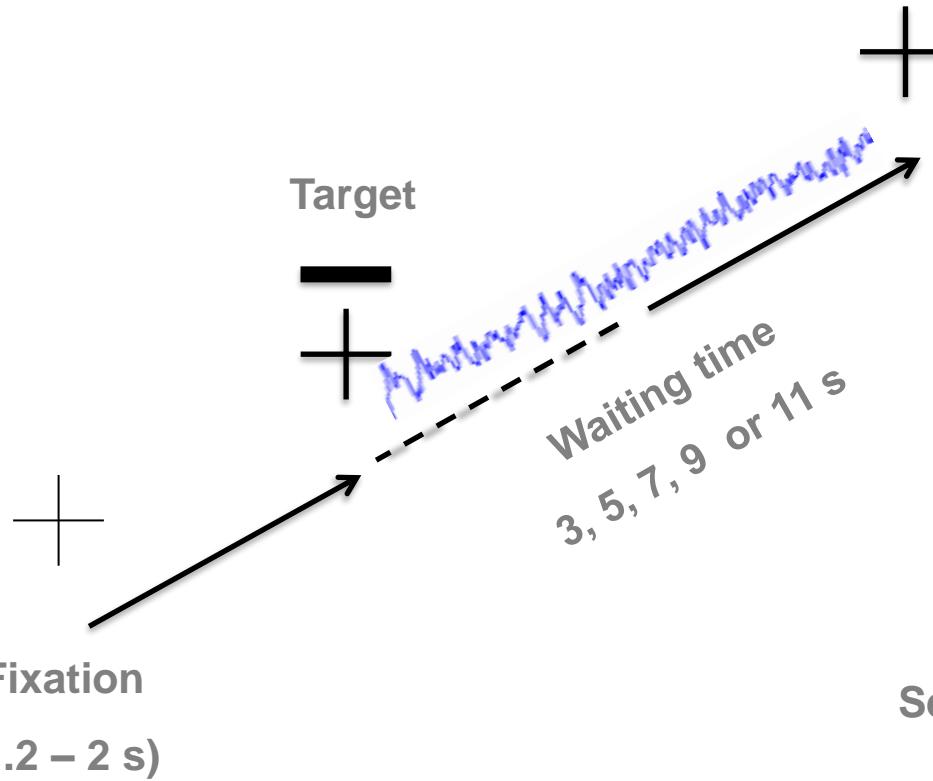
FFT



Wavelet

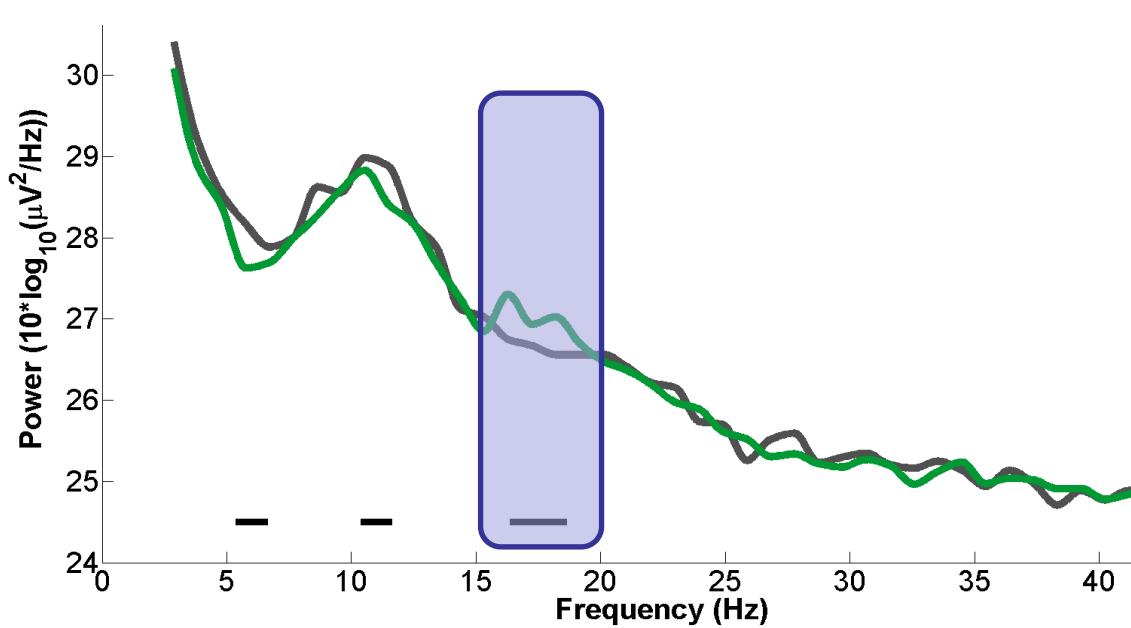


# Procedure



- With or without cue (50%)  
Sey if the target was preset in the target matrix or not.  
If you are not sure do not response.
- Self adapting time of presentation:**
- 5% after correct response
  - + 10% after incorrect response

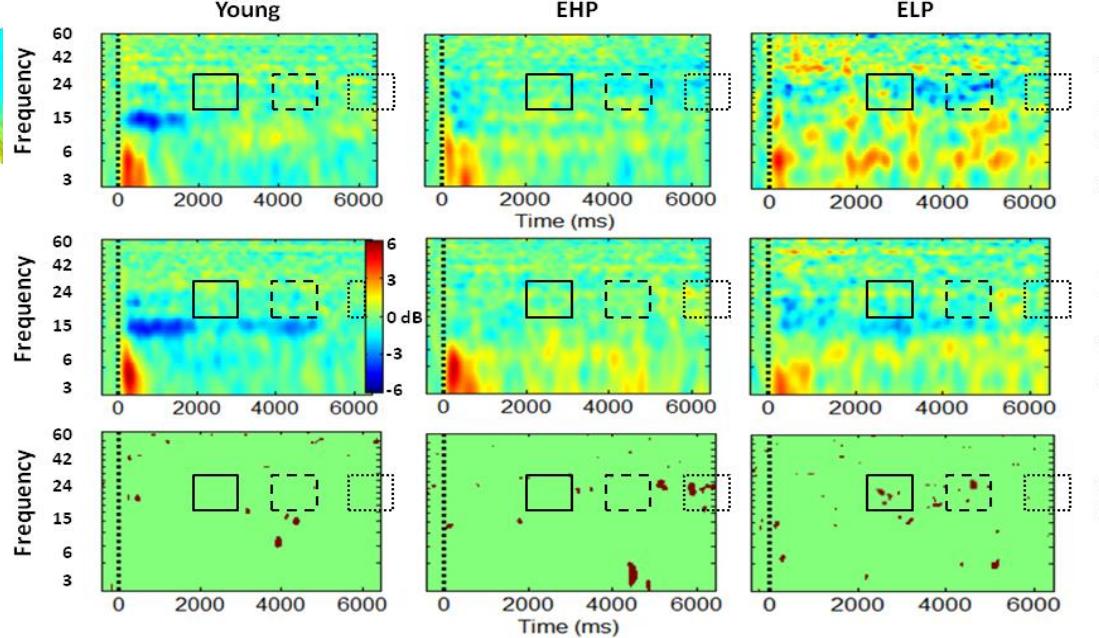
Gola et al., 2013



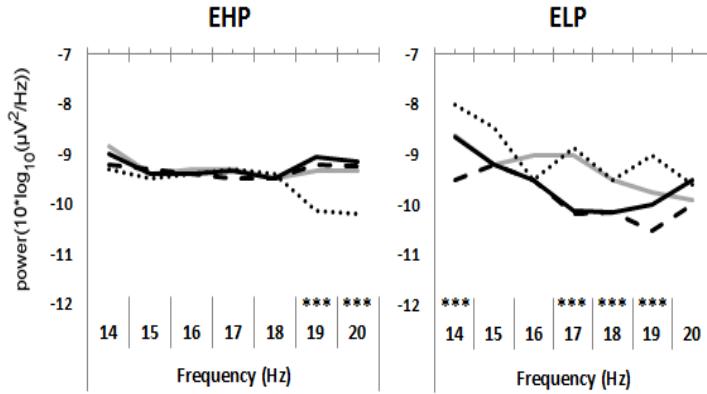
Cls 4 (23 Ss, 63 ICs)



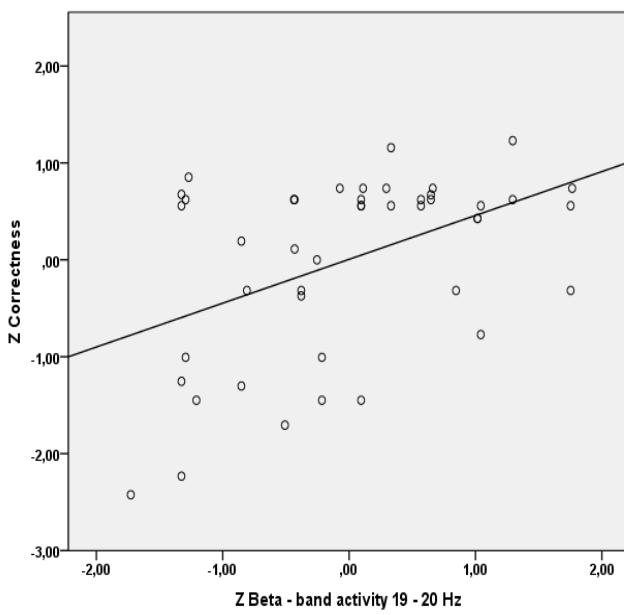
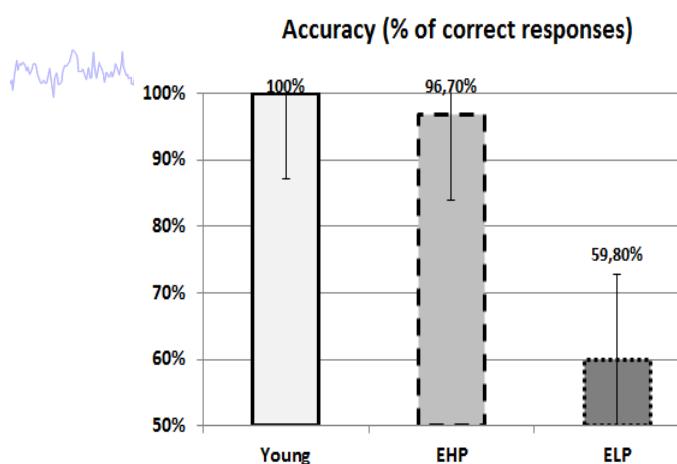
Gola et al., 2013



Resting  
— 3 sec  
- - - 5 sec  
..... 7 sec  
\*\*\* p<0,001



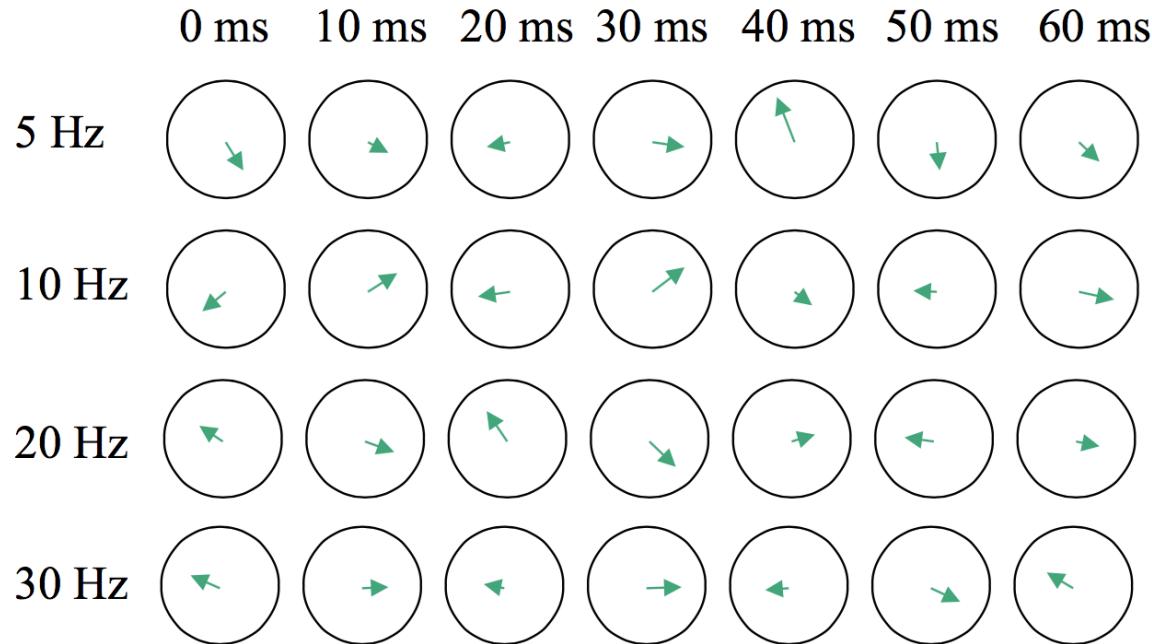
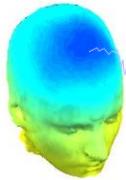
Visual attention Resting state p<0.01



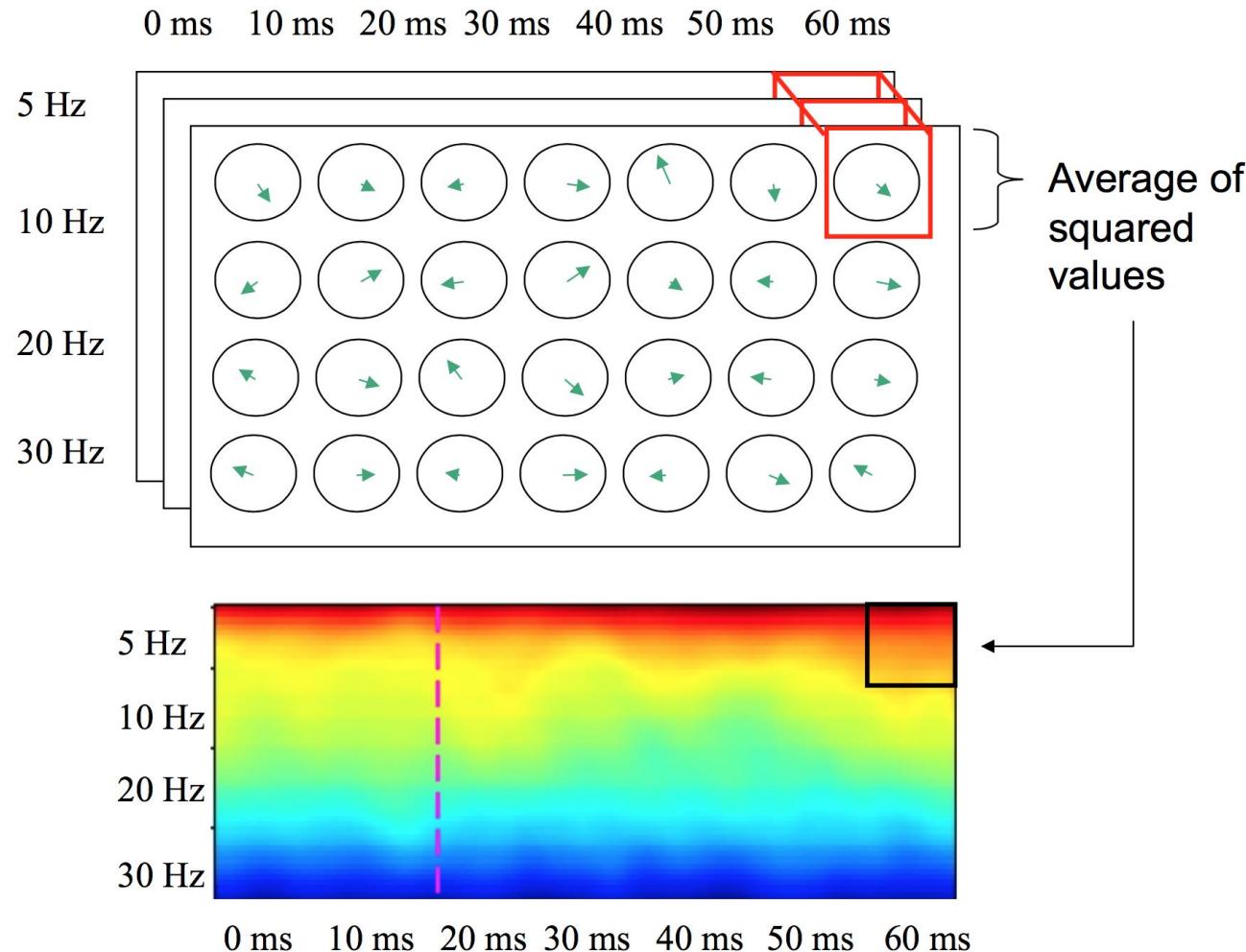
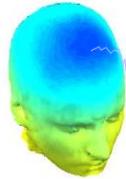
$$r = 0.456; p < 0.01$$

Gola et al., 2012

# Spectrogram of one epoch of data



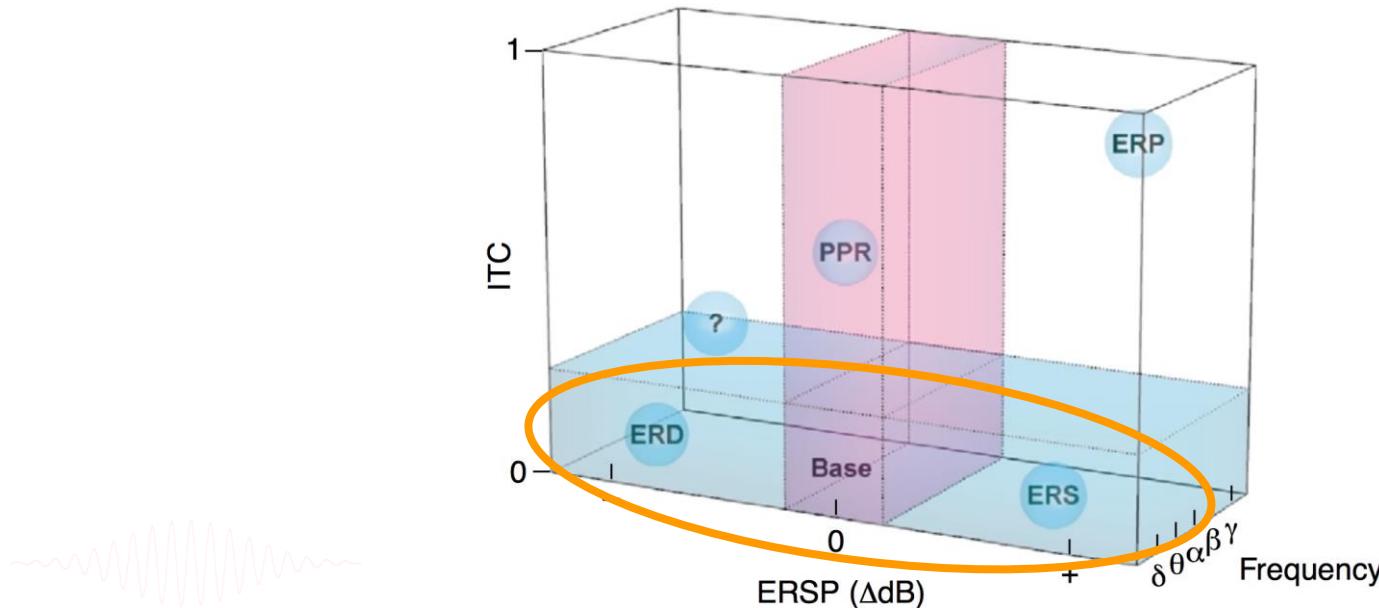
# Computing Spectrogram Power



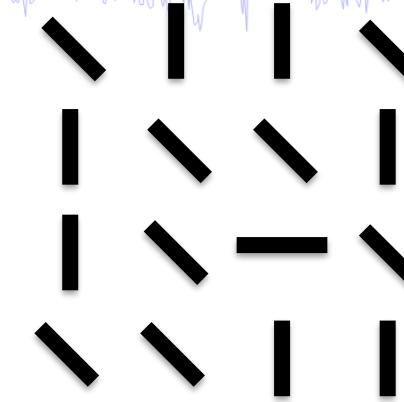
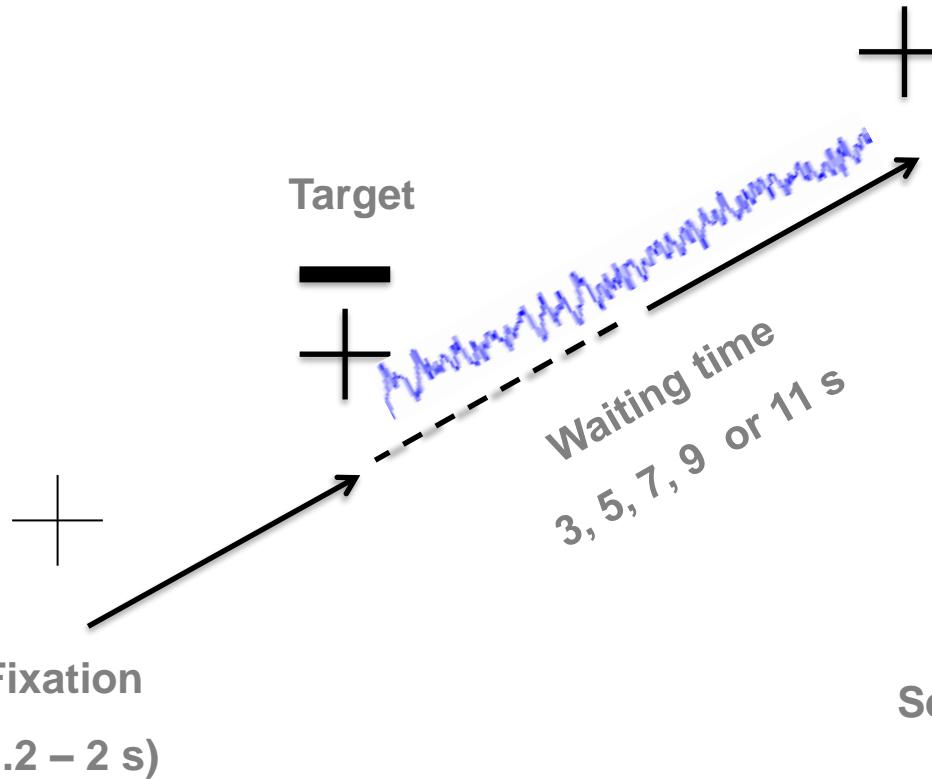
# Definition: ERSP



- Event Related Spectral Perturbation
- Change in power in different frequency bands relative to a baseline. ERS (Event-Related Synchronization), ERD (Event-Related Desynchronization)



# Procedure



With or without cue (50%)

Say if the target was preset in the target matrix or not.

If you are not sure do not response.

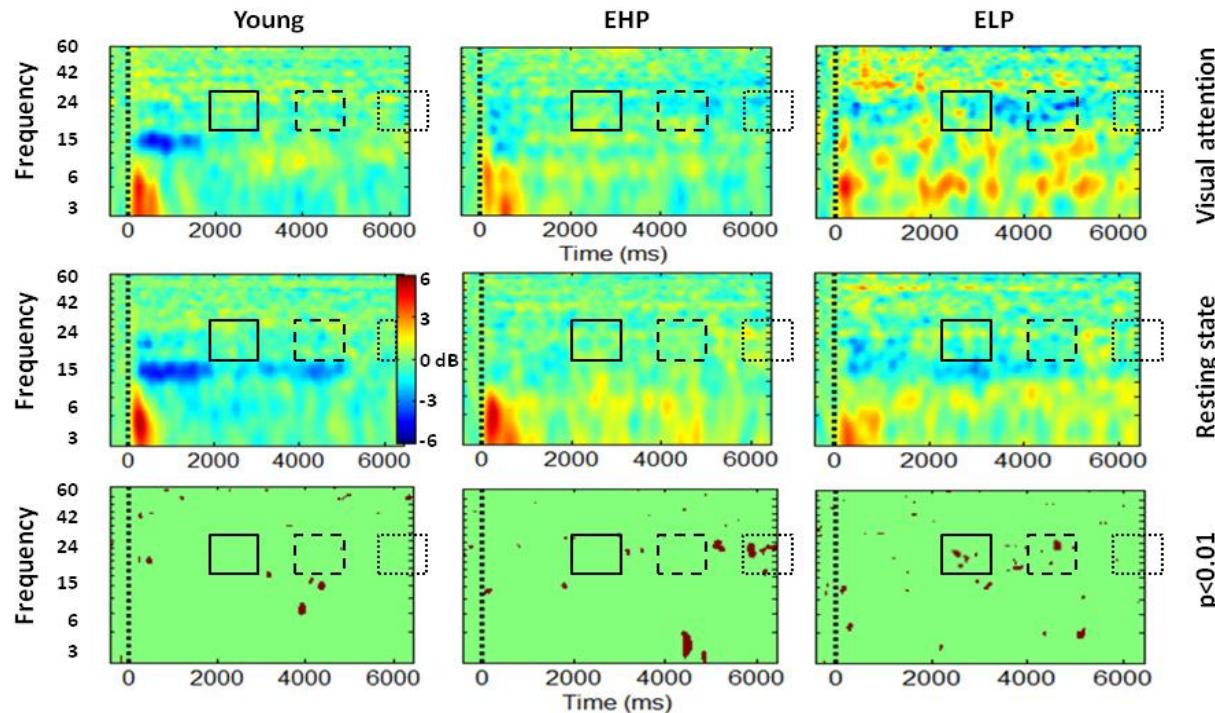
**Self adapting time of presentation:**

- 5% after correct response

+ 10% after incorrect response

Gola et al., 2013



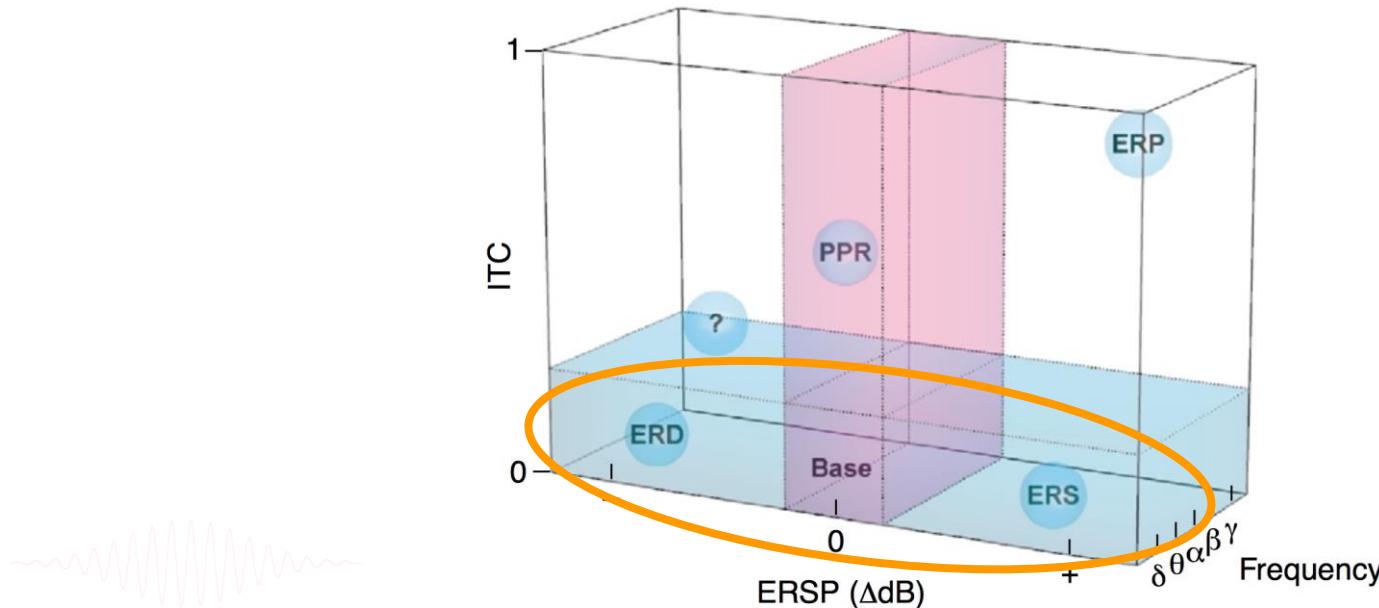


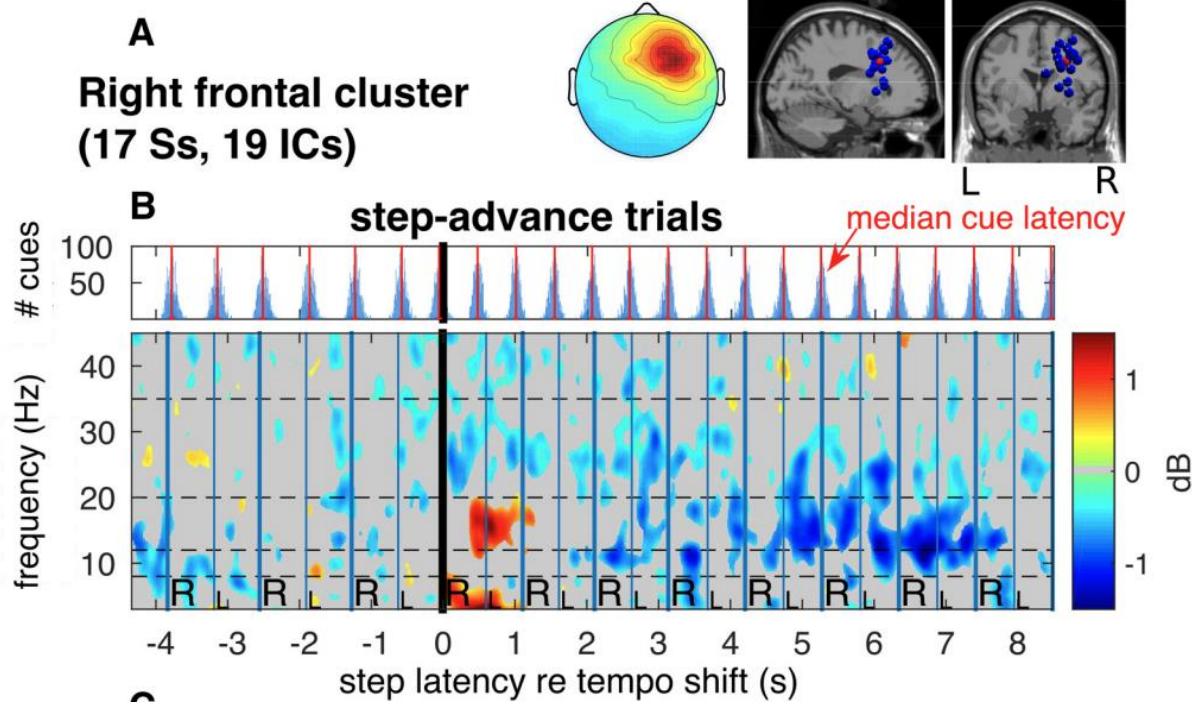
Gola et al., 2012

# Definition: ERSP



- Event Related Spectral Perturbation
- Change in power in different frequency bands relative to a baseline. ERS (Event-Related Synchronization), ERD (Event-Related Desynchronization)

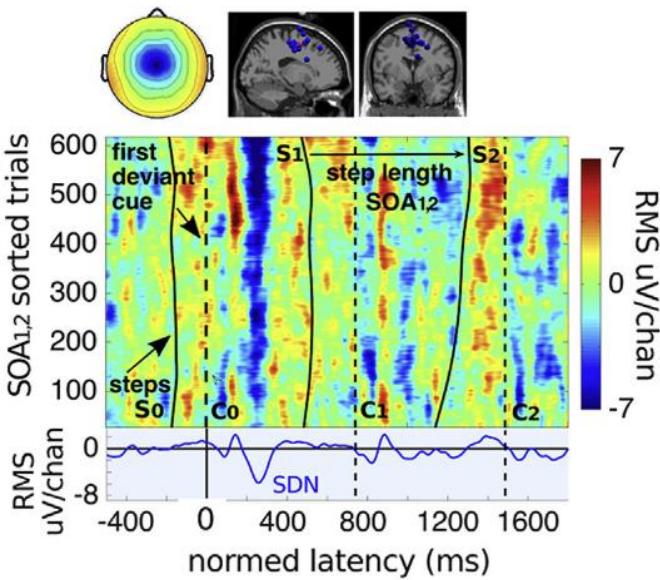




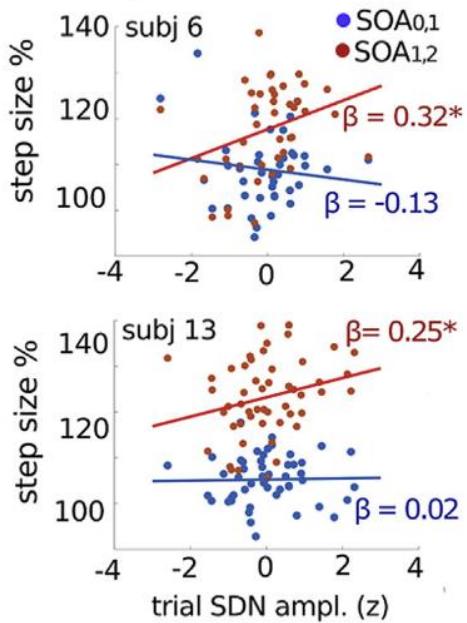
Wagner et al., 2016



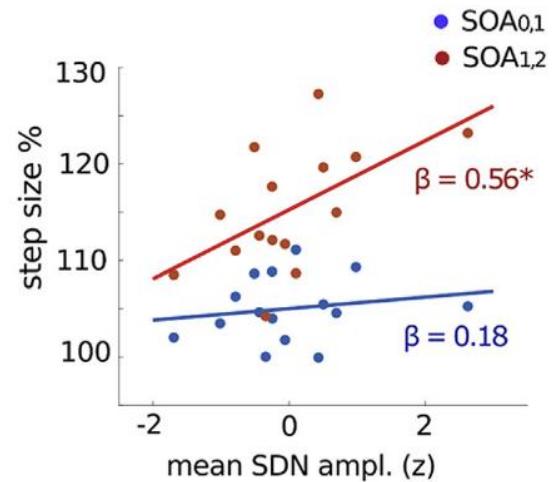
### A step delay trials sorted by adaptation step size



### B single trials

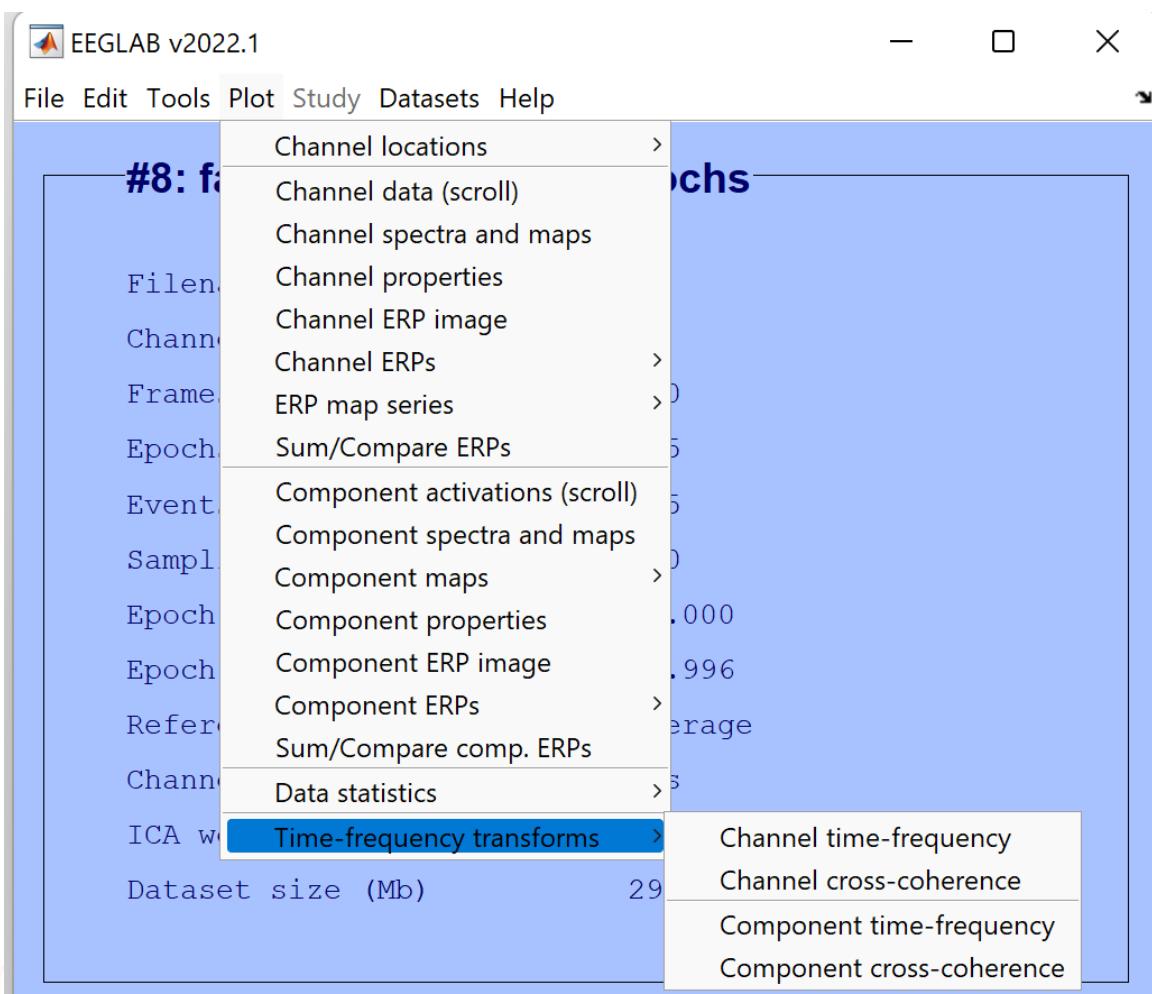


### C trial-averages



Wagner et al., 2019

# Try it out



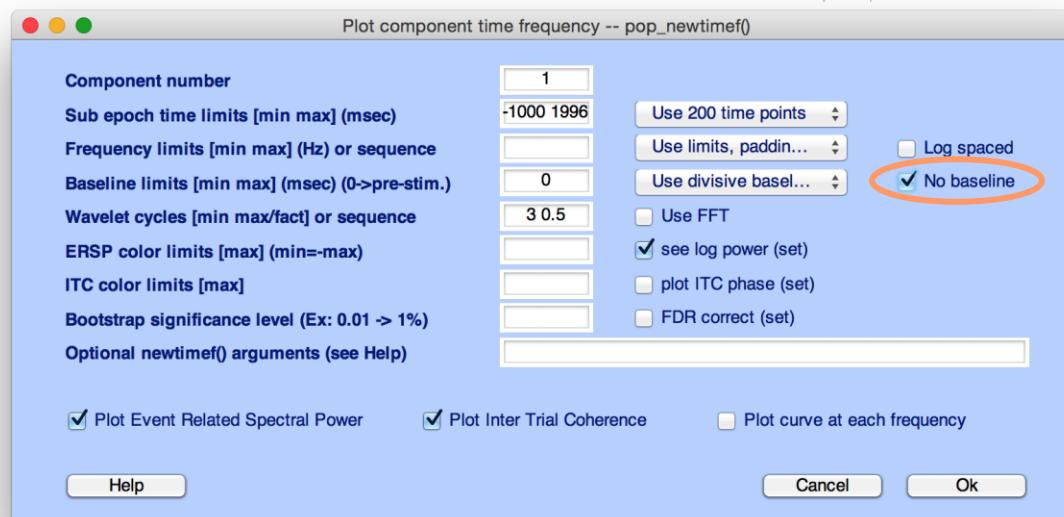
The image shows the EEGLAB v2022.1 software interface. The main window title is "EEGLAB v2022.1". The menu bar includes File, Edit, Tools, Plot, Study, Datasets, and Help. A context menu is open over a data series labeled "#8: faces", showing options like Channel locations, Channel data (scroll), Channel spectra and maps, etc. A sub-menu for "Time-frequency transforms" is open, listing Channel time-frequency, Channel cross-coherence, Component time-frequency, and Component cross-coherence. A red box highlights the "Time-frequency transforms" option in the sub-menu. In the bottom right corner, there is a red-bordered box containing the text "(Load faces\_4.set" and "Epoch on 'face' event)".

(Load faces\_4.set  
Epoch on 'face' event)

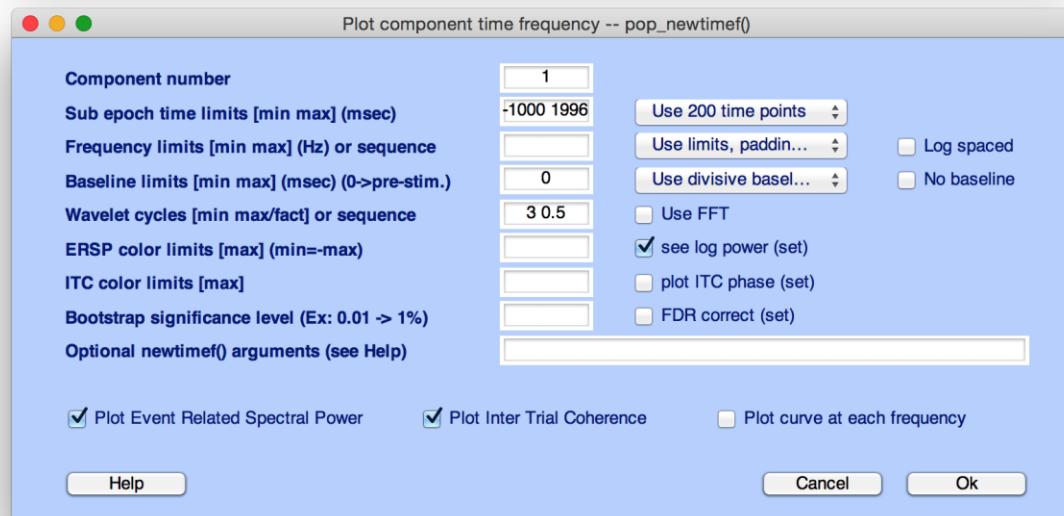
# Display ERS vs. ERSP



## Event-related Spectrogram

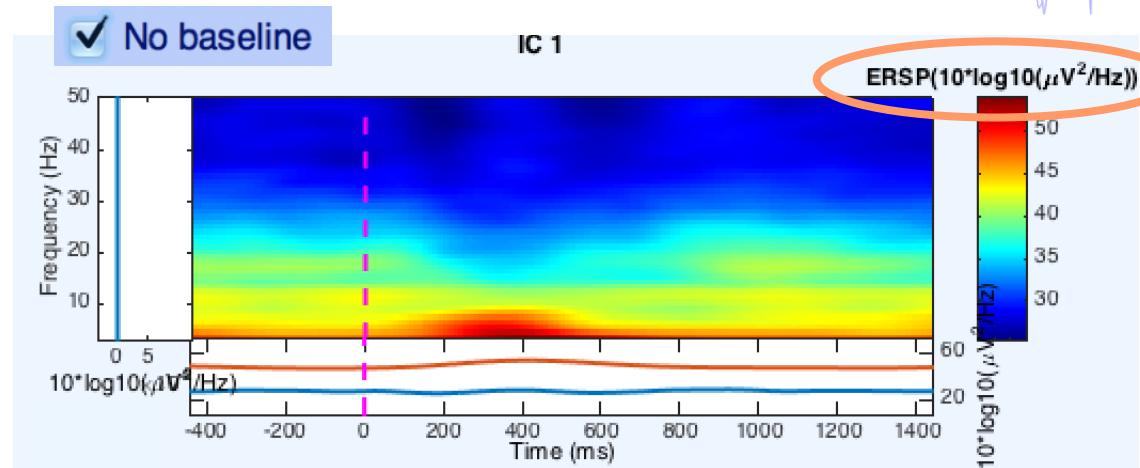


## Event-Related Spectral Perturbation (ERSP)

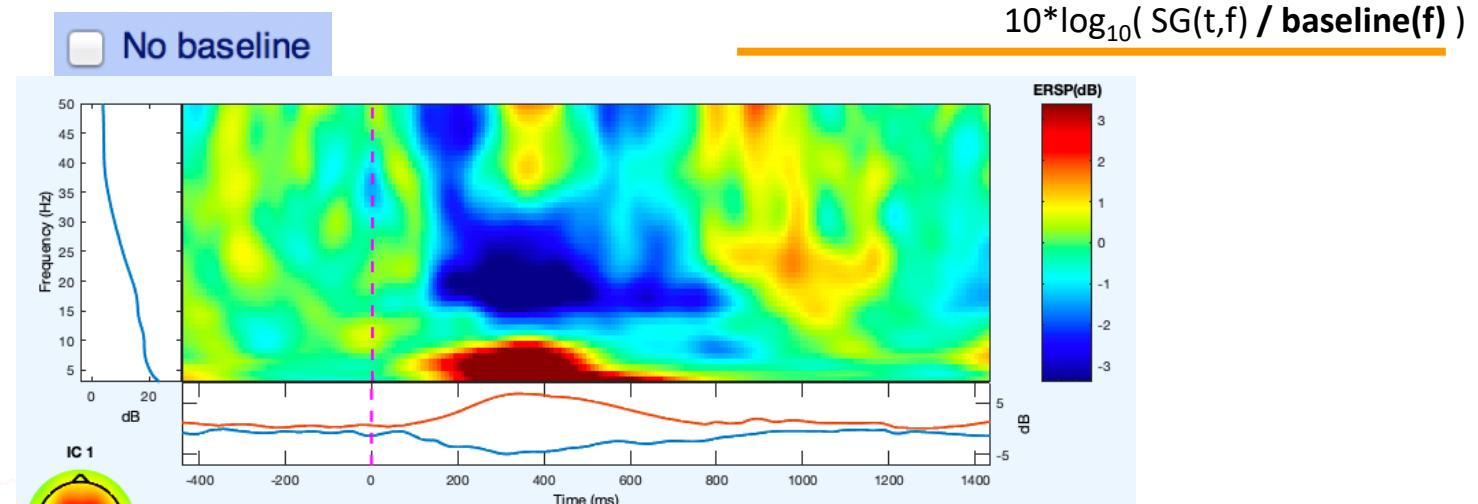




Event-related  
Spectrogram  
 $SG(t,f)$



Event-Related  
Spectral Perturbation  
(ERSP)



# Exercises



- Try different wavelet specifications

Wavelet cycles [min max/fact] or sequence

3 0.8

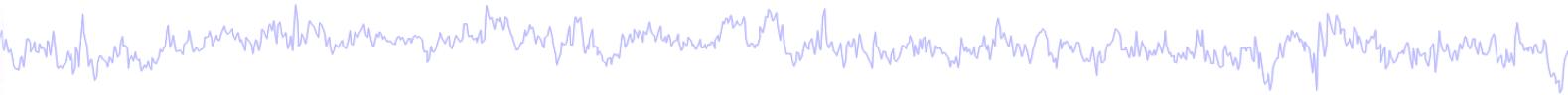
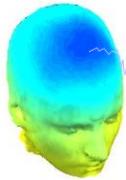
- Default: 3 0.8
    - 3 cycles. Try 2. How do the time limits of the plot change?
    - What is the 0.8? Try 0. Try 1...what do you observe?

- Try different low-frequency limit

Frequency limits [min max] (Hz) or sequence

- what is the effect on the time limits of the ERSP?
- Try different baseline methods
  - divisive
  - standard deviation (express spectral perturbations in #sd relative to baseline sd)

# Wavelet Specification



Wavelet cycles [min max/fact] or sequence

3 0.8

Answer: The first #cycles controls the basic duration of the wavelet in cycles.

The second factor controls the degree of shortening of time windows as frequency increases

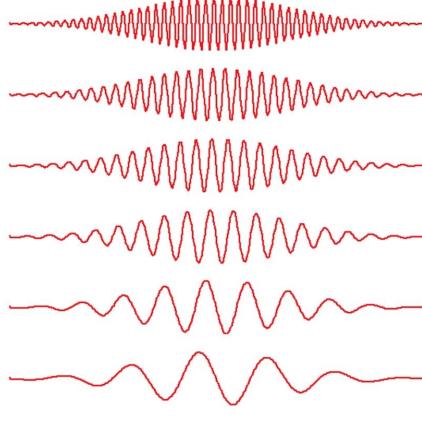
0 = no shortening = FFT (duration remains constant with frequency)

1 = pure wavelet (#cycles remains constant with frequency)

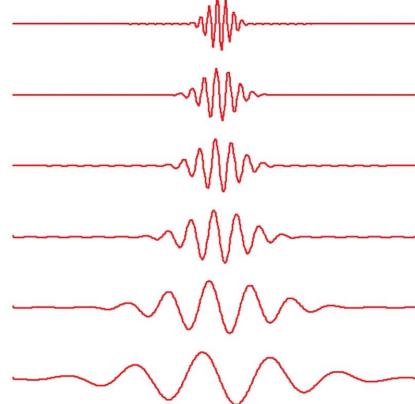
0.5 = *intermediate, a compromise that reduces HF time resolution to gain more frequency resolution.*

0.8 = *EEGLAB default—higher HF time resolution*

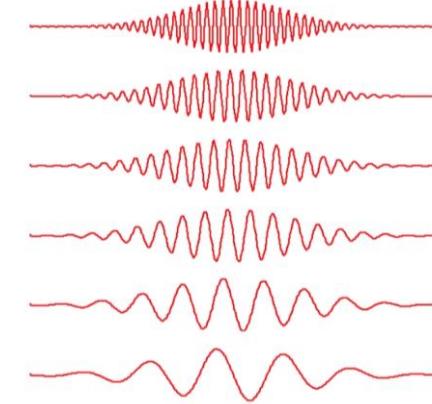
3 0



3 1



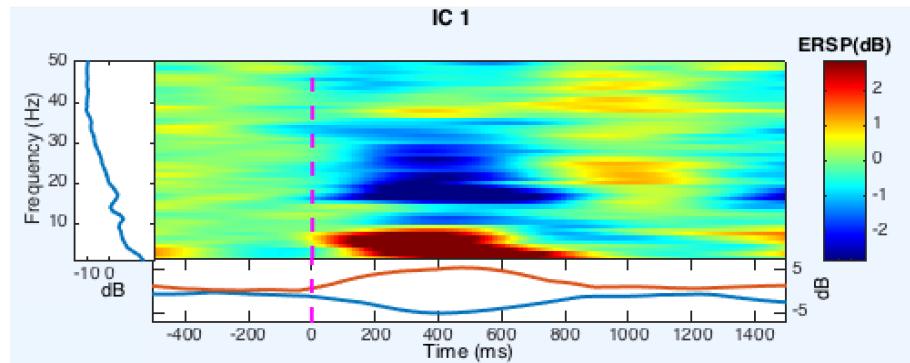
3 0.5



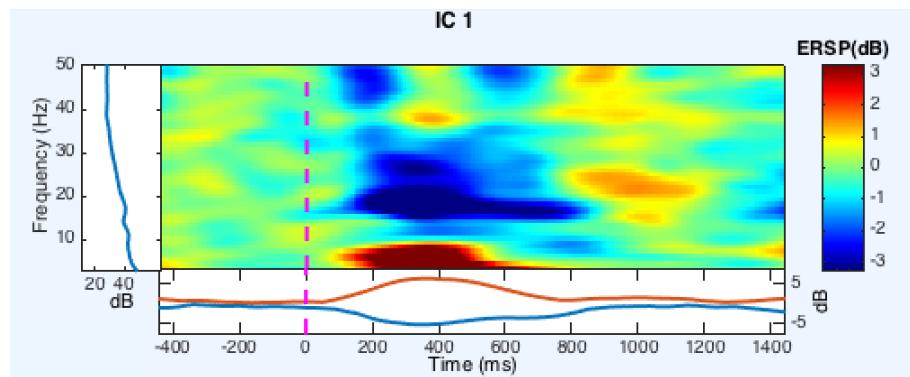
# Comparison of FFT & Wavelet



[3 0] (FFT)



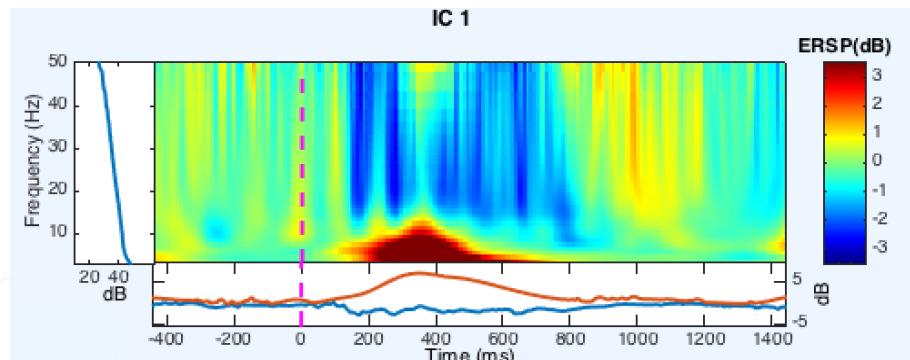
[3 0.5] Wavelet



A reasonable choice:

Notice: features have  
similar time and frequency  
resolution

[3 1] Wavelet

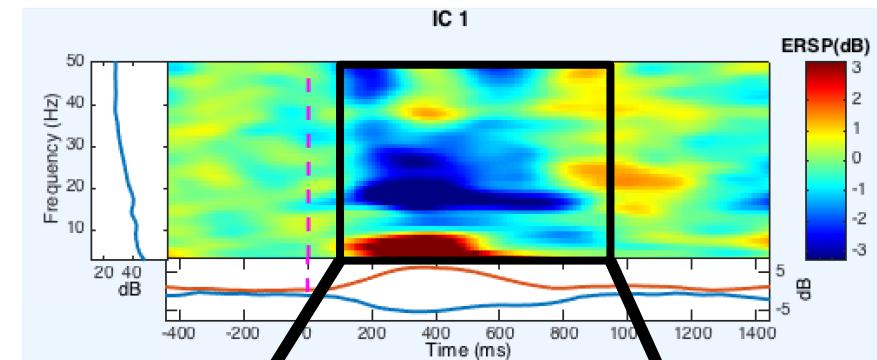


# Time loss at edge of ERSP

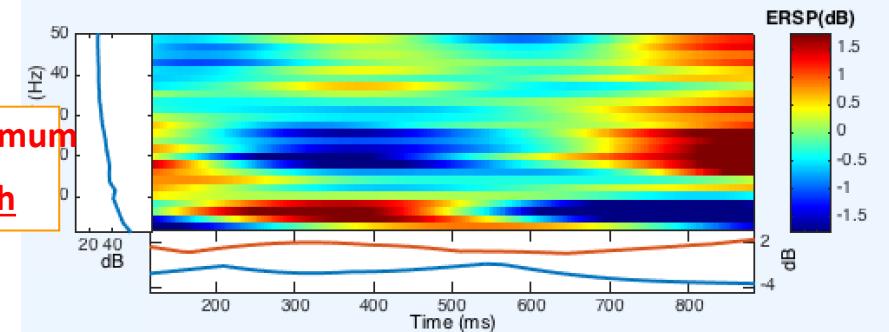


- Settings for 1) *wavelet cycles* and 2) *lowest frequency* impact the time limits of analysis

MIN FREQ: 3 Hz



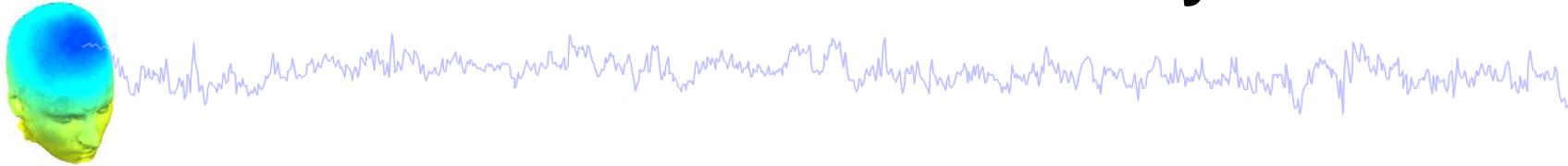
MIN FREQ: 1 Hz



\*more wavelet cycles, or a lower minimum frequency loses time at edges of epoch

**Solution: If you need low frequencies in your ERSP, be sure to extract longer epochs to counteract this. If you can't re-epoch, then try reducing the number of wavelet cycles.**

# Part 3: Coherence Analysis

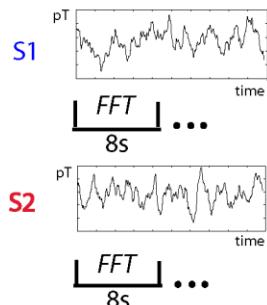
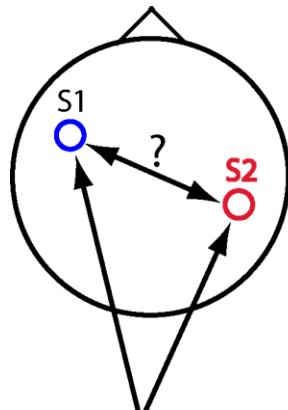


- Goal: How much do two signals resemble each other?
- Coherence = complex version of correlation: how similar are power and phase at each frequency?
- Variant: phase coherence (phase locking, etc.) considers only phase similarity, ignoring power
  - Regular coherence is simply a power-weighted phase coherence
  - Inter-trial coherence is useful!
- NOTE: For **understanding** connectivity between regions, *channel* coherence is a poor choice due to volume conduction. For IC connectivity, directional, 'causal' measures of connectivity have been developed (See SIFT lecture).

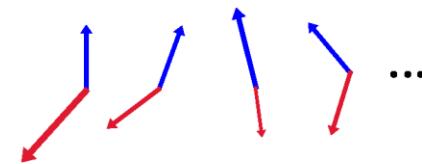
# Coherence



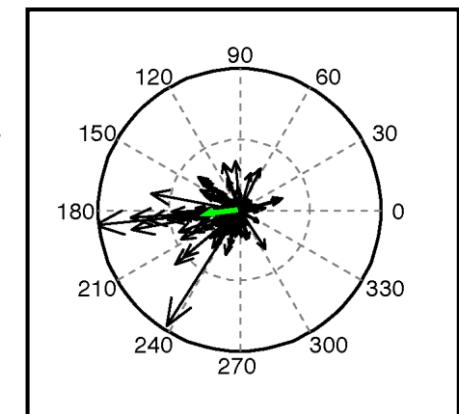
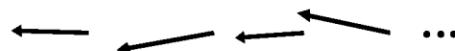
$$C(f, t) \mu \underset{k=trials}{\circ} F1_k(f, t) \overline{F2_k(f, t)}$$
$$a_1 e^{iq_1} a_2 e^{-iq_2} \boxed{\mu e^{i(q_1 - q_2)}}$$



Fourier time series  $F_{S1}$  and  $F_{S2}$

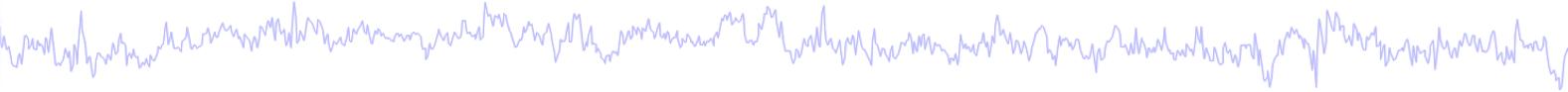


Phase difference between  $S1$  and  $S2$ ,



...

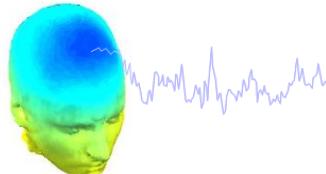
# Part 3a: Inter-Trial Coherence



- Goal: How much do different trials resemble each other?
- Phase coherence not between two processes, but between multiple trials of the same process
- Defined over a (generally) narrow frequency range

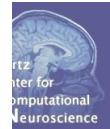
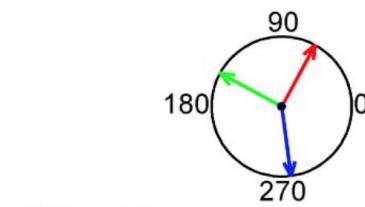
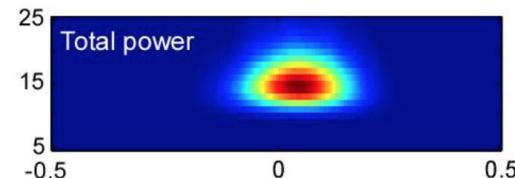
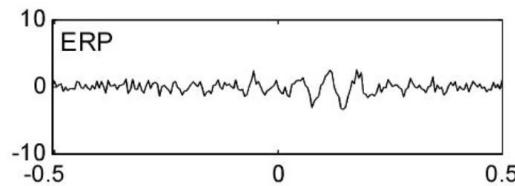
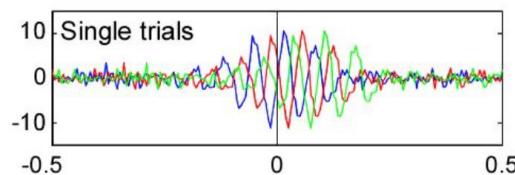


# ITC Example (3 trials)



**Increased power,  
no phase alignment**

Intertrial Coherence (ITC)



**Increased power,  
AND phase alignment**

**small ERP**

**Large ERP**

**'Induced' power**

**Same power**

**Low ITC**

**High ITC**

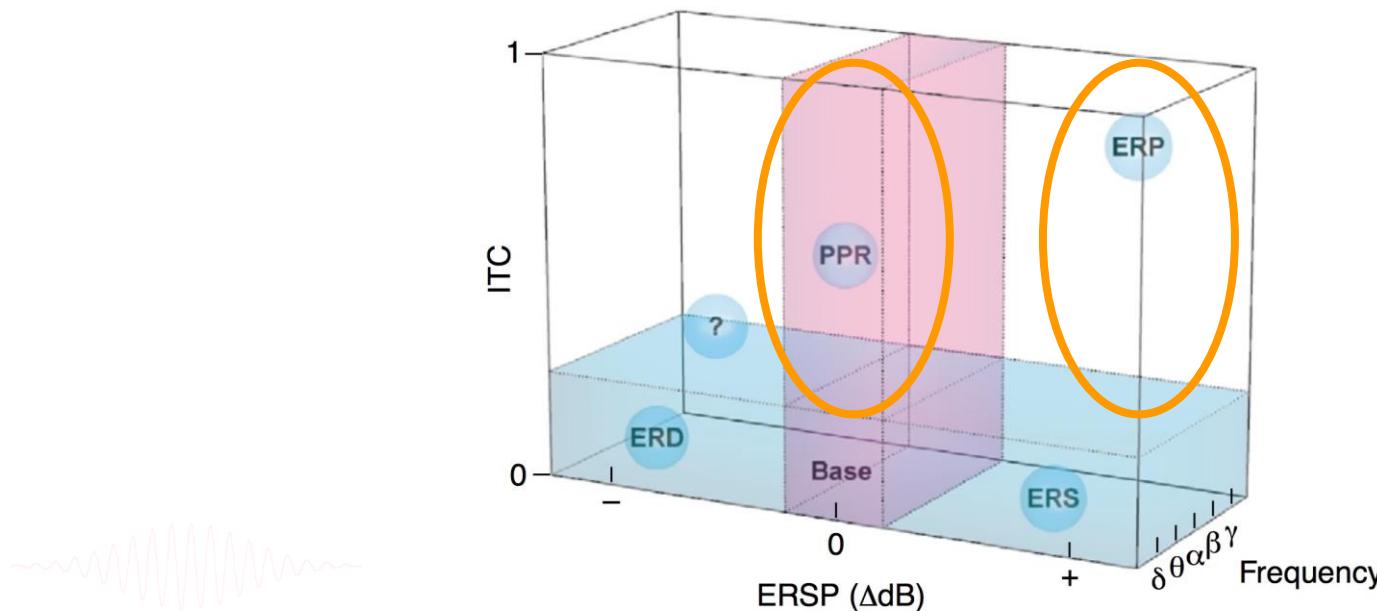


Slide courtesy of Stefan Debener

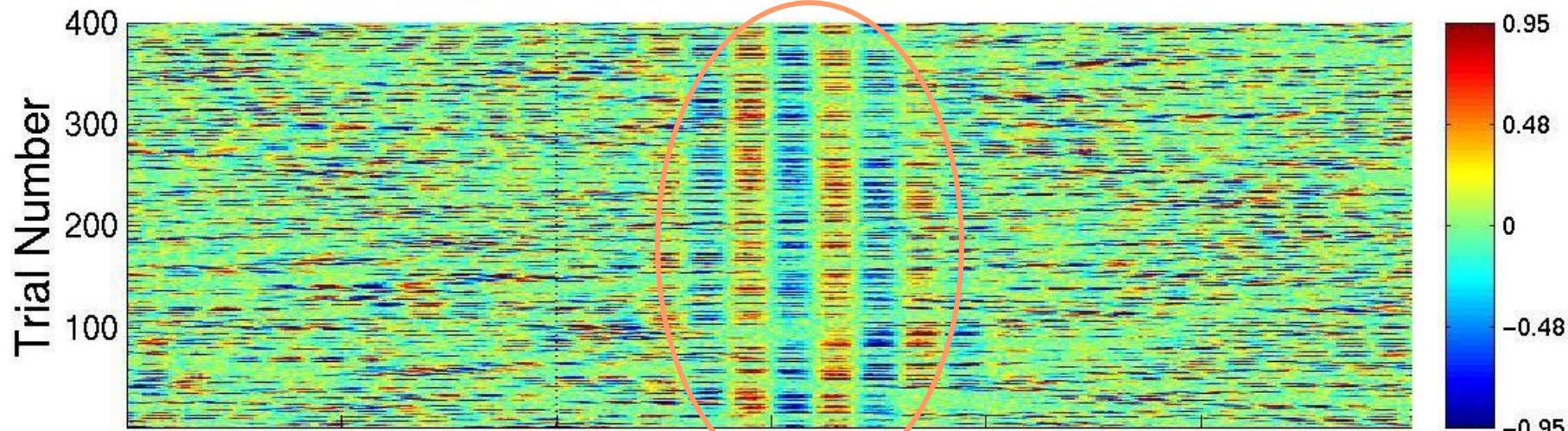
# \*\* Several possible origins of an ERP \*\*



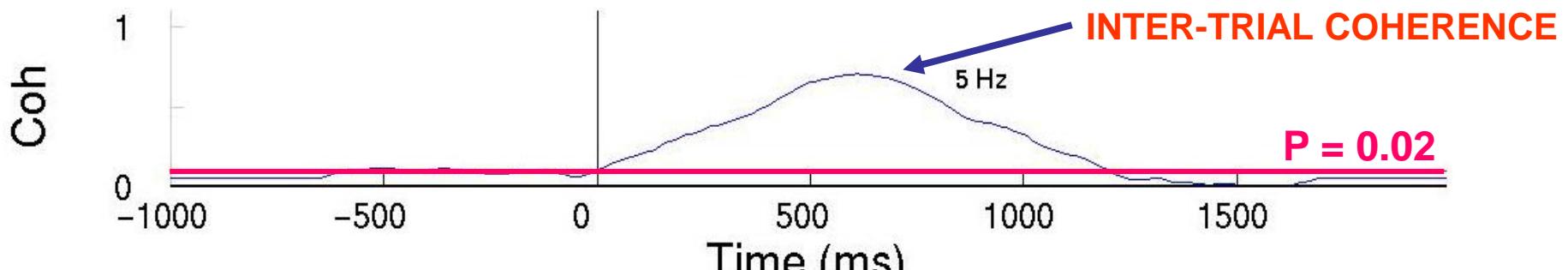
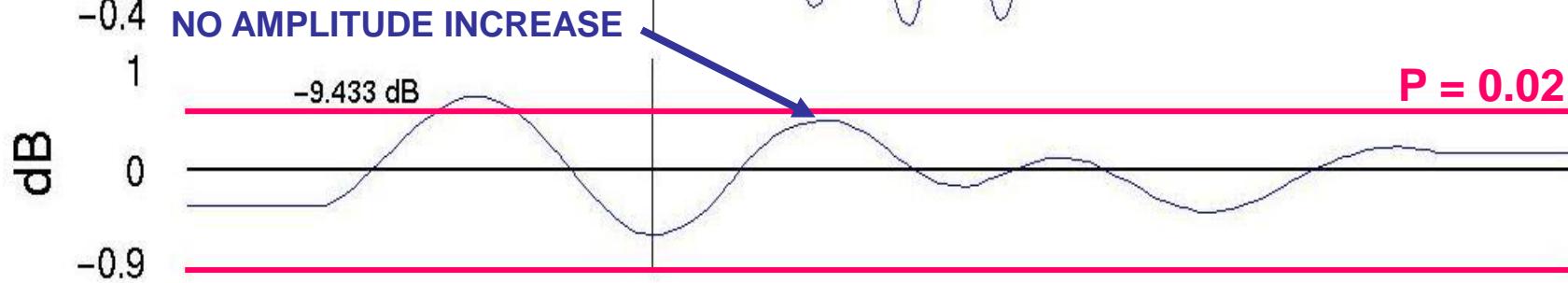
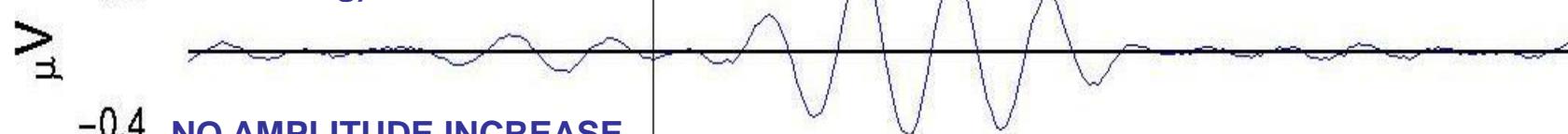
- Event Related Potential can result from
  - ITC increase (with no change in power)
  - ITC & Power change



### ERP-IMAGE PLOT



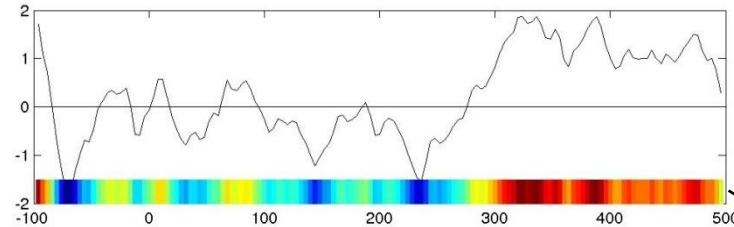
INTER-TRIAL COHERENCE (phase resetting)      AVERAGE ERP      400 SIM. TRIALS ...



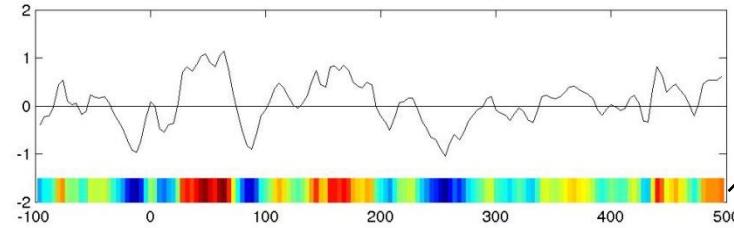
# (ERP Image basics → Johanna Wagner [Wednesday AM] )



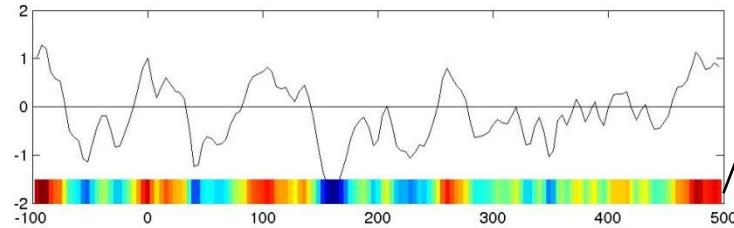
Trial 1



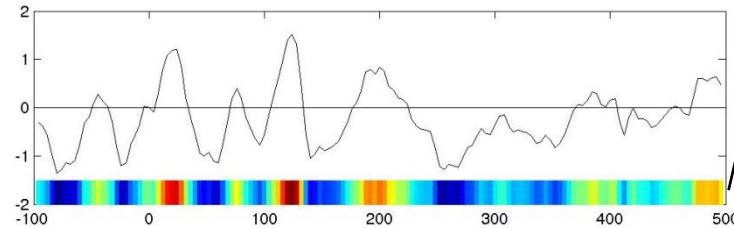
Trial 2



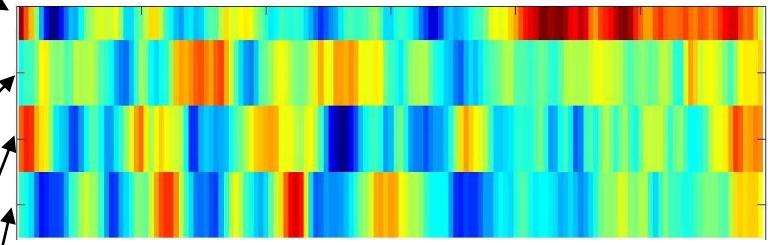
Trial 3



Trial 4

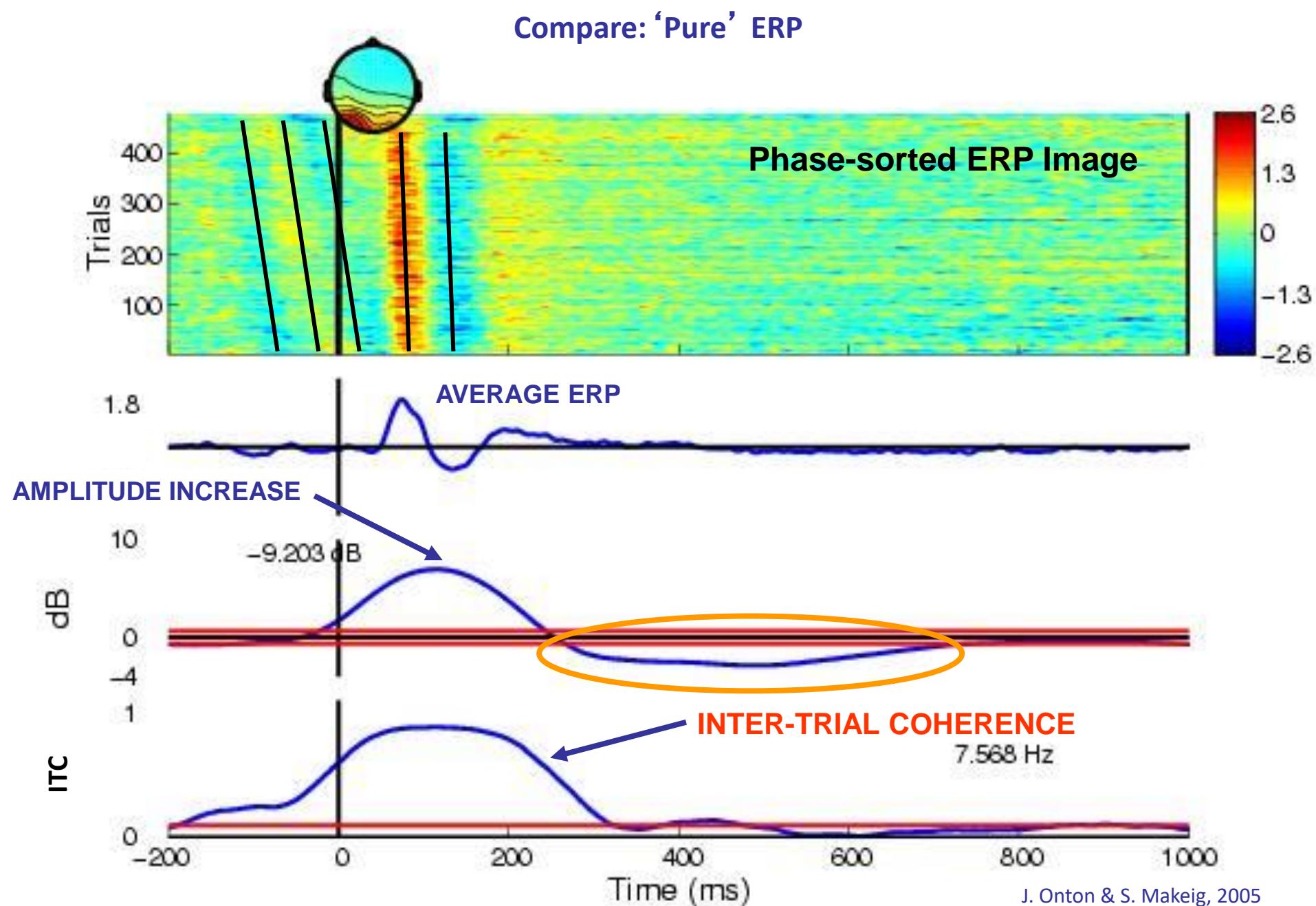


ERP Image

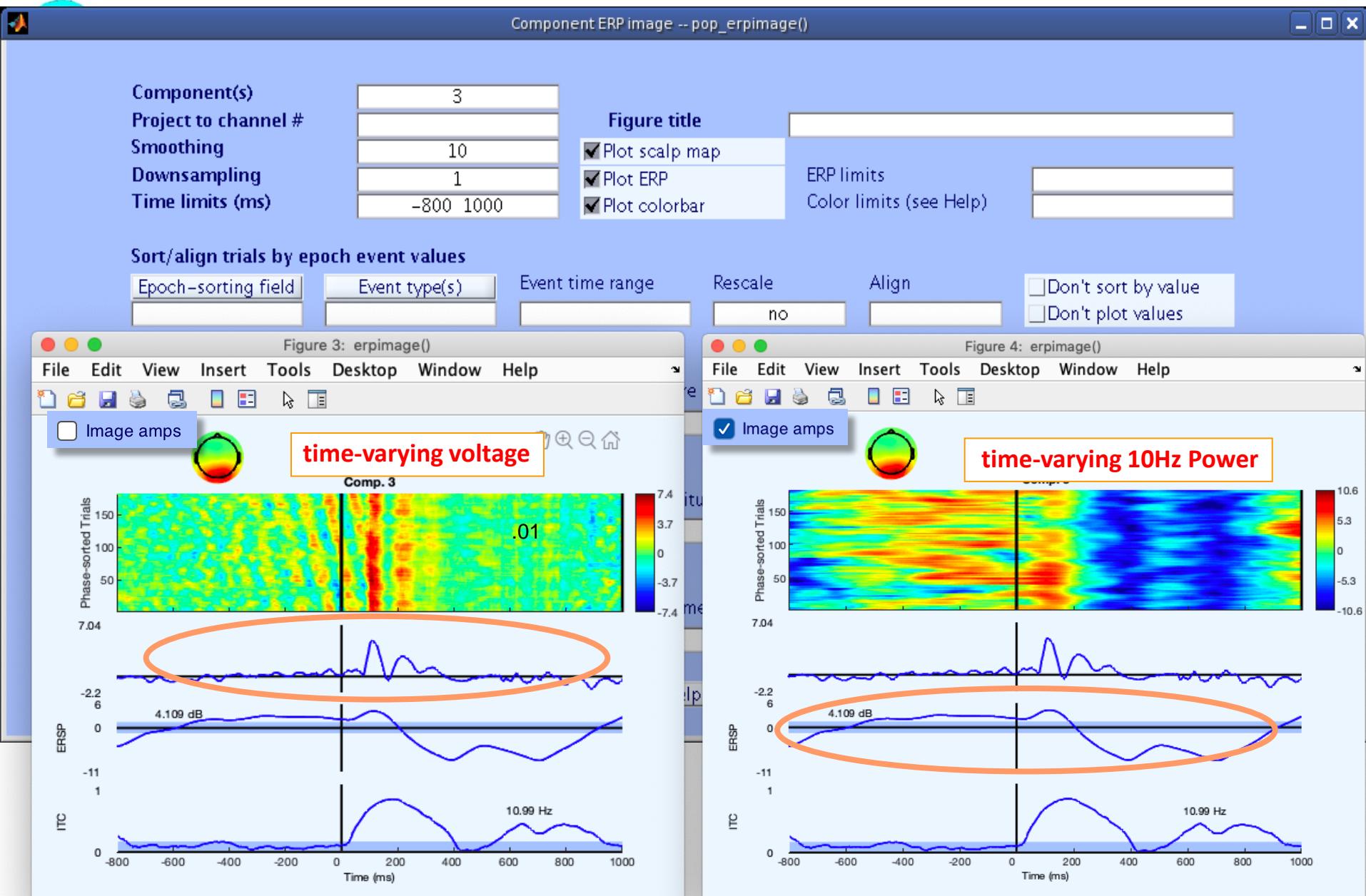


by default, sorted by  
time-on-task  
(1<sup>st</sup> trial, 2<sup>nd</sup> trial, ...)

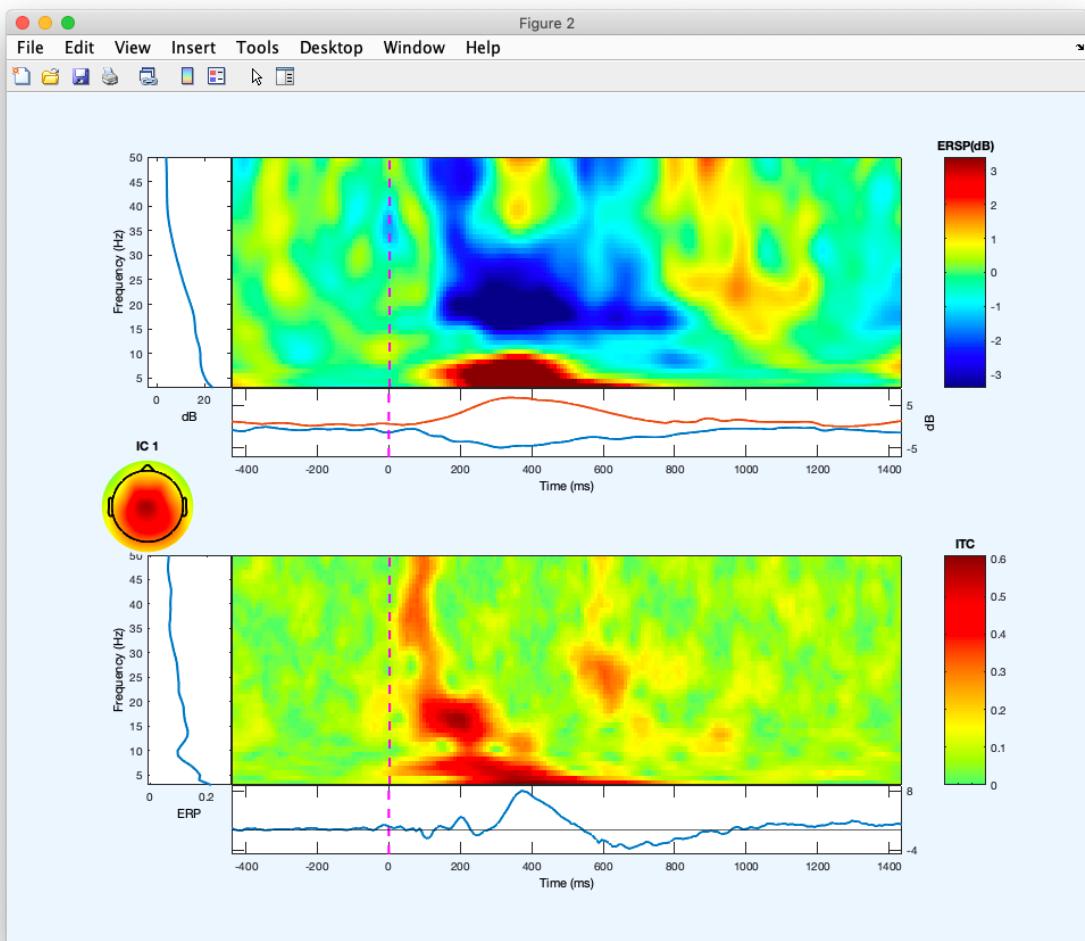
## Compare: 'Pure' ERP



# Component ERP Image: Activation vs. Amplitude



# Putting it all together



## Exercise

All: Compute ERSP/ITC for a component of your choice

Compute ERP Image (with ERSP and ITC displayed\*)

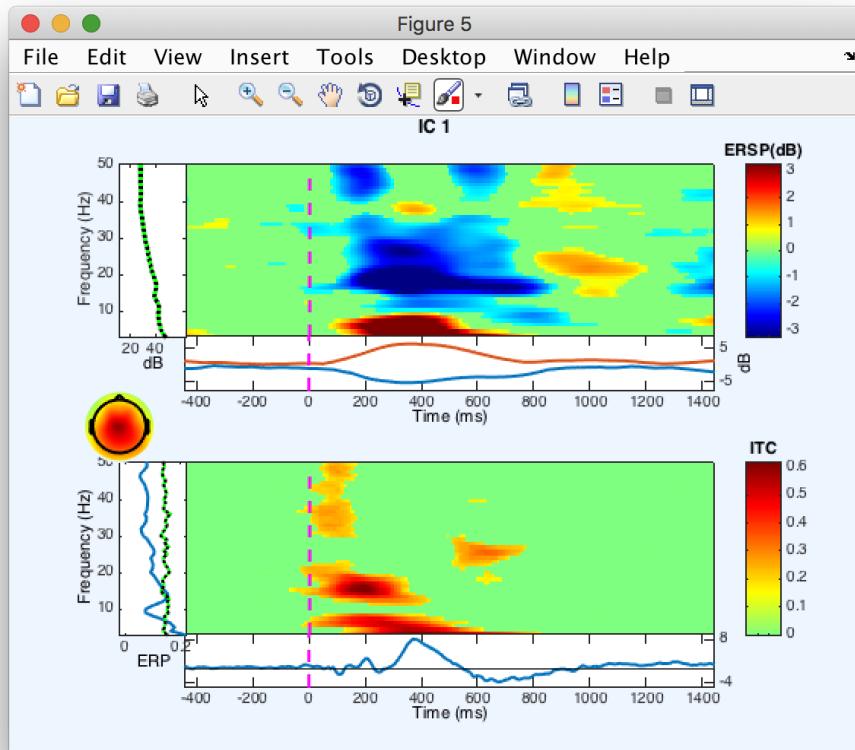
Use all of this information to explain the origin of the Evoked Response

Question: Which changes are significant? Use the options in ERP Image and ERSP dialogs to set significance threshold e.g. 0.01. Do the results survive?

# Significance Testing



- Keep in mind: "is this significant?"



**Method: Bootstrap**

**Green areas are not significant.**

**Scale of ERSP & ITC values also give a clue:**

**Large values are often encouraging of a significant effect**

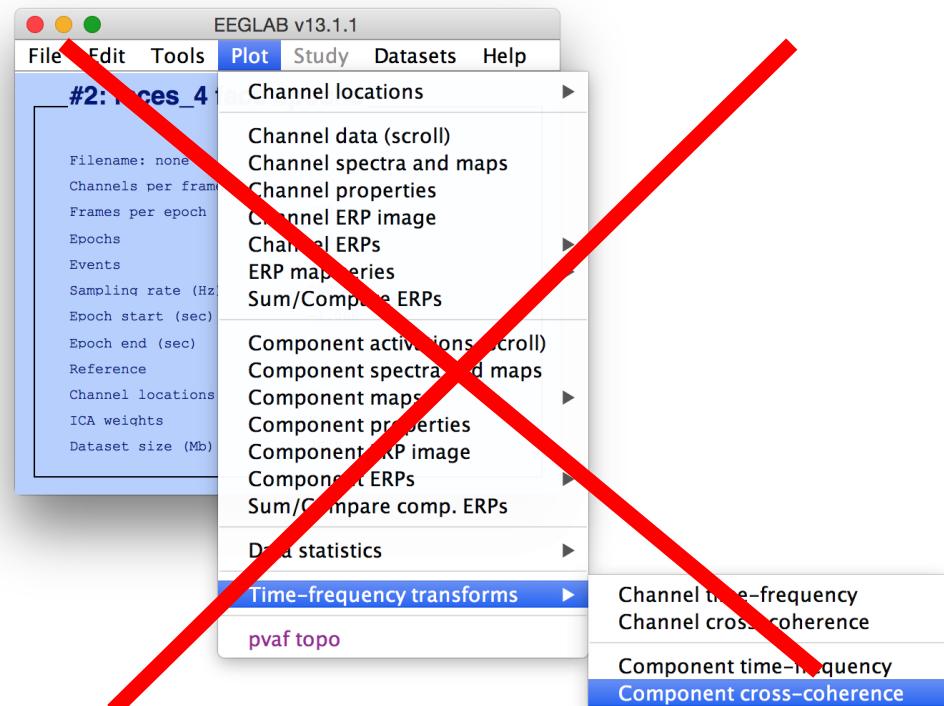
**(Large  $\approx$   $> 1\text{dB}$  for ERSP;  $> 0.5$  for ITC)**

**For exploratory purposes, can try 0.01 without FDR correction**

# Bonus: Part 3b: Event Related Coherence



- Goal: How similar is the event-related response of two signals?
  - Between channels (problematic due to volume conduction)
  - Between ICs
  - Useful to quickly begin to understand relationships between components
  - SIFT provides more complete solution

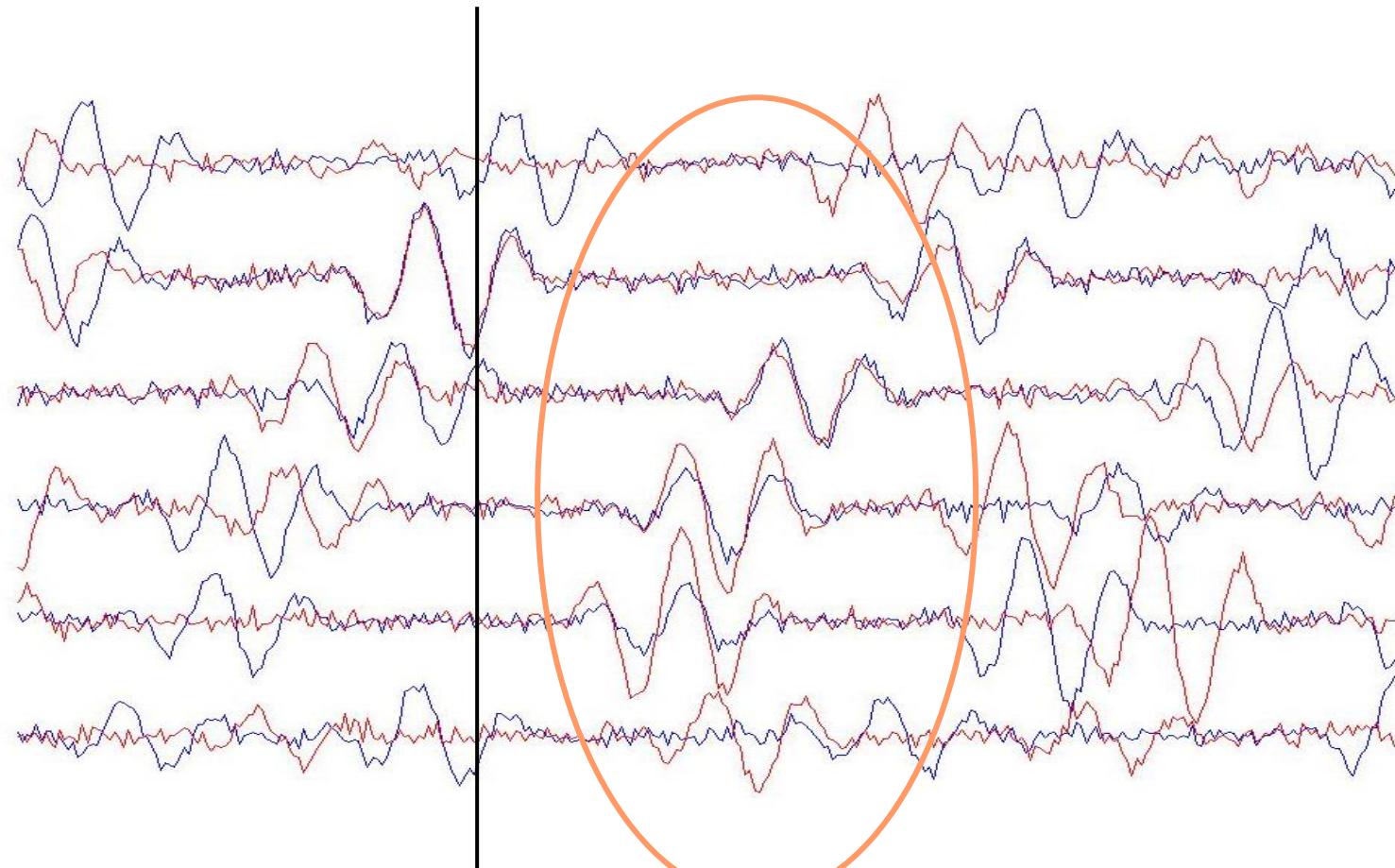


No longer accessible through GUI

`pop_newcrossf(EEG, 0);`



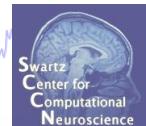
# TWO SIMULATED THETA PROCESSES



**Event-related  
Coherence**

# Try it!

m



Plot component cross-coherence -- pop\_newcrossf()

**First component number**

**Second component number**

**Epoch time range [min max] (msec)**

**Wavelet cycles (0->FFT, see >> help timef)**

**[set]->log. scale for frequencies (match STUDY)**

**[set]->Linear coher / [unset]->Phase coher**

**Bootstrap significance level (Ex: 0.01 -> 1%)**

**Optional timef() arguments (see Help)**  [Help](#)

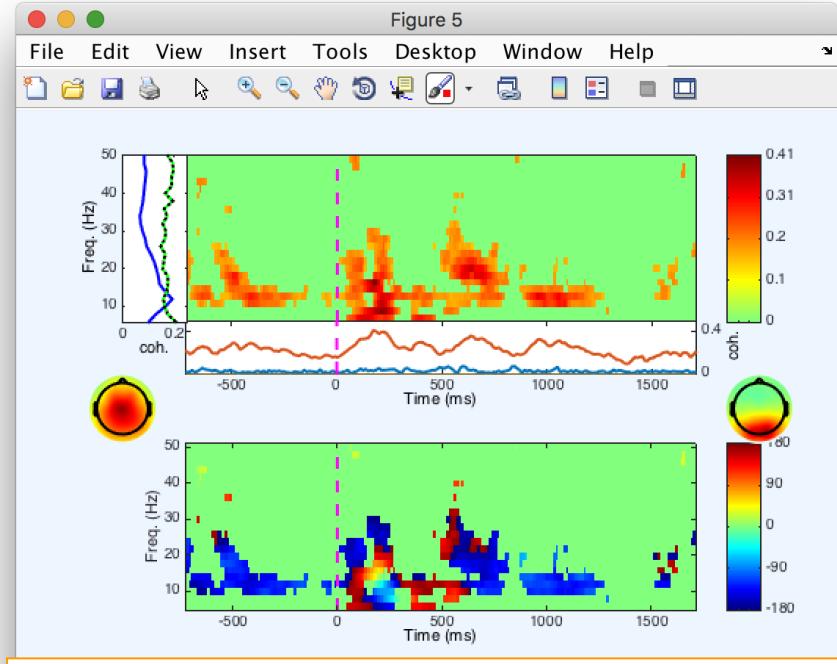
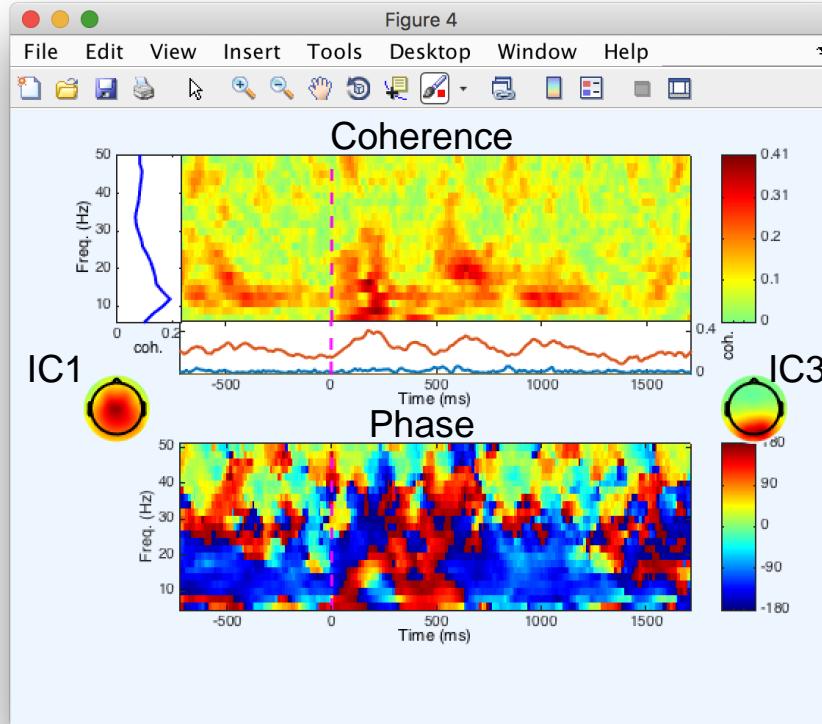
Plot coherence amplitude  Plot coherence phase

[Help](#) [Cancel](#) [Ok](#)

# Cross coherence between IC 1 and IC 3



$$\alpha = 0.01$$

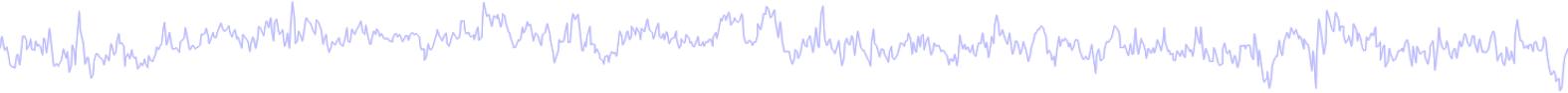


Significant event-related coherence (as well as tonic coherence) in alpha/beta bands

IC 1 tonically leads IC 3 (negative phase), but phase relationships are changed post-stimulus

More advanced, directional, measures of effective connectivity are present in the SIFT toolbox (a later lecture).

# Event-Related Coherence Exercise



- Examine event-related coherence between two ICs
  - Which pair did you pick, and why? What do you predict?
  - What did you learn?
- Explore other options:
  - Significance threshold
  - Figure out how to subtract a baseline
  - Phase vs. Linear Coherence



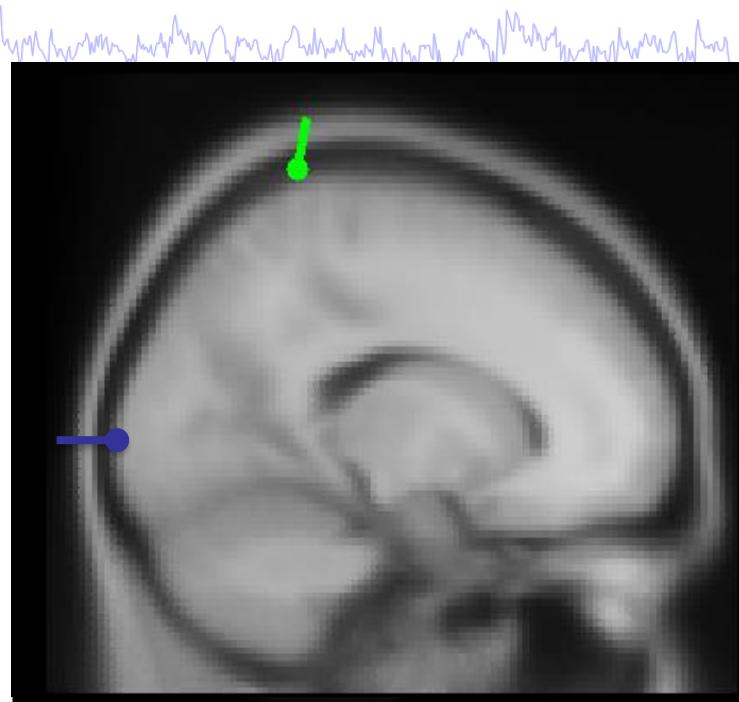
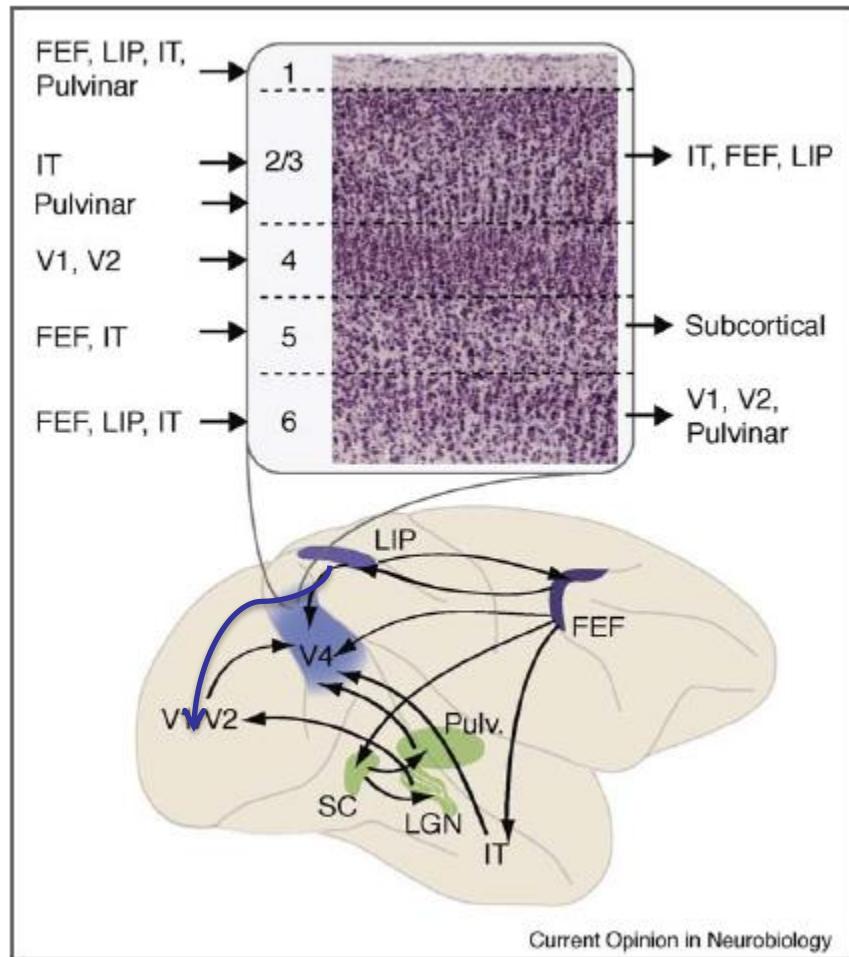
# Part 4: Other Applications



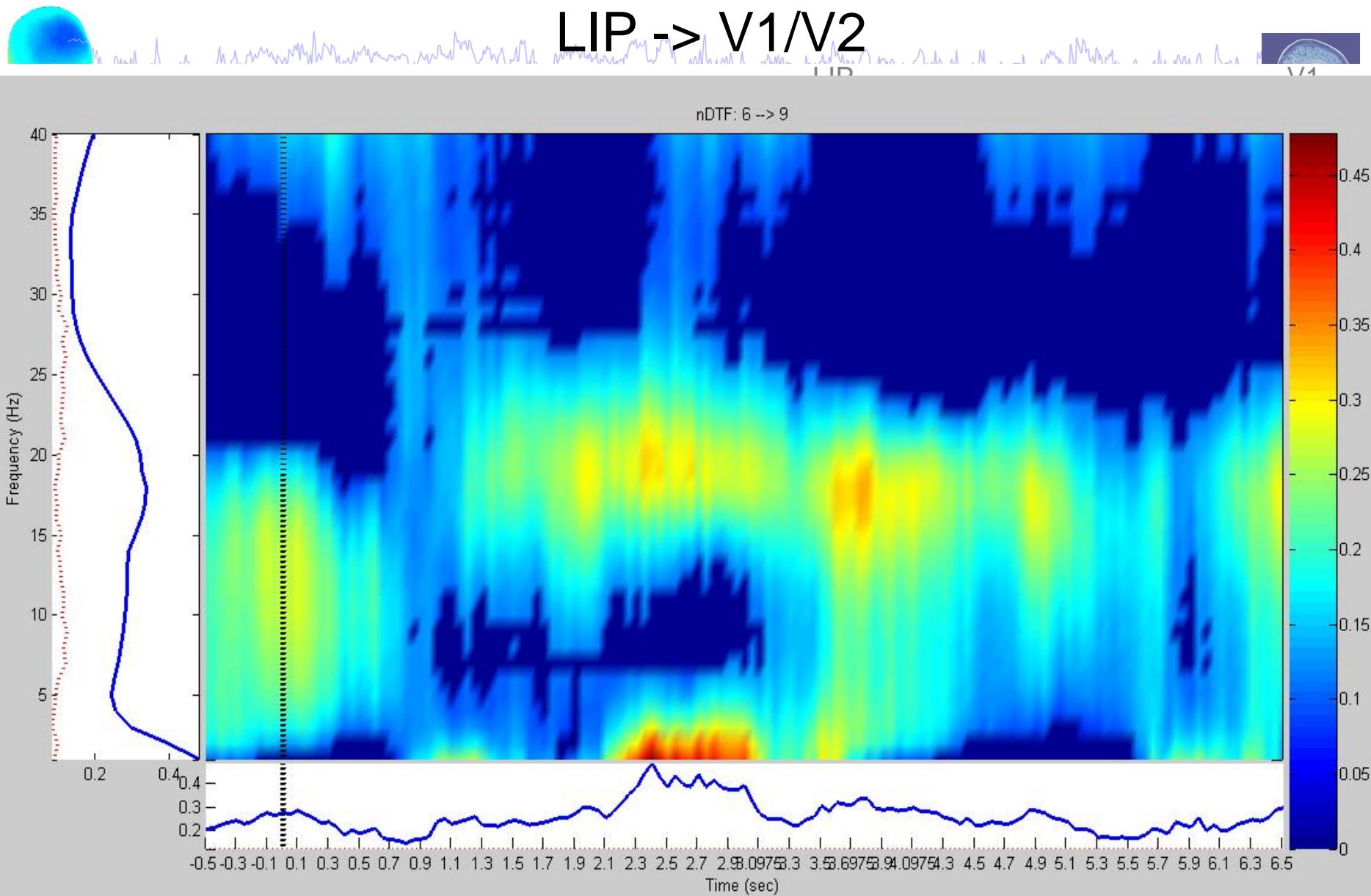
- Information Flow: Autoregressive modeling → time/frequency resolved directed information flow
  - **SIFT** – Tim Mullen [Tomorrow, Connectivity Analysis Track]
- Cross-frequency Analysis
  - **Phase/amplitude coupling (PAC)** - Ramón Martínez-Cancino [Right after this talk!]



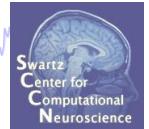
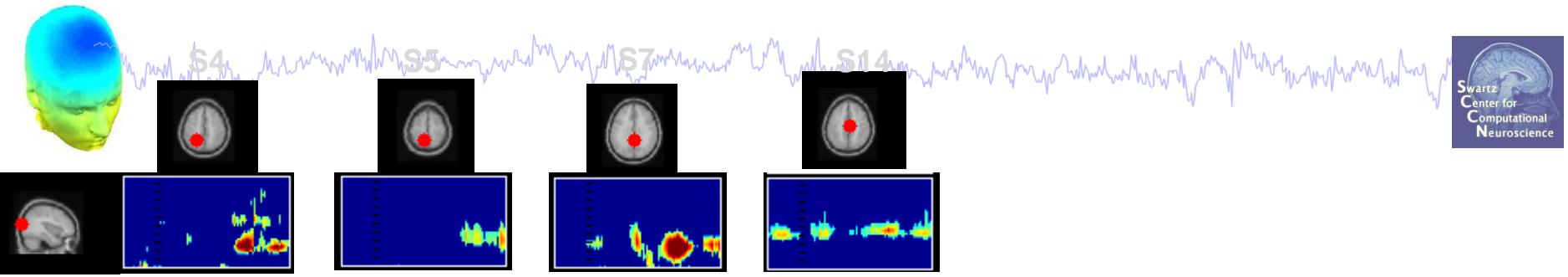
# Aktualne badania



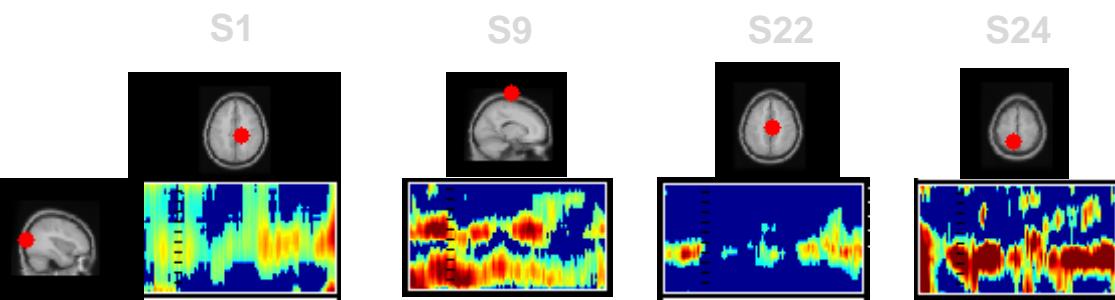
# Aktualne badania: matoda



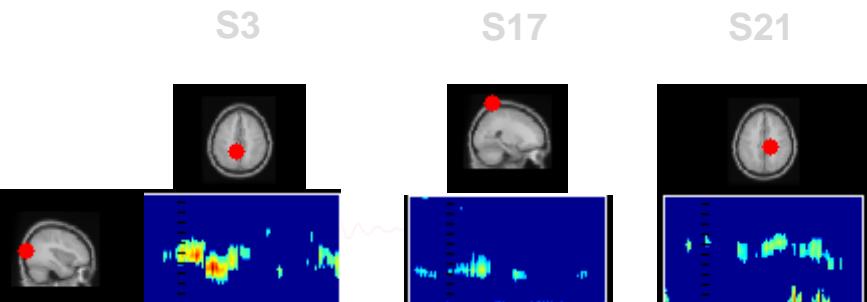
## Young



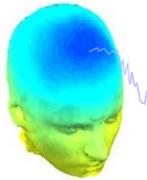
## EHP



## ELP



# PRACTICUM



- Follow the red bordered slides, using the faces\_4.set, epoched on the 'face' event.

