Forward and Inverse Models - The DIPFIT tools

Robert Oostenveld

Donders Institute, Radboud University, Nijmegen, NL
NatMEG, Karolinska Institute, Stockholm, SE

Overview

- Motivation and background
- Forward modeling
  - Source model
  - Volume conductor model
- Inverse modeling - biophysical models
  - Single and multiple dipole fitting
  - Distributed source models
  - Beamforming methods
- Summary

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Motivation 1

Strong points of EEG and MEG

- Temporal resolution (~1 ms)
- Characterize ERP components, like N100 or P300
- Oscillatory activity
- Disentangle dynamics of cortical networks

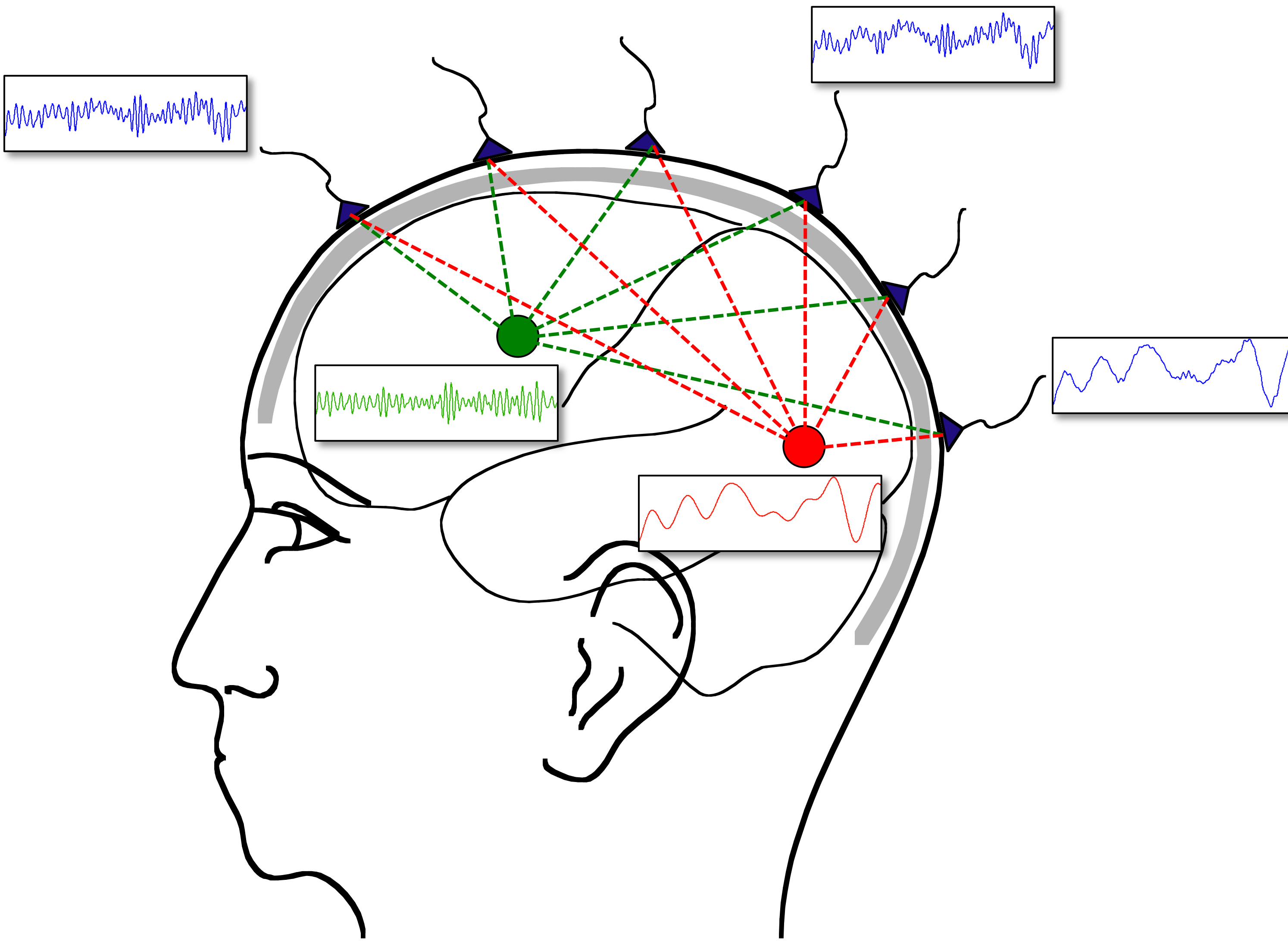
Weak points of EEG and MEG

- Measurement on outside of brain
- Overlap of ERP components
- Low spatial resolution

Motivation 2

- If you find a ERP component like the N100, you want to characterize it in physiological terms
  - Time and amplitude or frequency are the “natural” characteristics
  - “Location” requires interpretation of the scalp topography
- Forward and inverse modeling helps to interpret the topography
- Forward and inverse modeling helps to disentangle overlapping source timeseries

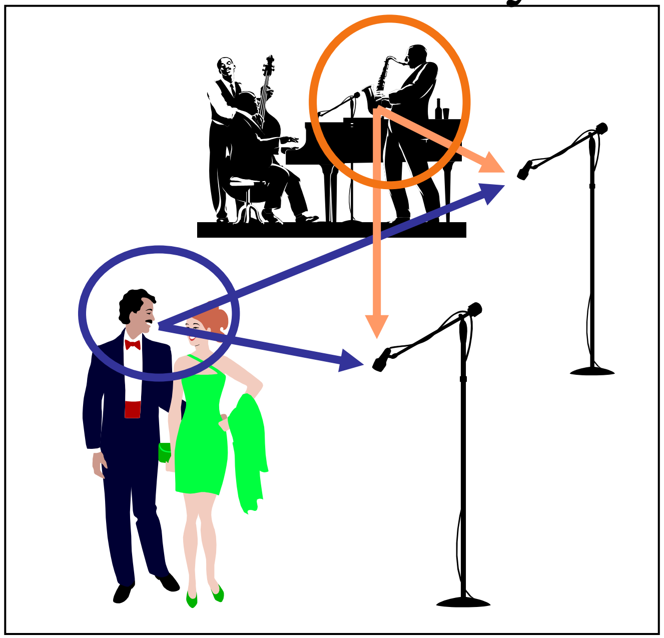
Superposition of source activity

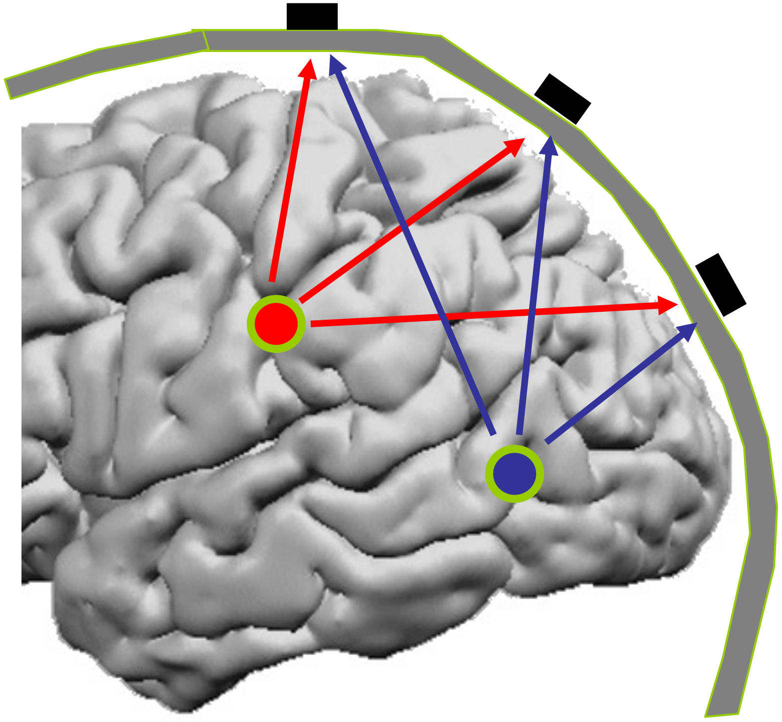


Superposition of source activity

Mixture of Brain source activity

Cocktail Party

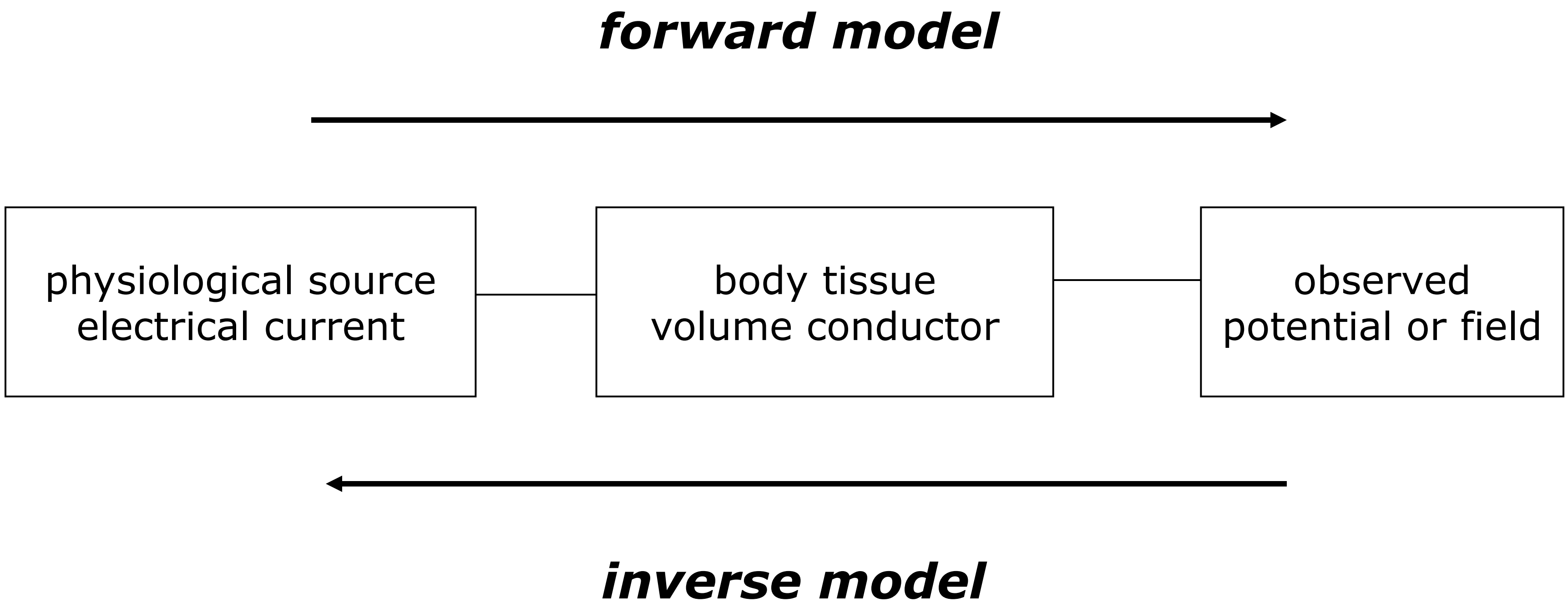




Different source analysis methods

- Blind source separation, such as ICA
  - Assumption on temporal independence
  - Spatially stationary over time
- Biophysical source models
  - Assumption on geometry and conductivity
  - Maxwell equations for electromagnetism
- Not mutually exclusive, can be applied in succession

Biophysical source modelling: overview



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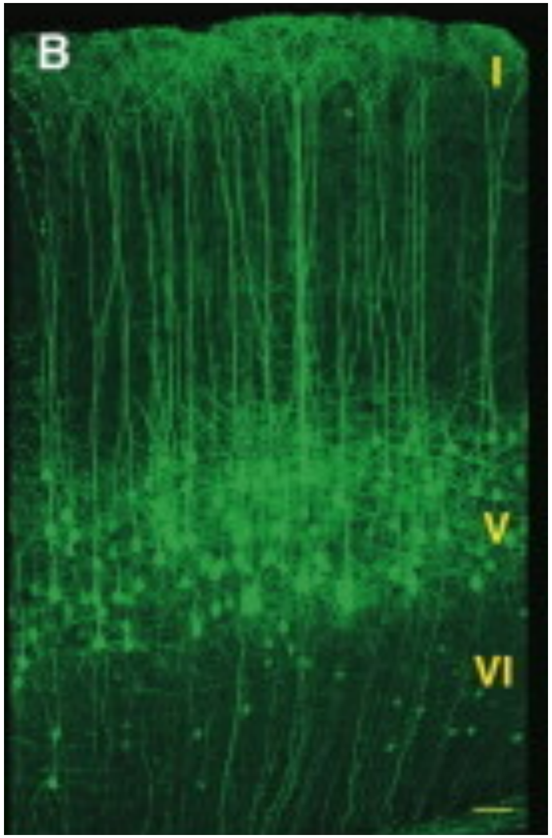
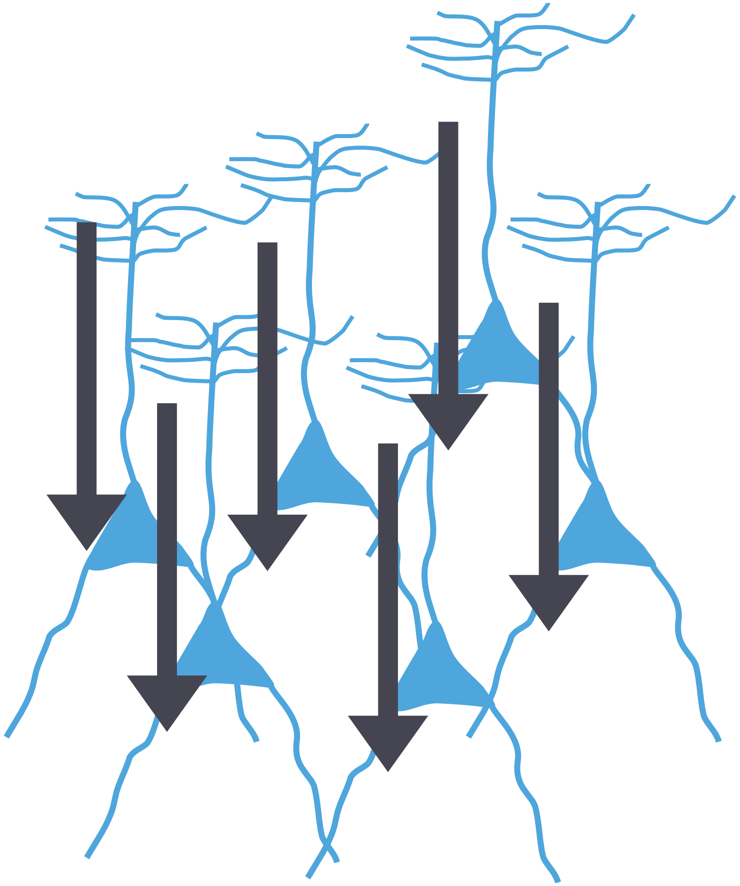
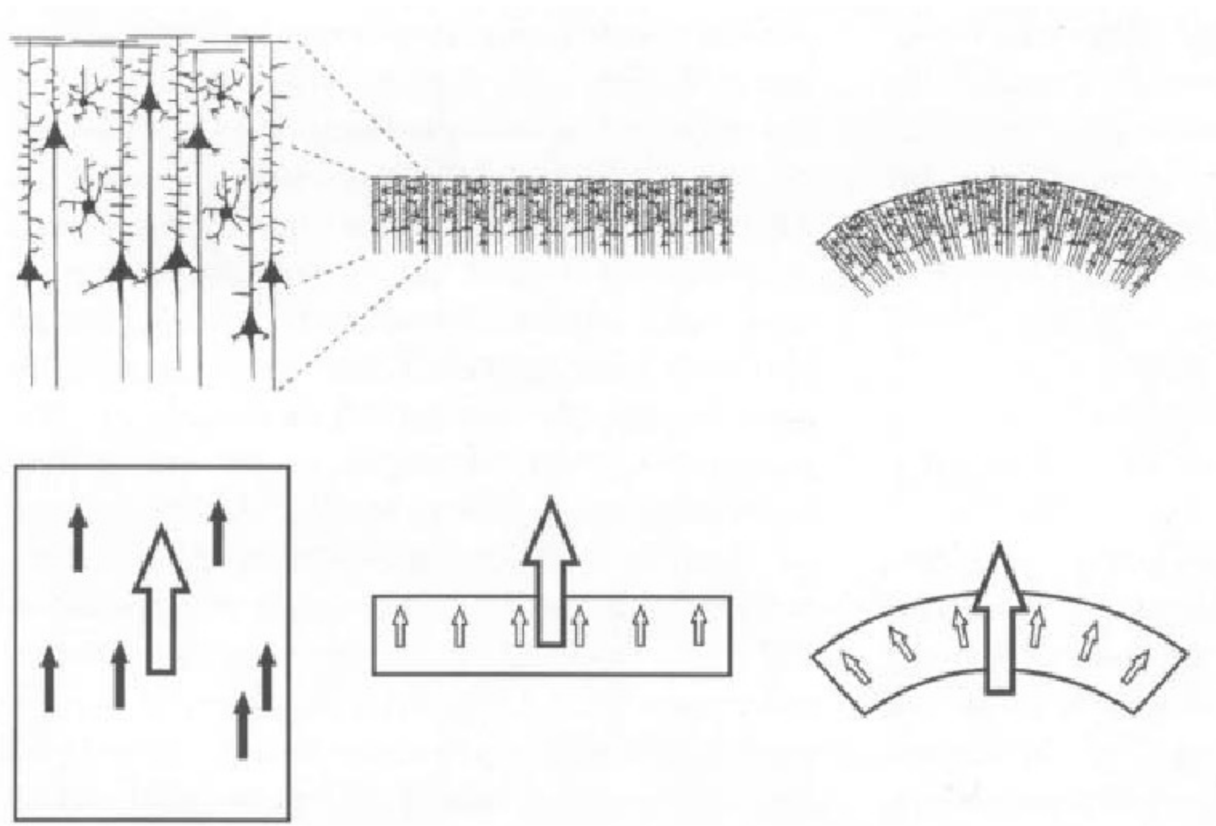
What produces the electric current





Equivalent current dipoles



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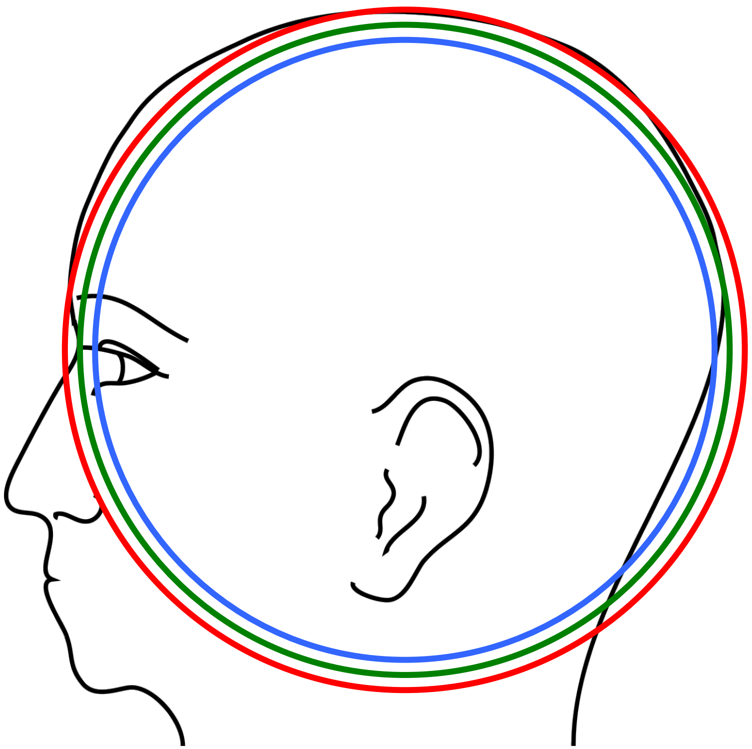
Volume conductor

described electrical properties of tissue

describes geometrical model of the head

describes **how** the currents flow, not where they originate from

same volume conductor for EEG as for MEG, but also for tDCS, tACS, TMS, …



Volume conductor

Computational methods for volume conduction problem that allow for realistic geometries

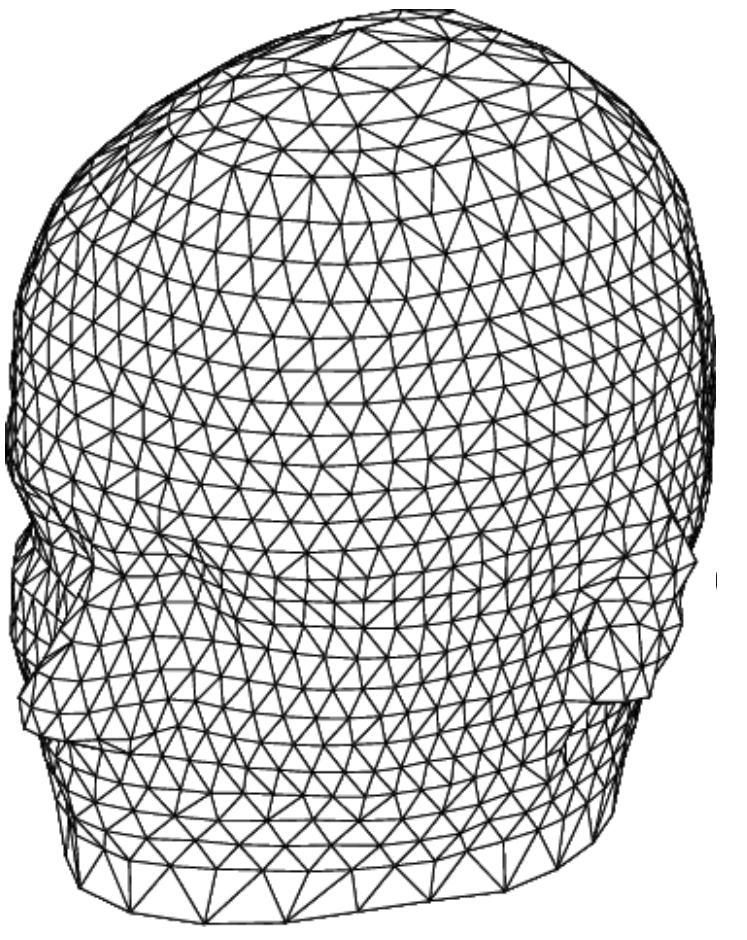
BEM	*Boundary Element Method*
FEM	*Finite Element Method*
FDM	*Finite Difference Method*

Volume conductor: Boundary Element Method

Each compartment is
- homogenous
- isotropic

Important tissues
- skin
- skull
- brain
- (CSF)

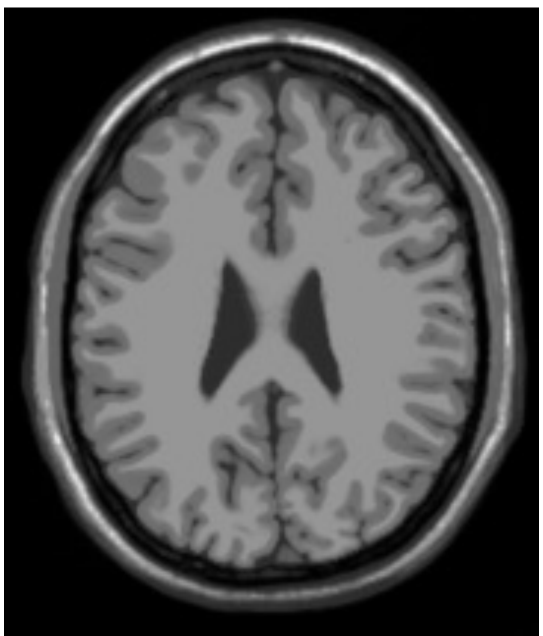
Triangulated surfaces describe boundaries

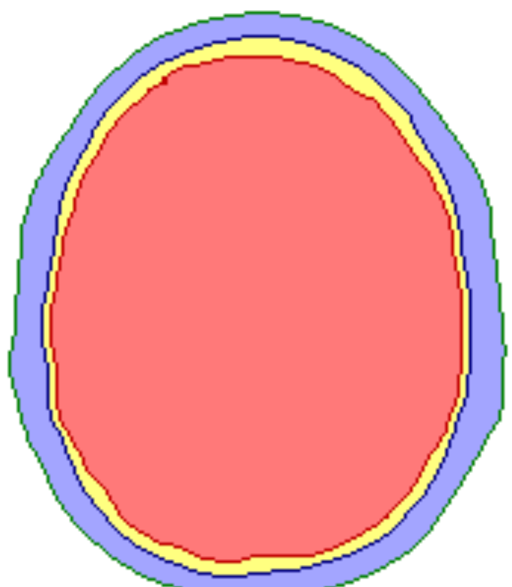


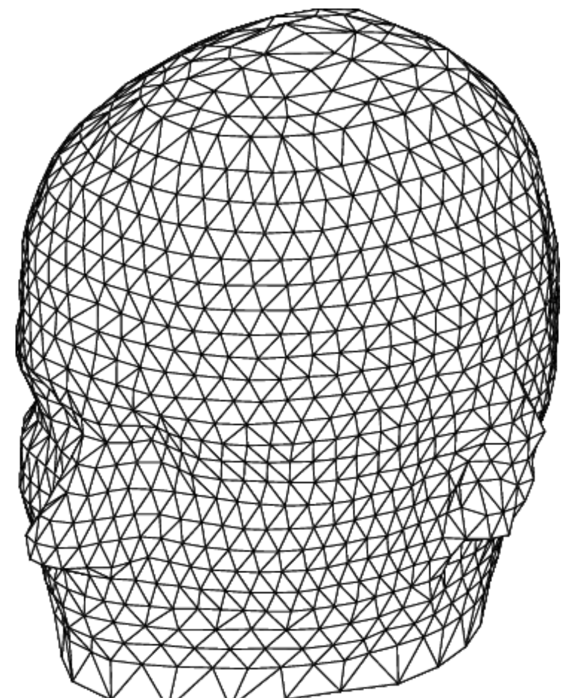
Volume conductor: Boundary Element Method

Construction of geometry from anatomical MRI

- segmentation in different tissue types
- extract surface description
- downsample to reasonable number of triangles





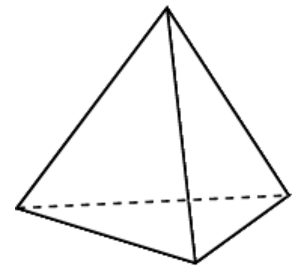


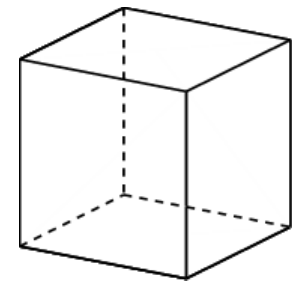
Volume conductor: Boundary Element Method

- Construction of geometry
  - segmentation in different tissue types
  - extract surface description
  - downsample to reasonable number of triangles
- Computation of model
  - independent of source model
  - only one lengthy computation
  - fast during application to real data
- Can also include more complex geometrical details
  - ventricles
  - holes in skull

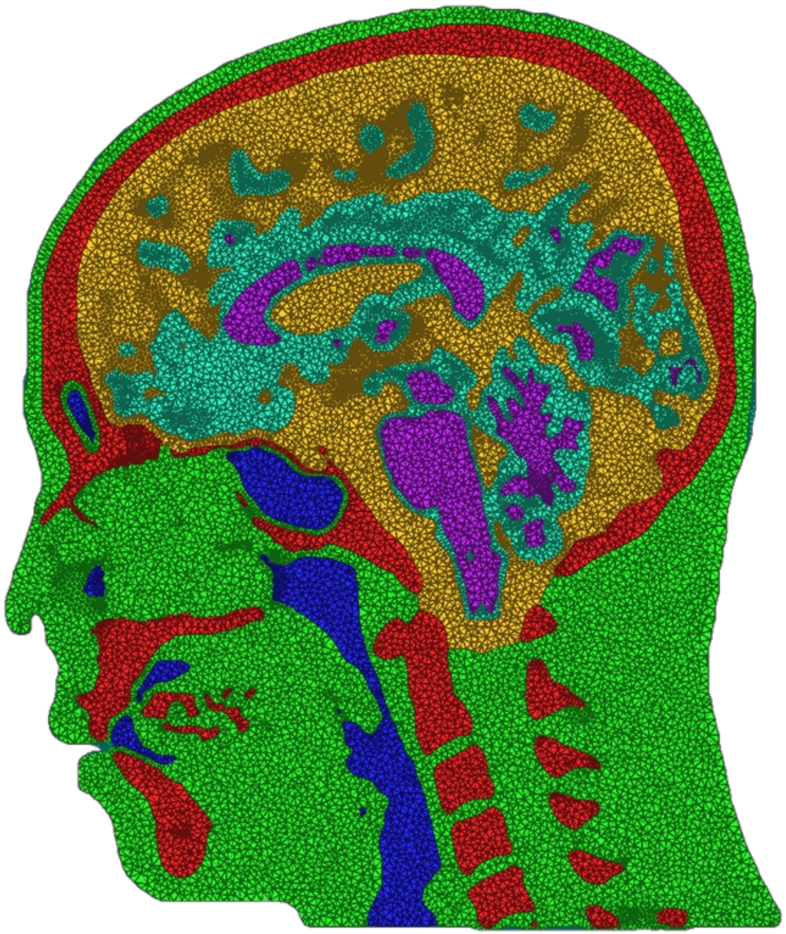
Volume conductor: Finite Element Method

Tesselation of 3D volume in
tetraeders or hexaheders

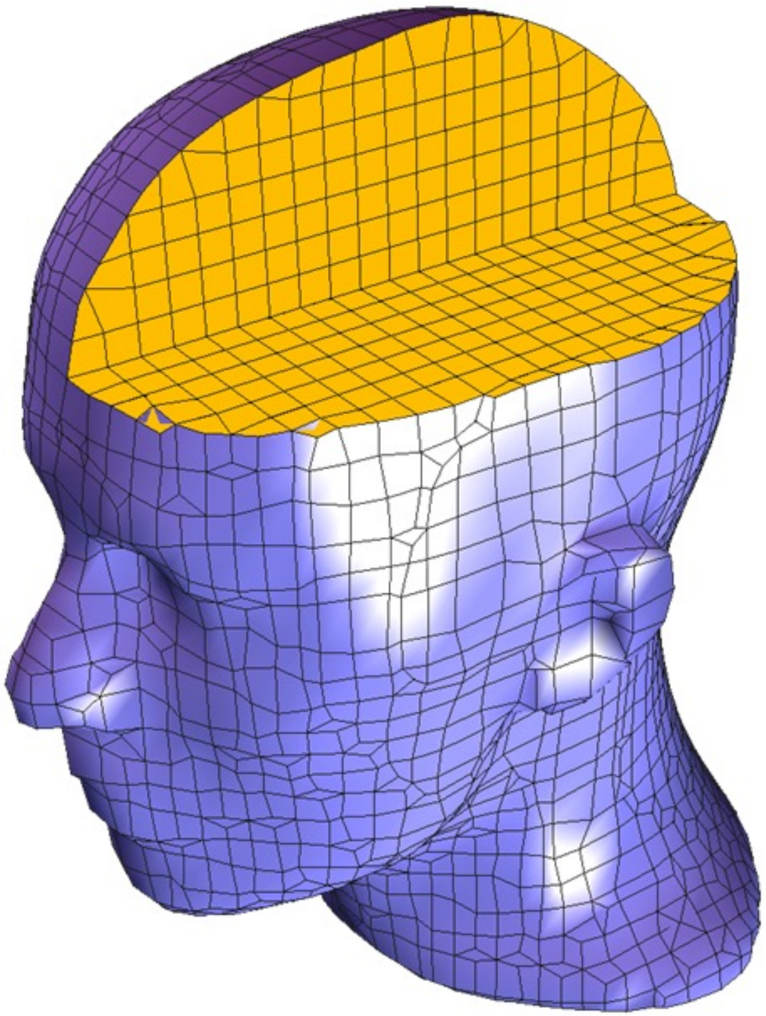




Volume conductor: Finite Element Method



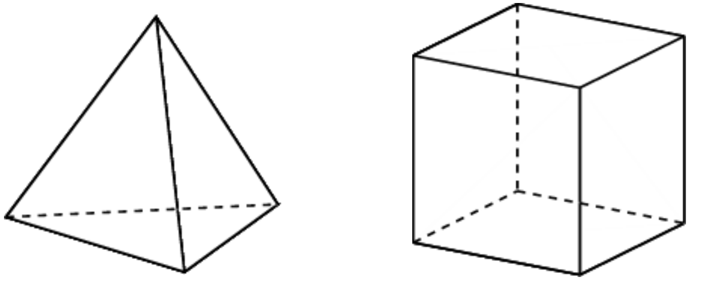
tetraeders



hexaheders

Volume conductor: Finite Element Method

Tesselation of 3D volume in tetraeders or hexaheders



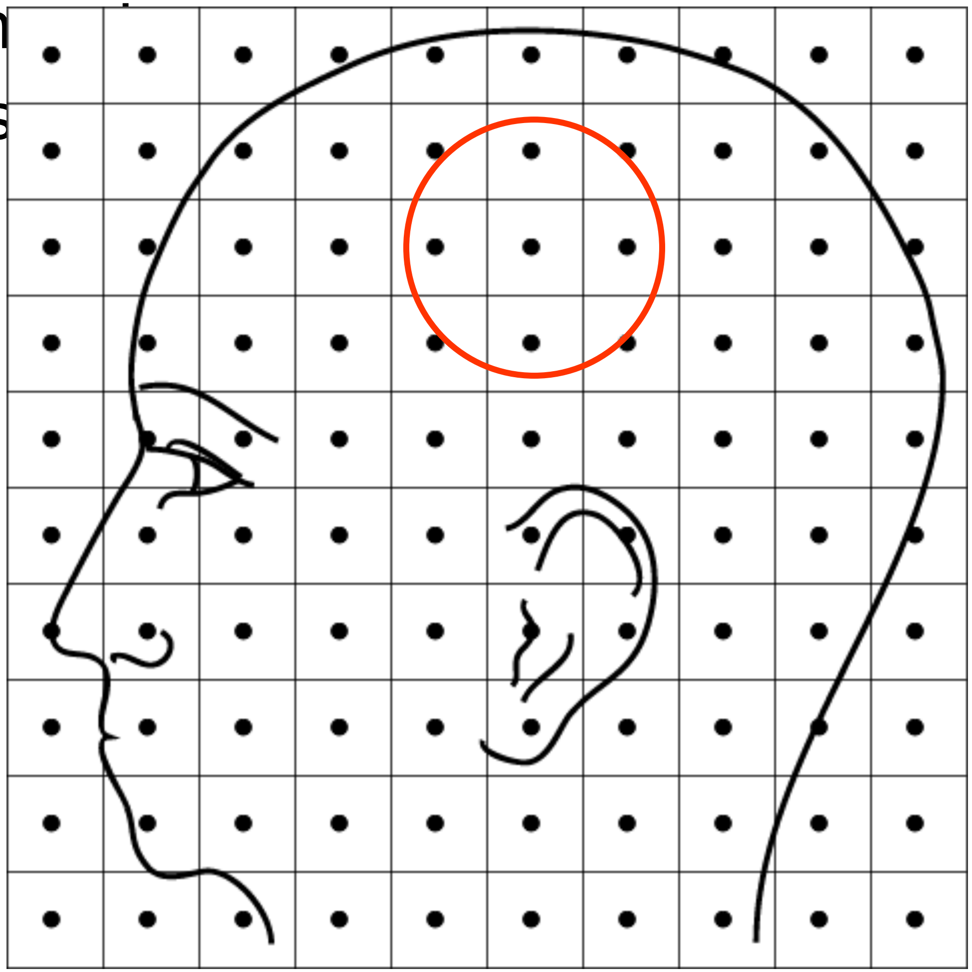
Each element can have its own conductivity

FEM is the most accurate numerical method but computationally quite expensive

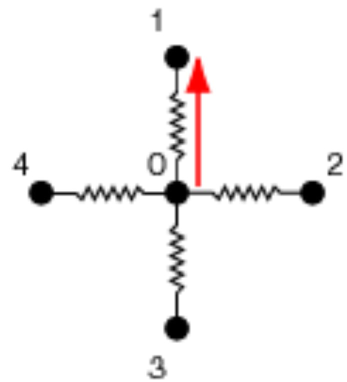
Geometrical processing not as simple as BEM

Volume conductor: Finite Difference Method

- Easy to con
- Not very us



Volume conductor: Finite Difference Method



$$\left.\begin{array}{l} I_1 + I_2 + I_3 + I_4 = 0 \\ V = I*R \end{array}\right\} \Longrightarrow$$

$$\Delta V_1/R_1 + \Delta V_2\,/R_2 + \Delta V_3\,/R_3 + \Delta V_4\,/R_4 = 0 \quad \Longrightarrow$$

$$(V_1\text{-}V_0)/R_1 + (V_2\text{-}V_0)/R_2 + (V_3\text{-}V_0)/R_3 + (V_4\text{-}V_0)/R_4 = 0$$

Volume conductor: Finite Difference Method

Unknown potential Vi at each node

approx. 100x100x100 = 1.000.000 unknowns

Linear equation for each node

approx. 100x100x100 = 1.000.000 linear equations

Add a source/sink

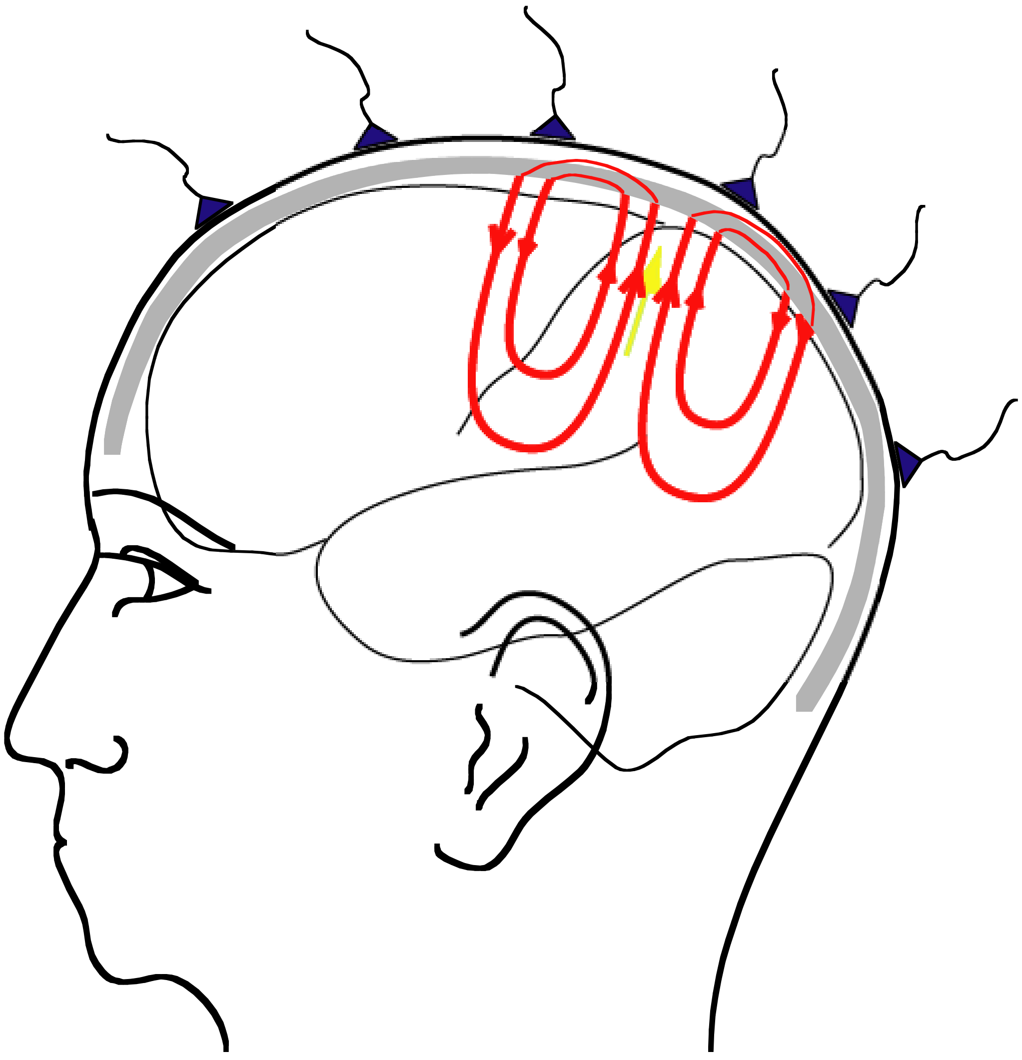
sum of currents is zero for all nodes, except

sum of current is I+ for a certain node

sum of current is I- for another node

Solve for unknown potential

EEG volume conduction



EEG volume conduction

Potential difference between electrodes corresponds to current flowing through skin

Only tiny fraction of current passes through skull

Therefore the model should describe the skull and skin as accurately as possible

MEG volume conduction

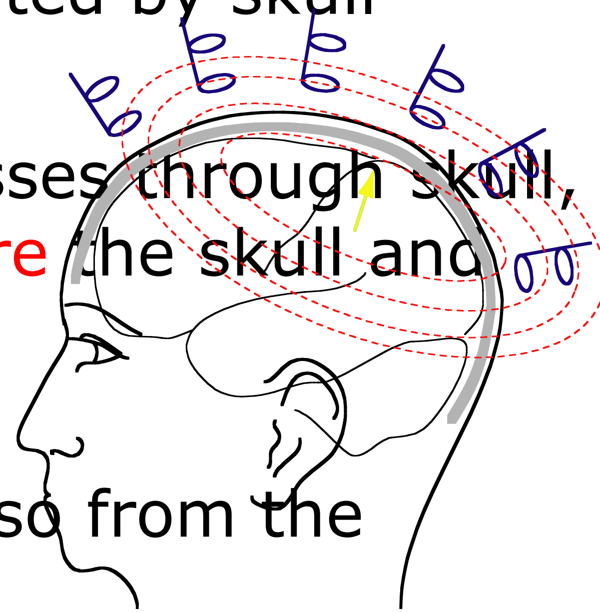
MEG measures magnetic field over the scalp

M istorted by skull

O passes through skull,

gnore the skull and

M ut also from the

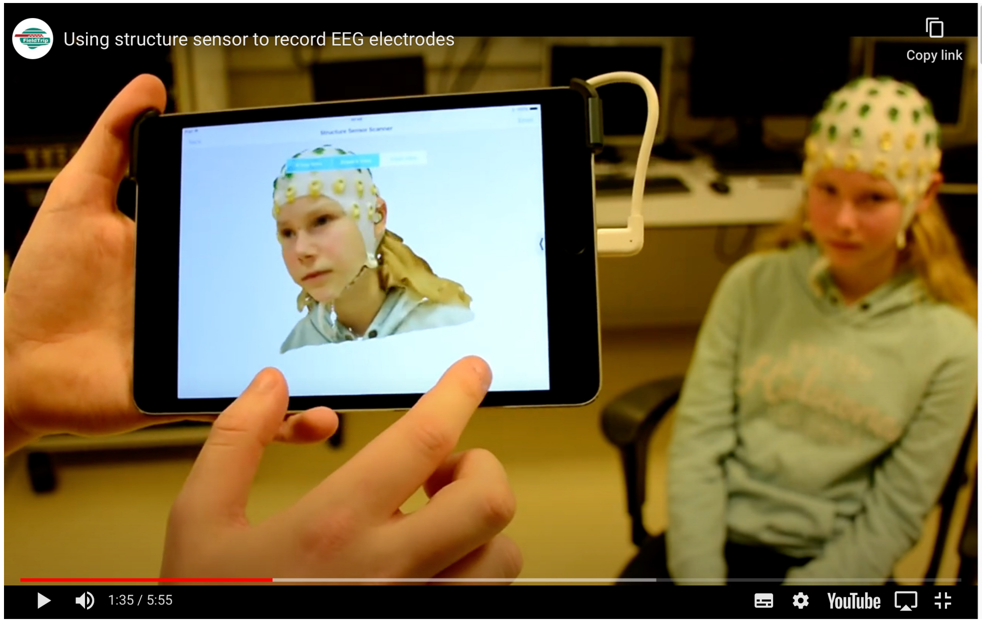


Practical considerations for EEG/MEG head models

- Best is to make model for each participant based on individual MRIs
- Use a template MRI and/or a template head model
  - EEG electrodes scale with the head size (since different caps), same head model for all participants
  - MEG sensors are fixed in a helmet, so the model needs to be scaled to the subject’s head size
- With a template head model, you still need to get the electrodes in the right position
  - Use a Polhemus or 3D optical scanner
  - Use template electrode positions

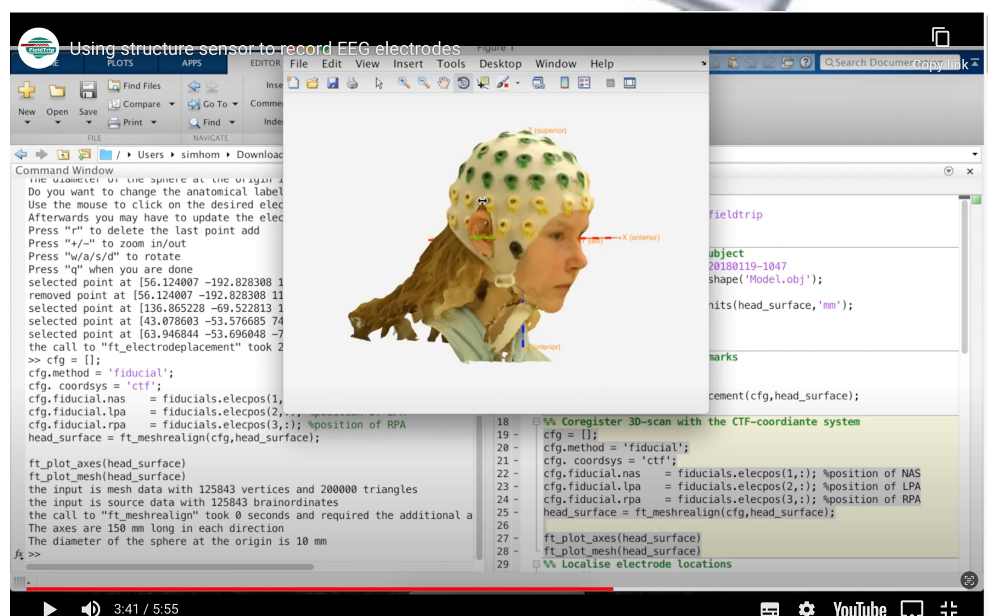
Simon Homölle, Robert Oostenveld.
Using a structured-light 3D scanner to improve EEG source modeling with more accurate electrode positions.
J Neurosci Methods. 2019

Getting the EEG electrodes right with a 3D scanner



Structure Sensor, http://structure.io/
Alternatively an iPhone 13 pro or iPad pro

See https://www.fieldtriptoolbox.org/tutorial/electrode/ and
https://eeglab.org/tutorials/09_source/Custom_head_model.html



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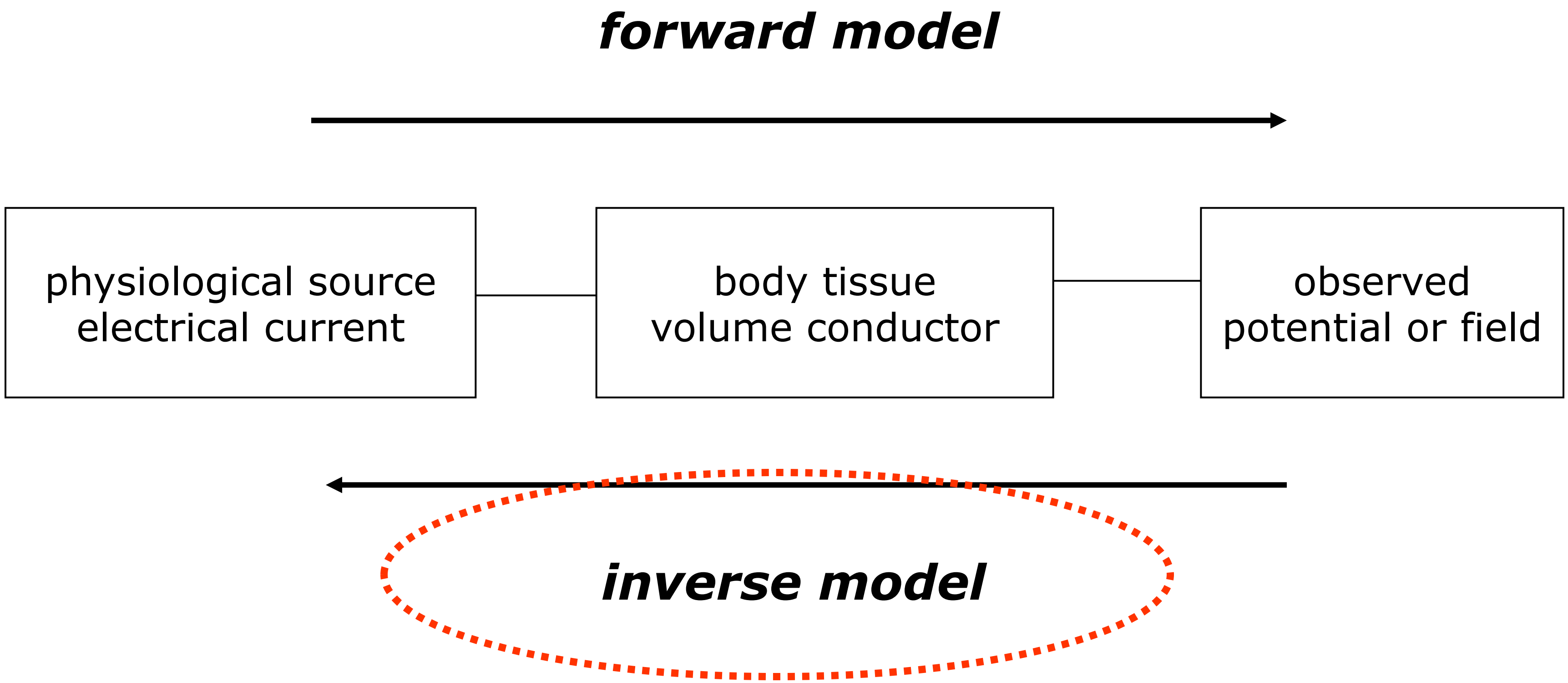
Biophysical source modelling: overview



Inverse localization: demo

Inverse methods

Single and multiple dipole models

Minimize error between model and measured potential/field

Distributed source models

Perfect fit of model to the measured potential/field

Additional constraint on source smoothness, power or amplitude

Spatial filtering

Scan the whole brain with a single dipole and compute the filter output at every location

Beamforming (e.g. LCMV, SAM, DICS)

Multiple Signal Classification (MUSIC)

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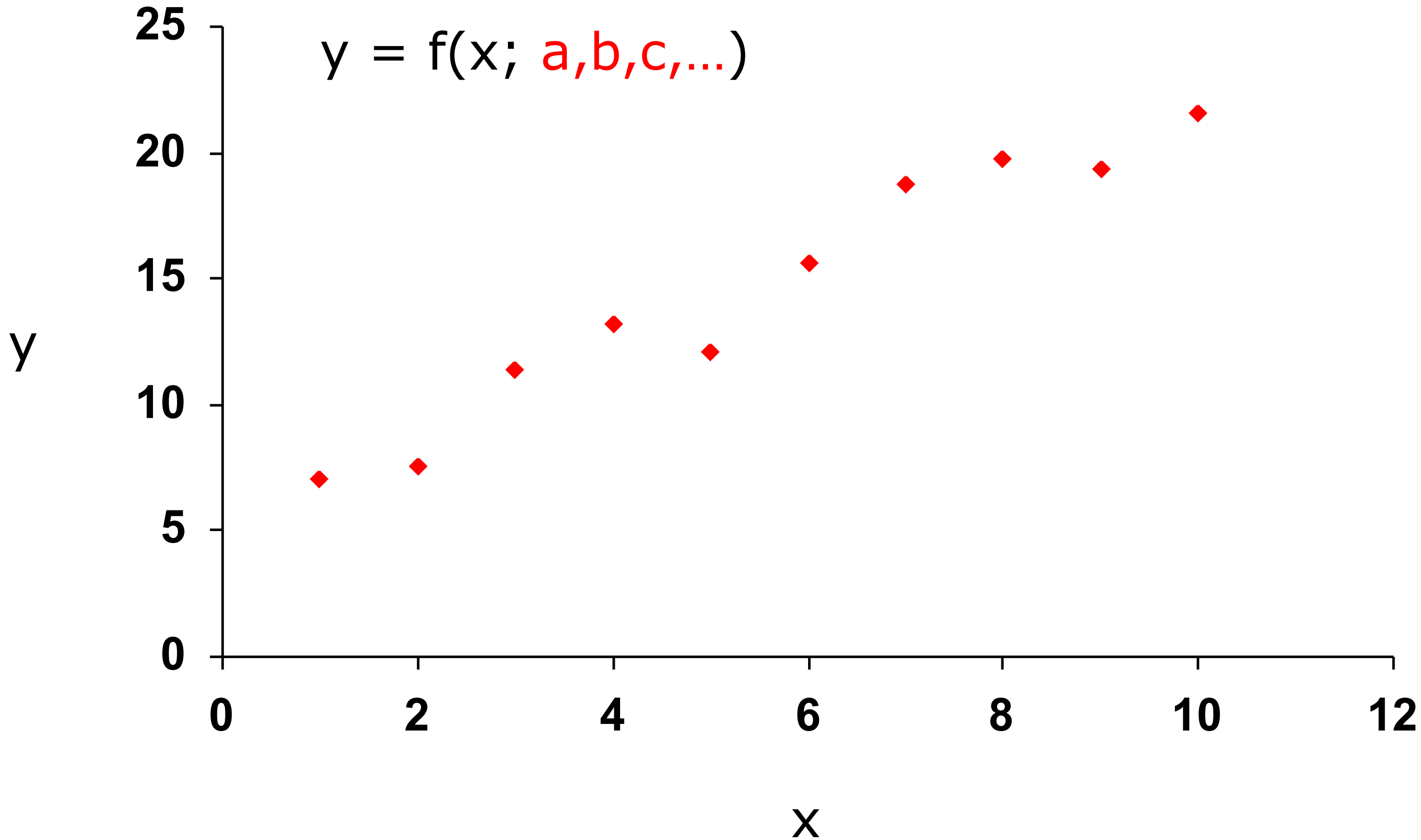
Single or multiple dipole models - Parameter estimation



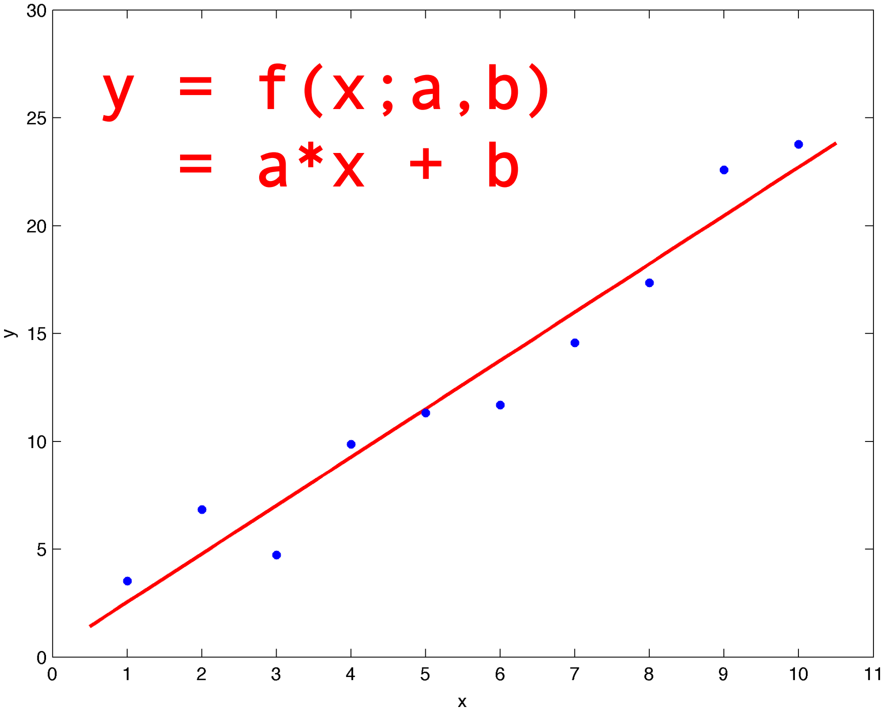
Parameter estimation: dipole parameters

source model with few parameters

- position
- orientation
- strength

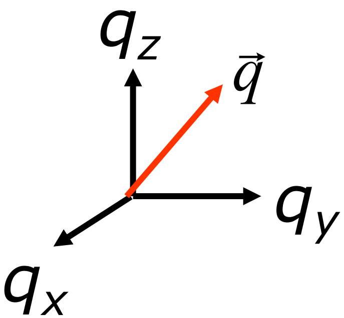
compute the model data

minimize difference between actual and model data



Linear parameters: estimation just like GLM

$$Y = G_x q_x + G_y q_y + G_z q_z = \begin{bmatrix} G_{x,1} & G_{y,1} & G_{z,1} \\ G_{x,2} & G_{y,2} & G_{z,2} \\ \vdots & \vdots & \vdots \\ G_{x,N} & G_{y,N} & G_{z,N} \end{bmatrix} \cdot \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} = \mathrm{G} \cdot \vec{q}$$



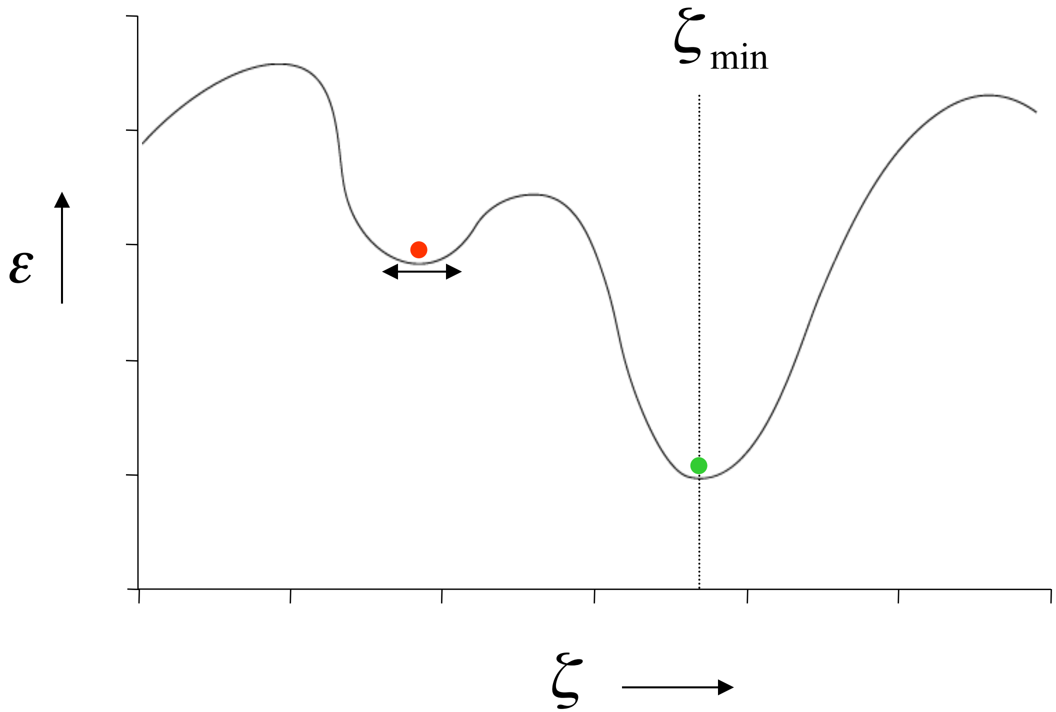
$$Y = \mathrm{G} \cdot \vec{q}$$
$$= \mathrm{G}(\zeta) \cdot \vec{q}$$

$$\vec{q} = \mathrm{G}^{-1} \cdot Y$$

Non-linear parameters

$$\varepsilon rror(\zeta) = \sum_{i=1}^{N} \left(Y_i(\zeta) - V_i\right)^2 \quad \Rightarrow \quad \min_{\zeta}\left(\varepsilon rror(\zeta)\right)$$

$$\zeta = a, b, c, \ldots$$



Non-linear parameters: grid search

One dimension, e.g. location along medial-lateral

100 possible locations

Two dimensions, e.g. med-lat + inf-sup

100x100=10.000

Three dimensions

100x100x100 = 1.000.000 = 10^6

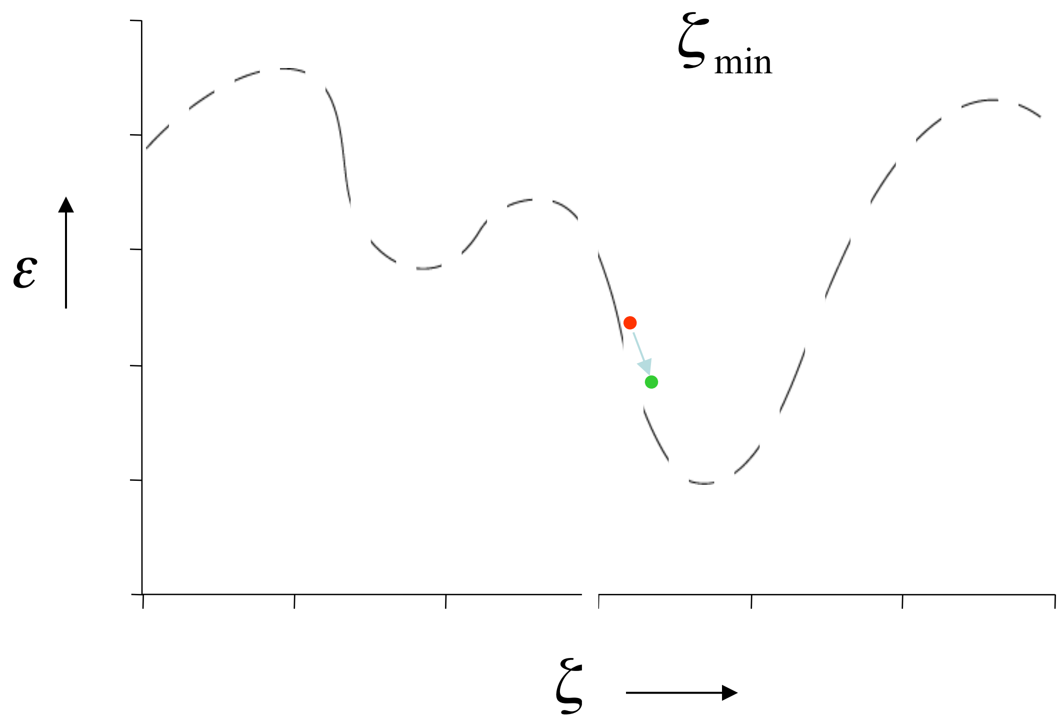
Two dipoles, each with three dimensions

100x100x100x100x100x100 = 10^{12}

Non-linear parameters: gradient descent optimization

$$\varepsilon rror(\zeta) = \sum_{i=1}^{N} \left(Y_i(\zeta) - V_i\right)^2 \quad \Rightarrow \quad \min_{\zeta}\left(\varepsilon rror(\zeta)\right)$$

$$\zeta = a, b, c, \ldots$$



Single or multiple dipole models - Strategies

Single dipole:

scan the whole brain, followed by iterative optimization

Two dipoles:

scan with symmetric pair, use that as starting point for iterative optimization

More dipoles:

sequential dipole fitting, add dipoles to the model one-by-one

Fitting dipoles to ERP timecourses – Sequential fit

- Assume that activity starts “small”
  - explain earliest ERP component with single equivalent current dipole
- Assume later activity to be more widespread
  - add ECDs to explain later ERP components
  - estimate position of new dipoles
  - re-estimate the activity of all dipoles
- Iterative and interactive (hence subjective) process, difficult to determine how many dipoles are needed

Fitting dipoles to ICA component topographies

ICA unmixes the sources and gives topographies and timeseries

Use a single dipole (or two) to explain each of the ICA topographies

Arnaud Delorme, Jason Palmer, Julie Onton, Robert Oostenveld, Scott Makeig
Independent EEG Sources Are Dipolar
PLOS One (2012) doi: 10.1371/journal.pone.0030135

Note: You can also fit distributed source models to ICA topographies

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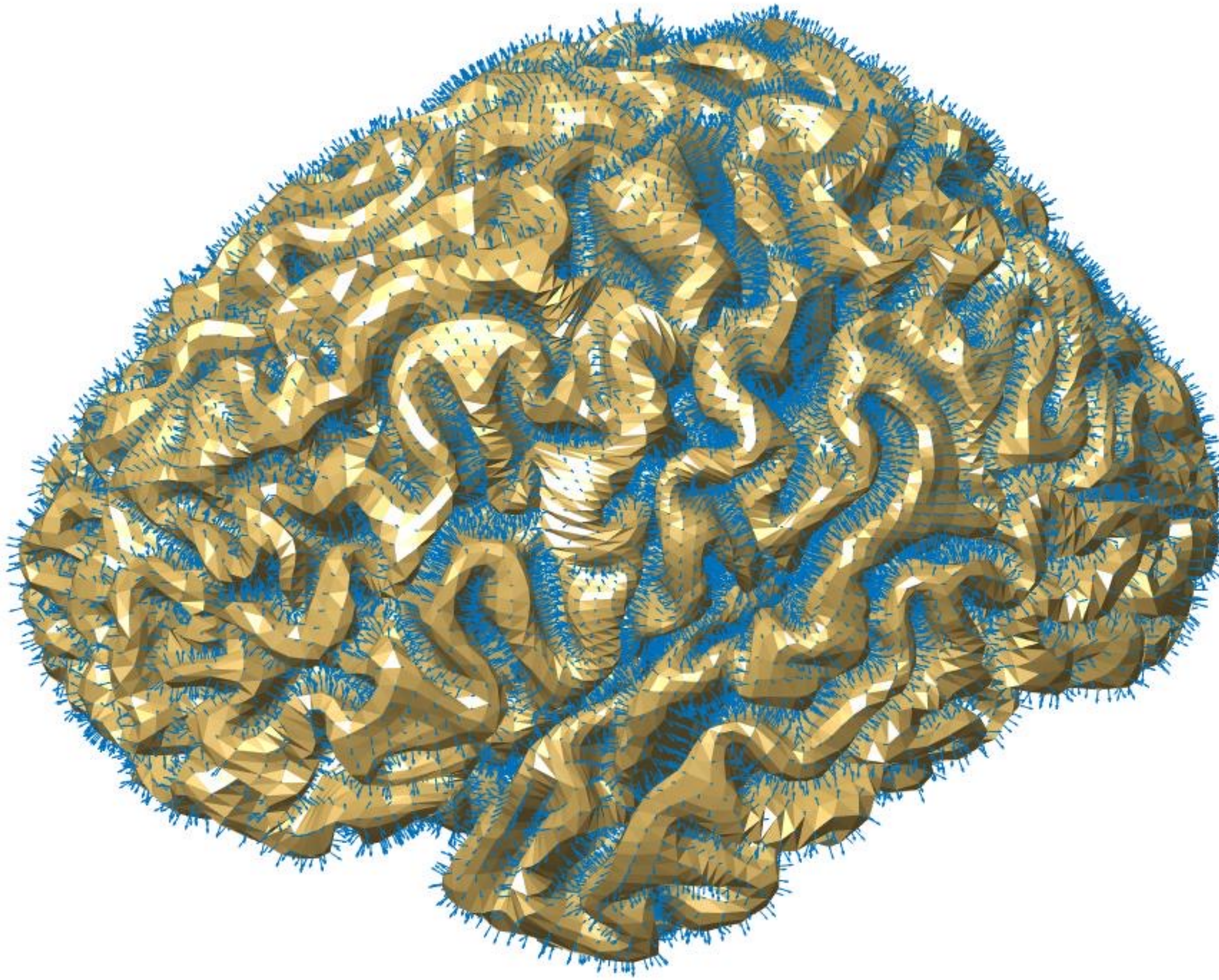
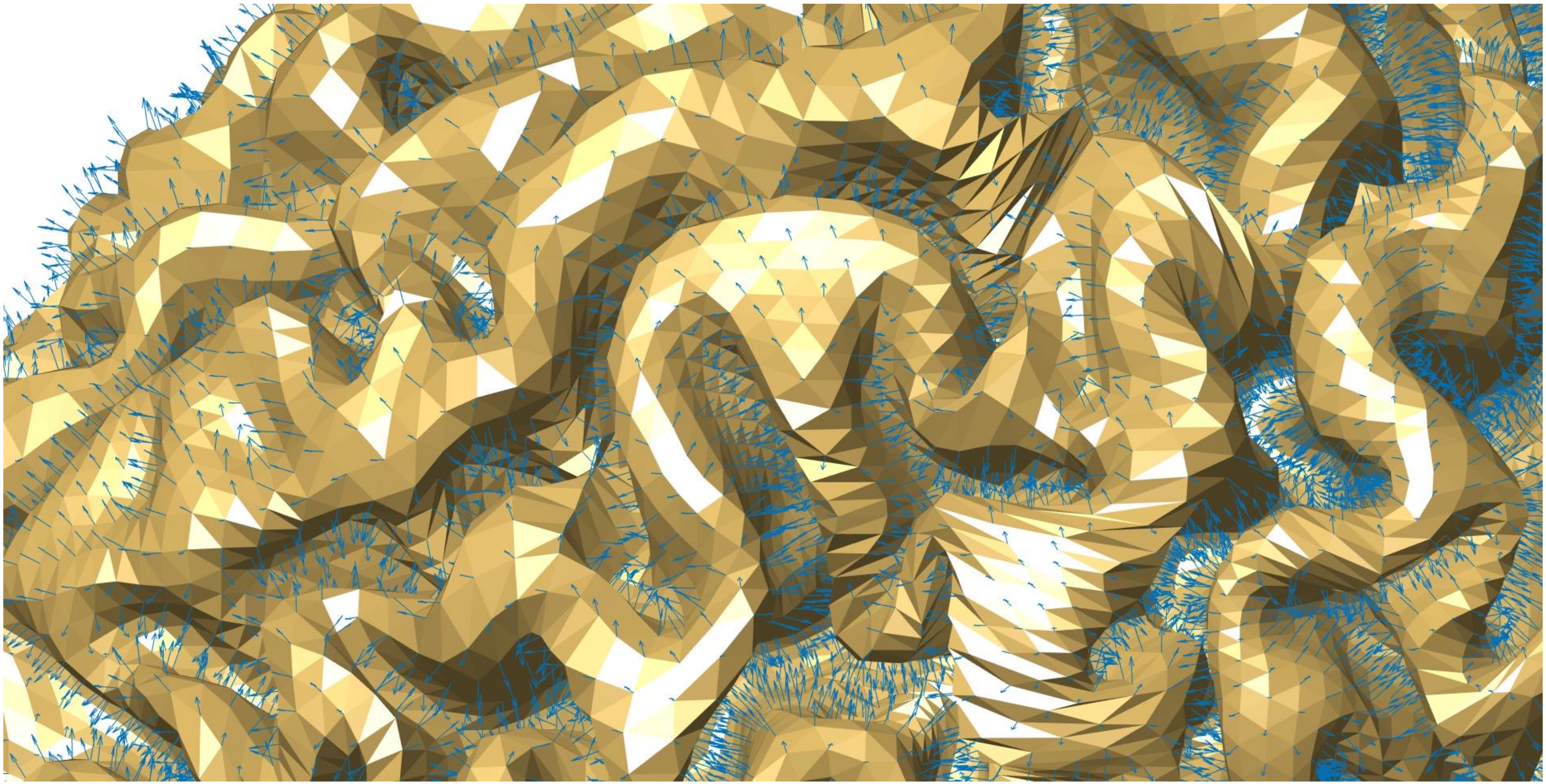
Distributed source model

- Position of the source is not estimated as such
  - Pre-defined grid (cortical sheet or 3D volume)
- Strength is estimated
  - In principle easy to solve, however…
  - More “unknowns” than “knowns”
  - Infinite number of solutions can explain the data perfectly
  - Additional constraints required
  - Linear estimation problem

Distributed source model



Distributed source model



Distributed source model: linear estimation like GLM

$$Y = G_1 q_1 + G_2 q_2 + \ldots = \begin{bmatrix} G_{1,1} & G_{2,1} & \cdots \\ G_{1,2} & G_{2,2} & \cdots \\ \vdots & \vdots & \ddots \\ G_{1,N} & G_{2,N} & \cdots \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ \vdots \end{bmatrix} = \mathrm{G} \cdot \vec{q}$$

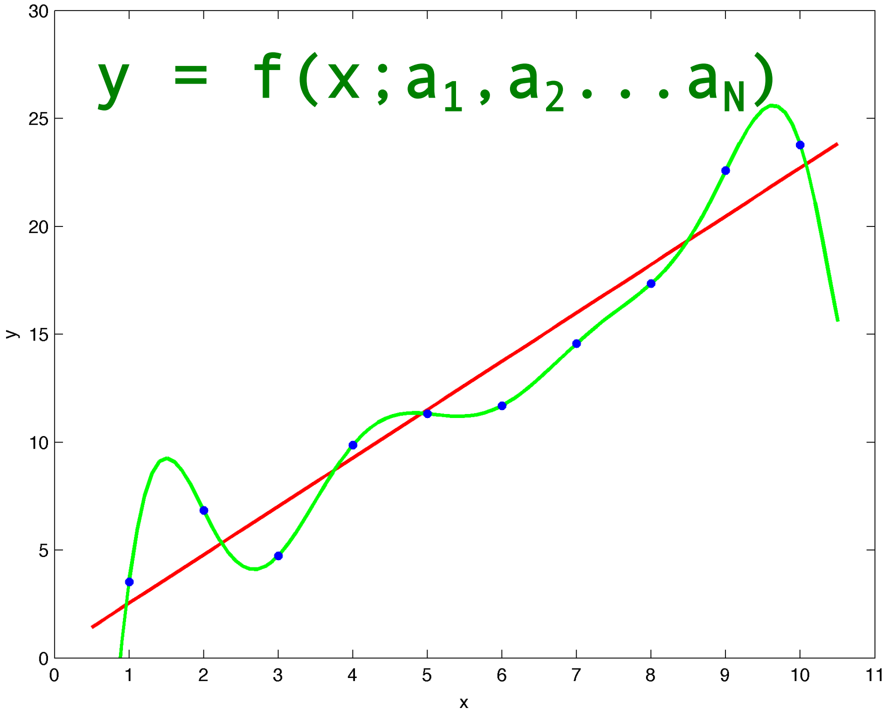
$$\vec{q} = \mathrm{G}^{-1} \cdot Y$$

Distributed source model: linear estimation

distributed source model with **many dipoles** throughout the whole brain

estimate the strength of all dipoles

data and noise can be perfectly explained



Distributed source model: regularization

$$V = G \cdot q + Noise$$

$$\min_q \{ \| V - G \cdot q \|^2 \} = 0 \;\; !!$$

Regularized linear estimation:

$$\rightarrow \min_q \{ \underbrace{\| V - G \cdot q \|^2}_{\text{mismatch with data}} + \lambda \cdot \underbrace{\| D \cdot q \|^2}_{\text{mismatch with prior assumptions}} \}$$

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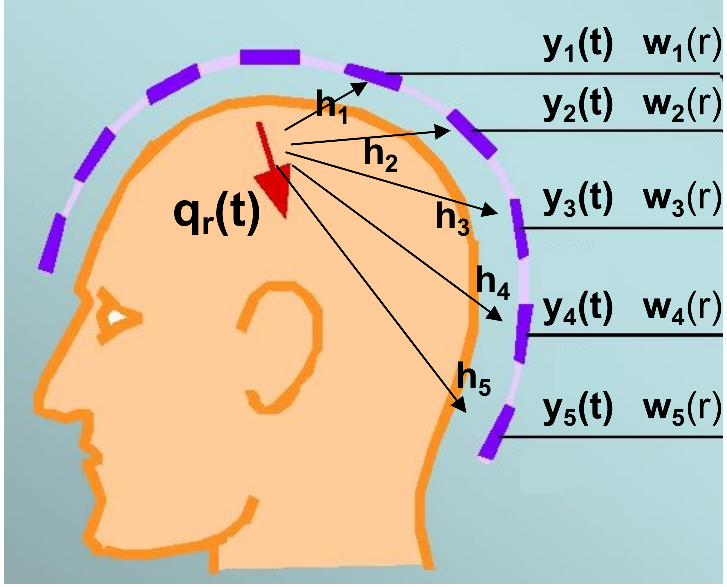
Spatial filtering with beamforming

- Position of the source is not estimated as such
  - Loop over a pre-defined grid
- Manipulate filter properties, not source properties
  - No explicit assumptions about source constraints (implicit: single dipole)
  - Assumption that sources that contribute to the data should be uncorrelated

Beamformer: the question

What is the timecourse of activity of a source **q**, at a location **r**, given the data **y**?

We estimate **q** with a spatial filter **w**



$$\hat{q}_r(t) = \mathbf{w}(r)^T \mathbf{y}(t)$$

Beamformer: the solution

Two simultaneous constraints on the spatial filter:

1) The source in the region of interest should be visible with 1x gain, i.e., no amplification, no attenuation
2) All other contributions to the data should be filtered out as much as possible

Spatial filter is computed from the data covariance matrix and the leadfield. Assumes reasonably uncorrelated sources (not independent).

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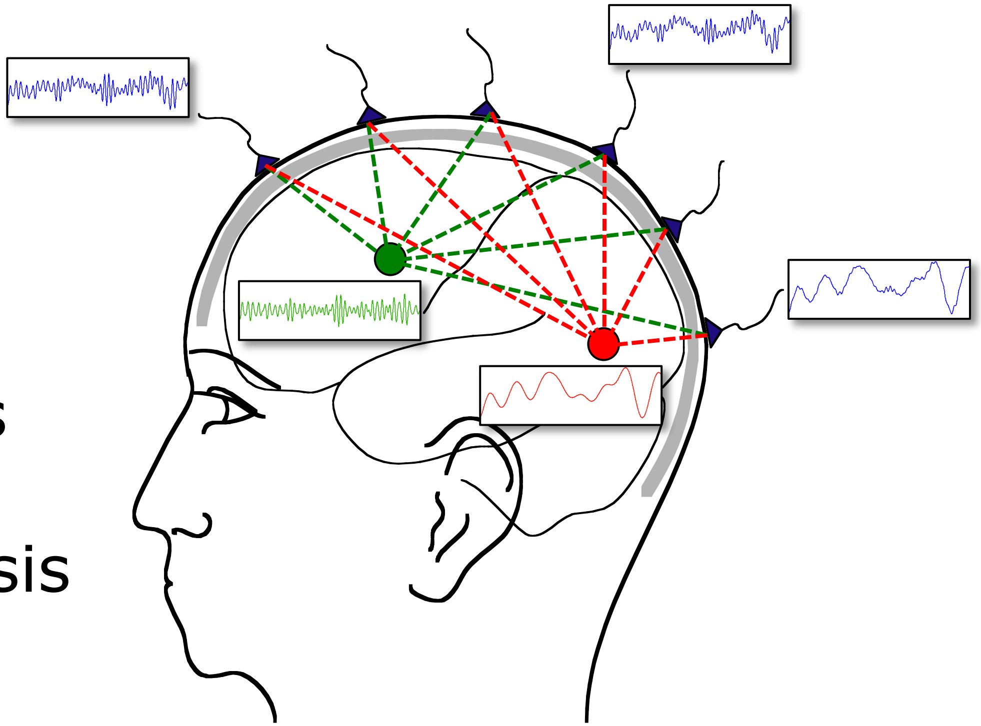
Summary 1

- Forward modelling
  - Required for the interpretation of scalp topographies
  - Different methods with varying accuracy
- Inverse modelling
  - Estimate source location and timecourse from data
- Assumptions on source locations
  - Single or multiple point-like source
  - Distributed source
- Assumptions on source timecourse
  - Uncorrelated (and dipolar)
  - Independent

Summary 2

Source analysis is not only about the "where" but also about untangling the "what", "when" and "how".

Source analysis gives you locations



The source timecourses can come from ICA, or from source analysis on the ERPs

Estimating source timecourse activity

$$Y = G_1X_1 + G_2X_2 + \ldots + G_nX_n + \text{noise}$$

$$X'(t) = W\,Y(t)$$

few sources

distributed sources

one at a time

all brain (and artifact) sources

dipole fitting

minimum norm estimate

beamforming

independent component analysis

Estimating source timecourse activity

$$Y = G_1X_1 + G_2X_2 + ... + G_nX_n + noise$$

Estimating source timecourse activity using dipole fitting

$$Y = G_1X_1 + G_2X_2 + \dots + G_nX_n + \text{noise}$$

n is typically small

$$(Y - G_1X_1 - \qquad\qquad = \text{residual}$$

measured data model data

$X' = W\,Y$, where $W = G^T (G\,G^T)^{-1}$

Estimating source timecourse activity using distributed source models

$$Y = G_1X_1 + G_2X_2 + \dots + G_nX_n + \text{noise}$$

n is typically large (> # channels)

$$Y = (G_1X_1 + G_2X_2 + \dots + G_nX_n) + \text{noise}$$

$$Y = G\,X + \text{noise}$$

$X' = W\,Y$, where W ensures $\min_X \{\| Y - G \cdot X \|^2 + \lambda \cdot \| X \|^2\}$

Estimating source timecourse activity using spatial filtering

$$Y = G_1X_1 + G_2X_2 + \dots + G_nX_n + \text{noise}$$

any number of n

$$Y = (G_1X_1 + G_2X_2 + \dots) + G_nX_n + (\text{noise})$$

$$X'_n = W_n\,Y, \quad \text{where} \quad W^T = [G_n^{\ T}\,C_Y^{\ -1}\,G_n]^{-1}\,G_n^{\ T}\,C_Y^{\ -1}$$

Estimating source timecourse activity

$$Y = G_1X_1 + G_2X_2 + \ldots + G_nX_n + \text{noise}$$

few sources
distributed sources
one at a time

$$X'(t) = W\,Y(t)$$

dipole fitting
minimum norm estimate
beamforming

Estimating source timecourse activity using independent component analysis

$$Y = G_1X_1 + G_2X_2 + \ldots + G_nX_n + \text{noise}$$

n typically the same as the number of channels

$$Y = G\ (X\ \ + \text{noise})$$

includes line-noise, EOG, ECG and other noise that is visible on all channels

$X' = W\,Y$, where W maximizes the independence of X'

rows of W^{-1} correspond to G_1, G_2, …